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Fostering students' mathematical reasoning through a cooperative learning model

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ABSTRACT

This study sought to ascertain whether the student teams-achievement division (STAD) model of cooperative learning is effective in fostering students' mathematical reasoning. Using the cluster random sampling technique, 301 eleventh-grade students between the ages of 14 and 20 were chosen from six public secondary schools within one district in Zambia. Students were given tasks on quadratic equations and functions both before and after the intervention. A robust analysis of the covariance test revealed that students' mathematical reasoning abilities were significantly higher for the group that received instruction using the STAD approach than for the group that was taught using conventional methods of instruction at each of the five design points where regression slopes were comparable. A Chi-square test of independence further revealed that the STAD learning approach was associated with a greater proportion of students who demonstrated an appropriate degree of mathematical reasoning ability for each of the three indicators (conjecturing, justifying, and mathematizing). These results demonstrate that enhancing students' mathematical reasoning abilities through the integration of classroom activities that engage students intellectually, physically, and socially is beneficial.

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1. INTRODUCTION

Logic, critical thinking, precision, decision-making, and problem-solving are all mental skills that may be developed with the help of mathematical knowledge. As a result, mathematics occupies a central place in the school curriculum not just in Zambia but also in other settings. For instance, South Africa's Department of Basic Education [1] asserts that solving mathematical problems teaches us to think creatively and helps us comprehend the social, economic, and physical contexts around us. According to Sidhu [2], "mathematics is pursued for a variety of practical purposes, including a person's intellectual development in numeracy, reasoning, thinking, and problem-solving skills".

Ensuring that learners display appropriate mathematical reasoning is one way through which various goals of mathematics education could be achieved. This is attributed to a variety of reasons. First, mathematical reasoning is used to illustrate mathematical behavior, which suggests or reveals mathematical proficiency [3]. Second, mathematical reasoning provides insights into how students process information, solve problems, and make connections among mathematical concepts [4]. Third, enhanced mathematical reasoning can help students make informed decisions both within and outside the mathematics classroom [5].

This demonstrates why the development of students' mathematical reasoning skills is a goal of several high school mathematics curricula around the world [3], [5]–[8]. As a result, research in this area can help in the development of effective teaching strategies and curricula that emphasize mathematical reasoning.

Mathematical reasoning is a key skill that students should develop through their mathematics education, but it is often challenging to foster in many classroom settings [9], [10]. One of the factors that influences the development of mathematical reasoning is the type of teaching strategies that teachers employ in their lessons. This paper focuses on the situation in Zambia, where teaching strategies have been found to be largely ineffective in promoting mathematical reasoning among students [3].

One of the benefits of cooperative learning is that it enhances students' ability to reason mathematically by allowing them to share their thoughts with peers [11]. Students can construct their own understanding of mathematical concepts by combining new and existing ideas with valid and persuasive arguments, which emerge from their interactions with peers during cooperative group discussions [12]. In the absence of such interactions, there is a possibility that the learned content may be superficially fixed. This suggests, to some extent, that the nature of the mathematical activities that students engage in and the methods they use to solve them are essential for the development of their mathematical reasoning skills.

The aim of this research was to examine the impact of the student teams-achievement division (STAD) model of cooperative learning on the development of mathematical reasoning skills among grade 11 students from selected schools. Slavin [13] defined STAD as "a cooperative learning method in which learners with different abilities work in small groups to achieve a common learning goal". The main features of STAD are team rewards, individual accountability, and equal opportunities for success. Team rewards are the certificates or other incentives that may be awarded to the team(s) that perform better than a set standard. Previous studies have shown that different cooperative learning models can enhance students' achievement in mathematics [11], [14]–[16]. Therefore, it was expected that a well-implemented STAD model of cooperative learning would also improve students' mathematical reasoning skills. This expectation was also based on the evidence that STAD has been effective in mathematics classrooms where it has been used [17], [18]. Therefore, the following research question guided this study: How effective is the STAD model of cooperative learning for improving students' mathematical reasoning skills?

It was anticipated that answering this question would highlight the value of the STAD model of cooperative learning in mathematics classrooms. The study was also motivated by the lack of research that have specifically focused on understanding the impact of the STAD model of cooperative learning regarding the development of students' mathematical reasoning skills. Moreover, the use of cooperative learning has become more relevant since it promotes collaboration, one of the 21st century skills that has been emphasized in various policy documents including Zambia's 2013 curriculum framework at all levels of education [19].

2. DEFINING MATHEMATICAL REASONING

Mathematical reasoning has not been clearly defined in existing literature since different scholars have used varied definitions of the term depending on the context. For instance, Ball and Bass [20] consider mathematical reasoning as nothing less than a basic skill, in contrast to Lithner [21] who views it as a trait with a strong deductive-logical quality. Others have just provided a general definition of reasoning as a line of thought taken to make claims and achieve conclusions in task solving [21]–[23]. Even though a need to foster students' mathematical reasoning has been emphasized in various curriculum reform documents, the way the term has been described in those documents "tends to be vague, unsystematic, and even contradictory from one document to the other" [5]. The four key elements of mathematical reasoning that Jeannotte and Kieran [5] discovered from their study and analysis of the literature are the activity-product dichotomy, its inferential character, it is objective and functions, and the structural and process aspects. These four components were combined to characterize mathematical reasoning as "a process of communication with others or with oneself that allows for inferring mathematical utterances from other mathematical utterances." This perspective extends the concept of mathematical reasoning by incorporating its structural and process aspects. In other words, the model highlights both the practical and theoretical aspects of mathematical reasoning.

Therefore, the mathematical reasoning referred to in this study relates to how well students can relate the mathematics they learn in class to the real-life situations [3], as well as their ability to draw justifiable inferences with justification and generalization serving as the central components [21], [24]. Given this context, the current study's assessment of the students' mathematical reasoning skills focused on three mathematical abilities: conjecturing, justifying, and mathematizing. Conjecturing in the context of high school mathematics refers to the process of making a mathematical statement or claim that has not yet been rigorously proved. According to Aaron and Herbst [25], conjecturing is an important step in problem-solving, as it helps students develop their mathematical thinking and reasoning skills through the analysis of problem structure, examination of cases, and confirmation. Research on conjecturing in high school mathematics education has focused on understanding how students engage in this process and how it can be effectively taught [25]–[27]. In the context of this study, questions 1(a) and partly 3(a) in Figure 1 are samples of questions that were used to assess students conjecturing skills.

Justification, in the context of high school mathematics refers to the process of providing evidence or reasoning to support a mathematical statement or claim [27]. Research in this area focuses on understanding how students engage in this process and how it can be effectively taught [4], [27]. In the context of this study, students' ability to justify mathematical statements was evaluated using question such as 1(b), 2(a), 3(a), and 3(b) that have been provided in Figure 1.

Mathematization is the process of translating a real-world problem into a mathematical problem, and then using mathematical reasoning to solve it. According to the Department of Basic Education [28], "mathematization involves identifying the relevant mathematical concepts and relationships, representing them using mathematical symbols and language, and then using mathematical reasoning to solve the problem." Therefore, mathematization is an important aspect of mathematical reasoning, as it helps students develop their ability to apply mathematics to real-world situations [3]. In the context of this study, question 2 in Figure 1 is a sample of those that required students to put a real-world scenario into mathematical terms, and vice versa.

1. Consider the statement *"$x^2 + 1$ can never be zero"*.
   a. If x is a real number, state whether the above statement is true or false.
   b. Justify your choice in (a) above.
2. A boy buys x eggs at $(x-8)$*kwacha* each and $(x-2)$ notebooks at $(x-3)$ *kwacha* each. If the total bill is 76 *kwacha*;
   a. Show that $2x^2 - 13x - 70 = 0$
   b. Hence determine the number of eggs and the number of notebooks that he bought.
3. Given the function $A = 60x - 2x^2$
   a. State whether this function will have a *maximum* or *minimum* value. Give a reason for your choice.
   b. Sketch the graph of $A = 60x - 2x^2$ taking values of x from 0 to 30.
   c. Based on your graph in (b), do the coordinates of the turning point justify why the graph has a minimum or maximum value? Give a reason.

Figure 1. Sample questions from the MRT test items on quadratic functions and equations

3. RESEARCH METHOD

3.1. Research design

The work reported in this paper is part of the research whose aim was to develop students' mathematical reasoning alongside their self-efficacy beliefs using the STAD model of cooperative learning. This learning approach was considered appropriate due to its focus on engaging learners physically, socially, and intellectually in meaning and knowledge creation. After a baseline study whose aim was to establish the prevailing mathematics teaching practices in selected secondary schools, quasi-experimental research was administered to examine the effectiveness of STAD on students' mathematical reasoning. Among the several quasi-experiments, a pretest-posttest control group design was used in the present study. The baseline survey and lesson observations that were conducted earlier, revealed that most teachers did not use all the available opportunities to foster students' mathematical reasoning, even though they claimed to have tried hard to do so [3]. The survey also showed that most teachers avoided cooperative learning strategies because they found them difficult to manage, assess, and fit into the bulky syllabus [29]. Based on the finding of the survey conducted prior to the intervention, the teachers in the experimental group received training on how to implement the STAD cooperative learning model effectively to improve students' mathematical reasoning on quadratic equations and functions.

3.2. Participants

Cluster random sampling was used to choose participants for the study. Based on their average performance from national examinations, twenty public secondary schools from one district in Zambia's Copperbelt Province were grouped into three categories: high, moderate, and low. Schools with pass rates of 75% and higher were coded as high performing, while those with pass rates from 50% to 74 % were coded as moderate performing and those below 50% pass rates were coded as low performing. Then, two schools were chosen at random from each cluster. Assignment of the two schools from each cluster to the control or experimental group was also random. This means that each group (control and experimental) was allocated with one high, one moderate, and one low performing school. This was done to make sure that both the control group and the experimental group had representation from each of the three different types of schools. Each of the six selected schools had one grade 11 class that was randomly picked, and all of those students were included in the sample, making up a total of 301 participants.

The sample size employed in this study was deemed sufficient for conducting an analysis of covariance (ANCOVA) that compared two independent groups. The sample was divided approximately equally between the two groups. Specifically, 150 participants were randomly allocated to the control group, while the experimental group comprised 151 participants. Although these numbers diminished slightly during the post-test phase, the distribution remained nearly balanced, with 146 participants in the control group and 148 in the experimental group. A notable limitation in our sample size estimation was the absence of a power analysis to ascertain the appropriate sample size prior to data collection. This was largely due to the inherent nature of the study, which involved intact classes with a fixed number of students. Nonetheless, it is worth noting that our sample size significantly exceeds the minimum suggested sample size of 15 to 19 per group for achieving a power of 0.8 to 0.9, as recommended by Shieh [30].

3.3. Topic selection

For six weeks, students from both groups were taught quadratic equations and quadratic functions. The contents of the two topics did not only focus on knowledge that prepares students for undertaking advanced mathematics courses but also to improve their logical reasoning, accuracy, and decision making as prescribed in the school mathematics curriculum [19]. It was further anticipated that engaging students in applications of quadratic equations, graph construction and interpretation would introduce them to various kinds of representations and real-world experiences, which in turn would increase their mathematical reasoning skills. Besides that, students' performance in these topics has been below the expected standard not only in Zambia [31]–[33], but also in other settings [34]–[37].

3.4. Intervention

Two instructional approaches were used in this research: Expository teaching (regarded as a traditional method of teaching in this study) and cooperative learning (STAD). Before and during the intervention, most of the techniques observed in the control group supported the expository approach. In this method, students were required to sit in rows and columns with a teacher in front utilizing question-and-answer procedures, mostly using chalk and talk. At the end of the first cycle which lasted for 3 weeks, a written quiz was administered in which students answered questions individually. However, scores that students obtained in this quiz were not analyzed. Those scores were primarily used for monitoring learners' progress in terms of their ability in conjecturing, justifying, and mathematising. Week 6 was characterized by whole class revisions and the administration of the post-test. During week 6 revisions teachers were encouraged to revisit certain concepts that appeared more challenging to the learners.

The STAD model of cooperative learning, on the other hand, focused primarily on learner-centered approach and was distinguished by small group discussions. Before commencement, teachers received a 3-day orientation session on STAD implementation, which was organized by a researcher. The following procedures were implemented in all three classes belonging to the experimental group in accordance with earlier research on the STAD model of cooperative learning [14], [16], [38]. The researcher's role during the intervention was to observe lessons in both control and experimental group to ensure adherence to prescribed procedures. Materials to use such as lesson notes, exercises, quizzes and tests were provided by the researcher to ensure that both groups were taught the same material.

3.4.1. Step I: whole class presentation

This step involved a teacher presenting the material to the whole class using lectures and demonstrations. This phase usually lasted for 10 to 30 minutes depending on the nature of the activities involved for teaching a particular concept. For instance, explaining the procedure for graphing a quadratic function of the form $f(x) = ax2 + bx + c$ took longer than explaining the concept of solving quadratic equations by factorization method.

3.4.2. Step II: small group discussion

After the whole class presentation, students were split into heterogeneous groups of four with differing mathematical ability and gender. Students' ability levels were judged based on their performance in the previous test quadratic equations and functions were taught. A teacher's knowledge of each student's ability was also used as a basis for group formation. Students worked within their groups to make sure that every member understood the material. Within those small groups, students were encouraged to ask for clarifications from their peers. They were also encouraged to debate fault reasoning and justify their ideas to other group members. This was done to promote a sense of interdependence among group members and individual accountability to the whole group. It was assumed that constituting groups with students of varying ability levels would enable the more knowledgeable students to explain concepts to their peers. After group discussions, each group was requested to present their solutions to the entire class. Group representatives during whole-class presentations were appointed at random by the teacher to encourage the participation of all members of a particular group regardless of their ability or gender. About 40 minutes were allocated to small group discussions and presentations while the remaining 10 to 25 minutes were assigned to lesson evaluation including teacher's and/or students' clarifications as well as giving of class exercises, and homework where it was necessary.

3.4.3. Step III: quiz/test administration

Quizzes and tests were not given in every lesson as most lessons ended at step II. In addition to what occurred in the control group in week 3 and week 6, students in the experimental group were also encouraged to meet in their respective groups to hold discussions even during their free time after class sessions. After marking of the test scripts, the highest performing group was identified and awarded. Determination of students' improvement and their contribution to the group average was done using the "improvement score conversion table and the test score sheet" [39]. An achievement test score for each student was compared to his previous test score (base score), and points were awarded to the group based on the degree to which a particular student met or exceeded his previous score following the criteria prescribed in Table 1. The individual improvement score was then calculated by comparing the difference between the new score and the old score using the improvement score conversion sheet as shown in Table 1. It suffices to mention that the improvement score conversion table proposed by Li and Lam [38] was not strictly followed as some modifications were made to suit the current scenario and context. Besides the criteria outlined in Table 1, every new score of 85% or more was classified as outstanding and 25 points were awarded to the group regardless of a participant's previous score. For instance, a new score of 88% compared to a base score of 92% would still fall in the category of outstanding performance and 25 points would be allocated despite a drop by 4%. The justification behind this classification is that such a student still managed to maintain high level performance despite a decrease in marks obtained compared to the previous score.

The rationale behind this method was to give equal opportunities to group members to contribute points to the group whenever their new score was better than the previous one. As such, it was assumed that low-achieving students would be motivated to improve their scores because they were also able to see their contribution to group success. High-achieving students were equally motivated to help their peers to understand the material to boost the group average score. This collective responsibility resulted in individual learning benefits for all group members regardless of their reasoning ability levels.

Table 1. Improvement score conversion sheet

Improvement score	Points earned
Less than (below) the previous score	-5
Equal to the previous score	0
More than the previous score by 1 to 5	5
More than the previous score by 6 to 10	10
More than the previous score by 11 to 15	15
More than the previous score by 16 or more/Outstanding performance	25

3.4.4. Step IV: group recognition

Points contributed by individuals to the group were summed up and the average for each group was computed. The group with the highest average points was recognized and presented with a certificate for being the best performing group. In some cases, rewards were also given to groups that reached a pre-determined level of performance.

3.5. Mathematical reasoning test item formulation and validation

A mathematical reasoning test (MRT) was administered to all the research participants before and after the intervention. Formulation of test items was anchored on the notion of mathematical reasoning for school mathematics [5], [9]. Besides that, all the included items conform to the aims and objectives of the Zambian mathematics curriculum for secondary schools [8]. The 2013 curriculum framework [19] outlines the learning outcomes for school mathematics students, such as clear mathematical thinking, logical reasoning, problem-solving, and real-world application of the learned content. For example, students working with quadratic equations and functions should be able to use the quadratic formula correctly, identify and interpret quadratic equations and functions in real-world contexts, relate the concept of turning points to maximum and minimum values, and generate and interpret the graphs of quadratic functions accurately.

Before administering the MRT, 13 experts including secondary school mathematics teachers, and mathematics teacher educators from colleges of education and universities were contacted for instrument validation [39]. These experts were chosen because of their experience and expertise in teaching and learning secondary school mathematics in Zambia. The experts were asked to score each test item in terms of sufficiency, clarity, coherence, and relevance to ensure that all the included test items were valid. Additionally, they were asked to comment on how each item may be made better to meet the study's goals and context.

3.6. Data analysis techniques and procedures

The robust analysis of covariance (robust ANCOVA) and the Pearson Chi-square tests were performed to provide answers to the research question. The classic analysis of covariance (ANCOVA) was originally planned as the ideal statistical technique for testing the experimental effect (students' exposure to STAD) on the dependent variable (students' mathematical reasoning ability). Before performing the actual data analysis, ANCOVA assumptions were checked in accordance with the recommended approach. Based on the Kolmogorov-Smirnov (K-S) test, a normality assumption was violated for the control group at both levels of measurement (pre-test and post-test). Only the post-test scores of the experimental group had a distribution that was not significantly different from normal, $D(148)=.065$, $p=.200$. A further analysis of the skewness and Kurtosis revealed similar results. It was also established that both the homogeneity of regression slopes and the homoscedasticity assumptions were equally violated.

Due to violations of these three assumptions, an ordinary ANCOVA was deemed unfit for this analysis. Instead, a robust ANCOVA was performed as recommended in existing literature [40]–[43]. A robust ANCOVA with the recommended bootstrap method of 20% data trimming was performed using R version 3.6.1

It was also deemed necessary to determine whether there was any association between the instructional approaches to which students were exposed (control group vs experimental group) and the MR ability level for each of the three MR indicators. To determine this association, a Pearson Chi-square test was performed followed by computation of the Odds ratio to measure the effect size of the association [40]. This test was based on a 2×2 contingency table of group (control and experimental) against a student's level of reasoning (inadequate and adequate) for each of the three MR indicators. Students were categorized as having "inadequate reasoning" if their scores on each of the MR indicators were less than 50%, and as having "adequate reasoning" if their scores were 50% or higher. According to the standards established by the Examinations Council of Zambia, a secondary school graduate must receive at least 50% in each of the relevant subjects to be admitted to the chosen program at a college or university.

4. RESULTS

4.1. Descriptive statistics on students' MRT scores

Table 2 gives a summary of the distribution of students' MRT scores for both the pre-test and post-test. These results indicate that the mean score for the control group (*M*=10.97, *SD*=6.33) was slightly higher than that of the experimental group (*M*=10.18, *SD*=5.86) before the intervention. After exposing the experimental group to STAD model of cooperative learning and the control group to the traditional methods of teaching, the experimental group (*M*=43.50, *SD*=21.89) outperformed the control group (*M*=22.11, SD=11.33). Results displayed in Table 2 further indicate that the "0" mark persisted in the control group even after students were taught the two topics.

Table 2. Descriptive statistics on students' MRT scores

Measure	Group	N	Minimum	Maximum	Mean	SD
Pre-test	Control	150	0	30	10.97	6.33
	Experimental	151	0	28	10.18	5.86
Post-test	Control	146	0	66	22.11	11.33
	Experimental	148	6	96	43.50	21.89

4.2. Robust ANCOVA results

The RSW package in R was used to compare the trimmed means between the groups at 5 design points on the post-test score with pre-test score as a covariate. The 'ancova' and 'ancboot' functions were performed. The R script (available at https://data.mendeley.com/datasets/3472zggczv/2) gives a detailed robust ANCOVA procedure that was performed alongside the associated the output, and the dataset.

Table 3 displays the results of the ANCOVA output whereas Table 4 shows the results from the ancboot output, with a focus on testing the following hypothesis as in (1).

$$H_0: m_1(x_k) = m_2(x_k)\ for\ k = 1, 2, 3, 4, 5 \quad (1)$$

In this case, m1 and m2 represent 20% trimmed means for the control and experimental groups respectively. In both Tables 3 and 4, the *X* column represents the five design points of the covariate at which the regression lines of the two groups are comparable. Similarly, n_1 and n_2 represent the number of pre-test (covariate) scores (very close to *X*) for the control and experimental groups respectively. The column labelled "DIF" indicates the trimmed mean-differences at each of the five design points while standard errors are stored in the column labelled "SE". Dividing the DIF value by SE value produces the test statistics that appear in the column labelled "TEST". The 95% confidence intervals (CIs) for the trimmed means have been included.

Table 3. Output from the ancova function

X	n_1	n_2	DIF	TEST	SE	C.I Lower	C.I Upper	*p*-value	Crit. value
0	41	46	17.42	6.29	2.77	9.83	25.02	.000	2.74
5	89	79	20.40	9.5	2.15	14.75	26.06	.000	2.63
10	116	109	24.92	9.39	2.66	18.01	31.83	.000	2.60
15	105	96	30.87	10.67	2.90	23.35	38.39	.000	2.60
20	57	65	39.25	10.85	3.62	29.72	48.78	.000	2.63

Table 4. Output from the ancboot function

X	n_1	n_2	DIF	TEST	C.I Lower	C.I Upper	*p*-value
0	41	46	17.42	6.29	9.78	25.07	.000
5	89	79	20.40	9.50	14.48	26.33	.000
10	116	109	24.92	9.39	17.60	32.24	.000
15	105	96	30.87	10.67	22.9	38.85	.000
20	57	65	39.25	10.85	29.27	49.23	.000

Results displayed in Tables 3 and 4 show that the p-values were less than .05 at all the five design points at which regression slopes were comparable. This is confirmed by the fact that the test statistic is greater than the critical value at each of the five design points as reflected in Table 3. These results show significant differences between trimmed means of the control group and those of the experimental group at all the five design points. This implies that students from the control group and those from the experimental group were significantly different in their mathematical reasoning abilities after the intervention, while controlling for the effect of students' prior mathematical reasoning skills.

It has been noted that the confidence intervals displayed in Table 3 are not the same as those reported in Table 4. This difference is attributed to the fact that the *ancboot* output in Table 4 is based on a bootstrap method. Results presented in Tables 2, 3, and 4 all point to the conclusion that the group that was exposed to STAD learning mode exhibited a significantly higher mathematical reasoning ability than the group that was taught using the traditional methods.

4.3. Results of the chi-square test of independence

Table 5 illustrates post-intervention results of a 2×2 contingency table of group (control and experimental) against the students' MR ability levels for each of the three indicators. Results displayed in Table 5 indicate that the 'inadequate' MR ability level for each of the three indicators was more prevalent in the control group than that of the experimental group. On the other hand, higher proportions of students who exhibited an adequate MR ability level for each of the three indicators was associated with the experimental group.

To establish the statistical significance of the associations displayed in Table 5, a chi-square test was performed and odds ratios for all the three indicators were computed. Results from chi-square test reflect a significant association between the teaching method to which students were exposed and their conjecturing ability levels, $\chi^2(1)=67.9$, $p<.05$. Based on the computed odds ratio it was found that the odds of students' conjecturing ability were 3.12 times higher when exposed to the STAD model of cooperative learning (experimental group) than when exposed to traditional methods of teaching (control group). In terms of students' ability to mathematize the learned algebraic concepts to real world experiences or vice versa, results show a significant association between the method of teaching and students' MR ability level, $\chi^2(1)=58.6$, $p<.05$. It was further established that the odds of students' mathematising ability levels were 11.9 times higher when exposed to the STAD than when exposed to traditional methods of teaching. Similarly, a statistically significant association between the method of teaching to which students were exposed and their ability to justify their reasoning, $\chi^2(1)=37.5$, $p<.05$. Results further indicate that the odds of students' justification abilities were 5.71 times higher when exposed to the STAD than when exposed to traditional methods of teaching.

Table 5. Cross-tabulation of group versus students' MR ability levels

MR indicator	MR ability level	Group		Total
		Control	Experimental	
Conjecturing	Inadequate	117 (70.9%)	48 (29.1%)	165
	Adequate	29 (22.5%)	100 (77.5%)	129
Justifying	Inadequate	128 (61%)	82 (39%)	210
	Adequate	18 (21.4%)	66 (78.6%)	84
Mathematising	Inadequate	137 (62.8%)	81 (37.2%)	218
	Adequate	9 (11.8%)	67 (88.2%)	76

Note. Percentages are calculated within MR ability levels for each for each indicator.

5. DISCUSSION

It has been established that students' mathematical reasoning for the experimental group was significantly higher than that of the control group at each of the five design points in the robust ANCOVA test. A Pearson Chi-square analysis and the odds ratio further revealed that higher proportions of students in the experimental group exhibited an adequate MR ability level compared to their counterparts in the control group for each of the three MR indicators. These results demonstrate that STAD is an effective cooperative learning model for fostering students' mathematical reasoning.

By having structured groups consisting of students with differing levels of academic performance and gender, findings have demonstrated that students were able to co-construct ideas. The idea of group rewards also motivated students to work collaboratively in formulating and investigating conjectures based on their own observations. This finding provides evidence of why group rewards or group goals maximizes the achievement effects of cooperative learning as pointed out in previous studies [14], [16]. It is also evident that the implemented classroom activities in the experimental group did not only help students to understand quadratic equations and functions but also improved their communication skills as they interacted with peers of varying aptitude and gender. This is consistent with the observation by Brodie *et al.* [9] that allowing students to work collaboratively on mathematical tasks would enable them to start viewing mathematics as a worthwhile human activity.

A baseline study conducted before the intervention found that teachers were reluctant to introduce cooperative learning in their classrooms because it was challenging for them to control classes with a lot of adolescents [6]. It was discovered that STAD was one method of managing large classes because knowledge and meaning construction was decentralized to groups, as opposed to individualized learning where a teacher is regarded as a knowledge authority. The classroom environment that Blatchford *et al.* [44] referred to as "more giving and more receiving help, more joint construction of ideas, and more sustained interactions in groups" was created by allowing students to express their ideas and their reasons to group members. Findings of the present study further highlights that holding students accountable for their learning and allowing them to discuss and recognize opposing points of view can increase learning quality.

Consistent with the findings of previous studies, this study also established that exposing students to tasks requiring them to formulate and investigate conjectures, justify and validate algebraic statements and arguments, and apply or mathematise contextual problems into mathematical terms, significantly improved their understanding of quadratic equations and functions [3], [24], [45], [46]. Other researchers have also emphasized that teachers need to use the right instructional strategies to enable students to participate in activities that foster higher-order thinking [47], [48]. STAD is one of these instructional strategies that allows learners to engage deeply with mathematics while working together in a socially constructed classroom environment.

However, we are aware that STAD may not always result in improved students' mathematical reasoning, even though STAD was successful in the current research. This could be due to the challenge of teaching teenagers in cooperative group settings, especially in large classes. The teacher must put in a lot of effort and make thorough preparations. For students to participate in fruitful mathematical conversations, teachers must make sure that their students are well-versed in the relevant abilities (such as listening to peers, explaining, and sharing ideas with others). In STAD, the concept of collective rewards should also be approached with caution. Slavin [14] offers advice that should be heeded, "*There is no motivation for group members to explain concepts to one another if awards are granted based on a single group product (e.g., the team completes one worksheet or solves one task). This is due to the fact that one or two group members may handle all the work*."

Based on the advice, the current study found that a good strategy to reward diligent groups is to give each member of a given group the same mark (score), and to post quiz or test results on the classroom notice board. Being aware that each group member would earn the same grade (the group average score) is likely to increase fruitful group discussions. Additionally, it may cause each member to make a significant contribution to group success.

The results of this study have also demonstrated that STAD is a successful method for handling big classrooms, particularly because the teacher may interact with a lot of students through their groups. However, if there are too many groups, teachers need to be mindful that it can take a long time to reach out to them all. This issue might not be resolved by evenly increasing the size of the group since group discussions become less efficient as the group size increases. Most sub-Saharan African countries and other parts of the world are experiencing population growth; therefore, it is expected that the student-teacher ratio will continue to rise, especially in low-income areas. This will continue to pose a challenge for most teachers on how to effectively engage students intellectually, physically, and socially. As such, there is a need to ensure that schools are adequately equipped with both infrastructure and human resources [49].

6. CONCLUSION

The main argument of this article is that high school students benefit from mathematical reasoning, as it helps them to comprehend, apply, and evaluate mathematical concepts in various situations. The authors suggest that the STAD model of cooperative learning is an effective way to foster mathematical reasoning skills, as it involves learners in active and social construction of meaning. The article also implies that mathematics teachers should be trained in cooperative and other learner-centered methods to improve the quality of mathematics education. Moreover, the article recommends that governments should invest more in expanding the access and availability of education, especially in Sub-Saharan Africa. Finally, the article proposes that future research should explore other cooperative learning models and their impact on students' mathematical reasoning, using both quantitative and qualitative methods.

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