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Finite-Time Synchronization of the Rabinovich and Rabinovich-Fabrikant Chaotic Systems for Different Evolvable Parameters

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ABSTRACT

This paper addresses the challenge of synchronizing the dynamics of two distinct 3D chaotic systems, specifically the Rabinovich and Rabinovich-Fabrikant systems, employing a finite-time synchronization approach. These chaotic systems exhibit diverse characteristics and evolving chaotic attractors, influenced by specific parameters and initial conditions. Our proposed low-cost finite-time synchronization method leverages the signum function's tracking properties to facilitate controlled coupling within a finite time frame. The design of finite-time control laws is rooted in Lyapunov stability criteria and lemmas. Numerical experiments conducted within the MATLAB simulation environment demonstrate the successful asymptotic synchronization of the master and slave systems within finite time. To assess the global robustness of our control scheme, we applied it across various system parameters and initial conditions. Remarkably, our results reveal consistent synchronization times and dynamics across these different scenarios. In summary, this study presents a finite-time synchronization solution for non-identical 3D chaotic systems, showcasing the potential for robust and reliable synchronization under varying conditions.

1. Introduction

Chaos, a nonlinear phenomenon observed in a class of dynamic systems, has recently garnered significant research interest due to its expanding applications in both real-world and hypothetical scenarios. These applications span a wide spectrum, encompassing secure communication [1], navigation systems [2], robotics [3], and power system dynamics [4]. Over the last decade, the literature has seen the emergence of various chaotic systems with diverse dimensions and unique characteristics, ranging from three-dimensional systems [5], four-dimension [6], five-dimension [7], six-dimension [8], seven-dimension [9]-[11], to even higher-dimensional counterparts [12]. However, the utility of chaotic systems is greatly enhanced when they can be controlled. Consequently, researchers have adopted various control strategies tailored to systems of different dimensions. These strategies encompass feedback control [13], [14], adaptive control [15], passive control [16], sliding mode control, and active control [17], among others. Among the intriguing phenomena associated with chaotic systems is synchronization, where the dynamics of two chaotic systems, whether identical or non-identical, are coupled through appropriate control laws within finite time. Numerous synchronization schemes have been developed to facilitate this coupling, including adaptive control-based synchronization [18], adaptive hybrid synchronization [19], functional projective synchronization [20], and sliding mode control-based synchronization [21], [22], fuzzy synchronization [23], invariant ellipsoids method [24]. Each scheme serves specific purposes in different contexts and applications. In this study, we delve into the synchronization of two particular chaotic systems: the Rabinovich chaotic system [25]-[27], and the Rabinovich-Fabrikant system [28]. These systems, characterized by sets of three-coupled differential equations, possess unique properties that have attracted significant attention in diverse fields, such as image encryption, power system dynamics, and control. The Rabinovich-Fabrikant system, in particular, is renowned for its complexity owing to the presence of quadratic and cubic terms [29], making it a subject of extensive analysis and exploration [30]-[32].

In the following sections, we will focus on the finite-time synchronization of these intriguing chaotic systems, presenting our approach and numerical findings. This study aims to contribute to the understanding of chaos synchronization, with practical implications in various interdisciplinary domains.

2. Dynamics of the Rabinovich and Rabinovich-Fabrikant Systems

2.1. Rabinovich System

The architecture of the Rabinovich system consists of a set of first-order, three-coupled ordinary differential equations given by

$$\begin{aligned} \dot{x_1} &= -ax_1 + bx_2 + x_2x_3 \\ \dot{x_2} &= cx_1 - dx_2 - x_1x_3 \\ \dot{x_3} &= -ex_3 + x_1x_2 \end{aligned} \tag{1}$$

where a, b, c, d, and $e > 0$ are positive parameters, whose values determines the type of attractor that is evolved, and $x \in [x_1, x_2, x_3]$ are the state variables that gives the order of the system. When $a = 4, b = 6.75,\ c = 6.75, d = 1, e = 1$, the Rabinovich system evolves the attractors shown in Fig. 1.

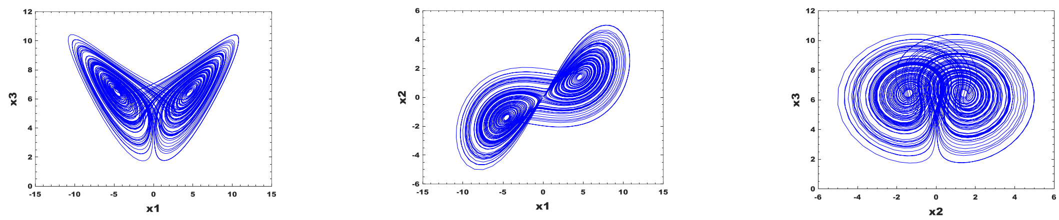

Fig. 1. Chaotic attractors of the Rabinovich system - I

When the parameters are changed to the following, $a = 1.5, b = 0.04,\ c = 0.04, d = -0.3, e = 1.67,$ a distinct set of attractors are evolved as shown in Fig. 2.

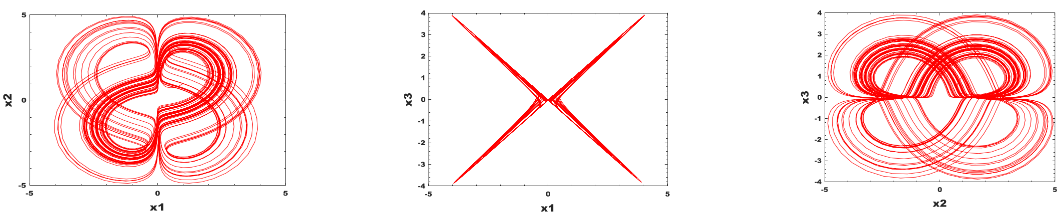

Fig. 2. Chaotic attractors of the Rabinovich system - II

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2.2. The Rabinovich – Fabrikant System

The Rabinovich-Fabrikant system consists of a set of three-coupled differential equations represented by

$$\begin{aligned} \dot{y_1} &= y_2(y_3 - 1 + y_1^2) + \delta y_1 \\ \dot{y_2} &= y_1(3y_3 + 1 - y_1^2) + \beta y_2 \\ \dot{y_3} &= -2y_3(\varphi + y_1 y_2) \end{aligned} \tag{2}$$

Where $\delta, \beta, \varphi > 0$ are parameters and $y \in [y_1, y_2, y_3]$ are the state variables. When $\delta = -1.0, \beta = -1.0, \varphi = -0.2$, the Rabinovich-Fabrikant system evolves the attractors shown in Fig. 3.

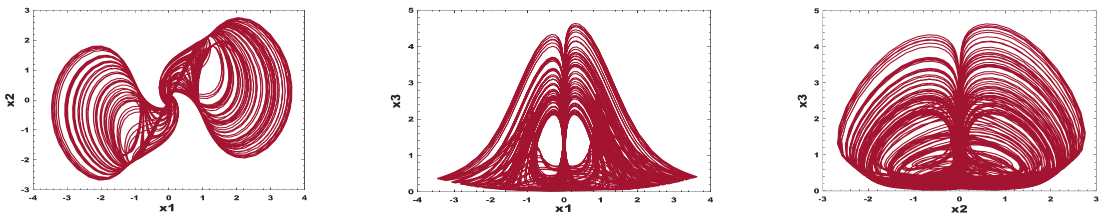


Fig. 3. Chaotic attractors of the Rabinovich – Fabrikant system - I

When $\delta = 0.87, \beta = 0.87, \varphi = 1.1$ the Rabinovich-Fabrikant system evolves the attractors shown in Fig. 4.

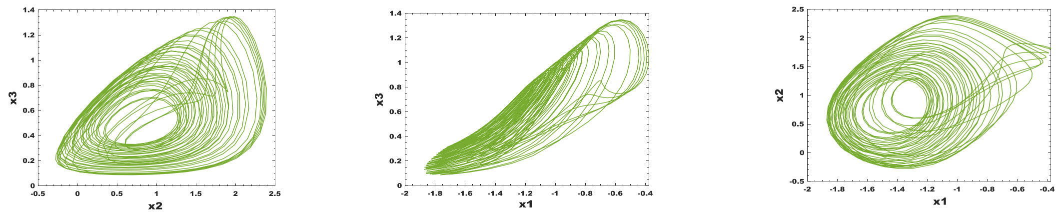


Fig. 4. Chaotic attractors of the Rabinovich – Fabrikant system - II

3. Summarization of the Finite-Time Control Scheme

Conceptually, two identical or non-identical chaotic systems are said to synchronize in finite time, when for a given initial conditions, their coupling trajectories settle uniformly after a specified time. Finite-time synchronization is essential for time-critical applications to avoid overshoots and other undesirable consequences [33]. Finite-time schemes are realized with the application of Lyapunov stability criteria in conjunction with definitions and lemmas. In recent years, finite-time control and synchronization schemes have attracted much attention they have been applied to the synchronization of chaotic systems with different dimensions and uncertainties [34]-[36], coronary artery chaotic systems [37], memristor chaotic systems [38] and hyperchaotic systems [33] amongst others. In this section, we summarized a body of definitions and lemmas suitable for the synchronization of the Rabinovich and Rabinovich-Fabrikant systems.

Definition 1 [39], [40]. The origin of (1) is a finite-time stable equilibrium if the origin is Lyapunov stable and there exist a function $T: \mathfrak{R}^n \to \mathfrak{R}^+$ called the settling time function, such that for every $x_0 \in \mathfrak{R}^n$, the solution $x(t, x_0)$ of (1) is defined on $[0, T(x_0), x(t, x_0) \in \mathfrak{R}^n \ \forall t \in [0, T(x_0)]]$ and $\lim_{t \to T(x_0)} x(t, x_0) = 0$.

Lemma 1 [41], [42]. If there exist a differential and continuous positive definite function $V(t): \mathfrak{R}^n \to \mathfrak{R}^n$, such that $V\big(x(t)\big) \to \infty$ as $x(t) \to \infty$) and satisfies the following differential inequality:

$$V(t) \leq -\chi\big(V(t)\big)^{\phi}, \quad \forall t \geq 0, \quad T \geq t_0, \quad V(t_0) \geq 0, \tag{3}$$

Where $\chi > 0$ and $0 < \phi < 1$ are two positive numbers. For any $t_0, V(t)$ satisfies the inequality:

$$V^{1-\phi}(t) \leq V^{1-\phi}(t_0) - \chi(1-\phi)(t-t_0), \quad t_0, \quad t_0 \leq t \leq T \tag{4}$$

And $V(t) \equiv 0, \forall t \geq T$. Thus, the origin of (1) is globally stable in finite time, and the settling time T is given as:

$$T \leq t_0 + \frac{V^{1-\phi}(t_0)}{\chi(1-\phi)} \tag{5}$$

Assumption 1: [43], [44]. Assume $\xi_1, \xi_2, \dots, \xi_n \in R^n$ and $0 < \gamma < 1$ are all real numbers. Then the following inequality holds:

$$(|\xi_1| + |\xi_2| + \cdots + |\xi_n|)^{\gamma} \leq |\xi_1|^{\gamma} + |\xi_2|^{\gamma} + \cdots + |\xi_n|^{\gamma} \tag{6}$$

Definition 2: If system (1) and (2) are master and slave chaotic systems respectively, and $e_i = y_i - x_i$ $(i = 1, 2, 3)$, are synchronization error functions, then (1) and (2) can achieve finite-time synchronization, if there exists a settling time $T(e_i(0)) > t_0$, such that $\lim_{t \to T} |e_i| = 0$ and $|e_i| \equiv 0$, for $t > T$ for any initial conditions $x_1(0), x_2(0), x_3(0) \neq y_1(0), y_2(0), y_3(0)$.

3.1. Architecture of the Finite-Time Synchronization Scheme

The architecture of the proposed Finite-time synchronization scheme is shown in Fig. 5.

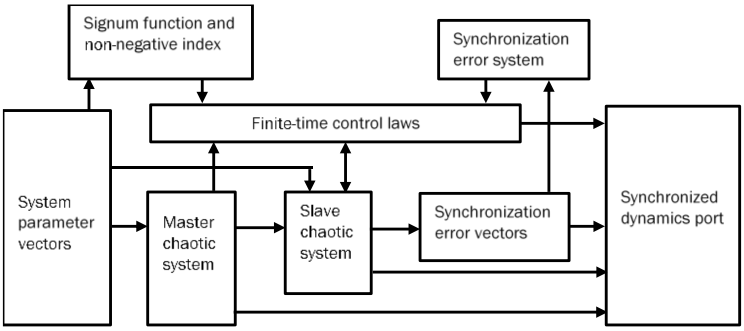


Fig. 5. Architecture of the Finite Time Synchronization scheme

The master chaotic system is the Rabinovich system which is used to drive the slave system. The slave chaotic system is the Rabinovich Fabrikant system that is controlled to couple the dynamics of the two systems. The synchronization error system is used to track the dynamics of the master and slave systems to bring them into a state of synchrony in finite time. The parameter vector are the parameters whose values evolves the set of attractors. The finite time control laws are the set of functions based on the Lyapunov stability criteria that provides the sufficient conditions to couple the dynamics of the chaotic systems in finite time. The signum function and non-negative indices constitutes the weighted parameters that constrains overshoots of the asymptotically stable coupled dynamics to keep them within the finite time. The synchronization error dynamics of the systems (1) and (2) using the Definition 2 is given by

$$\begin{aligned} \grave{e}_1 &= -ae_1 - e_2 + (a+\delta)x_1 + (1+b)y_2 - x_1x_3 + y_2y_1^2 + y_2y_3 + u_{e1} \\ \grave{e}_2 &= \beta y_2 + (\beta + d)x_2 + x_1 - ce_2 + de_2 - x_1x_3 + 3y_1y_3 - +y_1^3 + u_{e2} \\ \grave{e}_3 &= -2\gamma e_3 - (2\gamma - e)y_3 - x_1x_2 - 2y_1y_2y_3 + u_{e3} \end{aligned} \tag{7}$$

Where u_{ei} are vectors of the associate control laws of the synchronization error system, and are given by

$$\begin{gathered} u_{e1} = ae_1 + e_2 - (a+\delta)x_1 - (1+b)y_2 + x_1x_3 - y_2y_1^2 - y_2y_3 \\ u_{e2} = -\beta y_2 - (\beta + d)x_2 - x_1 + ce_2 - de_2 + x_1x_3 - 3y_1y_3 + y_1^3 \\ u_{e3} = 2\gamma e_3 + (2\gamma - e)y_3 + x_1x_2 + 2y_1y_2y_3 \end{gathered} \tag{8}$$

We proposed a global finite-time control law that combines the associate control laws and control strength coefficients that comprise weighted parameters signum functions and non-negative indices and given by

$$U_i = u_{ei} - \Pi_{U_i}^{\mu}\text{sgn}(e_i)|e_i|^{\mu} \tag{9}$$

Where $\Pi_{U_i}^{\mu}$ are control strength coefficients, μ is non-negative index and *sgn* is a Signum function and e_i has been defined in Definition 2. By replacing u_{ei} in (7) with (9) and using (8), results in the following

$$\begin{aligned} \grave{e}_1 &= -\Pi_{U_1}^{\mu}\text{sgn}(e_1)|e_1|^{\mu} \\ \grave{e}_2 &= -\Pi_{U_2}^{\mu}\text{sgn}(e_2)|e_2|^{\mu} \\ \grave{e}_3 &= -\Pi_{U_3}^{\mu}\text{sgn}(e_3)|e_3|^{\mu} \end{aligned} \tag{10}$$

Theorem. The dynamics of the master chaotic system (1) and the controlled slave chaotic system (2) can be synchronized in finite time, if the proposed global finite-time controller given by (9) is applied. Proof: Consider the quadratic Lyapunov function candidate.

$$V_e(t) = \sum_{i=1}^{3} \frac{1}{2} e_i^2 \tag{11}$$

The time derivative of (10) along the trajectories of (7) is given by

$$\dot{V}_e(t) = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 \tag{12}$$

Using (10) in (12) results in the following

$$\begin{aligned} \dot{V}_e(t) &= e_1(-\Pi_{U_1}^{\mu}\text{sgn}(e_1)|e_1|^{\mu}) + e_2(-\Pi_{U_2}^{\mu}\text{sgn}(e_2)|e_2|^{\mu}) + e_3(-\Pi_{U_3}^{\mu}\text{sgn}(e_3)|e_3|^{\mu}) \\ &= -\Pi_{U_1}^{\mu}\text{sgn}(e_1)|e_1|^{\mu}e_1 - \Pi_{U_2}^{\mu}\text{sgn}(e_2)|e_2|^{\mu}e_2 - \Pi_{U_3}^{\mu}\text{sgn}(e_3)|e_3|^{\mu}e_3 \end{aligned} \tag{13}$$

By using the convention:

$$\text{sgn}(e_i) = \frac{|e_i|}{e_i}, \text{sgn}(e_i)|e_i|^{\mu} \Leftrightarrow \frac{e_i|e_i|}{e_i} = |e_i|, e_i\text{sgn}(e_i)|e_i|^{\mu} \Leftrightarrow |e_i|^{1+\mu} \tag{14}$$

Inserting (14) into (13) gives

$$\dot{V}_e(t) = -\Pi_{U_1}^{\mu}|e_1|^{1+\mu} - \Pi_{U_2}^{\mu}|e_2|^{1+\mu} - \Pi_{U_3}^{\mu}|e_3|^{1+\mu} \leq 0 \tag{15}$$

Eq. (15) is negative-definite in $\mathbb{R}^3$. Thus Lemma 1 is satisfied, which implies that the synchronization error system (7) can be uniformly asymptotically stabilized in finite time while simultaneously synchronizing the dynamics of the master and slave chaotic systems. If the control strength coefficients are set to $\Pi_{U_1}^{\mu} = \Pi_{U_2}^{\mu} = \Pi_{U_3}^{\mu} = \Pi_{U_G}^{\mu}$, where "G" is a prefix for global control strength coefficient, then (15) reduces to

$$\dot{V}_e(t) = -\sum_{i=1}^{3} \Pi_{U_G}^{\mu} |e_i|^{1+\mu} \tag{16}$$

Based on Assumption 1,

$$\sum_{i=1}^{3} \Pi_{U_G}^{\mu} |e_i|^{1+\mu} \geq \left(\sum_{i=1}^{3} e_i^2\right)^{\frac{1+\mu}{2}} = \|e\|^{1+\mu} \tag{17}$$

By virtue of Assumption 1,

$$\begin{aligned} \dot{V}_e(t) &\leq -\Pi_{U_G}^{\mu} \|e\|^{1+\mu} = \Pi_{U_G}^{\mu} (\|e\|^2)^{1+\mu} \\ &\leq -2^{\frac{1+\mu}{2}} \Pi_{U_G}^{\mu} \left(\frac{\|e\|^2}{2}\right)^{\frac{1+\mu}{2}} = -2^{\frac{1+\mu}{2}} \Pi_{U_G}^{\mu} V^{\frac{1+\mu}{2}} \end{aligned} \tag{18}$$

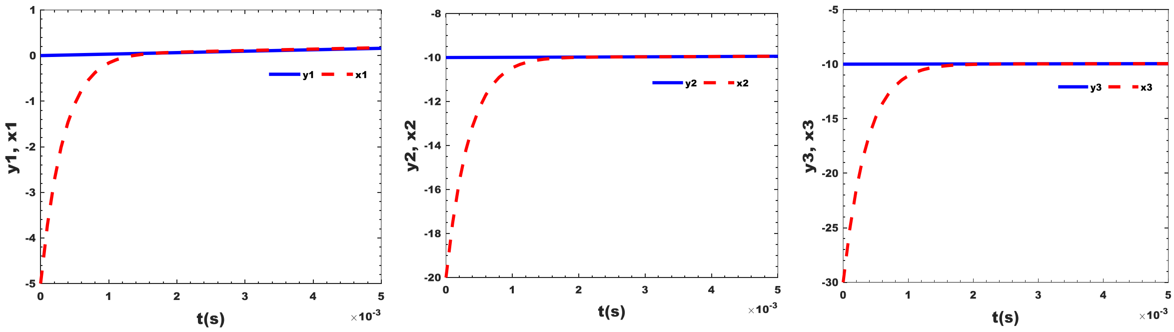
According to Lemma 1, the two chaotic systems, the time of synchronization is estimated as

$$T \leq \frac{(V(t_0))^{\frac{1-\mu}{2}}}{2^{\frac{1-\mu}{2}} \Pi_{U_G}^{\mu} (1-\mu)} \tag{19}$$

Overall, the negative-definiteness of the partial derivative of the Lyapunov function along the trajectories of the coupling systems proves that the proposed control laws can synchronize the systems in finite time.

4. Result and Discussion

Few cases which comprise the coupling of the master and slave systems for different parameter sets were explored to test the robustness of the synchronization scheme. When the Master system: $a = 4, b = 6.75, c = 6.75, d = 1, e = 1$ and Slave system: $\delta = 0.87, \beta = 0.87, \varphi = 1.1$, the plots are shown in Fig. 6.



(a) (b) (c)

Fig. 6. Synchronized dynamics of the state variables

Fig. 6 (a), (b) and (c) depicts the synchronization of the dynamics of the master ad slave system. it can be seen that the trajectories coupled in less than 0.002s. these are relatively fast. The stabilized dynamics of the synchronization error functions and the finite time control law are shown in Fig. 7 and Fig. 8 respectively.

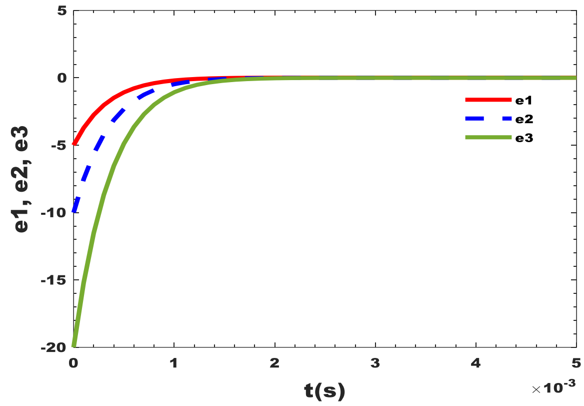


Fig. 7. Stabilized dynamics of the synchronization error functions

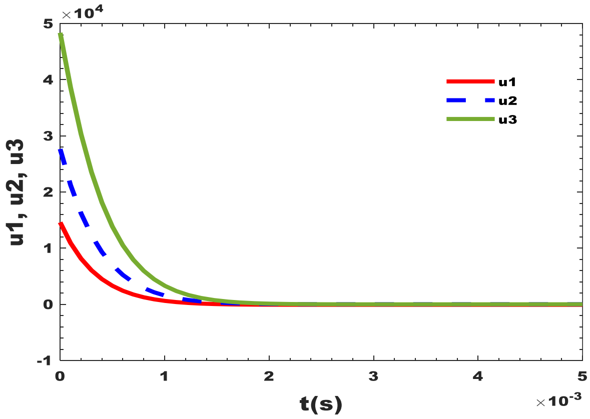


Fig. 8. Stabilized dynamics of the finite-time control laws

In finite time control scheme, stabilized dynamics of the finite-time control laws and synchronization error functions are sufficient proof of the viability of a proposed control architecture. Thus, Fig. 7 and Fig. 8 confirmed the effectiveness of the proposed synchronization architecture. There was no significant difference in the synchronized dynamics of the master and slave systems for other parameters and initial conditions because all the varied parameters. Thus, the finite-time synchronization scheme is robust in the face of parametric disturbances.

5. Conclusion

A finite-time synchronization scheme was proposed to couple two non-identical chaotic systems. the dynamics of the two systems are highly sensitive to initial conditions and parameter variations. We derived a set of control laws that coupled the dynamics of the two systems, based on Lyapunov stability criteria. Robustness to parameter variations is an indication of effectiveness of the scheme. To examined the robustness of the synchronization scheme, the system parameters and initial conditions were varied. The results shows that the variations in system parameters had no noticeable impact on the synchronization time. It was observed that the signum function and non-negative indices played a crucial role in accelerating the synchronization process and preventing overshoots. The higher the non-negative index, the shorter the synchronization time.

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References

[1] C. Nwachioma, J. H. Perez-Cruz, A. Jimenez, M. Ezuma, and R. Rivera-Blas, "A New Chaotic Oscillator—Properties, Analog Implementation, and Secure Communication Application," *IEEE Access*, vol. 7, pp. 7510–7521, 2019, https://doi.org/10.1109/ACCESS.2018.2889964.

[2] C. Nwachioma and J. H. Perez-Cruz, "Analysis of a new chaotic system, electronic realization and use in navigation of differential drive mobile robot," *Chaos, Solitons and Fractals*, vol. 144, p. 110684, 2021, https://doi.org/10.1016/j.chaos.2021.110684.

[3] S. Nasr, A. Abadi, K. Bouallegue, and H. Mekki, "Chaos engineering and control in mobile robotics applications," *15th International Conference on Informatics in Control, Automation and Robotics (ICINCO 2018)*, vol. 2, pp. 364–371, 2018, https://doi.org/10.5220/0006867103640371.

[4] Q. Hu, "Chaotic Oscillation Control Model of Power System Under Electromechanical Power Disturbance," *Front. Energy Res.*, vol. 10, p. 887561, 2022, https://doi.org/10.3389/fenrg.2022.887561.

[5] A. J. M. Khalaf, T. Kapitaniak, K. Rajagopal, A. Alsaedi, T. Hayat, and V.-T. Pham, "A new three-dimensional chaotic flow with one stable equilibrium: dynamic properties and complexity analysis," *Open Phys.*, vol. 16, no. 1, pp. 260–265, 2018, https://doi.org/10.1515/phys-2018-0037.

[6] E. A. Umoh and O. N. Iloanusi, "Algebraic structure, dynamics and electronic circuit realization of a novel reducible hyperchaotic system," *2017 IEEE 3rd International Conference on Electro-Technology for National Development (NIGERCON 2017),* pp. 483–490, 2017, https://doi.org/10.1109/NIGERCON.2017.8281917.

[7] Y. Wang, X. Li, X. Li, Y. Guang, Y. Wu, and Q. Ding, "FPGA-Based Implementation and Synchronization Design of a New Five-Dimensional Hyperchaotic System," *Entropy*, vol. 24, no. 9, p. 1179, 2022, https://doi.org/10.3390/e24091179.

[8] L. Yi, W. Xiao, W. Yu, and B. Wang, "Dynamical analysis, circuit implementation and deep belief network control of new six-dimensional hyperchaotic system," *Journal of Algorithms & Computational Technology*, vol. 12, no. 4, pp. 361-375, 2018, https://doi.org/10.1177/1748301818788649.

[9] W. Yu *et al.*, "Design of a new seven-dimensional hyperchaotic circuit and its application in secure communication," *IEEE Access*, vol. 7, pp. 125586–125608, 2019, https://doi.org/10.1109/ACCESS.2019.2935751.

[10] S. N. Lagmiri, M. Amghar, and N. Sbiti, "Hyperchaos based cryptography: new seven dimensional system to secure communication," *Circ. Comput. Sci.*, vol. 2, no. 2, pp. 20–30, 2017, https://doi.org/10.22632/ccs-2017-251-59.

[11] Q. Yang, D. Zhu, and L. Yang, "A New 7D Hyperchaotic System with Five Positive Lyapunov Exponents Coined," *Int. J. Bifurc. Chaos*, vol. 28, no. 5, p. 1850057, 2018, https://doi.org/10.1142/S0218127418500578.

[12] K. Benkouider *et al.*, "A new 10-D hyperchaotic system with coexisting attractors and high fractal dimension: Its dynamical analysis, synchronization and circuit design," *Plus One*, vol. 17, no. 4, p. e0266053, 2022, https://doi.org/10.1371/journal.pone.0266053.

[13] R. T. Fotsa, A. R. Tchamda, A. S. K. Tsafack, and S. T. Kingni, "Microcontroller Implementation, Chaos Control, Synchronization and Antisynchronization of Josephson Junction Model," *Int. J. Robot. and Control System*, vol. 1, no. 2, pp. 198–208, 2021, https://doi.org/10.31763/ijrcs.v1i2.354.

[14] L. Makouo, A. S. K. Tsafack, M. M. Tingue, A. R. Tchamda, and S. F. Kingni, "Synchronization and chaos control using a single controller of five-dimensional autonomous homopolar disc dynamo," *Int. J. Robot. Control Syst.*, vol. 1, no. 3, pp. 244–255, 2021, https://doi.org/10.31763/ijrcs.v1i3.380.

[15] P. P. Singh and B. K. Roy, "Comparative performances of synchronisation between different classes of chaotic systems using three control techniques," *Annu. Rev. Control*, vol. 45, pp. 152–165, 2018, https://doi.org/10.1016/j.arcontrol.2018.03.003.

[16] U. E. Kocamaz, Y. Uyaroglu, and H. Kizmaz, "Controlling Hyperchaotic Rabinovich System with Single State Controllers : Comparison of Linear Feedback , Sliding Mode , and Passive Control Methods," *Optik*, vol. 130, pp. 914–921, 2017, https://doi.org/10.1016/j.ijleo.2016.11.006.

[17] P. P. Singh, J. P. Singh, and B. K. Roy, “Nonlinear Active Control Based Hybrid Synchronization between Hyperchaotic and Chaotic Systems,” *IFAC Proceedings Volumes*, vol. 47, no. 1, pp. 287-291, 2014, https://doi.org/10.3182/20140313-3-IN-3024.00068.

[18] S. Li, Y. Wu, and G. Zheng, “Adaptive synchronization of the hyperchaotic Liu system,” *Front. Phys.*, vol. 9, p. 812048, 2021, https://doi.org/10.3389/fphy.2021.812048.

[19] E. A. Umoh, “Adaptive Hybrid Synchronization of Lorenz-84 System with Uncertain Parameters,” *TELKOMNIKA Indones. J. Electr. Eng.*, vol. 12, no. 7, pp. 5251-5260, 2014, https://doi.org/10.11591/telkomnika.v12i7.5853.

[20] Z. Xianren, Y. Shihui, Z. Wentao, L. Zhenbo, and L. Linmei, “Function Projective Synchronization of Chaotic Systems with a New Kind of Scaling Function,” *Chem. Eng. Trans.*, vol. 75, pp. 583–588, 2019, https://doi.org/10.3303/CET1975098.

[21] S. Çiçek, U. E. Kocamaz, and Y. Uyaroğlu, “Secure chaotic communication with jerk chaotic system using sliding mode control method and its real circuit implementation,” *Iranian journal of science and technology, transactions of electrical engineering*, vol. 43, pp. 687-698, 2019, https://doi.org/10.1007/s40998-019-00184-9.

[22] E. A. Umoh and O. J. Tola, “Robust global synchronization of a hyperchaotic system with wide parameter space via integral sliding mode control technique,” *Int. J. Robot. Control Syst.*, vol. 1, no. 4, pp. 453–465, 2021, https://doi.org/10.31763/ijrcs.v1i4.485.

[23] J. Zaqueros-Martinez, G. Rodriguez-Gomez, E. Tlelo-Cuautle, and F. Orihuela-Espina, “Fuzzy Synchronization of Chaotic Systems with Hidden Attractors,” *Entropy*, vol. 25, no. 3, p. 495, 2023, https://doi.org/10.3390/e25030495.

[24] G. Fedele, “Invariant Ellipsoids Method for Chaos Synchronization in a Class of Chaotic Systems,” *Int. J. Robot. Control Syst.*, vol. 2, no. 1, pp. 57–66, 2022, https://doi.org/10.31763/ijrcs.v2i1.533.

[25] A. S. Pikovsky, M. I. Rabinovich, and V. Y. Traktengerts, “Onset of stochasticity in decay confinement of parametric instability,” *Sov. Physics. JETP*, vol. 47, no. 4, pp. 715–719, 1978, https://www.osti.gov/biblio/6426388.

[26] O. Chis and M. Puta, “The dynamics of Rabinovich system,” *Differ. Geom. - Dyn. Syst.*, vol. 10, pp. 91–98, 2007, https://doi.org/10.48550/arXiv.0710.4583.

[27] E. A. Umoh, "Chaos control of the complex Rabinovich system via Takagi-Sugeno fuzzy controller," *2013 IEEE International Conference on Emerging & Sustainable Technologies for Power & ICT in a Developing Society (NIGERCON)*, pp. 217-222, 2013, https://doi.org/10.1109/NIGERCON.2013.6715659.

[28] Y. F. Alharbi, M. A. Sohaly, M. A. Abdelrahman, “Fundamental solutions to the stochastic perturbed nonlinear Schrödinger’s equation via gamma distribution,” *Results in Physics*, vol. 25, p. 104249, 2021, https://doi.org/10.1016/j.rinp.2021.104249.

[29] M. F. Danca and G. Chen, “Bifurcation and chaos in a complex model of dissipative medium,” *Int. J. Bifurc. Chaos*, vol. 14, no. 10, pp. 3409–3447, 2004, https://doi.org/10.1142/S0218127404011430.

[30] M.-F. Danca, M. Fečkan, N. Kuznetsov, and G. Chen, “Looking More Closely at the Rabinovich–Fabrikant System,” *Int. J. Bifurc. Chaos*, vol. 26, no. 2, p. 1650038, 2016, https://doi.org/10.1142/S0218127416500383.

[31] M. F. Danca, N. Kuznetsov, and G. Chen, “Unusual dynamics and hidden attractors of the Rabinovich–Fabrikant system,” *Nonlinear Dyn.*, vol. 88, pp. 791–805, 2017, https://doi.org/10.1007/s11071-016-3276-1.

[32] M. F. Danca, “Lyapunov exponents of a class of piecewise continuous systems of fractional order,” *Nonlinear dynamics*, vol. 81, pp. 227-237, 2015, https://doi.org/10.1007/s11071-015-1984-6.

[33] E. A. Umoh and O. N. Iloanusi, “Synchronized Dynamics of Hyperchaotic Flows via an Improved Finite-Time Adaptive Controller Design,” *Int. J. Eng. Res. Africa*, vol. 48, pp. 49–62, 2020, https://doi.org/10.4028/www.scientific.net/JERA.48.49.

[34] L. Dekui and W. Xingmin, “Finite-Time Generalized Synchronization and Parameter Identification of Chaotic Systems with Different Dimensions,” *Wuhan Univ. J. Nat. Sci.*, vol. 27, no. 2, pp. 135–141, 2022, https://doi.org/10.1051/wujns/2022272135.

[35] S. Pang, Y. Feng, and Y. Liu, “Finite-Time Synchronization of Chaotic Systems with Different Dimension and Secure Communication,” *Mathematical Problems in Engineering*, vol. 2016, 2016, https://doi.org/10.1155/2016/7693547.

[36] J. Li and J. Zheng, “Finite-time synchronization of different dimensional chaotic systems with uncertain parameters and external disturbances,” *Sci. Rep.*, vol. 12, p. 15407, 2022, https://doi.org/10.1038/s41598-022-19659-7.

[37] C. Chantawat and T. Botmart, “Finite-time H-inf synchronization control for coronary artery chaos system with input and state time-varying delays,” *PLoS One*, vol. 17, no. 4, p. e0266706, 2022, https://doi.org/10.1371/journal.pone.0266706.

[38] L. Wang, T. Dong, and M.-F. Ge, “Finite-time synchronization of memristor chaotic systems and its application in image encryption,” *Appl. Math. Comput.*, vol. 347, pp. 293–305, 2019, https://doi.org/10.1016/j.amc.2018.11.017.

[39] S. P. Bhat and D. S. Bernstein, “Finite-time stability of continuous autonomous systems,” *SIAM J. Control Optim.*, vol. 38, no. 3, pp. 751–766, 2000, https://doi.org/10.1137/S0363012997321358.

[40] M. Defoort, K. Veluvolu, M. Djemai, A. Polyakov, and G. Demesure, “Leader-follower fixed-time consensus for multi-agent systems with unknown non-linear inherent dynamics,” *IET Control Theory Appl.*, vol. 9, no. 14, pp. 2165–2170, 2015, https://doi.org/10.1049/iet-cta.2014.1301.

[41] Y. Feng, L. Sun and X. Yu, "Finite time synchronization of chaotic systems with unmatched uncertainties," *30th Annual Conference of IEEE Industrial Electronics Society, 2004. IECON 2004*, vol. 3, pp. 2911-2916, 2004, https://doi.org/10.1109/IECON.2004.1432272.

[42] H. Wang, Z. Han, and Q. Xie, “Finite-time chaos control of unified chaotic systems with uncertain parameters,” *Nonlinear Dyn.*, vol. 55, pp. 323–328, 2009, https://doi.org/10.1007/s11071-008-9364-0.

[43] M. P. Aghababa, S. Khanmohammadi, and G. Alizadeh, “Finite-time synchronization of two different chaotic systems with unknown parameters via sliding mode technique,” *Appl. Math. Model.*, vol. 35, no. 6, pp. 3080–3091, 2011, https://doi.org/10.1016/j.apm.2010.12.020.

[44] G. McCartney, F. Popham, R. McMaster, and A. Cumbers, “Defining health and health inequalities,” *Public health*, vol. 172, pp. 22-30, 2019, https://doi.org/10.1016/j.puhe.2019.03.023.