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Combining dimensional analysis with model based systems engineering

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Abstract

The model based systems engineering (MBSE) approach describes a system using consistent views to provide a holistic model as complete as possible. MBSE methodologies end with the physical architecture of the system, but a physical model is clearly incomplete without the study of its associated physical laws and phenomena related to the whole system or its parts. However, the computational demands could be excessive even for modest projects. Dimensional analysis (DA) is common in fluid dynamics and chemical engineering, but its application to systems engineering is still limited. We describe an engineering methodological process, which incorporates DA as a powerful tool to understand the physical constraints of the system without the burden of complex analytical or numerical calculations. A detailed example describing a microantenna is presented showing the benefits of this approach. The selected example describes a problem rarely covered in modern expositions of DA in order to show the wide benefit of these techniques. The information provided by this analysis is very useful to select the best physically realizable architectures, testing design, and conduct trade-off studies. The complexity of modern systems and systems of systems demands new testing procedures in order to comply with increasingly demanding requirements and regulations. This can be accomplished through research in new DA methods. Finally, this article serves as a fairly comprehensive guide to the use of DA in the context of MBSE, detailing its strengths, limitations, and controversial issues.

KEYWORDS

dimensional analysis, MBSE, physical constraints

1 | INTRODUCTION

Systems engineering as a branch of engineering is a holistic and multidisciplinary approach for the design, realization, technical management, operations, and retirement of a system, where a system is defined as the combination of elements that function together to produce the capability required to meet a need.[1]

Nowadays the paradigm applicable in systems engineering is model based systems engineering (MBSE) where the model or abstract representation of the system is considered as the single source of truth instead of a collection of documents and diagrams as it was the current situation in traditional systems engineering.

To develop systems using MBSE, we have to consider three main aspects: a methodological process, methods or best practices associated to the MBSE process, and engineering tools supporting standard notations to represent the system model and its views.

Here we use integrated systems engineering and pipelines of processes in object oriented architectures (ISE&PPOOA),[2] as the

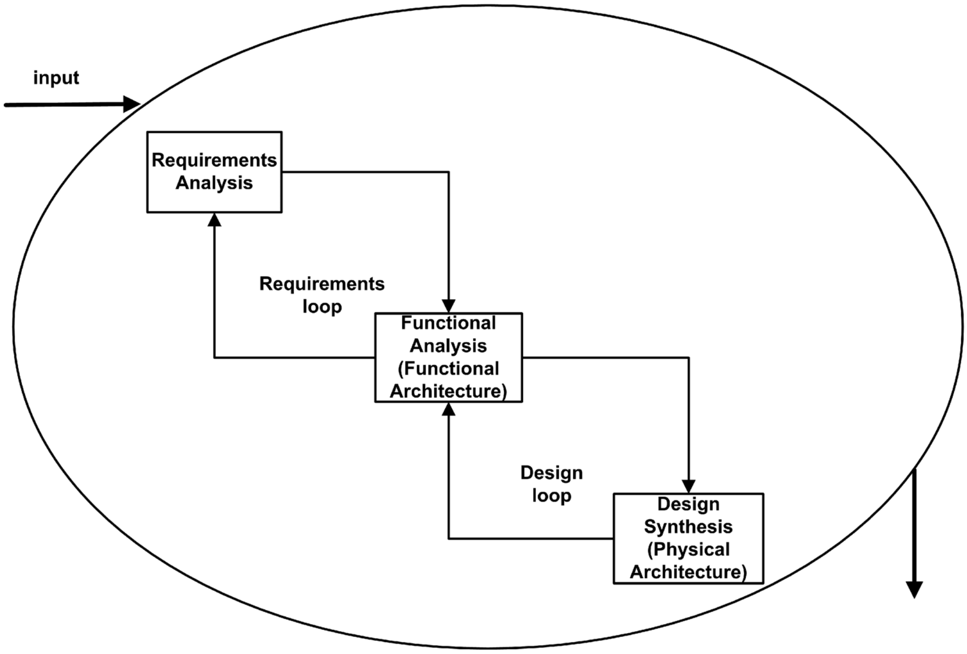

FIGURE 1 Requirements and design loops in the SE process

methodology for the development of systems which the main deliverables are those related to the concept of operations, requirements, and architecture. So the system model developed with ISE&PPOOA has three main dimensions: the mission, the system, and the software. For each of these dimensions some views of the system model are proposed. Particularly for the system dimension of the model two views with their corresponding diagrams are required. One is the behavioral view of the system representing its functions, interfaces, and the functional flows. The other is the structural view of the system representing its hierarchies of building blocks and how these building blocks interact using interfaces represented as ports and connectors in SysML notation.[3]

The ISE&PPOOA MBSE process has its foundations on a systems engineering principle borrowed from building architecture, as stated by architect Louis Sullivan[4]: "Form follows function," that means that we create the solution physical architecture considering the allocation of the functions to be provided by the system to the physical building blocks that implement them, see design loop of Figure 1. This allocation of functions to physical blocks is applied in document-centric traditional systems engineering and in some current MBSE methodologies as well.

The main outcome of the above process is a physical architecture that allocates the functions obtained by previous functional analysis. This approach needs some additional guidelines describing the constraints that apply to the physical building blocks of the system. In many cases, these constraints are due to complex physical phenomena in a real world context. In these cases, where the equations describing such phenomena are not clearly known or are prohibitively complex, reasoning based on general principles, like the physical laws of mass, energy, and momentum conservation, are applicable. However, in many situations, even the application of general conservation laws is not clear or provides little information. This is common in fast evolving fields which force the engineers to research new technologies even without a complete theoretical framework. A classical example would be radio engineering in the first half of the 20th century and a modern example could be quantum computing, without forgetting the extremely important but unsolved problem of turbulence in fluid and plasma dynamics, where DA particularly shines. MBSE can offer only the modeling of the main functional and physical interfaces. Engineers can identify all or many physical magnitudes related to the system, but the mathematical equations and boundary conditions constraining it and how to solve them is not provided by systems engineering methods and tools. MBSE notations such as SysML standard provide the way to consider these equations for the physical blocks of the system. SysML provides constraint blocks and parametric diagrams for modeling the mathematical equations but no method is supported to obtain these equations.

Here we propose an approach that uses dimensional analysis (DA) for the exploration of the mathematical equations, scaling properties, and orders of magnitude of the physical phenomena related either to the system or its parts. We use the inputs and outputs identified in the functional interfaces described in the functional architecture as inputs of the DA process. The approach is summarized here and described with more detail in the following sections.

This approach is not new, because DA has a long history of applications from the times of Rayleigh[5] to the present, but the steps of a systematic and smooth integration with MBSE has not been described, as far as we know. In a more general context, DA can be considered the first and most abstract step in the new research area of System-level Modeling,[6] which uses advanced mathematical techniques and physical analogies in order to reduce the complexity of the physical description of systems, while preserving their most important features and behavior. This mathematical reduction of the system complexity is not trivial at all and known methods are only useful as far as the physical equations and the relevant boundary conditions are completely known. In this regard, DA is as useful or more than ever.

In order to properly use DA in a systems context, we must select the variables that define the physical phenomena associated with the lowest level of the system functional hierarchy, making the following decisions:

- If analytical solutions exist, use the variables of these solutions.
- If analytical solutions are not possible, but a complete mathematical description of the problem exists in form of equations, use the variables of the equations.
- If a complete mathematical model is not known, but partial results exist, use these variables and complete them using experimental data.
- If even a partial model is not known, use physical analogies, mathematical exploratory techniques, and functional interfaces in order to formulate an approximate model. DA as an exploratory tool can be improved in a number of ways. For example, directional and angular information in vector or matrix quantities can be used to refine the equations in greater detail.[7,8] Unfortunately, this technique, which increases the number of dimensions with spatial information and permits exploring further the form of the equations terms is only valid in linear systems.

After identifying the physical dimensions and measurement units of the model variables, a dimensional matrix is produced which contains the exponents of every physical dimension (length, mass, time, electrical current, temperature, etc.) for each physical or chemical system variable. Finally, this matrix is reduced using the Buckingham Pi theorem. See, for example, Barenblatt[9] for a mathematical discussion and complete proof and Lemons[10] for a very good explanation with examples. In essence, this theorem permits the reduction of the model variables as much as the difference between the number of initial variables and the number of fundamental physical dimensions involved. The new variables are dimensionless and reflect the underlying scaling properties of the system. Thus, the mathematical model of the physical phenomena is redefined using the obtained dimensionless quantities in a way such that essential properties of the studied system are now obvious. One apparent limitation of DA is that the selection of dimensionless quantities is not unique. However, the clever exploration of several dimensionless combinations can provide important physical insights. The correct selection of the best dimensionless numbers is an art in itself, because many combinations are possible, and demand a deep knowledge of the problem. Fortunately, in fluid mechanics, chemistry and thermodynamics, many dimensionless numbers are already known and their application is fairly evident. In a fundamental sense, the equations obtained by means of DA correspond to the long term or stationary behavior of the system, providing important clues about the limiting system properties.

Usually, DA is not able to reduce the system complexity to the point that all equations are explicit. This is not as bad as it seems, because this result implies that complex system behaviors are expected, like intermediate asymptotics in the Barenblatt sense.[11] In such cases, very interesting emergent properties can happen, like fractal structures, self-organization, chaos, and irreversibility. Bejan offers an alternative perspective which permits the unification of the mathematical view of Barenblatt with the modern thermodynamic approach based on the profound and powerful constructal theory,[12] which deserves to be much more known in the systems engineering community. For example, constructal theory, combined with DA, has been used to explain pattern formation and natural design in several natural systems, including living beings, arguably the most complex.[13,14] But the theory is not restricted to natural phenomena and its possible applications to engineering are manifold.[15–17] Thus, in order to reduce the uncertainty of physical models, the problem should be simplified using reasonable hypotheses, based on fundamental laws and theories, experimental results, analogies, or numerical simulations. However, all these approaches should be guided by the previous DA study.

The main benefits of this DA approach are described later illustrating it with an example of application of a microantenna for wireless power transfer to a robotic device inside the human body.

The paper is organized into four sections beyond this introduction. Section 2 describes briefly the system architecture views proposed by ISE&PPOOA. Section 3 describes the physical and mathematical foundations of DA. Section 4 explains the DA process main steps. Section 5 describes the application of the DA process to the transfer of energy to a miniature wireless micro-robotic system for micro-surgery and in vivo interventions. We follow up with a section of lessons learned and conclusions.

2 | SYSTEM ARCHITECTURE VIEWS IN ISE&PPOOA

The ISE&PPOOA MBSE methodology proposes two views of the architecture of the modeled system. These views are the functional architecture and the physical architecture. Below, we will describe briefly each of them. A detailed description can be found in the references.[2]

The functional architecture represents using diverse SysML diagrams and tables the system functions, understanding a function as a transformation to be performed by the system that receives mass, energy, or signals and generates new ones or transforms them.

The functional architecture (Figure 2) represents the functional hierarchy using a SysML block definition diagram. The N^2 chart is a table used as an interface description where the main functional interfaces are identified. A textual description of the system functions is provided as well. The functional hierarchy is complemented with activity diagrams for the main system functional flows to represent the system behavior.

The physical architecture or architecture of the solution (Figure 3) is represented by the system decomposition into subsystems and parts using a SysML block definition diagram. This diagram is complemented with SysML internal block diagrams representing the system physical blocks with either logical or physical connectors for each subsystem identified, and activity and state diagrams for behavioral description as needed. A tabular description of the system parts may be provided

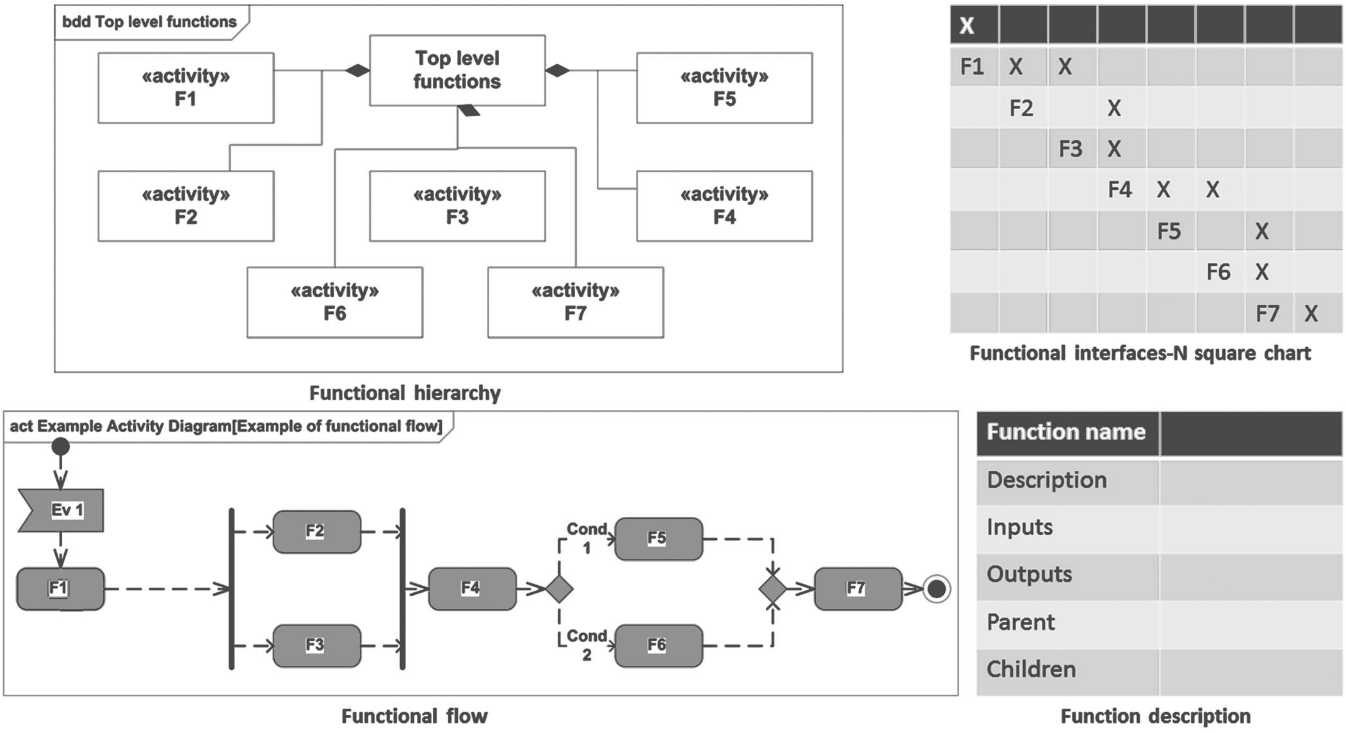

FIGURE 2 Functional architecture diagrams and tables

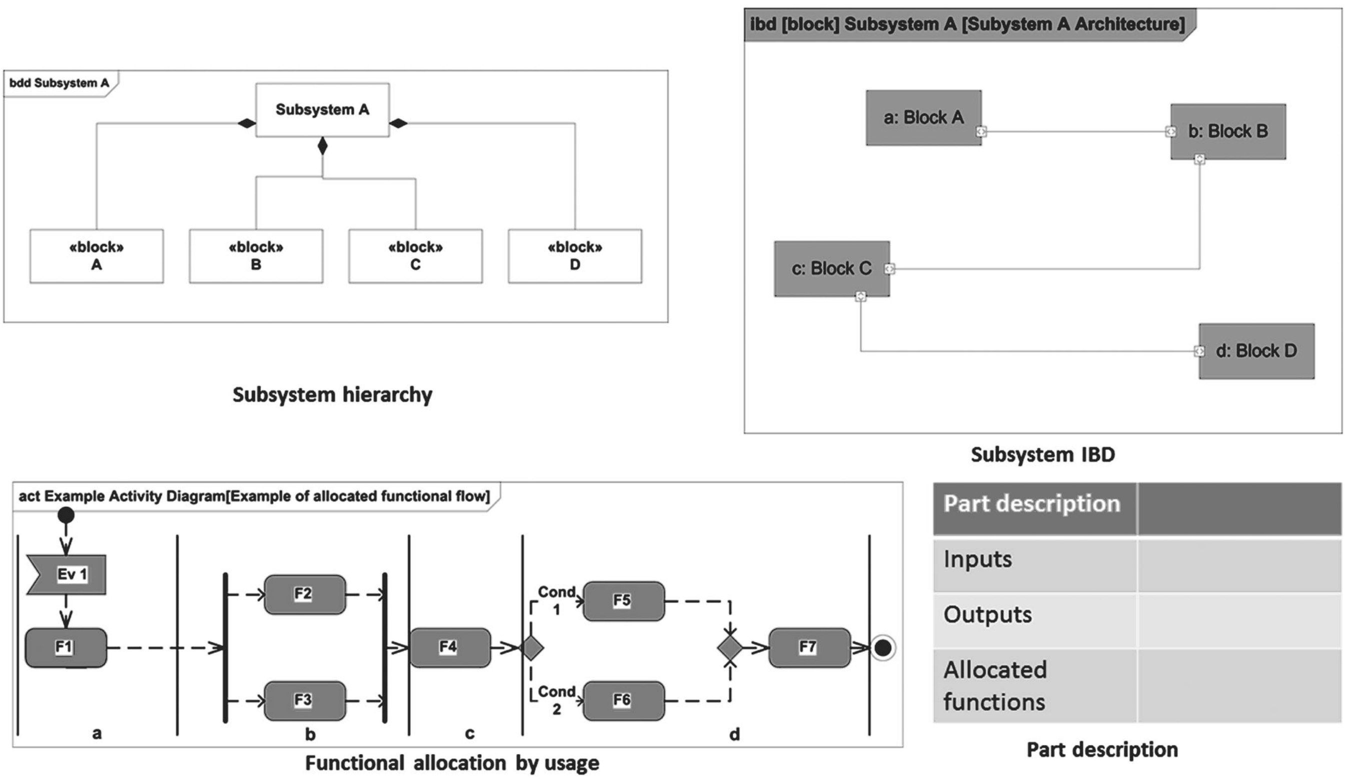

FIGURE 3 Physical architecture diagrams and table

as well. Functional allocation may be represented either in tabular form or at the system blocks, allocation by definition, or as partitions in the activity diagrams, allocation by usage, represented using SysML notation.

Besides the functional and physical blocks used in the system architecture described above, the SysML constraint block encapsulates a constraint on the properties of a physical block of the system. So constraint blocks are used to represent the equations, equalities, and correlations related here to the physical phenomena associated to the system or one of its parts. Engineers can compose complex constraint blocks from existing constraint blocks on a block definition diagram. These diagrams are used to define constraint blocks in a similar way to which they are used to define the physical blocks of the system.

3 | FOUNDATIONS OF DA

DA is a mathematically simple, but very powerful technique, to study the complexity of a physical or chemical process. It is rooted in an obvious, but deep principle: the laws of Physics cannot depend on the units or system of measurement. An important consequence is: the laws of Physics do not depend on the scale of the magnitudes which describe a phenomenon when DA provides a complete solution of it. Thus, the only really meaningful quantities in Physics are dimensionless numbers. In fact, even "dimensional" quantities are measured as quotients of numbers with respect a standard and every physical dimension can be made dimensionless by a suitable choice with respect to other dimensions. Thus, the choice of base dimensions is not unique and many different systems of units are possible, making their definition a matter of convention, as can be easily seen in the recent redefinition of the kilogram.[18] All physical laws can be put in dimensionless form and in certain natural systems of units important physical constants are not only dimensionless but even equal to 1. For some deep conceptual problems associated with the definition of physical dimensions, see, for example.[19,20] DA is related with such important concepts as dimensional homogeneity of equations, symmetry, scaling, asymptotics, renormalization groups, and geometric algebra.

One of the first things every physicist or engineer learns is that both sides of a physical equation must have the same dimensions. This is called "dimensional homogeneity" and seems a very easy task to perform, but, actually, when complex partial differential equations and tensors are involved, or even more advanced mathematical formulations are used, it can be a very complex task.

Symmetry is the conceptual core of contemporary Physics and perhaps it's most abstract foundation. Some prominent physicists think that, in the end, Physics can be reduced to "the study of symmetry."[21] Symmetry is a kind of invariance under a mathematical transformation like translation, rotation, reflection, or scaling. As we have said, DA is derived from a scaling symmetry.

Asymptotic analysis is the study of the limiting behavior of a mathematical, computational, or physical model. This limiting behavior depends not only on the governing equations, but also on the initial and boundary conditions. When a system evolves to the point where its state does not change anymore, we have the static or stationary asymptotic limit of the system. If the system is completely dependent on the initial and boundary conditions during the first instants of its evolution, we have another asymptotic limit, the exact opposite of the first one. However, generally, the most interesting behavior is precisely the intermediate one, when the system still evolves, but it does not "remember" its initial and boundary conditions.[11] This reflects the scaling behavior of the system under study. This intermediate asymptotic behavior permits a panoramic view of the most important properties of the system, without being lost in the details, "to see the forest and not the trees." Obviously, this approach has an enormous interest from the point of view of systems engineering, because this is just the perspective which is searched for. Most times, the intermediate asymptotic regime cannot be found by DA alone, because the system exhibits more complex scaling features, but DA is always the first step to analyze it, providing crucial clues about the complexity of its behavior.

Very related with the concept of intermediate asymptotics are renormalization groups. A detailed study can be seen in the work of Goldenfeld et al.[22] Renormalization group techniques allow the study of a physical system at different scales, so that they are intimately related with scaling symmetry, DA, and self-similarity.

Geometric algebra (GA) is a very general approach to the formulation of mathematical physics also related to DA. An excellent source to learn GA is the work of Doran et al.[23] It represents one of the most ambitious unification approaches to the concepts of Vector Analysis, Algebra, and Geometry, allowing an intermediate perspective between purely synthetic (axiomatic geometry) and purely analytic (coordinate based geometry) formulations trying to preserve the advantages of both approaches.

Sometimes, a clever combination of the election of variables of a problem, its simplification hypotheses, and DA is enough to obtain the complete mathematical dependence of the solution of the problem, except some undefined constants of the order of unity in most cases. The aim of physical theories is, in fact, to refine the value of these constants and to provide a conceptual framework to understand the mathematical formulation.

The most important result in DA is the Buckingham Pi theorem, which says[11]: every physical equation with a certain number n of physical variables, can be rewritten in terms of a set of $p = n - k$ dimensionless parameters $\pi_1, \pi_2, \ldots, \pi_p$ constructed from the original variables, where k is the number of physical dimensions involved. This means that a physical problem, or any other which can be expressed as a physical one, can reduce its dimensional complexity as much as the number of the base quantities used to formulate the problem. If the resulting number $p = 1$, then the dimensionless quantity can be equated to a constant and the complete functional dependence among the original variables can be obtained, even if the equations of the problem are not known. However, most times the problem is underdetermined after DA is applied and a full solution is not possible. Sometimes the number of dimensions can be increased using spatial information, as previously commented, but this demands a careful study of the possible nonlinearities of the system. Also alternative formulations of the problem can result in more variables that seem to increase the knowledge of the system and reduce the uncertainty of the equations. However, such reformulations, if possible, demand a very cautious approach and great expertise in the use of DA.

As a famous example of the insights provided by DA and the subtleties of its application, which permits the discovering of deep results, we will summarize the Rayleigh problem explained in detail in Sedov (pp. 41–43).[24] Rayleigh used DA to attack the Boussinesq problem of heat transfer from a body to a fluid flowing around it. He chose length, time, temperature, heat, and mass as fundamental dimensions, and the heat emitted by the body, the characteristic length of the body, the velocity of the fluid far from the body, the temperature gradient between the fluid and the body, the specific heat of the fluid, and the coefficient of heat conduction of the fluid as variables.

He obtained a very compact mathematical dependence giving the heat emitted by the body as a function of only one dimensionless number and the product of the rest of variables. However, this formulation raised questions from other physicists, notably Riabouchinsky, who objected that if temperature was interpreted as a mechanical variable explained by kinetic theory, a less informative result was obtained. This constituted a perplexing paradox. How a more fundamental microscopical explanation of Nature could give less information than a phenomenological (macroscopic) one? There are very deep issues at stake here.

Being brief, the solution involves an intelligent reflection about the reducibility (scaling behavior) of macroscopic phenomena to microscopic ones: “the whole is greater than the sum of its parts,” which is the main statement of Systems Thinking. If viscosity of the fluid is ignored, then heat transfer and kinetic energy can be decoupled and this information can be used to simplify the problem, obtaining the simpler Rayleigh equation. But, if viscosity is important, Riabouchinsky is right and we must be content with a more complex, less informative result.

Even with all its limitations, DA is so powerful, that it is one of the few tools which allows us to explore uncharted territory in Physics, Biology, and many other branches of Science. An extreme example is this, at present there is not a successful theory of Quantum Gravity, but DA arguments provide us important results and constraints about its formulation via Planck Units[25] and Black Hole Thermodynamics.[26] DA is at the core of many important concepts in Physics, Mathematics, and other branches of Science. The most abstract formulation of DA is via toric ideals,[27] but we need not such powerful and abstract formulations for systems analysis in general. This approach is cited only as an example of the profound implications of DA in the study of systems complexity paralleled by recent developments in the mathematical foundations of systems engineering derived from Category Theory.[28,29] In this way DA can be seen as an elementary application of the theory of Lie groups and invariants, when the group is the scale group defined by multiplication, reinforcing the unifying role of DA in Mathematical Physics via de concept of scaling.

There are many classical and recent general references of the application of DA and scaling to Physics, Mechanical Engineering, Chemical Engineering, and Thermodynamics. The most relevant and available are listed in Table 1 by area of application, although some cover several areas, so that this is not an exclusive classification and

TABLE 1 Summary of literature about dimensional analysis by domain of application

Dimensional analysis application domain	Reference
Astrophysics and cosmology	Dolan[26] Kurth[30] Wesson[25]
Chemical engineering	Dobre and Marcano[31] Herschbach et al.[32] Worstell[33] Zlokarnik[34]
Control engineering	Balaguer[35] Brennan[36]
Differential equations	Alhama et al.[37] Petter Langtangen and Pedersen[38] Sánchez Pérez et al.[39]
Experimental design, similitude, and modeling	Albrecht et al.[40] Kline[41] Kuneš[42] Samarskii and Mikhailov[43] Szücs[44] Westine et al.[45]
Fluid mechanics	Islam[46] Sposito[47] Yarin[48]
Food processes	Delaplace et al.[49]
Historical	Macagno[5]
Mathematical foundations	Atherton[27] Curtis et al.[50] Siano[7,8]
Mechanics	Hutter and Jöhnk[51] Sedov[24] Tan[52]
Mechatronics	Ewing[53] Sell and Christophe[54]
Physical foundations and general description	Anderson[21] Bhaskar and Nigam[55] Bridgman[56] Gattus and Karamitsos[20] Gibbings[57] Goldreich et al.[58] Ipsen[59] Lemons[10] Simon et al.[60] Szirtes[61] Weisskopf[62] Zohuri[63,64]
Production processes	Miragliotta[65]
Scaling, asymptotic analysis, and renormalization	Badii[66] Barenblatt[9] Barenblatt et al.[11] Batterman[67] Cercignani and Sattinger[68] Goldenfeld[22] Henriksen[69] Lesne and Laguës[70]

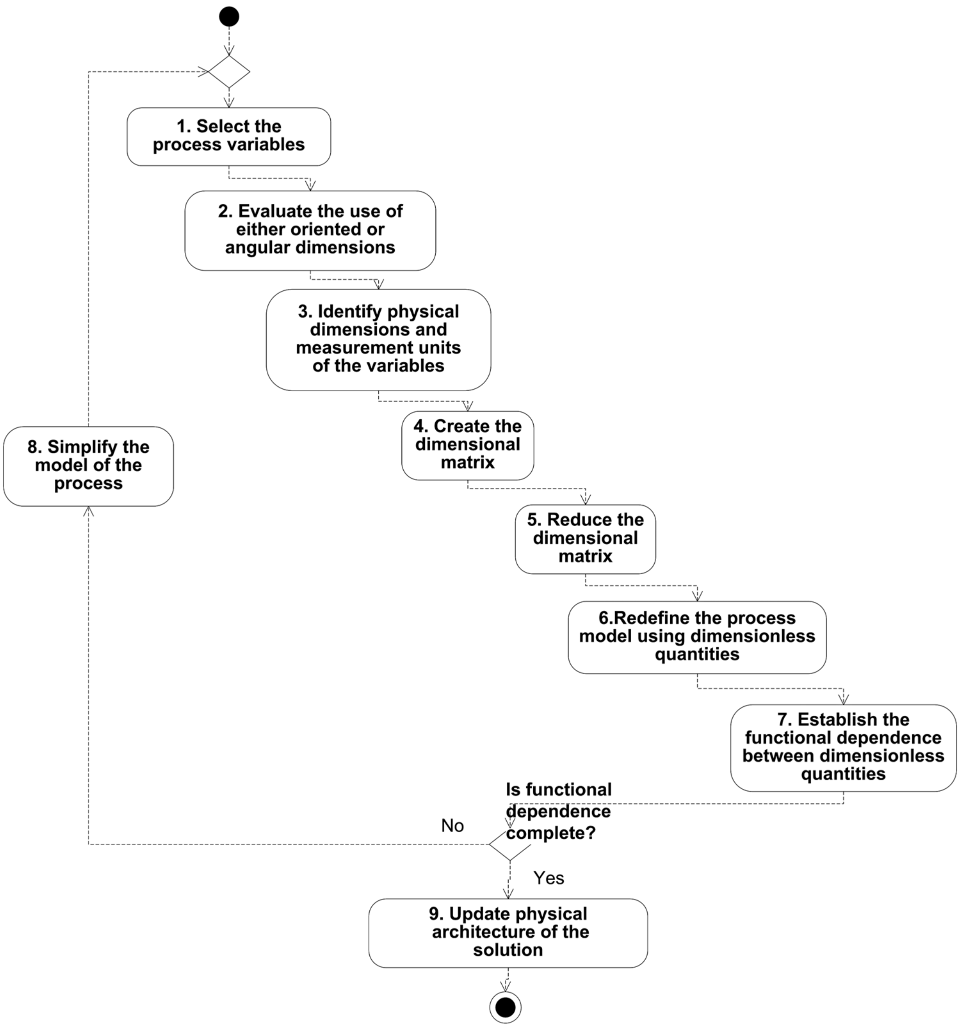

FIGURE 4 Main steps of ISE&PPOOA + DA study of a physical or chemical process

it is intended only as a general guide. Of course, many more books, articles, and handbooks have more or less detailed descriptions of DA, but we have to restrict our literature survey here to the most directly related DA monographs for brevity sake. However, the use of DA in Systems Engineering is much more limited and practically reduced to control engineering[35,36] and mechatronics.[53,54] This is surprising, because from a conceptual point of view, Systems Engineering can be understood essentially as the applied study of the scaling behavior of manmade systems. The main aim of this paper is to fill this gap.

4 | PHASES OF DA IN THE CONTEXT OF SYSTEMS ENGINEERING

The main advantage of DA in Systems Engineering is to reduce the complexity of the model, to gain information about the behavior of the system, and to find the mathematical functions connecting the variables which physically constraint either the system or its parts, represented as blocks in SysML notation.

We can see in Figure 4 the main steps of the application of DA to Systems Engineering extending the ISE&PPOOA system architecting process.

1. *Select the process variables.* Select the variables that define the physical or chemical problem associated with the lowest level of the functional architecture, making the following decisions:
   - If analytical solutions exist, use the variables of these solutions.
   - If analytical solutions are not possible, but a complete mathematical description of the problem exists in form of equations, use the variables of the equations.
   - If a complete mathematical model is not known, but partial results exist, use these variables and complete the others using experimental data.
   - If a partial model is not known, use physical and mathematical intuition and experience in order to formulate an approximate model.

   This is the most difficult step of all, coupled with the appropriate simplifying hypothesis (step 8). A very detailed recent example can be studied in the work of Alhama et al.[37] We will solve later a new problem outside the traditional areas to show the power of DA with a system perspective.

a) Input: List of all variables and parameters involved in the physical or chemical process.

b) Output: List of relevant variables and parameters.

Complete automation of this step would be desirable, but it is not possible at present with the existing artificial intelligence algorithms. Human ingenuity is necessary to reduce systems complexity and use the best approximations. In fact, Physics has been proven to be undecidable from a mathematical point of view, so computers can help to deal with complexity, but only partially.[71,72]

2. *Evaluate the use of either oriented or angular dimensions.* Some dimensions can be oriented, because they describe spatial information. For example, length can be described in 3D space as three cartesian orientations: L_x, L_y, L_z. Study the possibility of using oriented or angular dimensions to simplify the model further. However, this step can be very subtle. For example, nonlinear equations like Navier–Stokes do not allow DA operations with oriented quantities, because nonlinearity destroys their direct comparison, but their linearized approximations could use them. This points to the important fact that fully developed nonlinear systems, like turbulence, cannot be described in an asymptotic, stationary limit. Another problem is the disputed status of angular quantities as "dimensionless." Many experts think, both from a conceptual and practical point of view, that angular quantities have some kind of "dimension."[19,73,74] The best formulation of this technique is Siano's Orientational Analysis.[7,8]
   a) Input: Selected variables.
   b) Output: Oriented variables (if possible and necessary).
3. *Identify physical dimensions and measurement units of the variables.* Write all physical dimensions and measurement units of the model variables. Measurement units are not needed for DA, but they are very important from the practical point of view. Infamous disasters due to mismatching of measurement units in both hardware and software are well known. For example, the loss of the Mars Climate Orbiter.[75]
   a) Input: Oriented variables (if possible and necessary).
   b) Output: Physical dimensions and measurement units.
4. *Create the dimensional matrix.* Write the dimensional matrix using the exponents of the physical dimensions for every variable as matrix data. In fact, there are two general methods to perform DA: the matrix based reduction and the Rayleigh method. Both are equivalent and equally powerful. The Rayleigh method is very intuitive and simple, as can be seen, for example, in Lemons.[10] However, most recent publications use the matrix based approach, because it seems more abstract in the context of linear algebra, which is used to prove the Buckingham Pi theorem. In order to be consistent with the literature, the matrix based approach is followed here, but this does not imply any preference. Actually, the venerable Rayleigh algorithm is more direct in most cases.
   a) Input: Variables and their physical dimensions.
   b) Output: Dimensional matrix.
5. *Reduce the dimensional matrix.* Reduce it using the Buckingham Pi theorem and diagonalization techniques. A very powerful method is explained in Atherton et al.,[27] but in practice intuition and experience are used to obtain the most informative and physically meaningful dimensionless numbers. Many of them are already tabulated in areas like Fluid Dynamics or Chemical Engineering. Try to check first if some of these dimensionless numbers suit your problem.
   a) Input: Dimensional matrix.
   b) Output: Number of reduced variables.
6. *Redefine the process model using dimensionless quantities.* Once the relevant dimensionless numbers have been obtained, the problem is reformulated in terms of these new quantities. Nondimensionalization of the differential equations that describe the system physics is a very important step in any mathematical simulation or modeling effort.[37] Nondimensional quantities allow to estimate the order of magnitude of each term in the equations, which is an essential step for order reduction, simplification, numerical approximations, linearization, and asymptotic approaches. Many nondimensional quantities in fluid mechanics, chemical engineering, and thermodynamics are so important that they have special names, usually their discover's name, like the Reynolds number, for example.
   a) Input: Dimensional variables.
   b) Output: Dimensionless quantities.
7. *Establish the functional dependence between dimensionless quantities.* Calculate the mathematical functional dependence between the relevant dimensionless numbers as accurately as possible. If DA is enough to solve the problem, go to step 9. If not, go to step 8. If even new simplifying hypothesis are not able to solve the problem, then try more complex mathematical techniques like intermediate asymptotics, nondimensionalization of the governing equations searching for symmetries and conserved quantities and approaches based on Lie groups. However, some of these techniques are mathematically very advanced and too difficult for a seamless systems engineering application, so that a physicist or mathematician should be consulted if necessary. Even if these methods are not considered in its full potential for an engineering project, some exploration should be beneficial before numerical simulation is used. Many previously "untractable" problems, even for the most powerful computing resources, are now feasible thanks to discoveries based on these advanced techniques. The previously cited research in the recent area of systems-level modeling is very related to this approach.[6] If these mathematical studies are not successful, then try to obtain relevant information through experiments or numerical simulations. Even if powerful computing resources are available, DA should always used first as a checking reference, estimating the order of magnitude of the variables as constraints. A direct numerical simulation is not always the better approach, thus traditional best practices should always complement the numerical efforts.
   a) Input: Dimensionless quantities.
   b) Output: Mathematical dependence of the selected dimensional variables.

8. *Simplify the model of the process*. Reduce the uncertainty of the model simplifying the problem with reasonable hypotheses and previous knowledge or using experimental results, other analog models and numerical simulations. This is one of the most difficult steps in the whole study of the problem. Even if the problem is well known and constrained, systems of interest are usually so complex that simplifying hypothesis are unavoidable. Here, human ingenuity and experience are irreplaceable.
9. *Update physical architecture of the solution*. Enhance the physical architecture of the system adding the obtained constraint blocks and using the results from DA and the order of magnitude estimations of the approximate models.

Today DA is less popular than years ago, when computational power was lower, and it is generally restricted to fluid dynamics and related fields, where it has passed the test of time with flying colors due to the complexity of the phenomena and the lack of a definitive theory of turbulence, one of the hardest problems in Physics. The same concern is openly discussed by the relatively few DA experts that continue to publish today, who attribute this progressive oblivion to the conceptual difficulty of its application and its lack of appeal in a digital environment. In this sense, the apparent ease of the mathematics involved in most elementary descriptions of DA is very deceptive. See, for example, the opinions of Alhama,[37] Klein,[76] or even Terence Tao,[77] one of the best mathematicians in the world.

5 | EXAMPLE OF A REAL ISE&PPOOA + DA APPLICATION

As a real application of the previously described method, we will use it to design a patch microantenna for microwave energy harvesting inside the human body. The main goal is to power a remote millimeter sized medical device inside an artery.[78] The design of micromedical devices are at the forefront of MEMS (micro-electro-mechanical systems) technology and demand a systems engineering approach from the beginning. So, this analysis represents an excellent example of a real complex research project.

Several energy sources are possible. Near infrared radiation, ultrasounds and microwaves are the most common options. A trade-off analysis of these alternatives can be seen in our recent work.[79] For the present example, we have selected the microwave energy option because it has not a mechanical origin like most published DA examples.

For the sake of brevity, the following figures represent the SysML diagrams that represent partially the functional and physical breakdowns of the system. Figure 5 represents a SysML BDD diagram showing the main functions of the system. For the purpose of DA we are interested in representing the children functions of F3. These children functions are represented as well using a SysML BDD shown in Figure 6. Figure 7 is a BDD diagram representing the physical breakdown where the main subsystems are identified. Figure 8 is the physical breakdown of the main parts of the Internal Power and Communication subsystem. These diagrams represent only hierarchies. A description of the interactions between the system parts is also a project deliverable and the SysML IBD is used for that purpose, but it is shown in the references.[80]

We complement the above diagrams with a N^2 chart for defining the functional interfaces between the F3 children functions. The N^2 chart for the functional architecture can be seen in Table 2. N^2 chart is a powerful schematic view of the interfaces.

Table 3 represents the allocation table of the F3 children functions and the Internal Power and Communication subsystem parts. As described in the SysML literature there are two kinds of allocation: allocation by definition, used here, and allocation by usage. The allocation table is a good way to check if all functions are allocated to the solution building blocks and all the solution building blocks implement some of the functions identified in the functional architecture.

The outputs of the lowest level functions, identified by systems engineers, are the inputs of the DA in order to check if these outputs are constrained by physical laws. In our case, the output of the F3.1 function is the electric energy delivered by the antenna. DA shows that this must be embodied as electric current and voltage, constrained by the mathematical dependence between electric current and the incident electric field and the antenna parameters.

We select a circular patch antenna as a first physical approximation to the functional problem to be analyzed by DA. In order to solve this problem, we follow the previously described steps.

1. Select the process variables.
   Input: List of all variables and parameters involved in the process.
   Output: List of relevant variables and parameters.

- Input: List of all variables, before any simplification.
  - Radiation pattern: E (electric field, generally expressed in polar coordinates).
  - Gain: G
  - Frequency: f
  - Electric current: I

  Geometric parameters of the patch antenna:
  - Antenna radius: r
  - Substrate thickness: z_{sub}
  - Superstrate thickness: z_{sup}
  - Conductor thickness: z_c
  - Ground plane thickness: z_m
  - Conductor conductivity: s_c
  - Substrate conductivity: s_{sub}
  - Superstrate conductivity: s_{sup}
  - Substrate permittivity: ε_{sub}
  - Superstrate permittivity: ε_{sup}
  - Position of the feed point: x, y

- Output: List of relevant variables (this can be defined later as a refinement in step 8, “Simplify the model of the process”).
  * Radiation pattern: E (electric field in polar coordinates)
  * Electric current: I
  * Antenna radius: r

TABLE 2 F3 functional N^2

Microwave energy			Internal heat	Microwave energy			
F3.1. Receive remote power signal inside the patient's body.	Electric energy				Electromagnetic energy		
	F3.2. Generate internal power.	Electric energy			Electromagnetic energy		
		F3.3. Regulate internal system power consumption and distribution.			Electromagnetic energy	Electric energy	Electric energy, control data (current and voltage values)
			F3.4. Dissipate heat produced by internal electronics and electrical devices.				Radiated heat
			Heat	F3.5. Minimize direct transfer of energy to other internal subsystems.			
			Heat		F3.6. Minimize coupling between internal subsystems.		
		Stored electric energy	Heat			F3.7. Store energy inside the internal system.	

TABLE 3 Allocation table

	Internal μwave antenna	Internal μwave demodulator	Internal rectifier	Internal voltage converter	Internal power controller	Internal thermal manager	Internal isolating encapsulation	Internal electrical ground plates	Internal Faraday cages	Internal battery
F3.1.	X	X								
F3.2.			X	X						
F3.3.					X					
F3.4.						X				
F3.5.							X	X	X	
F3.6.							X	X	X	
F3.7.										X

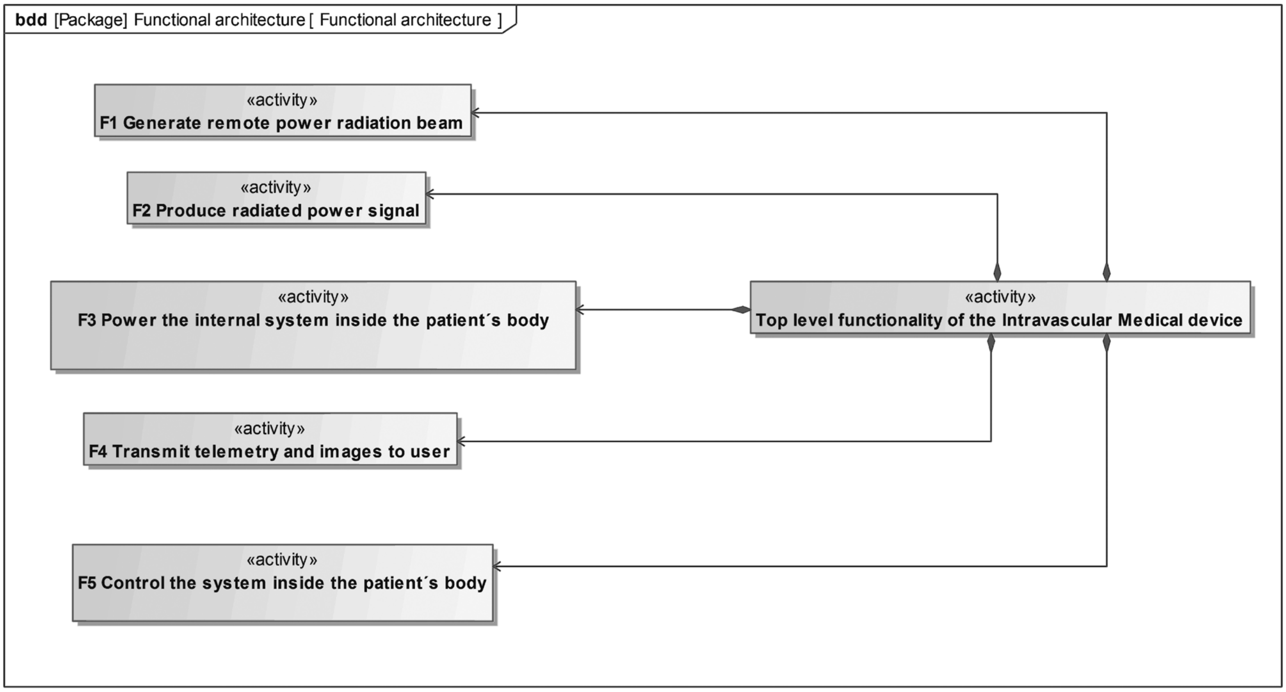

FIGURE 5 BDD of the functional architecture

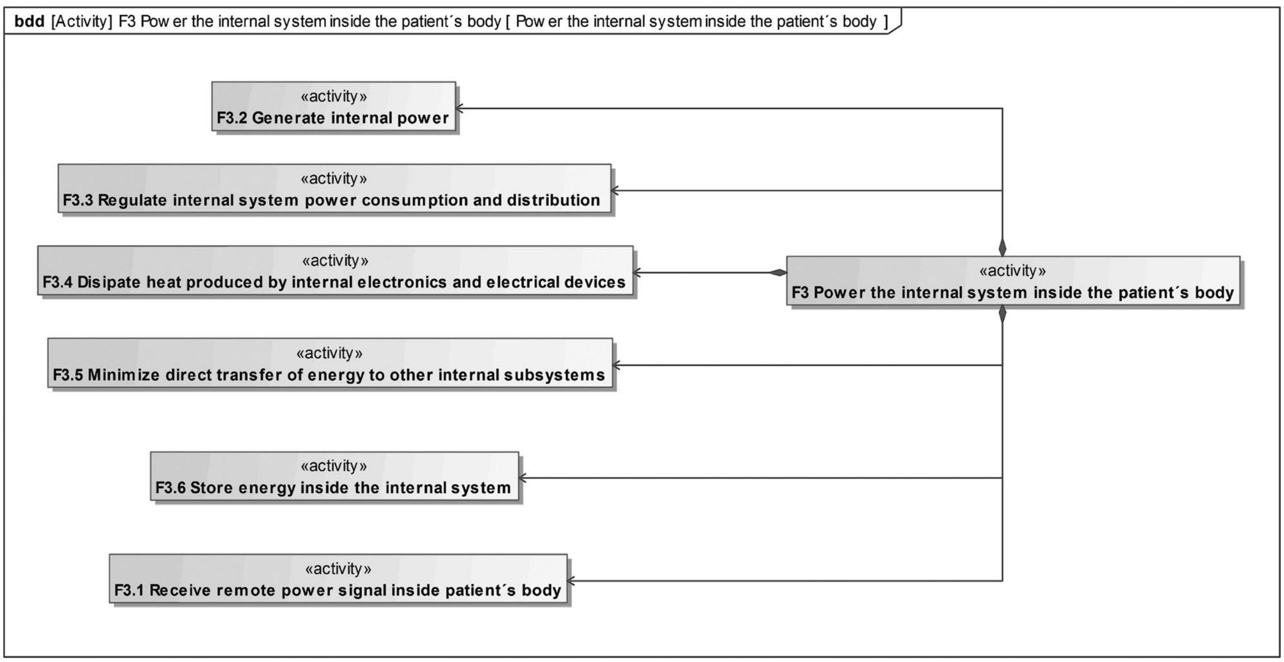

FIGURE 6 Detailed BDD of the F3 subfunctions

  * Substrate thickness: z_{sub}
  * Substrate permittivity: ε_{sub}

- Simplifying hypotheses, in order to make the problem tractable without losing the essential system behavior:
  * There are no superstrate.
  * Conductor thickness is negligible.
  * Ground plane thickness is negligible.
  * Conductor is perfect (its conductivity is infinite).
  * The substrate is a perfect dielectric (its conductivity is negligible).
  * The feed point is fixed (it does not count as variable).

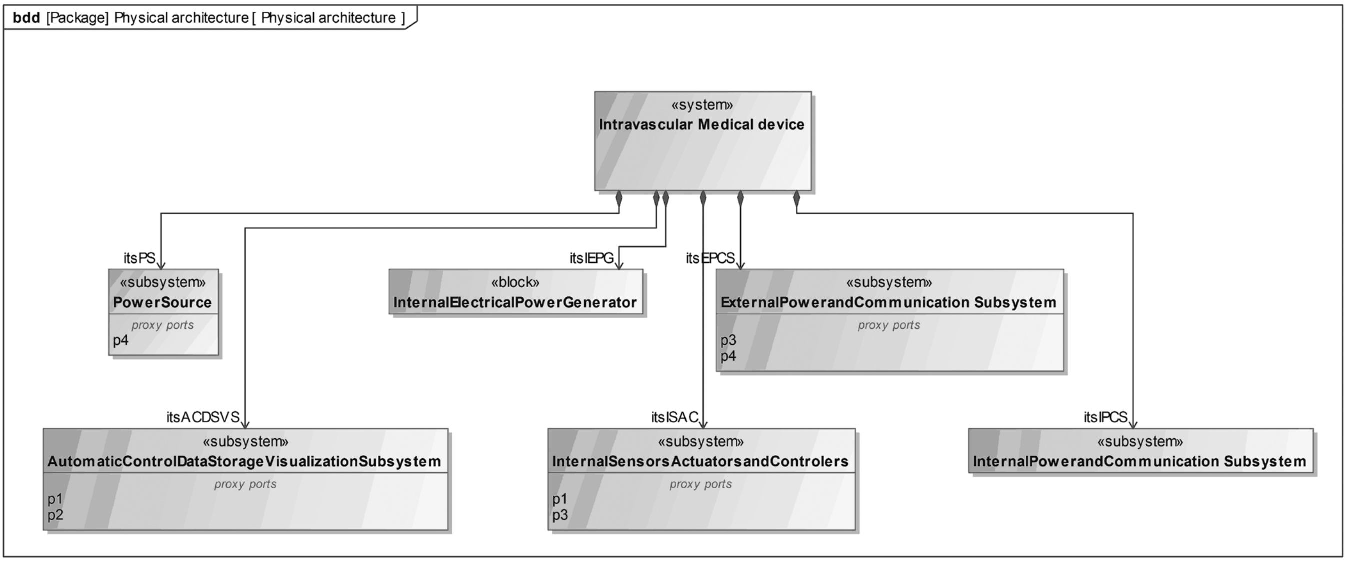

FIGURE 7 BDD of the physical arquitecture

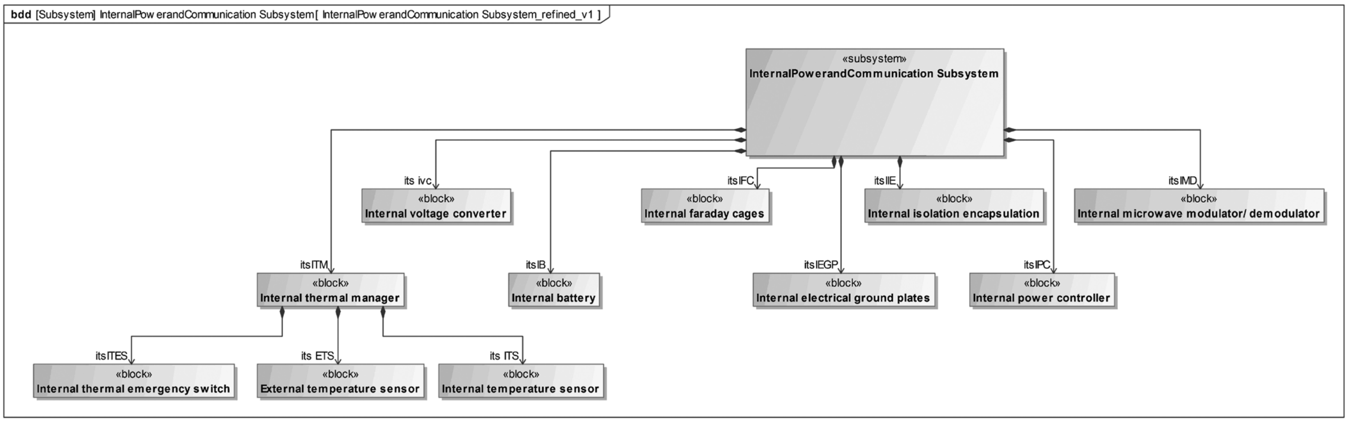

FIGURE 8 Detailed BDD of the internal power and communication subsystem

2. Evaluate the use of either oriented or angular dimensions.
   Input: Selected variables.
   Output: Oriented variables (if possible and necessary).

- List of oriented variables: None, because the selected variables are unidimensional (radius, thickness). The rest are not orientable. The electric field could be oriented, because it is a vector quantity, but in this case this is not advantageous for a DA approach, so only its scalar magnitude is considered.

3. Identify physical dimensions and measurement units of the variables.
   Input: Oriented variables (if possible and necessary).
   Output: Physical dimensions and measurement units.
   - Radiation pattern: M L T^{-3} I^{-1} (V m^{-1}, it is the radiated electric field)
   - Frequency: T^{-1} (Hz)
   - Electric current: I (A)
   - Antenna radius: L (m)
   - Substrate thickness: L (m)
   - Substrate permittivity: M^{-1} L^{-3} T^4 I^2 (F m^{-1})

4. Create the dimensional matrix. The dimensional matrix contains the same information as the previous list but in a more mathematically rigorous form, as previously explained. If the matrix approach is not desired, the Rayleigh algorithm could be used.
   Input: Variables and their physical dimensions.
   Output: Dimensional matrix (Table 4).

5. Reduce the dimensional matrix.
   Input: Dimensional matrix.
   Output: Number of reduced variables.

TABLE 4 Dimensional matrix

	M	L	T	I
E	1	1	−3	−1
f	0	0	−1	0
I	0	0	0	1
r	0	1	0	0
z_{sub}	0	1	0	0
ε_{sub}	−1	−3	4	2

We use the Buckingham Pi theorem in order to reduce the matrix and find the dimensionless numbers which best describe the problem. As we have commented, this step cannot be completely automated, because there are several possible dimensionless numbers from different initial variable combinations. In other areas, like fluid mechanics, many useful numbers already exist that can be used for certain classes of problems, but this is not always the case. Actually, we have selected a problem from another physical domain to better illustrate this fact. In this case:

$p = NV - ND = 6 - 4 = 2$. The problem can be reduced to two dimensionless numbers.

6. Redefine the process model using dimensionless quantities. Dimensionless quantities should maximize the physical information (intuition) about the system, constituting so the foundation of later approximations and numerical work.
   Input: Dimensional variables.
   Output: Dimensionless quantities.

$$\Pi_1 = \frac{\varepsilon_{sub}\, r\, z_{sub}\, f\, E}{I} \tag{1}$$

$$\Pi_2 = \frac{r}{z_{sub}} \tag{2}$$

We verify that both numbers are dimensionless:

$$\Pi_1 : \left(\mathrm{M\, L\, T^{-3}\, I^{-1}}\right)\left(\mathrm{M^{-1}\, L^{-3}\, T^{4}\, I^{2}}\right)\mathrm{L\, L\, I^{-1}\, T^{-1}} = \mathrm{M^0\, L^0\, T^0\, I^0} = 1$$

$$\Pi_2 : \mathrm{L/L} = 1$$

7. Establish functional dependence between dimensionless quantities.
   Input: Dimensionless quantities.
   Output: Mathematical dependence of the selected dimensional variables.

The solution to the simplified patch antenna problem will be a function of the form:

$$F\,(\Pi_1, \Pi_2) = 0 \Rightarrow F\left(\frac{\varepsilon_{sub}\, r\, z_{sub}\, f\, E}{I}, \frac{r}{z_{sub}}\right) = 0 \tag{3}$$

As an implicit function, it can be written also as:

$$\frac{\varepsilon_{sub}\, r\, z_{sub}\, f\, E}{I} = F\left(\frac{r}{z_{sub}}\right) \tag{4}$$

In this form, we can easily study the relationship between the electric field and the current. We should arrange the terms of the resulting DA equation in the form that allows us to isolate our variables of interest. We are interested in the mathematical dependence of the electric current, because we want to use the antenna for microwave energy harvesting:

$$I = \varepsilon_{sub}\, r\, z_{sub}\, f\, E\, F\left(\frac{r}{z_{sub}}\right) \tag{5}$$

In this case, the mathematical dependence is almost complete and further reasonable simplifications cannot be made, so that the DA process is finished.

We can also estimate the order of magnitude of the variables of the solution as attributes of the physical blocks representing the system parts.

Input: Functional dependence of the dimensional variables and their estimated values in the prescribed measurement system (SI system).
Output: Order of magnitude of the solution.

Estimation of the electrical current in a MEMS antenna:

- Antenna conductor radius: 1 mm = 10^{-3} m
- Substrate thickness: 1 mm = 10^{-3} m
- Substrate permittivity: 100 $\varepsilon 0 = 100 \cdot 8{,}85 \cdot 10^{-12}$ F/m $\approx 10^{-9}$ F/m
- Electric field frequency: 1 GHz = 10^9 Hz
- Electric field safety limit (1 GHz = 10^3 MHz): 15.60 $f(\mathrm{MHz})^{0.25} \approx 10 \cdot 10^{3/4} \approx 10^2$ V/m
- The unknown function $F(\frac{r}{z_{sub}})$ is of the order of 1.
- **Electric current** $\approx I = \varepsilon_{sub}\ r\ z_{sub}\ f\ E\ F(\frac{r}{z_{sub}}) \approx (10^{-9}$ F/m$)(10^{-3}$ m$)(10^{-3}$ m$)(10^{9}$ Hz$)(10^{2}$ V/m$)(1) \approx 10^{-4}$ A.

This result is compatible with experimental values found in the literature for similar microantennas.[81]

The previous SE model is updated with the constraint blocks related to the microwave antenna, represented in Figure 9. They may be used as inputs for SysML parametric diagrams or simulation tools. The functional architecture offers more information than DA uses for a macroscopic (thermodynamic) vision of the system interfaces. Such thermodynamic study would represent the next level of system analysis after DA and order of magnitude estimation.

6 | CONCLUSIONS

The MBSE approach describes the system using consistent views to provide a holistic model as complete as possible, including all relevant information from the stakeholder's perspective. Most MBSE

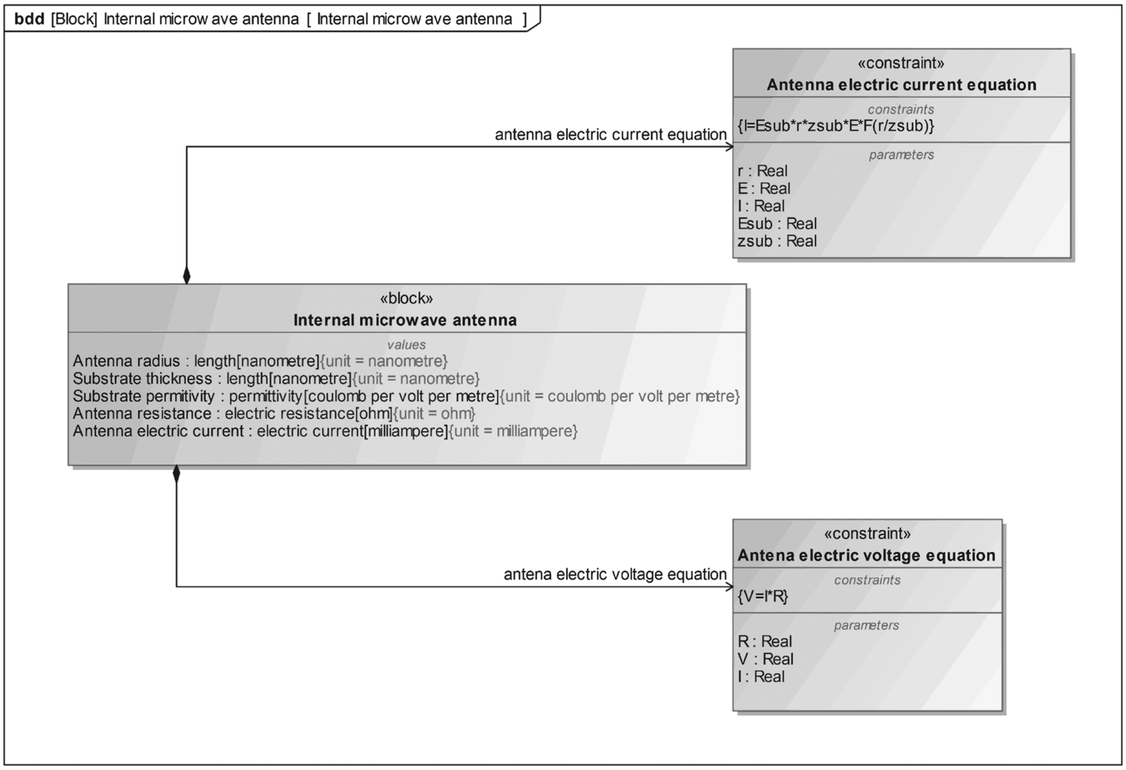

FIGURE 9 Constraint blocks related to the microwave antenna

methodologies end with the physical architecture of the system, but a physical model is clearly incomplete without the study of its associated physical laws and phenomena related to the whole system or its parts. However, a detailed simulation of the whole system is not the main job of a systems engineer and the computational demands would be excessive even for modest projects. Actually, most complex systems cannot be completely or accurately simulated. Thus, critical interactions affecting their performance would be ignored with deleterious effects.

The ISE&PPOOA + DA pragmatic methodological process goes a step further incorporating DA as a powerful instrument to understand the physical constraints of the system without the burden of complex analytical or numerical calculations. Even in the case that DA alone cannot provide a complete solution, it is invaluable as a best practice and it is required for the design of experimental tests and the studies of scaling behavior. Most tests are based on or constrained by analogies and similitude arguments which demand a careful DA study. The prototypical example is the design of experiments in wind tunnels. The practical application of the process explained in this paper forces the systems engineer to think in physical phenomena terms and to understand the system conceptually in depth. This information is critical to select the best physically realizable architectures. The simplification steps involved in the proposed process provide important insights about the interactions among system parts and suggest possible cost savings in later detailed numerical simulations, testing, and the final product realization.

We give a detailed example of the application of ISE&PPOOA + DA to the design of a MEMS microwave antenna to power remotely a millimeter size endoscopic capsule inside the human body. Even a simple antenna with all its parameters considered would demand a costly simulation to know if the harvested energy is enough to power the medical device. However, DA reveals that many of these parameters could be irrelevant and even allows an order of magnitude estimation, which is enough for a systems engineer for trade-off studies to compare the expected performance with other alternatives. Finally, from a theoretical point of view, DA can be seen as the unifying link between the functional and physical architectures, allowing a smooth transition from the lowest level of the functional hierarchy, described as physical interactions, unit operations and transport phenomena, and the corresponding elements of the physical architecture.

In a world increasingly dominated by digital computing, many scientists and engineers could think that DA is a thing of the past, when very limited computing resources demanded careful thinking of problems and their simplifications. In order to avoid repeating the same arguments, a compelling defense of DA can be read in Astarita.[82] Another possible criticism of DA is its lack of relevance for software projects or cyberphysical systems. However, this criticism is unfounded, because DA is far deeper than its application to physical magnitudes, as it has been previously explained. DA is at the core of scaling and similitude theories, from pure mathematics and theoretical physics to experimental testing of prototypes or complete systems. Reliable testing cannot be designed and performed without a previous complete study of similitude. Most modern systems are so large and complex that their direct testing is impossible, impractical, damaging, or prohibitively expensive. Therefore, reduced models must be used and DA must be applied. In fact, even systems level engineering disciplines like Electromagnetic Compatibility, especially in an aerospace context, where the use of DA has been almost inexistent, recently has made a strong case for more research in DA related to testing design of complete systems, as can be seen in page 7 of the Handbook of Aerospace Electromagnetic Compatibility[83]:

> "Ultimately, the ability to extend laboratory-scale results to realistic networks will likely require a variant of dimensional analysis theory."

DA is more relevant to software engineering than a superficial view suggests. From a conceptual perspective, there are intriguing and profound connections between DA and Type Theory in both Mathematical Logic and Computer Science, to the point that a unification seems possible in a more comprehensive framework. This is a promising avenue for future research. Thus, DA, through its parallelisms with computational type systems, can be seen also as a checking procedure for the correctness of the system physical behavior in the same way that type systems are used for the study of correctness and meaning of computer programs. Through Type Theory, DA would also be related with Category Theory, the most powerful and abstract mathematical theory at present, but this is a topic for future studies. However, it should be remarked that DA describes the functions relating physical variables (scaling-preserving functions), while Category Theory studies functions between mathematical structures (structure-preserving functions). The importance of exploring this analogy is evident, to the point that if Category Theory is an alternative axiomatic foundation of Mathematics, DA could be seen as a starting point for a rigorous foundation of Physics and Systems Science.

Finally, this work can be useful as a quick and fairly comprehensive reference of DA and order of magnitude calculations for the interested systems engineer in a more general way than is generally described in the literature, more focused in specific applications, like fluid mechanics.

DATA AVAILABILITY STATEMENT

Due to the nature of this paper, no data have been produced. No experiments or numerical simulations have been performed.

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