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Generalized soft multi connectedness and compactness

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Abstract

The significance of connectedness and compactness plays a vital role in studying the properties of topological spaces. Numerous authors explored several characterizations of topological spaces using these notions. Researchers had attempted to generalize the concepts of connectedness and compactness in various generalized topological spaces like ideal topological spaces, soft topological spaces, fuzzy topological spaces etc. In the year 2013, Babitha and John studied the concept of connectedness in soft multi topological spaces. Since then the study of these concepts has created a gap in the literature. The uniosity about the generalizations of connectedness and compactness in soft multi topological spaces explored this article.

Keywords: Soft multiset, soft multi topology, generalized semi closed soft multiset, generalized semi soft multi connectedness and generalized semi soft multi compactness.

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1 Introduction

In mathematics, a multiset is regarded as the generalization of a set. In the classical set theory, set is a well-defined collection of distinct objects. If a repeated occurrence of any object is allowed in a set, then it creates another mathematical structure known as multiset. A multiset differs from a set in the sense that each element has a multiplicity that indicates how many times it occurs in that collection.

Multiset theory was introduced by Cerf et al.[1971]. Yager [1986] and Blizard [1989] added further contributions to it. An algebraic structure for multiset space was introduced by Ibrahim et al. [2011]. Multiset relations and multiset functions were introduced by Girish and John [2009].

In 2012, Girish and Sunil [2012] introduced multiset topologies induced by multiset relations. Later they introduced the concepts of basis, sub-basis, closure, interior, continuity and related properties in multiset topological spaces in [2012]. S. A. El. Sheikh et al. introduced the concept of separation axioms on multiset topological space [2015] and they studied the generalization of open multisets and its properties [2015]. In 2017, the notion of compactness in multiset topological space [2017] was introduced by Sougata Mahanta et al.

J. Mahanta et al.[2014] have introduced the semi compactness of multiset topological space. Many interesting findings concerning with the multisets were examined in [2015, 2018].

In 1999, Molodstov[1999] initiated the theory of soft sets as a new Mathematical tool to deal with uncertainties while modelling problems in various fields like Engineering and Medical Sciences. P. K. Maji et al.[2003] defined some basic properties of soft sets. In 2002, P. K. Maji et al. [2002] applied the theory of soft sets to solve a decision making problem. The notion of soft mapping was first initiated by P. Majumdar et al. [2010]. In 2011, N. Caugman et al. [2011] defined the soft topology on soft sets and investigated related properties. In 2012, I. Zorlutuna et al. [2012] acquainted the concept of soft point, soft continuity and soft compactness.

Soft set is a mapping from a parameter set to power set of universal. But among many practical situations some situation may occur, where the respective counts of objects in the universe of discourse are not single. To handle the situation mathematically, in 2013 Babitha & John [2013] were introduced the concept of soft multiset as a combination of soft sets and multisets, in which the universal set is a multiset. The topological and algebraic structures of soft multi set have large number of applications in soft computing, decision making, data analysis and information aggregation and information measure. Moreover the soft multi topology and its basic properties were given in [2013]. The same authors initiated the concept of connectedness on soft multi topological spaces [2013].

In 2015, Deniz Tokat et al. [2015] acquainted the idea of soft multi continuous functions and also soft multi semi continuity by using soft multi function. After that El. Sheikh et al. [2016] introduced the concepts of generalizations of open soft multi sets and mappings in soft multi topological space. In 2015, they have introduced the concept of g-closed soft multisets in soft multi topological space. Ismail Osmanogluet al. [2014] acquainted the concept of compact soft multi spaces. In 2015, Deniz Tokat et al. introduced the concept of soft multi continuous functions by using soft multi function. Muhammed Raafat et al. presented the concept of semi-compact soft multi spaces. Muhammed Riaz et al. [2020] gave the properties of soft multi topological spaces and also gave some real life applications. Seydakaya Pezukl et al. [2021] provided an application of soft multisets to a decision making problem concerning side effects of COVID-19 vaccines. V. Inthumathi et al. [2022] acquainted the concept of generalized semi closed soft multisets and established several of their properties. Further they introduced the concepts of generalized semi operators and generalized semi continuous functions in soft multi topological spaces [2023].

In this paper, we generalized the work of Muhammed Raafat et al. as generalized semi soft multi compactness and generalized semi soft multi connectedness by introducing generalized semi soft multi separated sets, soft multi adherent point, generalized semi soft multi adherent point, soft multi open cover and generalized semi soft multi open cover and also we explored new characterizations of soft multi topological spaces.

2 Preliminaries

Throughout this paper SMTS denotes the soft multi topological spaces.

Definition 2.1. [2013] Let U be an universal mset, E be a set of parameters and $A \subseteq E$. Then, an ordered pair (F, A) is called a soft mset where F is a mapping given by $F: A \to P^*(U)$; $P^*(U)$is the power set of a mset$U$.

For all $e \in A$, $F(e)$ mset represents by count function $C_{F(e)}: U^* \to N$ where N represents the set of non-negative integers and U^* represents the support set of U.

Let $U = \{2/x, 3/y, 2/z\}$ be a mset. Then, the support set of U is $U^* = \{x, y, z\}$.

Definition 2.2. [2013] Let X be an universal mset and E be a set of parameters. Then, the collection of all soft msets over X with parameters from E is called a soft multi class and is denoted as X_E.

Definition 2.3. [2013] Let (X, τ, E) be a SMTS over X and (G, E) be a soft multi set over X and $x \in X$. Then, (G, E) is said to be a soft multi neighbourhood of x if there exists a soft multi open set (F, E) such that $x \in (F, E) \subseteq (G, E)$. The set of all soft multi neighbourhood of α, denoted by $\widetilde{N}(\alpha)$, is called the family of soft multi neighbourhoods of α,

i.e. $\widetilde{N}(\alpha) = \{(G, E): (G, E) \in \tau, \alpha \in (G, E)\}$.

Definition 2.4.[2022] Let f_E be a soft mset over X_E. f_E is called a soft multi point over X, if there exists $e \in E$ and $n/x \in X$, $1 \leq n \leq m$ such that

$$f(\epsilon) = \begin{cases} (n/x\} & if\ \epsilon = e,\ 1 \leq n \leq m \\ \phi & if\ \epsilon \in E - \{e\} \end{cases}$$

We denote the soft multi point f_E by $[(n/x)_e]_E$. In this case, x is called support point of $[(n/x)_e]_E$, $\{x\}$ is called support set of $[(n/x)_e]_E$ and e is called the expressive parameter of $[(n/x)_e]_E$. The family of all soft multi points over X is denoted by $P(X,E)$ or P.

i.e. $P(X,E) = \{[(n/x_i)_{e_j}]_E : x_i \in X, e_j \in E, 1 \leq n \leq m\}$.

Definition 2.5. [2022] A soft mset S_E in a SMTS (X,τ,E) is said to be generalized semi closed soft multiset (briefly $gscs$ mset) if $C_{scl(S)(e)}(x) \leq C_{U(e)}(x)$ whenever $C_{S(e)}(x) \leq C_{U(e)}(x)$ for all $x \in X^*$, $e \in E$ and $U_E \in OSM(X)_E$. The set of all $gscs$ mset is denoted by $gscs(X_E)$.

Definition 2.6. [2023] A sub soft mset S_E in (X,τ,E) is called generalized semi open soft multiset (briefly $gsos$ mset) if its complement is a $gscs$ mset. The set of all $gsos$ mset in (X,τ,E) is denoted by $gsos(X)_E$.

Definition 2.7. [2023] Let (X,τ,E) be a SMTS over X_E and S_E be a soft mset over X_E. Then the generalized semi soft multi closure of S_E, denoted by $gssm\text{-}cl(S_E)$ is the intersection of all $gscs$ mset containing S_E.

Definition 2.8. [2023] Let (X,τ,E) be a SMTS over X_E and S_E be a soft mset over X_E. Then the generalized semi soft multi interior of S_E, denoted by $gssm\text{-}int(S_E)$ is the union of all $gsos$ mset contained in S_E.

Definition 2.9. [2023] A sub soft mset N_E in SMTS (X,τ,E) is said to be generalized semi soft multi neighbourhood (briefly $gssm$-nbd) of a point $[(n/x)_e]_E$ in X_E if there exists an $gsos$ mset U_E such that $[(n/x)_e]_E \leq C_{U(e)}(x) \leq C_{N(e)}(x)$.

If N_E is a $gsos$ mset containing $[(n/x)_e]_E$, the N_E is called generalized semi soft multi open neighbourhood (briefly $gssm$-open nbd) of $[(n/x)_e]_E$.

Definition 2.10. [2020] Let $(\Omega_A, \tilde{\tau})$ be a SMTS and $\Omega_B \,\tilde{\sqsubseteq}\, \Omega_A$and $\alpha \in \Omega_A$. If every soft multi neighbourhood of α intersects Ω_B in some soft multi points other than α, then α is called a soft multi limit point of Ω_B.

Definition 2.11. [2014] Let (X_E, τ_1) and (X_E, τ_2) be SMTS. Then, the following hold:

(a) If $\tau_2 \sqsupseteq \tau_1$, then τ_2 is soft multi finer than τ_1.
(b) If $\tau_2 \sqsupset \tau_1$, then τ_2 is soft multi strictly finer than τ_1.
(c) If either $\tau_2 \sqsupseteq \tau_1$or $\tau_2 \sqsubseteq \tau_1$, then τ_1 is comparable with τ_2.

Definition 2.12. [2023] Let (X,τ,E) and (Y,σ,K) be two SMTSs and let $u: X \to Y$, $p: E \to K$ and $f: X_E \to Y_K$ be functions. Then the function f is called generalized semi soft multi continuous (briefly $gssm$-cts) if $f^{-1}(F_K)$ is a $gscs$ mset over (X,τ,E) for every closed soft msetF_K over (Y,σ,K).

Definition 2.13. [2014] A family of ψ of soft msets has the finite intersection property if the intersection of the members of each finite sub family of ψ is not the null soft mset.

Definition 2.14. [2013] Let (X_E, τ) be a SMTS over X. A soft multi separation of $\tilde{X}$ is a pair $((F, E), (G, E))$ of no-null soft multi open sets in X_E such that

$$\tilde{X} = (F, E) \tilde{\cup} (G, E), (F, E) \tilde{\cap} (G, E) = \phi.$$

Definition2.15. [2013] A SMTS (X_E, τ) is said to be soft multi connected if there does not exist a soft multi separation of $\tilde{X}$. Otherwise, (X_E, τ) is said to be soft multi disconnected.

Definition 2.16. [2014] A family ψ of soft multi sets is a cover of a soft multiset (F, A) if $(F, A) \subseteq \cup\{(F_i, A): (F_i, A) \in \psi, i \in I\}$. It is a soft multi open cover if each member of ψ is a soft multi open set. A subcover of ψ is a family of ψ which is also a cover.

Definition 2.17. [2014] Let (X_E, τ) be a SMTS and $(F, A) \in (X, E)$. Soft multi set (F, A) is called compact if each soft multi open cover of (F, A)has a finite subcover. Also SMTS (X_E, τ) is called compact if each soft multi open cover of $\tilde{X}$ has a finite subcover.

3 Generalized semi soft multi connectedness

In this section the notions of generalized semi soft multi separated sets, generalized semi soft multi connectedness are defined and some of their properties in SMTSs are studied.

Definition 3.1. Let (X, τ, E) be a SMTS. Two non-empty sub soft msets S_E and T_E are said to be generalized semi soft multi separated (briefly $gssm$-separated) if and only if $C_{(S \cap gssm-cl(T))(e)}(x) = C_{\phi(e)}(x)$ and $C_{(gssm-cl(S) \cap T)(e)}(x) = C_{\phi(e)}(x)$.

i.e. $C_{(S \cap gssm-cl(T)) \cup (gssm-cl(S) \cap T)(e)}(x) = C_{\phi(e)}(x)$.

Example 3.1. Let $X = \{2/x_1, 2/x_2\}$, $E = \{e_1, e_2\}$ and $\tau = \{\tilde{\phi}, \tilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$ where $F_1(e_1) = \{2/x_1\}$, $F_1(e_2) = \{\tilde{\phi}\}$, $F_2(e_1) = \{2/x_2\}$, $F_2(e_2) = \{\tilde{\phi}\}$, $F_3(e_1) = \{\tilde{X}\}$, $F_3(e_2) = \{\tilde{\phi}\}$. Let S_E, T_E and V_E be soft msets such that $S(e_1) = \{1/x_1\}$, $S(e_2) = \{\tilde{\phi}\}$, $T(e_1) = \{1/x_2\}$, $T(e_2) = \{\tilde{\phi}\}$ and $V(e_1) = \{1/x_1, 1/x_2\}$, $V(e_2) = \{\tilde{\phi}\}$. Then soft msets$S_E$ and T_E are $gssm$-separated but the soft msetsT_E and V_E are not $gssm$-separated.

Definition 3.2. Let (X, τ, E) be a SMTS and $C_{S(e)}(x) \leq C_{X(e)}(x)$. A soft multi point $[(n/x)_e]_E \in X_E$ is said to be soft multi adherent point of S_E if every open soft mset containing $[(n/x)_e]_E$, contains atleast one soft multi point of S_E.

Definition 3.3. Let (X, τ, E) be a SMTS and $C_{S(e)}(x) \leq C_{X(e)}(x)$. A soft multi point $[(n/x)_e]_E \in X_E$ is said to be generalized semi soft multi adherent point (briefly $gssm$-adherent point) of S_E if every $gsos$ mset containing $[(n/x)_e]_E$, contains atleast one soft multi point of S_E.

Proposition 3.1. Two $gssm$-separated sets are always disjoint.

Proof. Let S_E and $T_E$be $gssm$-separated sets. Then $C_{(S\cap gssm-cl(T))(e)}(x) = C_{\phi(e)}(x)$ and $C_{(gssm-cl(S)\cap T)(e)}(x) = C_{\phi(e)}(x)$. Now $C_{(S\cap T)(e)}(x) \leq C_{(gssm-cl(S)\cap T)(e)}(x) = C_{\phi(e)}(x)$. This implies that $C_{(S\cap T)(e)}(x) = C_{\phi(e)}(x)$. Hence S_E and T_E are disjoint.□

Proposition 3.2. Every soft multi separated sets are $gssm$-separated but the converse is not true.

Proof. It is follows from the definition. □

Example 3.2. In Example 3.1, the soft msetsS_E and T_E are $gssm$-separated but not soft multi separated.

Theorem 3.1. Two soft msets are $gssm$-separated if and only they are disjoint soft msets and neither of them contains soft multi limit point of the other.

Proof. Let S_E and $T_E$be $gssm$-separated sets if and only if$C_{(S\cap gssm-cl(T))(e)}(x) = C_{\phi(e)}(x)$ and $C_{(gssm-cl(S)\cap T)(e)}(x) = C_{\phi(e)}(x)$. Now $C_{(S\cap gssm-cl(T))(e)}(x) = C_{\phi(e)}(x) \Leftrightarrow C_{(S\cap(T\cup(n/x)))(e)}(x) = C_{\phi(e)}(x)$, where $[(n/x)_e]_E$ is the soft multi limit point of $T_E \Leftrightarrow C_{(S\cap T)(e)}(x) = C_{\phi(e)}(x)$ and $C_{(S\cap(n/x))(e)}(x) = C_{\phi(e)}(x) \Leftrightarrow S_E$ and $T_E$are disjoint soft msets and S_E contains no soft multi limit point of T_E. Similarly $C_{(gssm-cl(S)\cap T)(e)}(x) = C_{\phi(e)}(x)$ if and only if S_E and $T_E$are disjoint soft msets and T_E contains no soft multi limit point of S_E.□

Proposition 3.3. Sub soft msets of a $gssm$-separated sets are also $gssm$-separated.

Proof. Let S_E and $T_E$be $gssm$-separated of a SMTS (X, τ, E). Then $C_{(S\cap gssm-cl(T))(e)}(x) = C_{\phi(e)}(x)$ and $C_{(gssm-cl(S)\cap T)(e)}(x) = C_{\phi(e)}(x)$. Let $C_{U(e)}(x) \leq C_{S(e)}(x)$ and $C_{V(e)}(x) \leq C_{T(e)}(x)$. Then $C_{(U\cap gssm-cl(V))(e)}(x) = C_{\phi(e)}(x)$ and $C_{(gssm-cl(U)\cap V)(e)}(x) = C_{\phi(e)}(x)$. Thus U_E and V_E are $gssm$-separated. □

Theorem 3.2. Two $gscs$-sets of a SMTS (X, τ, E) are $gssm$-separated if and only if they are disjoint.

Proof. Since $gssm$-separated sets are disjoint, $gscs$-msets are disjoint. Conversely, let S_E and T_E be two disjoint $gscs$-msets. Then $C_{gssm-cl(S)(e)}(x) = C_{S(e)}(x)$, $C_{gssm-cl(T)(e)}(x) = C_{T(e)}(x)$ and $C_{(S\cap T)(e)}(x) = C_{\phi(e)}(x)$. Consequently, $C_{(S\cap gssm-cl(T))(e)}(x) = C_{\phi(e)}(x)$ and $C_{(gssm-cl(S)\cap T)(e)}(x) = C_{\phi(e)}(x)$. Hence S_E and $T_E$are $gssm$-separated.□

Theorem 3.3. Two sub $gsos$ msets of a SMTS (X, τ, E) are $gssm$-separated if and only if they are disjoint.

Proof. Since $gssm$-separated sets are disjoint, $gsos$-msets are disjoint. Conversely, let S_E and T_E be two disjoint $gsos$-msets. Suppose that $C_{(S\cap gssm-cl(T))(e)}(x) \neq C_{\phi(e)}(x)$. Let $[(n/x)_e]_E \leq C_{(S\cap gssm-cl(T))(e)}(x)$. Then $[(n/x)_e]_E \leq C_{S(e)}(x)$ and $[(n/x)_e]_E$ is a $gssm$-adherent point of T_E, since S_E is a $gsos$-mset containing $[(n/x)_e]_E$ and $[(n/x)_e]_E$

is a $gssm$-adherent point of T_E, S_E contains at least one soft multi point of T_E. Thus $C_{(S\cap T)(e)}(x) \neq C_{\phi(e)}(x)$. This contradicts the fact that S_E and T_E are two disjoint $gsos$-msets. Therefore $C_{(S\cap gssm-cl(T))(e)}(x) = C_{\phi(e)}(x)$. Similarly $C_{(gssm-cl(S)\cap T)(e)}(x) = C_{\phi(e)}(x)$. Hence S_E and $T_E$are $gssm$-separated.□

Definition 3.4. If $C_{X(e)}(x) = C_{S(e)}(x) \cup C_{T(e)}(x)$ such that S_E and T_E are non empty $gssm$-separated sets, then S_E, T_E form $gssm$- separation of X_E.

Definition 3.5. A SMTS (X, τ, E) is said to be generalized semi soft multi connected (briefly $gssm$-connected) if X_E cannot be written as a union of two disjoint non empty $gsos$-msets.

If X_E is not $gssm$-connected then it is $gssm$-disconnected.

Proposition 3.4. Every $gssm$-connected space is soft multi connected but not conversely.

Proof. Let (X, τ, E) be a $gssm$-connected space. Suppose that X_E is not soft multi connected. Then $C_{X(e)}(x) = C_{S(e)}(x) \cup C_{T(e)}(x)$, where S_E and T_E are disjoint non empty open soft msets in (X, τ, E). Since every open soft msets is $gsos$-mset, S_E and T_E are $gsos$-mset. Therefore $C_{X(e)}(x) = C_{S(e)}(x) \cup C_{T(e)}(x)$, where S_E and T_E are disjoint non empty $gsos$-msets in (X, τ, E). This contradicts the fact that X_E is $gssm$-connected and so that X_E is soft multi connected.□

Proposition 3.5. A SMTS (X, τ, E) is a $gssm$-disconnected space if there exists a non-empty proper sub soft mset of X_E which is both $gsos$-mset and $gscs$-mset.

Proof. Let S_E be a non-empty proper sub soft mset of X_E which is both $gsos$-mset and $gscs$-mset. Then clearly S_E^c is a non-empty proper sub soft mset of X_E which is both $gsos$-mset and $gscs$-mset. Thus $C_{(S\cap S^c)(e)}(x) = C_{\phi(e)}(x)$, and also $C_{X(e)}(x) = C_{S(e)}(x) \cup C_{S^c(e)}(x)$. Thus X_E is the union of two disjoint non empty $gsos$-msets. Hence $X_E is\ gssm$-disconnected.□

Theorem 3.4. A SMTS (X, τ, E) is a $gssm$-disconnected space if and only if X_E is the union of two disjoint non empty $gsos$-msets.

Proof. Let X_E be $gssm$-disconnected. Then there exists a non-empty proper sub soft mset of X_E which is both $gsos$-mset and $gscs$-mset. And therefore, S_E^c is a non-empty sub soft mset of X_E that is both $gsos$-mset and $gscs$-mset.Thus $C_{(S\cap S^c)(e)}(x) = C_{\phi(e)}(x)$, and also $C_{X(e)}(x) = C_{S(e)}(x) \cup C_{S^c(e)}(x)$. This shows that X_E is the union of two disjoint non empty $gsos$-msets.

Conversely, let X_E is the union of two disjoint non-empty $gsos$-msets S_E and T_E. Then $C_{T^c(e)}(x) = C_{S(e)}(x)$. Now T_E is a $gsos$-mset, S_E is a $gscs$-mset. Since T_E is non-empty S_E is a non-empty proper sub soft mset of X_E which is both $gsos$-mset and $gscs$-mset. Thus X_E is $gssm$-disconnected.□

Corollary 3.1. SMTS (X, τ, E) is a $gssm$-disconnected space if and only if X_E is the union of two disjoint non empty $gscs$-msets.
Proof. Obvious.□

Theorem 3.5. For a SMTS (X, τ, E), the following are equivalent

(a) (X, τ, E) is $gssm$-connected
(b) the only sub soft msets of (X, τ, E) which are both $gsos$-mset and $gscs$-mset are $\tilde{\phi}$ and $\tilde{X}$.

Proof. (a) ⇒ (b) Let U_E be a $gsos$-mset and $gscs$-mset of X_E. Then U_E^c is both $gsos$-mset and $gscs$-mset of X_E. Since X_E is disjoint union of $gsos$-msets U_E and U_E^c, by assumption one of these must be empty. i.e. $C_{U(e)}(x) = C_{\phi(e)}(x)$ or $C_{U(e)}(x) = C_{X(e)}(x)$.
(b)⇒ (a) Suppose that $X_E gssm$-disconnected. Then by Proposition 3.5, there exists a non-empty proper sub soft mset of X_E, which is both $gsos$-mset and $gscs$-mset, which is a contradiction. Thus X_E is $gssm$-disconnected.□

Proposition 3.6. If (X, τ_1, E) is a $gssm$-connected topological space and τ_1 finer than τ_2, then (X, τ_2, E) is also a $gssm$-connected space.

Proof. Suppose that (X, τ_2, E) is a $gssm$-disconnected space. Then there exists a non-empty proper sub soft msetS_E of X_E which is both $gsos$-mset and $gscs$-mset in (X, τ_2, E).

So S_E and S_E^c are both $gsos$-mset in (X, τ_2, E). Since $\tau_2 \sqsubseteq \tau_1$, it follows that S_E and S_E^c are both $gsos$-mset in (X, τ_1, E). This shows that S_E is a non-empty proper sub soft mset of X_E which is both $gsos$-mset and $gscs$-mset in (X, τ_1, E). This is contradiction. Therefore (X, τ_2, E) is a $gssm$-connected space.□

Proposition 3.7. Let (X, τ, E) and (Y, σ, K) be two SMTSs and let $u: X \to Y$, $p: E \to K$ be functions. If $f: X_E \to Y_K$ is $gssm$-continuous surjection and (X, τ, E) is $gssm$-connected then (Y, σ, K) is soft multi connected.

Proof. Suppose that (Y, σ, K) is not soft multi connected. Let $C_{Y(k)}(y) = C_{S(k)}(y) \cup C_{T(k)}(y)$, where S_K and T_K are disjoint non-empty open soft msets in Y_K. Since f is $gssm$-continuous surjection, $X_E = f^{-1}(S_K) \cup f^{-1}(T_K)$, where $f^{-1}(S_K)$ and $f^{-1}(T_K)$ are disjoint non-empty $gsos$-msets in X_E. This contradicts the fact that X_E is $gssm$-connected. Hence Y_K is soft multi connected.□

4 Generalized semi soft multi compactness

In this section, the notions of generalized semi soft multi compactness and locally generalized semi soft multi compactness are introduced. Some properties of generalized semi soft multi compact spaces are also discussed.

Definition 4.1. A collection $\{(S_E)_i : i \in \wedge\}$ of $gsos$-msets in SMTS (X, τ, E) is called generalized semi soft multi open cover (briefly $gssm$-open cover) of a sub soft mset B_E of X_E if $C_{B(e)}(x) \leq C_{\cup\{(S_i)(e_i): i\in\wedge\}}(x)$ holds.

Definition 4.2. A SMTS (X, τ, E) is generalized semi soft multi compact (briefly $gssm$-compact) if every $gssm$-open cover of X_E has a finite sub cover.

Example 4.1. In Example 3.1, the SMTS (X, τ, E) is $gssm$-compact.

Definition 4.3. A sub soft mset B_E of SMTS is said to be $gssm$-compact relative to X_E if for every collection $\{(S_E)_i : i \in \wedge\}$ of $gsos$-msets of X_E such that $C_{B(e)}(x) \leq C_{\cup\{(S_i)(e): i\in\wedge\}}(x)$, there exists a finite subset $\wedge_o$ of $\wedge$ such that $C_{B(e)}(x) \leq C_{\cup\{(S_i)(e): i\in\wedge_o\}}(x)$.

Proposition 4.1. Every $gssm$-compact space is soft multi compact space.

Proof. Let (X, τ, E) be a $gssm$-compact space. Let $\{(S_E)_i : i \in \wedge\}$ be a soft multi open cover of X_E. Since every open soft mset is a $gsos$-mset, $\{(S_E)_i : i \in \wedge\}$ is a $gssm$-open cover of X_E. Then there exists a finite sub set $\wedge_o$ of $\wedge$ such that $C_{X(e)}(x) \leq C_{\cup\{(S_i)(e): i\in\wedge_o\}}(x)$. Thus X_E is a soft multi compact space.□

Proposition 4.2. Every $gscs$-mset of a $gssm$-compact space is $gssm$-compact relative to (X, τ, E).

Proof. Let S_E be a $gscs$-mset of a $gssm$-compact space (X, τ, E). Then S_E^c is a $gsos$-mset in (X, τ, E). Let $C_{M(e)}(x) = C_{\cup\{(G_i)(e): i\in\wedge\}}(x)$ be a soft multi open cover of S_E by $gsos$-msets. Therefore $M_E \cup S_E^c$ is a $gssm$-open cover of X_E. Since X_E is $gssm$-compact, there exists a finite subset $\wedge_o$ of $\wedge$ such that $C_{(M\cup S^c)(e)}(x) \leq C_{\cup\{(G_i)(e): i\in\wedge_o\}}(x)$. Therefore $C_{S(e)}(x) \leq C_{\cup\{(G_i)(e): i\in\wedge_o\}}(x)$. Hence S_E is $gssm$-compact relative to (X, τ, E).□

Proposition 4.3. Let (X, τ, E) and (Y, σ, K) be two SMTSs and let $u: X \to Y$, $p: E \to K$ be functions. If $f: X_E \to Y_K$ is $gssm$-continuous and S_E is $gssm$-compact relative to X_E, then $f(S_E)$ is soft multi compact relative to (Y, σ, K).

Proof. Let $\{(T_E)_i : i \in \wedge\}$ be a soft multi open cover of $f(S_E)$. Then $C_{f(S)(e)}(x) \leq C_{(T_i)(e): i\in\wedge\}}(x)$ and so $C_{S(e)}(x) \leq C_{\{f^{-1}(T_i)(e): i\in\wedge\}}(x)$. Since f is $gssm$-continuous, $f^{-1}(T_E)_i$ is s $gsos$-mset for each $i \in \wedge$. Thus $\{f^{-1}(T_E)_i : i \in \wedge\}$ is a soft multi open cover of S_E by $gsos$-msets of X_E. Since S_E is $gssm$-compact relative to X_E, there exists a finite sub set $\wedge_o$ of $\wedge$ such that $C_{S(e)}(x) \leq C_{\{f^{-1}(T_i)(e): i\in\wedge_o\}}(x)$ and hence $C_{f(S)(e)}(x) \leq C_{\{(T_i)(e): i\in\wedge_o\}}(x)$. Thus $f(S_E)$ is soft multi compact relative to (Y, σ, K).□

Corollary 4.1. Let (X, τ, E) and (Y, σ, K) be two SMTSs and let $u: X \to Y$, $p: E \to K$ be functions. If $f: X_E \to Y_K$ is $gssm$-continuous surjection and X_E is $gssm$-compact, then Y_K is soft multi compact.

Definition 4.4. Let (X, τ, E) and (Y, σ, K) be two SMTSs and let $u: X \to Y$, $p: E \to K$ and $f: X_E \to Y_K$ be functions. Then f is said to be generalized semi soft multi irresolute (briefly $gssm$-irresolute) if the inverse image of every $gscs$-mset over (Y, σ, K) is $gscs$-mset over (X, τ, E).

Proposition 4.4. Let (X, τ, E) and (Y, σ, K) be two SMTSs and let $u: X \to Y$, $p: E \to K$ be functions. If a soft multi function $f: X_E \to Y_K$ is $gssm$-irresolute and a sub soft mset B_E of X_E is $gssm$-compact relative to (X, τ, E), then $f(B_E)$ is $gssm$-compact relative to (Y, σ, K).

Proof. Let $\{(S_E)_i : i \in \wedge\}$ be any collection of $gsos$-msets of (Y, σ, K), such that $C_{f(T)(e)}(x) \leq C_{\{(S_i)(e): i \in \wedge\}}(x)$. Then $C_{T(e)}(x) \leq C_{\{f^{-1}(S_i)(e): i \in \wedge\}}(x)$ holds. By assumption, T_E is $gssm$-compact relative to X_E, there exists a finite sub collection $\wedge_o$ of $\wedge$ such that $C_{T(e)}(x) \leq C_{\{f^{-1}(S_i)(e): i \in \wedge_o\}}(x)$. Therefore $C_{f(T)(e)}(x) \leq C_{\{(S_i)(e): i \in \wedge_o\}}(x)$. Hence $f(T_E)$ is $gssm$-compact relative to Y_K.□

Proposition 4.5. Let (X, τ, E) be a SMTS. If U_E and V_E are $gssm$-compact relative to X_E, then $U_E \cup V_E$ is $gssm$-compact relative to X_E.

Proof. Let $C_{F(e)}(x) = C_{\cup\{(S_i)(e): i \in \wedge\}}(x)$ be any soft multi open cover of $U_E \cup V_E$ by $gsos$-mset of X_E.Then F_E is a $gssm$-open cover of both U_E and V_E. By hypothesis, there exists a finite sub collection $\wedge_{U_E}$, $\wedge_{V_E}$ of $\wedge$ such that $C_{U(e)}(x) \leq C_{\cup\{(S_i)(e): i \in \wedge_{U_E}\}}(x)$ and $C_{V(e)}(x) \leq C_{\cup\{(S_i)(e): i \in \wedge_{V_E}\}}(x)$. Therefore $C_{U \cup V(e)}(x) \leq C_{\cup\{(S_i)(e): i \in \wedge_{U_E} \cup \wedge_{V_E}\}}(x)$. This shows that $U_E \cup V_E$ is $gssm$-compact relative to X_E.□

Proposition 4.6. Let (X, τ, E) be a SMTS. Let $C_{S(e)}(x) \leq C_{X(e)}(x)$ be $gssm$-compact relative to X_E and T_E be a $gscs$-mset of X_E. Then $S_E \cap T_E$ is $gssm$-compact relative to X_E.

Proof. Let $\{(U_E)_\alpha : \alpha \in \wedge\}$ be any soft multi open cover of $S_E \cap T_E$ be $gsos$-msets of (X, τ, E). Then $\{(U_E)_\alpha : \alpha \in \wedge\} \cup (X_E - T_E)$ is soft multi open cover of $gsos$-msets of (X, τ, E). Since S_E is $gssm$-compact relative to X_E, there exists a finite sub collection $\wedge_o$ of $\wedge$ such that $C_{S(e)}(x) \leq C_{(\cup\{(U_\alpha): \alpha \in \wedge_o\} \cup (X-T))(e)}(x)$. Then $\{(U_E)_\alpha : \alpha \in \wedge_0\}$ is a finite $gssm$-open sub cover of $S_E \cap T_E$, which implies that $S_E \cap T_E$ is $gssm$-compact relative to X_E.□

Theorem 4.1. A SMTS (X, τ, E) is $gssm$-compact if and only if every collection of $gscs$-msets of X_E with finite intersection property has a non-empty intersection.

Proof. Let (X, τ, E) be a $gssm$-compact and let $\{(F_E)_\alpha\}$ be a family of $gscs$-msets with the finite intersection property. Suppose that $C_{\cap_\alpha (F_\alpha)(e)}(x) = C_{\phi(e)}(x)$. Then $C_{\cap_\alpha (F_\alpha)^c(e)}(x) = C_{X(e)}(x)$. But X_E is $gssm$-compact, $C_{X(e)}(x) = C_{\cup_{i=1}^{n} (F_{\alpha_i})^c(e)}(x)$ and therefore $C_{\cap_{i=1}^{n} F_{\alpha_i}(e)}(x) = C_{\phi(e)}(x)$. This contradicts the finite intersection property of $\{(F_E)_\alpha\}$. Hence $C_{\cap_\alpha (F_\alpha)(e)}(x) \neq C_{\phi(e)}(x)$.

Conversely, suppose that every collection of $gscs$-msets with finite intersection property has a non-empty intersection. Let $\{(G_E)_\alpha\}$ be a $gssm$-open cover of X_E such that $C_{X(e)}(x) = C_{\cup_\alpha (G_\alpha)(e)}(x)$. Then $C_{\cap_\alpha (G_\alpha)^c(e)}(x) = C_{\phi(e)}(x)$, by assumption, this family does not have finite intersection property. That is $C_{\cap_{i=1}^n (G_{\alpha_i})^c(e)}(x) = C_{\phi(e)}(x)$ and so $C_{\cup_{i=1}^n G_{\alpha_i}(e)}(x) = C_{X(e)}(x)$. Hence (X, τ, E) is $gssm$-compact.□

Corollary 4.2. A SMTS (X, τ, E) is $gssm$-compact if and only if every collection of $gscs$-msets of X_E with empty intersection has a finite sub family with empty intersection.

Definition 4.5. A SMTS (X, τ, E) is said to be locally $gssm$-compact if every soft multi point in X_E has at least one $gssm$-nbd whose closure is $gssm$-compact relative to X_E.

Proposition 4.7. Every $gssm$-compact space is locally $gssm$-compact but not conversely.

Proof. Let (X, τ, E) be a $gssm$-compact. Let $[(n/x)_e]_E \in X_E$. Then X_E is an $gssm$-nbd of $[(n/x)_e]_E$ such that $C_{cl(X)(e)}(x) = C_{X(e)}(x)$ is $gssm$-compact. Thus every soft multi point in X_E has at least one$gssm$-nbd, namely X_E whose closure is $gssm$-compact. Hence (X, τ, E) locally $gssm$-compact.□

4 Discussion and Conclusions

The main focus of this work is the introduction of generalized semi soft multi connectedness and compactness by using generalized semi soft multi separated sets, soft multi adherent point, generalized semi soft multi adherent point and generalized semi soft multi open cover. Also, many related theorems and propositions are provided. As a continuation of this work, in future, more generalizations of soft multi connectedness and compactness will be investigated. Further we have an idea to extend the work in separation axioms and filters.

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