EFFECTIVE K–6 MATHEMATICS TEACHERS' MATHEMATICAL AND MATHEMATICS TEACHING SELF-EFFICACY, INSTRUCTIONAL BELIEFS, AND USE OF EFFECTIVE MATHEMATICS TEACHING PRACTICES

AMY E. ROEHRIG

237 Pages

The use of student-centered practices is influenced by several factors (Peterson et al., 1989). Specifically self-efficacy has been shown to influence teachers' self-reported teaching practices (Hadley & Dorward, 2011; Peterson et al., 1989; Skaalvik & Skaalvik, 2007). The purpose of this study was to determine possible relationships among effective teachers' mathematical and mathematics teaching self-efficacy, instructional beliefs, and the enacted use of effective practices in mathematics. The study involved two K–6 mathematics teachers who were identified as effective by recommendations from highly regarded mathematics teacher educators or administrators. To determine teachers' level of self-efficacies, instructional beliefs, and enacted teaching practices, I used self-efficacy surveys, multiple observations, and a stimulated recall end-of-study interview. Using a descriptive multi-case study methodology (Yin, 2003), I examined the relationships among the three factors (i.e., self-efficacy, instructional beliefs, and practices) of my participants. I found that the teachers' mathematical self-efficacy (MSE) influenced their mathematics teaching self-efficacy (MTSE). Additionally, teachers' self-efficacy interacted with their instructional beliefs and enactment of Standards of Mathematical Practices (NGA & CCSSO, 2010) and mathematical teaching practices (NCTM, 2014). Although teaching during a global pandemic was difficult, the teachers were able to adapt in ways that

reflected their instructional beliefs and allowed them to enact effective teaching practices. The resiliency of these effective teachers underscores the value of developing and supporting effective mathematics teachers.
KEYWORDS: effective teachers, effective teaching practices in mathematics, instructional beliefs, mathematical self-efficacy, mathematics teaching self-efficacy, pandemic.

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A.E.R.

CONTENTS

TABLES

FIGURES

CHAPTER I: INTRODUCTION

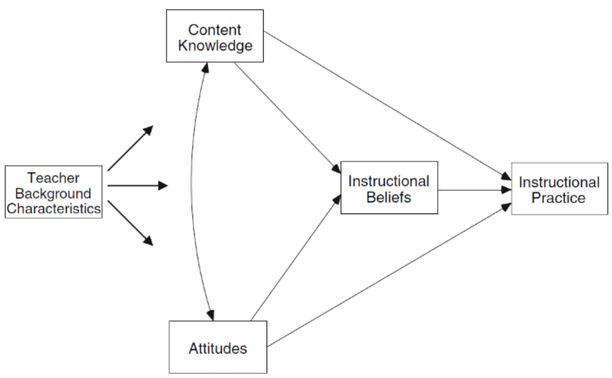
Effective mathematics teachers are essential to improving the mathematics education of students in the United States. Such teachers have the potential to help students see mathematics as more than a set of rules to memorize and algorithms to practice. Students who view mathematics as a flexible and creative subject are more likely to see a future in mathematics and related STEM careers (Azkiyah, 2017; Boaler, 1997, 2000; Boaler & Selling, 2017; Howard & Whitaker, 2011; Peterson et al., 1989; Stigler & Hiebert, 1999). For example, students who engaged positively with school mathematics stated that those experiences influenced their beliefs in the usefulness of mathematics (Boaler & Selling, 2017). Conversely, students who had negative experiences often lacked motivation and did not believe they could be successful in mathematics (Howard & Whitaker, 2011). Thus, a teacher's ability to engage students with mathematics and form positive experiences can have a powerful effect on students' beliefs concerning their mathematical ability (Boaler, 2000).

Several factors appear to be related to effective teaching. Those include various knowledge types referred to by some as content knowledge, pedagogical knowledge, and pedagogical content knowledge, among other terms (Ball et al., 2005; Ball et al., 2008; Peterson et al., 1989; Shulman, 1987). Likewise, affective factors such as beliefs about student potential, beliefs about teaching practices, and others influenced how teachers taught and, in turn, what students learned and how their own beliefs and identities as doers of mathematics developed (Boaler & Greeno, 2000; Brown, 2009; Gresalfi & Cobb, 2011; Wilkins, 2008).

Wilkins (2008) proposed a theoretical model that demonstrated the connections among teacher attributes—including some affective factors—and their choice of instructional practices (see Figure 1). Specifically, this model uses arrows to illustrate that teachers' background characteristics (e.g., years teaching, courses taken, degree earned) have the potential to influence instructional practices, mediated by their content knowledge, attitudes towards mathematics and mathematics teaching, and instructional beliefs.

Figure 1

Theoretical Model Relating Teachers' Background Characteristics to Instructional Practices



Note. From Wilkins, J. L. M. (2008). The relationship among elementary teachers' content knowledge, attitudes, beliefs, and practices. *Journal of Mathematics Teacher Education, 11*, 139–164. (https://doi.org/10.1007/s10857-007-9068-2)

Others have examined instructional practices, including characterizing them in different ways (see Bay-Williams & SanGiovanni, 2021; Boaler, 2022; Liljedahl, 2021; Smith et al., 2019). In the following sections, I will describe one way of distinguishing among more and less effective practices and then describe what is known about how various factors seem to be linked to these practices.

Effective Teaching Practices and Teachers

Instructional or teaching practices are actions teachers perform in the classroom to aid in student understanding and to achieve classroom goals (Stipek et al., 2001; Walshaw, 2013). In this section, I briefly describe effective mathematics teaching practices and what is known about effective teachers of mathematics.

Effective Mathematics Teaching Practices

One way to distinguish mathematics teacher practices is to classify them as indicative of teacher-centered approach or student-centered approach. A teacher-centered approach to teaching focuses on the teacher delivering a static collection of facts, procedures, and rules to a passive learner who receives or absorbs this fully formed knowledge (Romberg & Carpenter, 1986).

The National Council of Teachers of Mathematics (NCTM, 2014) described “research-informed practices” (p. 7)—which are consistent with student-centered practices—as those practices that focus on helping students construct their own knowledge through student explorations of concepts and lead those students to develop connections between new ideas and prior knowledge (Piaget, 1970; von Glasersfeld, 1989). One example of a student-centered practice was noted by Pehmer et al. (2015), who described how teacher questioning can activate student thinking by requiring students to explain their thinking.

Student-centered practices are advantageous because students build more connections and develop deeper understanding by expressing their own ideas (Webb et al., 2014), increase level of engagement among students (Toropova et al., 2019), and potentially decrease anxiety toward mathematics (Alsup, 2004). Consequently, researchers have concluded that teachers’ actions and choices have the potential to influence student achievement (Azkiyah, 2017; Pehmer et al., 2015). Echoing Dewey’s call for progressive education in the early 20th century (Santi & Gorghiu, 2017), mathematics education professional organizations have called for student-centered learning for the last 40 years (NCTM, 1991, 1997, 2014), However, research indicates that in most US classrooms, teachers primarily use teacher-centered models (Stigler & Hiebert, 1999).

Effective Mathematics Teachers

In this study, I focused on teachers labeled as effective by the administration or experts in mathematics education. Though closely interconnected, I distinguish between effective mathematics *teaching practices* and effective mathematics *teachers* (e.g., characteristics of the person).

Some studies of effective mathematics teachers—most using award-winning mathematics teachers as the participants—have focused on characteristics or aptitudes in using research-based mathematics teaching practices (e.g., Gay, 2012; Liang et al., 2012; Perry, 2007). Many of these effective teachers engaged students in student-centered practices (Gay, 2012; Perry, 2007), involved themselves in mathematics teaching research (Liang et al., 2012; Wang & Cai, 2007), and emphasized that effective teachers possess specialized mathematical knowledge for teaching (Cai & Wang, 2010; Gay, 2012; Liang et al., 2012; Perry, 2007; Wang & Cai, 2007). The studies that closely examine the works of effective teachers are particularly instructive because they bring to life effective mathematics teaching practices described by NCTM (1997, 2014) and others (see Bay-Williams & SanGiovanni, 2021; Boaler, 2022; Liljedahl, 2021; Smith et al., 2019).

Having effective teachers in classrooms potentially increases the likelihood that students engage in effective mathematics teaching practices. Therefore, it is imperative to enhance the research surrounding effective mathematics teaching and teachers (e.g., Ball, 2009; Lampert, 1992). Further, given that teaching practices can be so consequential for students and an apparent gap between what is recommended and what occurs most often in US classrooms, it is important to look more closely at the various factors that influence them, including instructional beliefs.

Instructional Beliefs and Teaching Practices

Instructional beliefs are a complex system that integrates teachers' beliefs concerning the nature of mathematics and beliefs about the relationship between the teaching and learning of mathematics (Ernest, 1989a, 1989b; O'Hanlon et al., 2015; Polly et al., 2013). These beliefs are views which teachers hold about best practices in teaching.

Teachers who have teacher-centered beliefs have beliefs that are aligned with teacher-centered practices or a traditionalist (i.e., behaviorist) approach to learning (Polly et al., 2013; Romberg & Carpenter, 1986; Woolley et al., 2004). For example, these teachers believe that classroom routines such as drill-and-practice (i.e., students practice skills to the point of automaticity) are effective methods of helping students attain learning objectives. Further, these teachers believe that student learning can be achieved by direct instruction in which teachers transfer their understandings to students by demonstration and explanation. These teachers believe that it is their role to do the teaching (i.e., telling) while the students are passively listening (Eisenberg, 1975; Raymond, 1997; Woolley et al., 2004).

Conversely, teachers who have student-centered beliefs ascribe to constructivist models of learning, in which they believe that direct transfer of knowledge from one person to another is not how learning occurs (Raymond, 1997; Skemp, 1978; Woolley et al., 2004). Rather, students must construct their own understanding by active engagement with the content (NCTM, 2014). Thus, they believe that the teacher should be designing purposeful learning experiences for students in which they will explore ideas, observe patterns, and reason about the mathematical structure behind the patterns they observe.

Many researchers have investigated the connections between teacher beliefs and practices (see Peterson et al., 1989; Polly et al., 2013; Stipek et al., 2001) often concluding that the relationship between beliefs and practices is complex. Stipek et al. (2001) found that teachers who held more teacher-centered beliefs adhered to more traditional teaching practices (e.g., direct-teaching, following a prescribed algorithm). Though, sometimes, there is a conflict between teachers' instructional beliefs and instructional practices (Raymond, 1997; Yurekli et al., 2020). Researchers have found that this conflict could be a result of time constraints, lack of resources, worries concerning high-stakes testing, or student behavior (Raymond, 1997). In addition, teachers could find it difficult to enact student-centered practices despite believing that student-centered practices are worthwhile and beneficial (Yurekli, et al., 2020). For example, using surveys and self-reported teaching practices, Yurekli et al. found that teachers who believed that it was important to focus on teaching for conceptual understanding (i.e., student-centered), did not consistently implement practices to support student conceptual understanding. With these conflicting reports concerning the correspondence between beliefs and practices, the relationship between beliefs and practices may not be so clear cut.

Self-Efficacy and Teaching Practices

Although not included in Wilkins' model (2008), other affective factors have been linked to effective teaching. One such factor is self-efficacy. In Bandura's (1977, 1997) description of social cognitive theory, he defined self-efficacy as beliefs in one's own capability "to organize and execute the courses of action to produce given attainments" (1997, p. 3). These beliefs stem from one's view of the likelihood of either succeeding or failing at a task. In the context of teaching, many researchers have studied teachers' beliefs about their capability to influence student learning in a broad sense and defined this belief as general teaching self-efficacy (i.e., Gulistan et al., 2017; Hackett & Betz, 1989; Toropova et al., 2019; Tschannen-Moran et al., 1998). Within many of these studies, researchers have shown that general teaching self-efficacy can influence students' learning outcomes. Ashton (1984) found that general teaching self-efficacy was the best predictor of teacher behavior and highly efficacious teachers retained more positive relationships with students showing a greater openness to student ideas. In a comparison study of self-concept and self-efficacy, Pajares and Miller (1994) highlighted the specificity of self-efficacy. Echoing Bandura's (1986) claim that self-efficacies are content specific assessments of one's own capabilities and therefore when evaluating those beliefs, one must inspect specific behaviors within specific scenarios.

Mathematical self-efficacy (MSE) is one's belief in their own capability to successfully complete a mathematical task (Kahle, 2008). In the development of one's MSE, positive or successful experiences often lead to higher self-efficacy. In contrast, low self-efficacy most likely stem from repeated negative experiences with mathematics (Bandura, 1986; Kahle, 2008). As one's MSE develops over time, these beliefs begin to influence how one performs on mathematical tasks (Hackett & Betz, 1989; Pajares & Miller, 1994).

Mathematics teaching self-efficacy (MTSE) is one's belief in their own capability to teach mathematics to others (Enochs et al., 2000; Kahle, 2008; Swars, 2005). Just as other self-efficacy, MTSE develops from one's past experiences teaching mathematics. MTSE has been linked to several aspects of teaching (Fives & Buehl, 2010; Gulistan et al., 2017; MacMillan, 2009; Marsh, 1986). For example, after collecting self-reported instructional practices from teachers, Toropova et al. (2019) found that teachers with increased MTSE were more likely to use student responses and inquiry during mathematics lessons. Researchers have also linked teachers' MTSE to their own fear of mathematics (Bates et al., 2013; Chen et al., 2014; Gulistan et al., 2017).

Statement of Problem

Although research has shown that student-centered teaching is an effective teaching practice, the use of these practices is not widespread in US schools (Stigler & Hiebert, 1999). Therefore, there is a need to learn more about characteristics or behaviors of effective teachers so that schools can create conditions that will enable teachers to use more student-centered approaches.

Wilkins' model (2008) illustrates some factors that may influence teaching practices, including the direct precursor, instructional beliefs. However, researchers have found that teachers may not enact practices that are consistent with their stated beliefs. Some researchers have noted that teachers' practices are aligned with their instructional beliefs (Peterson et al., 1989; Polly et al., 2013; Stipek et al., 2001) while others have found a conflict between beliefs and practices (Raymond, 1997; Yurekli et al., 2020).

Further, Wilkins' model (2008) does not account for self-efficacy, which others have found has the potential to contribute to enacted teaching practices, as well. Researchers have linked self-efficacy to teachers' beliefs about the most appropriate and effective teaching practices (McLeod, 1987; Opera & Stonewater, 1987; Raymond, 1997). Further, self-efficacy has been associated with selection of instructional practices (Peterson et al., 1989) and decision making in response to student actions (Tschannen-Moran & Woolfolk Hoy, 2007). These findings leave the overall picture unclear with respect to whether self-efficacies are mediated by beliefs or whether self-efficacies directly influence teaching practices. In addition, researchers have noted that when researching one's self-efficacy, those beliefs should be situation specific

(Bandura, 1993, 1997; Gibson & Dembo, 1984; Kahle, 2008). In a mathematics teaching context that would involve exploring both mathematical self-efficacy and mathematics teaching self-efficacy. There is limited research that focuses on effective teachers and their self-efficacy. Further, studies focused on the relationships among self-efficacy—specific to mathematics—teaching practices, and instructional beliefs of those effective teachers are scarce.

Lastly, the 2020 coronavirus pandemic caused an upheaval in most schools and dramatically increased the demands on teachers (Basilaia & Kvavadze, 2020; Clarkson et al., 2020). Although more research regarding the effect of COVID on teaching and learning is being released (see Echeverría et al., 2022; Zamarro et al., 2021), little of that research has examined the relationships among teacher self-efficacy and instructional practices during that time. Further, most teachers were ill prepared to provide effective instruction via video-conferencing software, hybrid delivery, in the circumstances that limited contact during in-person classes (Basilaia & Kvavadze, 2020). Teachers had to develop techniques for working in these circumstances as they were teaching. Because of the confluence of these events, my study offered an opportunity to shed light on the little that was known about how teachers might navigate these difficult circumstances. This opportunity provided an insight into the potential challenge on one's self-efficacy in extraordinary times, providing unique opportunities for teachers to reflect on their beliefs and experiences during the pandemic.

Research Questions

The following research questions guided this study:

1. How are mathematical self-efficacy and mathematics teaching self-efficacy related in mathematics teachers who have been labeled as effective?

2. How do teachers' instructional beliefs relate to their mathematical self-efficacy, mathematics teaching self-efficacy, and their use of effective teaching practices?
3. How did the spring 2020 coronavirus school shutdown, the immediate transition to remote learning, and the atypical fall 2020 semester, influence effective mathematics teachers' self-efficacies and instructional practices during the spring 2021 semester?

Rationale

Investigating effective teaching is important because the use of effective teaching practices may be positively related to learning (Jong et al., 2010) and such learning experiences may decrease negative affective factors, such as mathematics anxiety, in students (Alsup, 2004). Because effective, student-centered teaching practices are more rarely implemented than teacher-centered practices, it is important to learn more about factors such as self-efficacy and instructional beliefs, and how they connect to the use of effective teaching practices in real classrooms. To do so efficiently, this research should be conducted with teachers who have already been identified as effective.

Because the existing research which has primarily relied on self-reports via large scale surveys has led to conflicting results, qualitative research, which allows the researcher to collect richer, more detailed data, may be the most appropriate method for beginning to learn more about the mechanisms behind the factors influencing the use of effective practices. And, using outside observer to corroborate the use of effective practices may provide more reliable and complete evidence with respect to instructional practices and their connections to self-efficacy, instructional beliefs, and observed instructional practices.

There is evidence that self-efficacy is domain specific (Bandura, 1986; Kahle, 2008), and self-efficacy may vary by day or lesson. For this reason, I chose to focus on mathematical and mathematics teaching self-efficacy. Mathematical self-efficacy varies depending on the content (i.e., fractions, functions, division).

According to Echeverría et al. (2022), the pandemic "may serve as a window to reveal the deeply rooted conceptions or beliefs of teachers on what and how teaching should be provided" (p. 1). In this regard, my results may provide insight into the potential challenges on one's self-efficacy in extraordinary times, providing unique opportunities for teachers to reflect on their beliefs and experiences during the pandemic. In addition, I add to the recent body of research focused on teaching practices and routines teachers used to continue effective teaching during mask mandates, social distancing guidelines, and remote instruction.

Lastly, this study affords the potential to find out more about how the relationships among self-efficacy, instructional beliefs, and practices fit into Wilkins' (2008) theoretical model. This process of closely inspecting the relationships among these factors may provide evidence to support or identify adjustments to Wilkin's theoretical model. Last of all, my research can contribute to the growing body of research on both the self-efficacy of teachers and the influence on teachers' use of effective instructional practices.

CHAPTER II: LITERATURE REVIEW

In this chapter, I provide relevant background information to my study. I begin with a literature review of Wilkins' (2008) theoretical model (see Figure 1), which theorizes relationships among beliefs and practices. I then provide relevant literature concerning the primary focus of my study, the concept of self-efficacy. The following sections then focus on effective teachers, effective practices (e.g., student-centered), teachers' instructional beliefs, and the 2020 pandemic.

Theoretical Perspectives

The theoretical frameworks for this study were based on Wilkins' (2008) theoretical model (see Figure 1) and the theory of self-efficacy, which is a component of the broader social cognitive theory (Bandura, 1977, 1986).

Wilkins (2008) provided a theoretical model relating teachers' content knowledge, attitudes concerning mathematics and mathematics teaching, instructional beliefs, and practice. Further, Bandura's (1986) social cognitive theory conjectures that learning happens in a dynamic social context. In addition, the theory posits a reciprocal relationship among the person, their behavior, and the environment. In this way, the social context influences the acquisition and maintenance of one's behavior. Social cognitive theory contains six constructs. One of those constructs is self-efficacy which aids in understanding the initiation and maintenance of behavior. Using Wilkins' (2008) theoretical model and having a firm understanding of self-efficacy as both an affective factor and a component of social cognitive theory supported my interpretation of effective teachers, their instructional beliefs, and their enacted teaching practices.

Wilkins' (2008) Theoretical Model

Wilkins (2008) proposed a theoretical model (see Figure 1) which demonstrated the connections among teacher attributes and their choice of instructional practices. Based on Ernest's (1989b) instructional model, Wilkins (2008) explored the relationships among elementary teachers' subject knowledge, their beliefs about inquiry-based (i.e., student-centered) instructional methods and their effectiveness, their attitudes concerning mathematics and mathematics teaching, and their self-reported use of inquiry-based practices.

Wilkins' (2008) model illustrated his findings that teacher beliefs and attitudes towards mathematics and mathematics teaching mediated teachers' practice. Wilkins used a 13-item survey to measure inquiry-based practices. In the survey, participants used a 4-point Likert Scale to rate the importance of 10 mathematical teaching strategies (e.g., have students participate in appropriate hands-on activities, use calculators, use informal questioning to assess student understanding). Teachers reported the frequency with which they used 17 inquiry-based practices (e.g., use manipulatives to solve exercises or problems, share ideas or solve problems with each other in small groups). Interestingly, the author found that mathematical content knowledge—determined using a survey of 32 multiple choice items and four open-ended items—was negatively related to teachers' beliefs regarding inquiry-based practices and teachers' self-reported use of inquiry-based practices ($r = -.15$, $r = -.23$, $p < .001$). In other words, teachers whose content knowledge was measured to be relatively high were less likely to report using inquiry-based practices and they were also less likely to believe that inquiry-based practices were effective.

Social Cognitive Theory

In the following sections I describe the primary tenets of Bandura's social cognitive theory (1986) as they pertain to the current study.

Summary of the Theory

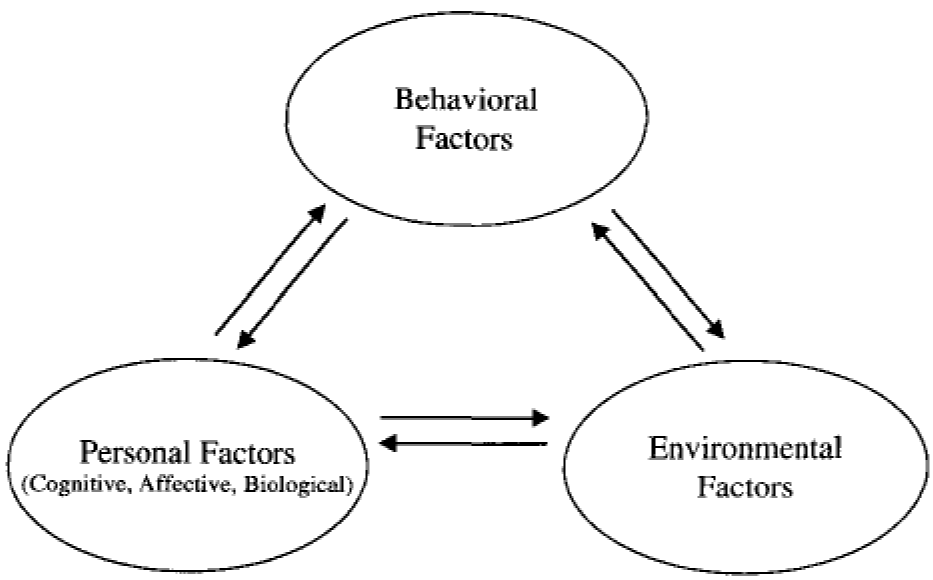
Social cognitive theory relies on an "agentic perspective" (Bandura, 2018). Bandura (2001) described the agentic perspective in terms of human agency, wherein people "are agents of experiences rather than simply undergoers of experiences" (p 4). In other terms, a person is someone who is "self-organizing, proactive, self-reflecting, and self-regulating" (Pajares & Usher, 2008, p. 392). In addition, according to Eisenberg (1995), a theory built on an agentic perspective has the potential to expand research focused on how social constructions influence the functioning of the human brain. Within this perspective, human functioning is the result of "a dynamic interplay of personal, behavioral, and environmental influences" (Pajares & Usher, 2008, p. 392). Bandura (1986, 2002) stated that humans, influenced by culture, have developed a set of capabilities—symbolizing, forethought, vicarious, self-regulatory, and self-reflective. Each of these capabilities plays a role in determining the individual and their behaviors.

Triadic Reciprocal Determinism

As stated in the previous section, triadic reciprocal determinism is based on the thought that human functioning is a result of personal, behavioral, and environmental influences (Bandura, 1977, 1986; Pajares & Usher, 2008). Each of these influences are not independent factors but instead have a dynamic relationship with the other. For example, an individual, through their own actions, constructs their own environmental conditions which in turn influence their behavior, retaining a level of reciprocity. Those experiences from the interplay of behavior and environment, can then potentially establish what a person becomes and can produce, which then influences subsequent behavior (Bandura, 1977, 1986). Figure 2 shows these reciprocal relationships among the three factors in the triad.

Figure 2

Bandura's Triadic Reciprocal Determinism Model



Note. From "Towards a Psychology of Human Agency: Pathways and Reflections," by A. Bandura, 2018, *Perspectives on Psychological Science, 13*(2), p131 (https://doi.org.10.1177/1745691617699280).

As mentioned in the previous section, an individual has a set of capabilities, one, which is distinctively human, is self-reflection (Bandura, 1977, 1986; Pajares & Usher, 2008). Within the process of self-reflection, one "makes sense of their experiences, explore their own cognitions and self-beliefs, engage in self-evaluation, and alter their thinking and behavior" (Pajares & Usher, 2008, p. 395). In this process, people construct judgements about their capabilities accomplishing and succeeding at a task (e.g., self-efficacy). Self-efficacy, which is thought to be the core of social cognitive theory and has the greatest influence on human functioning (Pajares & Usher, 2008), are covered in the following sections.

Theory of Self-Efficacy

One prominent component to social cognitive theory is the idea of one's self-efficacy. In the following sections, I provide an overview of Bandura's (19886) conception of self-efficacy.

Definition

To understand the theory of self-efficacy, it is important to begin by distinguishing two concepts: (a) self-efficacy—a person's perceived capability to exercising control over events in their lives; and (b) agency—a person's actual capability to exercise control, or intentionally take action. These ideas are not disjoint, because a person's agency is influenced by their self-efficacy. According to Bandura (1997), an individual has the capacity to control their actions based on how confidently they exercise that control. Bandura described a phenomenon in which individuals make choices based on how successfully they believe they can perform a particular task. For example, a teacher who does not have a high level of confidence in their capability to teach fractions, may not make a concerted effort, nor persevere in an effort, to produce specific outcomes, such as employing student-centered teaching practices during their lessons on fractions. Agency reflects a realization, which people develop over time, that their own actions produce effects, and that these "actions are part of oneself" (Bandura, 1997, p. 164).

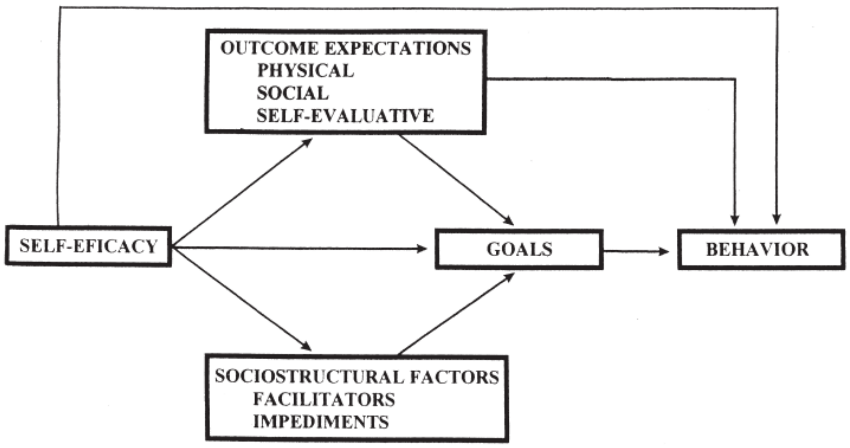
As children learn that their actions can also affect their environment or other outcomes (i.e., personal agency); they begin to develop a sense that they have the capacity to exercise control over their environment. As children recognize this ability to exercise control, they test their agency by taking actions that are intentional and planned. The results from these actions—whether intended or unintended—are the experiences that work to develop one's self-efficacy.

Throughout adolescence and into adulthood this cyclic relationship continues as people experience situations that affect their self-efficacies from four sources (i.e., verbal persuasion, vicarious experiences, physiological arousal, and mastery experiences; Bandura, 1997). Thus, a person's self-efficacy influences and is influenced by their actions, including (a) the amount of effort or motivation required to take an action, (b) the level of perseverance through hinderances and failures, (c) the extent to which thoughts are self-hindering or self-aiding, (d) the amount of stress or depression associated with demanding situations, and (e) their willingness to recognize their own accomplishments (Bandura, 2004). Bandura's (2004) model (see Figure 3) illustrates how a person's perceived self-efficacy, modified by several factors, may influence the kind of goals they set, which will lead to a behavior. Though personal agency is not an element of Bandura's original model, my interpretation of personal agency as one's capability to exercise control (i.e., enact a behavior) leads me to believe that personal agency is likely to have a role in mediating a person's goals and behavior.

When acting on one's personal agency there are outcomes. These outcomes are not always completely dependent on personal choices made but instead may depend on a combination of personal choices and events that are beyond one's control (i.e., sociostructural factors; Bandura, 2004; Vroom, 1964). For example, a teacher who felt successful teaching a lesson on decimals is not certain that the same lesson will be successful the following year because circumstances that are beyond the teacher's control can change from year to year (e.g., the motivation or response of the students). Expectancy-value models, such as Figure 3, in their most basic form, show the expectancy that certain behaviors can lead to specific outcomes and the more an individual values the outcomes, the more motivated they may be to ensure success (Bandura, 1997; Vroom, 1964).

Figure 3

Factors That Influence the Exercise of Control



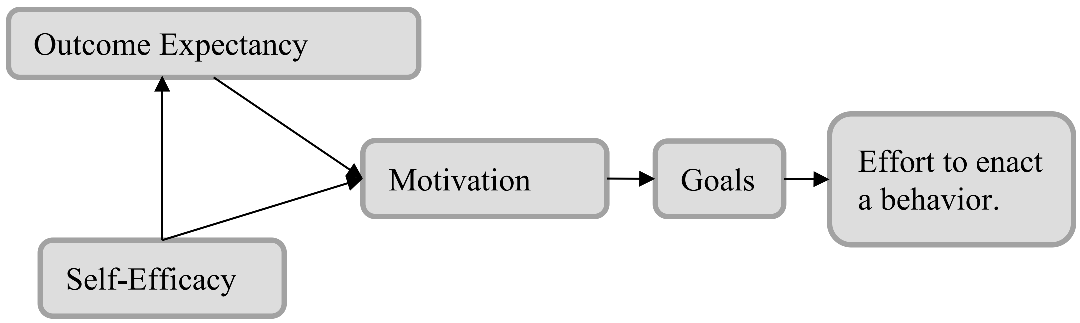
Note. This flow chart shows the overall nature of exercising control and those factors which influence one's behaviors. From "Health Promotion by Social Cognitive Means," by A. Bandura, 2004, *Health Education & Behavior, 31*(2), p. 146 (https://doi.org.10.1177/1090198104263660).

The Role of Self-Efficacy in Motivation

Many other factors rely on self-efficacy because self-efficacy influences the choices people make and level of motivation people feel, which, in turn, affects their learning outcomes (Bandura, 1997). If a person is confident about their ability to successfully take an action, they may be motivated to set goals that would require them to take that action (Bandura, 1993, 1997). Thus, self-efficacy influence the goals set by individuals, the amount of effort they plan to expend on the task, the level of perseverance they are willing to enact, and their resiliency in the face of mistakes or failures (Bandura, 1993). As part of this process, a person maintains beliefs about likely outcomes (i.e., expectancy outcomes). Motivation—a cognitive activity—is mediated by a person's expectation that certain outcomes will occur and the significance of those outcomes (Bandura, 1977, 1989; Vroom, 1964). In that way, motivation is influenced by outcome expectancies and self-efficacy (Bandura, 1997). These relationships and mechanisms are shown in Figure 4.

Figure 4

Self-Efficacy and Outcome Expectancy as Mediators of Motivation



Note. This flow chart shows motivation being influenced by a person's self-efficacy and outcome expectancy, meanwhile mediating those beliefs to a person's goals and enacted behavior.

Self-Efficacy Development

Self-efficacy—beliefs about one's capability to be successful in particular situations—have the potential to influence teachers' decision making concerning the use of particular practices to teach effectively (Bates et al., 2013; Perera & John, 2020; Swars, 2005). In addition, self-efficacy has the potential to mediate teachers' instructional beliefs as they choose whether to use student- or teacher-centered practices (Wilkins, 2008). It is important to recognize that self-efficacy is based on a person's interpretation of past experiences (Bandura, 1986, 1997; Hackett & Betz, 1989; Tschannen-Moran et al., 1998; Wilkins, 2008) and are constructed from four sources (i.e., verbal persuasion, vicarious experiences, physiological arousal, and mastery experiences; Bandura, 1986, 1997). Persons with high self-efficacy are more likely to: (a) attempt tasks they might find challenging; (b) expend more initial effort on successful completion of the task; and (c) be persistent when the task becomes difficult (Bandura, 1986). The following sections describe the role each source plays in the development of one's self-efficacy.

Verbal Persuasion. When one is doubting their own self-efficacy, an encouraging word expressed from another can interrupt the faltering self-efficacy judgements and positively influence self-efficacy (Bandura, 1997). It is also possible that verbal persuasion can increase the likelihood that the temporary boost in self-efficacy leads to a permanent positive change in self-efficacy judgements. In the study of 573 PSTs in their final year of a teacher preparation program, O'Neill and Stephenson (2012) administered a multi-scaled survey designed to measure teacher self-efficacies and their sources. To measure teacher self-efficacies, the authors used Tschannen-Moran and Woolfolk Hoy (2001) Teachers' Sense of Efficacy Survey (TSES), in which participants expressed their opinion of how much they can do when faced with challenges, such as "How much can you do to help students think critically?" and "How much can you do to get children to follow classroom rules?" thus, evaluating teachers' perceptions of their own capabilities related to instructional practices that affect student engagement and classroom management. To determine the PSTs' sources of self-efficacy, O'Neill and Stephenson used the Teaching Efficacy Sources Inventory (TESI; Poulou, 2007), in which PSTs responded to questions that explored the level of influence sources of self-efficacy had on the PSTs' self-efficacy in classroom management. The questionnaire had the item stem of "I attribute my confidence in classroom behavior management capabilities to…" (O'Neill & Stephenson, 2012, p. 538). To measure verbal persuasion the authors used statements such as, "feedback I received from my cooperating teacher during practicum" and "pupil enthusiasm about my teaching sessions during practicum." The authors determined that verbal persuasion from PSTs' cooperating teacher was one of the strongest (*r* = .49, *p* < .01) self-efficacy source.

Mohamadi and Asadzadeh (2012) surveyed 284 high school teachers' teacher self-efficacy, using TSES, the sources of their teacher self-efficacy using the Sources of Self-Efficacy Inventory (SOSI; Kieffer & Henson, 2000), and student achievement using classroom projects and assessments. On the SOSI, respondents rated their opinion on if a statement such as, "When people I respect tell me I will be a good teacher, I tend to believe them." is true on a 7-point Likert scale ranging from not true for me (1) to definitely true for me (7). The authors found that teachers' self-efficacy had a significant effect on student achievement and that in contrast to previous research, verbal persuasion had a larger influence on teachers' self-efficacy than vicarious experiences.

Vicarious Experiences. At times, one judges their own self-efficacy based on another's success or failure. This is what Bandura (1986, 1997) described as vicarious experiences. Self-efficacy can be further developed or diminished by vicarious experiences. For example, a person who attained a score on a test that is high above the average test score for the class can experience an increase in their self-efficacy, while in the same class one who scored considerably less than the average might feel a decrease in their self-efficacy. In O'Neill and Stephenson (2012), vicarious experiences were among the more influential sources of PSTs' teacher self-efficacy (*r* = .12, *p* < .01). Though the authors indicated a strong correlation coefficient (*r* = .46, *p* < .01) between vicarious experiences, verbal persuasion, and mastery experiences, there was evidence that the three sources of self-efficacy were more influential than the last source, physiological arousal.

Tschannen-Moran and McMaster (2009) conducted a quasi-experimental study which examined the role of different professional development (PD) models played in increasing teacher self-efficacy and the implementation of a new instruction strategy for reading. The four PD models included (1) a one-time 3-hour workshop in which verbal persuasion was the focus as the presenter demonstrated and teachers were provided with an opportunity to use the strategies outlined in the manual; (2) a one-time 3-hour workshop in which a vicarious experience— presenter used the instructional strategy with students during the workshop for teachers to observe— was added to the PD model; (3) a workshop that included the previous sources though teachers were also provided with a practice session in which they were able to decide how they would use the strategies, plan lessons for students, and practice implementation with their groups, this providing a mastery experience; and (4) a workshop similar to the third model with a follow-up coaching session with the presenter which included a review of the instructional strategy, one-on-one question and answer, and a coaching session in the teacher's classroom, providing the teacher with a more intense mastery experience. The TSES (Tschannen-Moran & Woolfolk Hoy, 2001) was used to measure teacher self-efficacy pre- and post-PD. In addition, the implementation of the instructional strategy was measured using a seven item pre- and post-PD questionnaire including questions such as, "To what extent do you use the Tucker Reading Strategies?" To the surprise of the authors there were substantial variance across (mean implementation scores ranged from 2.71 to 6.78) and within groups (mean implementation scores ranged from 1.67 to 2.43), though all treatment groups observed an increase in teaching self-efficacy. Interestingly, teachers in treatment groups 1 and 4 showed significant gains in a posttest on teacher self-efficacy one month later, while treatment groups 2 and 3 remained the same.

Physiological Arousal. Feeling physical factors (i.e., increased heart rate, sweating, emotional stress reactions) may indicate a "vulnerability to dysfunction" (Bandura, 1997, p. 106) for one's beliefs in their capabilities. To measure the physiological arousal, O'Neill and Stephenson (2012) asked questions focused on the anxiety, stress, or fatigue the PSTs felt during their student teacher practicum. Of all four sources of self-efficacy information, physiological arousal was the only source that was negatively associated ($r = -.15$, $p < .01$) with PSTs' TSES scores. The authors concluded that it is possible that physiological arousal functions more as a secondary predictor and source of teacher self-efficacy. Similarly, Mohamadi and Asadzadeh (2012) concluded that physiological arousal could not be assumed to be a self-efficacy information source because arousal was mediated through other sources, in particular mastery experiences.

Mastery Experiences. As the most influential self-efficacy information source, mastery experiences provide oneself with "authentic evidence" (p. 80) that one can be successful (Bandura, 1997). To develop a high sense of self-efficacy one must experience success through obstacles in which perseverance and effort were required. It is not necessary for one to experience success specific to each situation one encounters, instead self-efficacy information is developed through acquiring tools (e.g., cognitive, behavioral, and self-regulating) so that one can execute actions that produce success in different and similar situations. Tschannen-Moran and McMaster (2009) concluded that by teachers undergoing a mastery experience within the PD, the teachers were able to create subsequent points of mastery in their own classrooms, thereby developing a higher level of self-efficacy for implementation of a new instructional strategy. Mohamadi and Asadzadeh (2012) confirmed that mastery experiences are a highly influential source of self-efficacy information. Additionally, the authors supported the hypothesis that mastery experiences regulate one's physiological arousal, for they suggest that negative arousal is negated when one experiences mastery.

Self-Efficacy of Teachers

Definitions

Because self-efficacy is situation specific (Bandura, 1986), the following sections provide definitions of types of self-efficacies which are central to the current study.

Teacher Self-Efficacy

Self-efficacy, including those of teachers, are situation specific (Bandura, 1986). Teacher self-efficacy (TSE; Ashton, 1984) refers to specific beliefs a teacher holds about their capability to have a positive effect on student learning. Teaching self-efficacy, in general, has been shown to have an influence upon teachers' attitudes toward educational reform (De Mesquita & Drake, 1994), use of instructional practices (Allinder, 1994; Conroy et al., 2019; Depaepe & König, 2018; Gibson & Dembo, 1984), instructional quality (Holzberger et al., 2013; Perera & John, 2020), and academic achievement of students (Anderson et al., 1988; Shahzad & Naureen, 2017; Zee et al., 2016).

Mathematical Self-Efficacy

Kahle (2008) defined mathematical self-efficacy (MSE) as one's beliefs in their capability to successfully carry out a mathematical task. MSE can predict how one will cope with failure or difficulty (Bandura, 1986). As a person develops their MSE through their past experiences (i.e., vicarious or mastery), those beliefs begin to influence how one performs on mathematical tasks (Hackett & Betz, 1989; Pajares & Miller, 1994). Though, some experiences are negative and can potentially lead to fear and anxiety toward mathematics (Bandura, 1986) or provide a source of motivation to become successful at mathematical tasks—often through vicarious experiences or verbal persuasion.

Mathematics Teaching Self-Efficacy

Mathematics teaching self-efficacy (MTSE) has been defined as the beliefs in one's capabilities to teach others mathematics (Enochs et al., 2000; Kahle, 2008; Swars, 2005). For teachers, teaching self-efficacy influences choices they make in teaching practices, curriculum delivery, and task choices (Fives & Buehl, 2010; Gulistan et al., 2017; MacMillan, 2009; Marsh, 1986). More specifically, MTSE have been linked to: (a) teachers being more open to student responses and inquiry, including student engagement (Gibson & Dembo, 1984; Lin & Gorrell, 2001; Toropova et al., 2019); (b) instructional quality (Bruce & Ross, 2008; Perera & John, 2020); and (c) teachers' fear of mathematics (Bates et al., 2013; Chen et al., 2014; Gulistan et al., 2017).

Self-Efficacy in Literature

The following sections contain a review of pertinent literature to self-efficacy—mathematical and mathematics teaching—as they relate to the current study.

Mathematical Self-Efficacy

Performance. In their studies of undergraduate students, Hackett and Betz (1989) and Pajares and Miller (1994) found that MSE is strongly linked to mathematics performance. In a study of 262 undergraduate students enrolled in a psychology class, Hackett and Betz (1989) examined relationships among MSE, mathematics performance, and attitude toward mathematics. To measure MSE and performance, Hackett and Betz used the Mathematics Self-Efficacy Scale (MSES; Betz & Hackett, 1983), which included three subscales used to determine: (a) confidence toward everyday tasks (e.g., balancing your checkbook); (b) confidence in different mathematics courses (e.g., Basic College Math, Economics, Algebra II); and (c) confidence in their ability to solve mathematics problems involving arithmetic, algebra, and geometry. In addition to the last subscale of the MSES, American College Test (ACT) scores were also collected from all participants to determine mathematics performance. The authors found a moderately strong positive relationship with a correlation coefficient of .44 between MSE and mathematics performance. Further, Hackett and Betz (1989) determined that students with both high MSE and high mathematics performance had higher levels of confidence and motivation, interestingly, these were the same students who also reported that they had enrolled in high level mathematics courses while in high school. Though there was a relationship between attitudes towards mathematics and MSE, the relationship was weaker than the relationship between attitudes towards mathematics and mathematics performance.

Similar to Hackett and Betz (1989), Pajares and Miller (1994) used undergraduates in their path analysis to test previously stated hypotheses concerning the "predictive and mediational role" (p. 193) of MSE, though Pajares and Miller's participants were enrolled in courses within the College of Education and included 137 education majors and 213 from other university majors. To measure MSE, the Mathematics Confidence Scale (Dowling, 1978) was used, in which participants rated their confidence on tasks related to arithmetic, algebra, and geometry. Mathematics performance was measured using the Mathematics Problems Performance Scale (Dowling, 1978), a multiple-choice instrument containing mathematics tasks specific to the average college student. As with previous research, the authors found a significant correlation between MSE and performance with a .70 ($p < .0001$) correlation coefficient. Though Pajares and Miller found significant and positive correlations among gender ($r = .209$, $p < .0001$), high school mathematics experience ($r = .419$, $p < .0001$), and college mathematics experience ($r = .185$, $p < .0001$) on MSE, they determined that MSE affected performance directly rather than through mediated variables (e.g., gender, prior experiences). And, as a result, MSE is a better predictor of performance than gender and prior experience. Pajares and Miller concluded that teachers need to assess not only their own MSE but also their students' beliefs, as teachers work to increase the mathematics performance of their students.

Anxiety and Fear. Burns (1998) stated there is an *American Phobia* and it is mathematics. Burns noted that Americans' mathematics phobia is mostly a result of inconsistency between how mathematics is taught in schools—a senseless system of rules to be memorized—and how mathematicians see mathematics—a coherent and sensible logical system. While in school, mathematics is often taught in bits and pieces and rarely are connections made between those bits. Burns asserted that the lack of opportunities to make connections left students fearful of mathematics due to the perceived inability to make sense of complex mathematical ideas. This induced fear from school mathematics greatly influences a person's MSE, and teachers, many of whom learned mathematics in this disjointed manner, are not immune to this mathematics phobia or its potential effects on teachers' beliefs. To underscore this point, Ball (1990) noted that collegiate mathematics method courses are not the first mathematical learning experiences PSTs encounter. PSTs have spent many hours in mathematics classrooms, and those experiences act as apprenticeships for teaching mathematics which, in turn, shape PSTs' knowledge of and beliefs about the nature of mathematics teaching and learning.

Similarly, Hackett and Betz (1989) found that "mathematics avoidance" (p. 262) was a result of socialized negative attitudes and reactions to mathematics, which lead to issues such as mathematics anxiety. In a study of 189 PSTs, Coppola et al. (2013) used an open-ended questionnaire, which focused on PSTs' past experiences with mathematics and their future perspective on the teaching of mathematics. The authors found that though positive relationships with mathematics always were aligned with positive perspectives on the teaching of mathematics, the inverse was not always the case. PSTs who experienced negative relationships did not always have a negative perspective on the teaching of mathematics. The case that negative relationships with mathematics did produce anxiety for some so that a negative perspective on the teaching of mathematics was evident, many PSTs with a negative relationship with mathematics had positive feelings relating to the teaching of mathematics. The authors noted that many of the PSTs who had a negative relationship with mathematics and a negative perspective toward teaching mathematics had an "instrumental view" (p. 223) of mathematics—mathematics is basically computation and procedures. Those PSTs who were able to maintain a positive perspective toward the teaching of mathematics, tended to view mathematics as a means of cognitive development wherein one develops logical reasoning.

Mathematics Teaching Self-Efficacy

Student Engagement. In a large-scale study of early childhood and elementary PSTs' teaching self-efficacy before and after completion of the program, Lin and Gorrell (2001) found that early education PSTs were more confident in their capability to engage students in learning than elementary level PSTs. Additionally, at the conclusion of the program most PSTs, both early education and elementary, showed an increase in their teaching self-efficacy, which the authors noted involved the PSTs ability to adjust the content to students' needs and know when to intervene in students struggle. Furthermore, in a large-scale study, Perera and John (2020) found that Grade 4 students, who perceived high levels of positive student-teacher interactions, were more likely to have favorable mathematics self-concept beliefs ($r = .286$, $p < .001$).

Incidentally, students who rated student-teacher interaction at a high level had teachers who had high teaching self-efficacy.

Fear. Teachers who were fearful of mathematics due to low mathematical self-efficacy, tended to provide non-explorative lessons, as reported by teachers' self-reports of practices, in which students were expected to passively learn information, and often drew their lessons from a narrow curriculum—textbook created worksheets and problems—that did not support connections between mathematical concepts (MacMillan, 2009). People react differently to feared events than to non-feared events. They approach feared situations more apprehensively than those situations in which they feel more comfortable. The presence of safeguards can help people behave less apprehensively when they face situations which would otherwise generate a fear response (Bandura, 1977). One way that teachers add a safeguard to uncomfortable teaching situation is to rely on the textbook for instructional choices and lesson goals. Teachers who use student-centered approaches such as allowing students to develop methods or question relationships as they work through problems, must be confident enough in their own mathematics ability to be willing to face difficult or unpredictable student questions or they must be comfortable to let students see that they do not have all the answers. Not surprisingly, then, Wilkins (2008) found that teachers with high self-efficacy reported that they experiment with different pedagogies in their mathematics classrooms.

Effective Mathematics Teachers

According to Brophy and Good (1986), effective teachers are those who focus on providing socialization for students, helping students develop positive affective qualities, and advancing student knowledge in a chosen subject matter. Effective teachers have been described according to different perspectives: instructional, organizational, and emotional characteristics

(Holzberger et al., 2019). Some studies on effective mathematics teachers focused on finding commonalities among award winning mathematics teachers (Gay, 2012; Liang et al., 2012; Perry, 2007; Wang & Cai, 2007). While others, Cai and Wang (2010), worked to draw comparisons between distinguished teachers from both China (Mainland) U.S.

Attempting to tell the story of effective mathematics teachers, Gay (2012) purposefully selected three elementary teachers who received awards specific to their mathematics teaching. Completing a cross-case analysis using data from observations, interviews, teachers' notes, and a survey to determine mathematical knowledge for teaching, Gay found that collectively, the three effective teachers were similar in their "teaching philosophies and pedagogical decisions were relatively the same and grounded in research" (p. 135).

In a similar study, Perry (2007) interviewed 13 effective elementary teachers concerning effective mathematics teaching and learning. Each teacher was nominated for their excellence in teaching mathematics at the elementary level. Perry stated that all teachers "had strong opinions" about ways to implement effective lessons. The teachers' descriptions of effective lessons (Perry, 2007) were consistent with Gay (2012) observations of the effective elementary teachers. In both cases, the teachers valued or implemented student-centered practices (e.g., focus on student thinking, students interacting actively with mathematics, emphasis on worthwhile questioning and discussion).

Liang et al. (2012) collected in-depth interview data from 10 award winning middle or high school mathematics teachers to examine common characteristics among the teachers. Through analysis, Liang et al. determined that all the teachers in their study were active participants in pedagogical research and lesson collaboration with colleagues, used technology in their teaching, and purposefully engaged in research for the purpose of expanding professional opportunities. Similarly, Wang and Cai (2007) used interview data from 9 distinguished mathematics teachers from Mainland China and found that their participating teachers also actively engaged in and conducted pedagogical research for the purpose of improving their teaching.

In a comparison study between nine Chinese and 11 U.S. distinguished mathematics teachers, Cai and Wang (2010) found both commonalities and differences in U.S. and Chinese teachers' perceptions about effective teachers. Though both sets of teachers believed that an effective teacher should care about students, the U.S. teachers tended to describe the personality traits as important qualities while the Chinese teachers focused more on increased knowledge about mathematics and having a "thorough understanding of the textbook" (p. 277). The authors found that only two of 11 U.S. teachers stated that having strong mathematical knowledge was a quality of an effective teacher while all nine Chinese teachers found mathematical knowledge as a quality of an effective teacher.

Lastly, Perry (2007) found that many of the effective teachers highlighted the importance of having a strong "knowledge and understanding of both the subject itself and the syllabuses [content standards]" (p. 15). Perry specified that the subject knowledge to which the teachers were referring was primarily related to the mathematics they were teaching. Likewise, other authors have claimed that effective mathematics teachers require knowledge of mathematics for teaching (Cai & Wang, 2010; Gay, 2012; Liang et al., 2012; Wang & Cai, 2007). This knowledge of content for the purpose of teaching echoes the body of research focused on types of knowledge needed to teach mathematics (see Ball, 1988; Ball et al., 2008; Hill et al., 2005; Stylianides & Ball, 2008).

Teachers' Instructional Beliefs and Practices

The following sections contain literature pertaining to teachers' instructional beliefs and practices. Furthermore, I provide literature regarding the possible relationship between instructional beliefs and practices.

Instructional Beliefs

Instructional beliefs are views teachers hold about the best teaching practices to use when teaching students. These beliefs are a complex system that integrated teachers' beliefs concerning the nature of mathematics and beliefs about the relationship between the teaching and learning of mathematics (O'Hanlon et al., 2015). According to Ernest (1989a, 1989b), teachers' beliefs play a pivotal role in choices teachers make about their enacted teaching practices. These beliefs originate from three key components: (a) views concerning the nature of mathematics; (b) views on the teaching of mathematics; and (c) views on the learning of mathematics. In the following sections, I briefly describe each of these key components of teachers' instructional beliefs and the construction of belief surveys.

Beliefs About the Nature of Mathematics

Ernest (1989a, 1989b) described three basic views of the nature of mathematics. The first is an instrumentalist view, which portrays mathematics as "an accumulation of facts, rules and skills" (Ernest, 1989a, p. 250) which one applies to reach a pre-determined end (e.g., solution or answer). Using this perspective, mathematics is viewed as a collection of unrelated facts and rules that must be followed. The second view of the nature of mathematics is the Platonist view. In this view, mathematics is thought of as "discovered, not created" (Ernest, 1989a, p. 250). And mathematics itself consists of a set of rules static, yet integrated rules. Lastly, Ernest described the "problem-solving view of mathematics" (p. 250). This view requires one to see mathematics as ever changing and dynamic. Further, mathematics is seen as "a process of inquiry and coming to know, not a finished product, for its results remain open to revision" (Ernest, 1989a, p. 250).

Beliefs About the Teaching of Mathematics

Ernest (1989a) also described three views that influence teachers' beliefs about the teaching of mathematics. These views concern the roles that the teacher plays in the classroom and the intended outcome of these roles. The first perspective is that the teacher as an *instructor* and the intended outcome of the teacher's instruction is for students to master a skill with precision. Next, the teacher takes on the role of *explainer*. As an explainer, the teacher's intended outcome is "conceptual understanding with unified knowledge" (Ernest, 1989a, p. 251). Lastly, Ernest described the teacher's role as a *facilitator*. In the facilitator role, the teacher poses purposeful problems and allows students to engage in problem solving.

Because curricular materials are an important aspect of teaching, Ernest (1989a; 1989b) also stated that teachers' views on the teaching of mathematics incorporated how the teachers believe curricular materials should be used. According to Ernest, there are three modes in which teachers use curricula: (a) strictly adhering to tasks and sequence of content; (b) modifying the text as to enrich the material with extra problems and tasks; or (c) using teacher or school created curriculum materials.

Beliefs About the Learning of Mathematics

Previously, I described two views on the learning of mathematics, teacher-centered and student-centered. Ernest (1989a, 1989b) described that teachers' beliefs about the learning of mathematics depends on those teachers' beliefs about student learning. Teacher-centered instructional beliefs align with behaviorist perspectives on how students learn (Polly et al., 2013; Romberg & Carpenter, 1986; Woolley et al., 2004). Behaviorists believe that people learn through a stimulus and response process and look for an observable behavioral outcome at the conclusion of a lesson (Eisenberg, 1975). Learnings will repeat behaviors that are positively reinforced, and abandon behaviors that lead to negative consequences. In this way, learners are passive participants in a learning environment. Conversely, student-centered beliefs, which align with a constructivist viewpoint of learning, wherein the belief is that “learning is an active construction” (Ernest, 1989a, p. 251). In this viewpoint, students can construct their own understanding by actively engaging with the content (NCTM, 2014).

Determining Teachers’ Instructional Beliefs

Woolley et al. (2004) and O’Hanlon et al. (2015) developed surveys to aid teachers or PSTs, respectively, in creating an awareness of their own instructional beliefs. The authors’ purposes were to determine if the participants’ instructional beliefs aligned with a constructivist or traditional approach to teaching.

To develop their instructional beliefs survey, Woolley et al. (2004) began by posing open-ended interview questions to in-service teachers to determine the teachers’ philosophical views on teaching. By analyzing the teachers’ responses, seven common themes were established: learning environment, behavior management, curriculum, assessment, teaching strategies, student roles, and parent involvement. From these themes the authors developed the TBS to include questions that represented both constructivist and traditional teaching approaches within the context of the seven themes. A validation study confirmed that the TBS was a three-factor model which included Traditional Management, Constructivist Teaching, and Traditional Teaching as the embedded constructs.

Similarly, O’Hanlon et al. (2015) developed a beliefs survey for PSTs who participated in the Mathematics Research Experience for Pre-Service and In-Service Teachers (REU). The goals of the REU were to alter PSTs' beliefs about the teaching and learning of mathematics as well as beliefs about the nature of mathematics. During the REU experience, PSTs spent time engaging with mathematics through modeling and worthwhile tasks. The PSTs then were able to work with high school students in problem solving. To verify that PSTs experienced a change in beliefs about the nature of mathematics and the teaching and learning of mathematics, O'Hanlon et al. measured PSTs' beliefs at the beginning and end of the REU experience. O'Hanlon et al. based their model for the REU on Cuoco et al.'s (2010) habits of the mind (e.g., pattern analysis; creating and using representations; generalizing). O'Hanlon et al. chose categories, such as procedural versus conceptual understanding, mathematical authority and attitude, confidence, and efficacy, to include in their survey. To determine whether the REU program altered the beliefs of PSTs, O'Hanlon et al. calculated the effect size, using the group's average response pre- and post-REU. In their brief description of the results, O'Hanlon et al. described the changes in PSTs' beliefs in relation to discovery learning, problem solving, type of understanding (i.e., conceptual versus procedural), and the process of reasoning and proof. O'Hanlon et al. found that the REU experience had a positive effect size on PSTs' beliefs.

Instructional Practices

Instructional practices are ways in which teachers choose to engage students in the learning of academic content. One way to distinguish teaching practices is to classify them as indicative of a teacher-centered approach or a student-centered approach. A teacher-centered approach to teaching focuses on the teacher delivering a static collection of facts, procedures, and rules to a passive learner who receives or absorbs this fully formed knowledge (Romberg & Carpenter, 1986). In contrast, the NCTM (2014) described "research-informed practices" (p. 7)—which I refer to as student-centered practices—as those practices that focus on helping students construct their own knowledge through student explorations of concepts and lead those students to develop connections between new ideas and prior knowledge (Piaget, 1970; von Glasersfeld, 1989).

In addition, Skemp (1978) distinguished between the two types of understanding when describing teaching practices: relational and instrumental understanding. Relational understanding is "knowing both what to do and why" (p. 9) whereas instrumental understanding is "rules without reason" (p. 9). Student-centered practices align with relational thinking because in both cases students are provided with opportunities to construct their own knowledge and work to understand why they took the steps they did to complete a task. Teacher-centered practices align with instrumental understanding because in both cases the teacher is viewed as the owner of the knowledge and the one who imparts that knowledge to passive learners without reason.

Effective Teaching Practices in Mathematics

For the purpose of this study and because student-centered practices are regarded as an effective method for the teaching and learning of mathematics (NCTM, 2000, 2014; National Governors Association & Council of Chief State School Officers [NGA & CCSSO], 2010; National Research Council [NRC], 2001; Smith & Stein, 2018), I equated student-centered practices with effective teaching practices. In the following sections, I provide more context of what I mean by effective teaching practices in mathematics followed by relevant research to show the importance of using student-centered instruction.

National Standards and Frameworks. In *Principles and Standards for School Mathematics*, NCTM (2000) defined six principles (i.e., equity, curriculum, teaching, learning, assessment, and technology) that described high-quality mathematics. Within these principles, NCTM encouraged teachers to provide students with resources and teaching that allows them to actively construct new understanding from prior experiences and knowledge. In addition to the six principles, NCTM (2000) included content standards (e.g., number and operations, algebra, geometry) and process standards. The process standards focused on ways of "acquiring and using content knowledge" (p. 29). Problem solving, reasoning and proof, communication, connections, and representation were all included in the process standards that spanned grades K–12.

Whereas NCTM (2000) focused on student-centered learning from a student perspective, the NRC (2001) focused on the building of mathematical understanding through multiple strands of learning. The NCR described high-quality mathematics teaching not by what the teacher does independently, but by the interactions among teacher, students, and mathematics. Not only do effective teachers provide students with opportunities to learn but they also ensure that students recognize the importance and applicability of mathematical content. The NRC did not distinguish between the labels of student- or teacher-centered, instead their focus was on "the development of mathematical proficiency over time" (p. 315). The NRC defined *mathematical proficiency* as the interweaving of five strands of proficiency (i.e., adaptive reasoning, strategic competence, conceptual understanding, productive disposition, and procedural fluency), that aid in the successful learning of mathematics. For teachers to be effective, the NRC stated that they must utilize their knowledge of the content and the five strands to build worthwhile tasks, determine which student interactions to respond to, and monitor the engagement between students and mathematical tasks.

Similar to NCTM's (2000) focus on student-centered instruction, the Common Core State Standards for Mathematics (CCSS-M; NGA & CCSSO, 2010) maintained a focus on student-centered instruction because of the inclusion of the Standards for Mathematical Practices.

Teachers were to assist their students in mastering the mathematical practices in conjunction with content standards. The Standards for Mathematical Practices (SMPs) parallel the process standards (NCTM, 2000) mentioned earlier, both practices and process standards apply to all K–12 students. These practices included (a) making sense and persevering through problems, (b) constructing and critiquing arguments, (c) modeling with mathematics, (d) using appropriate tools, (e) attending to precision, (f) making use of structure, and (g) expressing regularity in repeated reasoning.

Following up on their 2000 standards and incorporating ideas expressed in the NRC (2001) and CCSS-M (2010), NCTM (2014) released eight teaching practices that promoted the "deep learning of mathematics" (p. 9). NCTM asserted that effective teaching occurs when students are engaged in worthwhile tasks that support the students' ability to make sense of mathematical ideas and reason with mathematics. Similar to the process standards (NCTM, 2000), the practices focused on the qualities of effective teaching that encouraged active participation of the students in their own construction of mathematical knowledge, as described in Chapter 1.

Lastly, similar to (and noted in) NCTM's (2000, 2014) standards and effective practices, the five practices (i.e., anticipating, monitoring, selecting, sequencing, connecting) to aid teachers in implementing effective mathematics teaching focused on student-centered instruction (Smith et al., 2019; Smith et al., 2020; Smith & Sherin, 2019; Smith & Stein, 2018). The five practices worked to help the teacher plan for and guide discussions during which students articulate their thinking and develop deep, connected understandings. Because anything can happen during unscripted discussions, the five practices were meant to provide mathematics teachers with a resource to employ "skillful improvisation" (Smith & Stein, 2018, p. 9) both before and during such discussion that would help teachers draw out connections (or disconnections) among concepts through the process of sharing and comparing student strategies.

In summary, although packaged in varying ways over time and by different authors, effective teaching practices are consistent in the sense that they all focus on helping students to actively construct their own understanding of mathematics. These recommendations also concurred that teachers have the responsibility to provide students with opportunities to interact with mathematics so that the student may become mathematically proficient. Though the authors and researchers previously described are similar, they do not all guide the teacher to effective practices in the same manner. For example, NCTM (2000), NRC (2001), and Smith and Stein (2018) all provide teachers with a framework in which they can function as mathematics teachers. These entities did not recommend a particular instructional strategy. Instead, their frameworks may be applied to different instructional practices. Specifically, NRC (2001) noted that the environment of the classroom can vary, and, therefore, it is not as important to inspect the type of instruction but instead focus on how the content, teacher, and students interact. Conversely, NCTM (2014), the five practices (Smith et al., 2019; Smith et al., 2020; Smith & Sherin, 2019; Smith & Stein, 2018), and the SMPs (NGA & CCSSO, 2010) provided teachers with specific strategies to help them teach mathematics effectively. NCTM (2014) and the five practices (Smith et al., 2019; Smith et al., 2020; Smith & Sherin, 2019; Smith & Stein, 2018) focused on teachers' instructional practices while the SMPs (NGA & CCSSO, 2010) focused on the development of qualities in students.

Effective Practices in Literature. The way a teacher interacts with students and tasks determines the level of effectiveness of the lesson. Webb et al. (2014) found that the use of student-centered practices related positively to student learning in mathematics in terms of higher test scores on curriculum-based, researcher-designed assessments (Jong et al., 2010), higher quality ratings of classroom instruction from both teachers and students (Toropova et al., 2019). The use of student-centered practices in a mathematics methods course for elementary level PSTs led to a decrease in mathematics anxiety and an increase of confidence in the capability to teach mathematics (Alsup, 2004).

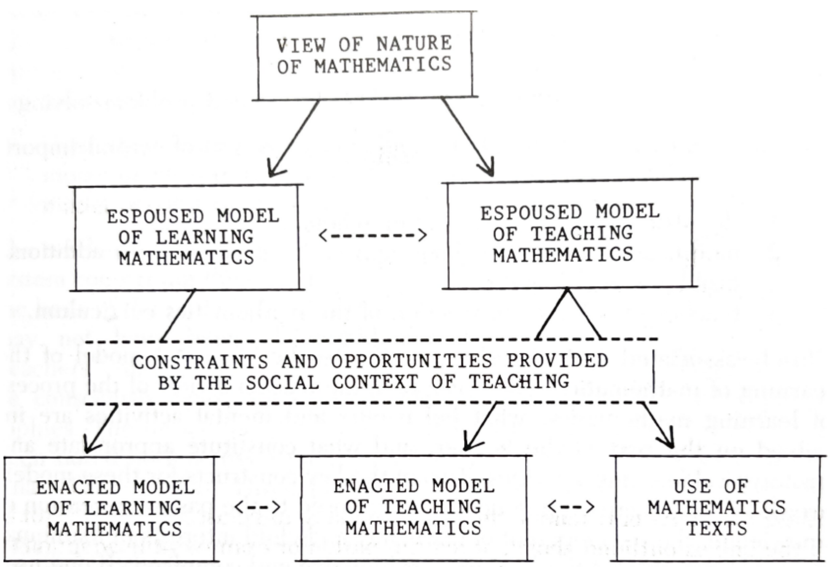
In a study to explore relationships among student participation, teaching practices, and student learning, Webb et al. (2014) observed six elementary school teachers during whole-class and small-group discussions during solving mathematical problems. Review of video and audio data from the observations were used to create codes from student participation, detail of student explanations, and student engagement. Students' previous years standardized test score was used as a pretest and a researcher-designed assessment focused on problem solving and mathematical reasoning was used as a post-test. The authors found that students who gave detailed explanations on the researcher-designed assessment were those who also had high achievement scores ($r = .30$, $p < .01$) and students who engaged with other students' ideas also had high achievement scores ($r = .44$, $p < .001$). Webb et al. also found that the quality of teachers' interactions—focused on encouraging student mathematical thinking explanation—with student ideas predicted the level of engagement the students had. For example, when a teacher encouraged students to elaborate on how their idea related to another student's idea, the level of student engagement with other students increased. The authors concluded that it is imperative for teachers to explicitly make expectations known to students and support students' engagement by emphasizing participation and detailed descriptions of the student's thinking. In a similar study, Jong et al. (2010) found that teachers who implemented the process standards, as described by NCTM (2000), had students who scored higher on a district mathematics assessment than students of teachers who did not implement the process standards. Jong et al. recruited 22 teachers who were in their first or second year of teaching and observed the teachers to determine the level on which each teacher implemented the process standards. Student assessments consisted of a curriculum-based, researcher-designed, pre-test and a district-developed test as a post-test measure. The correlation between the observed implementation of process standards and students' post-tests was .56 ($p < .05$), showing a significant and positive relationship between teaching practices and students mathematics learning. In contrast to Webb et al. (2014) and Jong et al. (2010) who focused on student achievement scores, Alsup (2004) focused on the level of students' mathematics anxiety. Alsup (2004) was the instructor for multiple courses in mathematics for PSTs, in which he taught two sections using a student-centered constructivist instructional model and one section using a teacher-centered approach. An Abbreviated Version of the Mathematics Anxiety Rating Scale (AMARS; Alexander & Martray, 1989) was used to measure PSTs' mathematics anxiety concerning anxiety linked to taking a test in mathematics, anxiety about mathematical content, and anxiety about taking a mathematics course. Teaching self-efficacy was measured using the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI; Enochs et al., 2000), in which students were assessed for their confidence in their ability to teach mathematics in a way that would positively affect students' learning. Both instruments were administered at the beginning and end of the courses. Alsup (2004) found that PSTs in the student-centered classes saw a decrease in their mathematics anxiety level and experienced an increase in their mathematics teaching self-efficacy.

Relationships Between Instructional Beliefs and Teaching Practices

Ernest (1989a) described how teachers "espoused models for teaching and learning mathematics" (p. 252) are then influenced by the social restrictions and opportunities within the context of education and the school, construct teachers' teaching practices. The author developed a model (see Figure 5) showing the relationships between teachers' beliefs and the influence on their practices. The downward arrows in the figure indicate how the teacher's views on the nature of mathematics influences their "espoused model" (i.e., beliefs) of the teaching and learning of mathematics, which in turn, mediated by social influences, produces teachers enacted practices and use of curricular materials.

Figure 5

Relationship Between Beliefs and Their Influence on Practice



Note. From "The Impact of Beliefs on the Teaching of Mathematics," by P. Ernest, 1989, *Mathematics Teaching: The State of Art,* p. 252. Copyright 1989 by P. Ernest.

Many researchers have studied the conflict that sometimes occurs between teachers' instructional beliefs and their enacted teaching practices (Raymond, 1997; Yurekli et al., 2020). And others have examined how the interplay between beliefs and practices can affect student learning (Peterson et al., 1989; Polly et al., 2013). In the following sections, I will review literature on these relationships between beliefs and practices and research focused on building instructional beliefs instruments.

Though teachers may be aware of their beliefs about instruction, it is possible that those beliefs are not present in teachers' enacted practices. Raymond (1997) investigated the relationship between teachers' beliefs and mathematics teaching practices. In a case study of Joanna, a first-year elementary school teacher, Raymond conducted six interviews and five observations in a 10-month time period, along with the collection of lesson artifacts. The focus of these instruments was on Joanna's beliefs about the nature of mathematics, mathematics teaching, and mathematics learning. Raymond documented Joanna's enacted teaching practices from classroom observations and interviews. Although Raymond had interviewed other first- and second-year teachers, Joanna became the focus of the study because of the inconsistency between her beliefs and practices. Raymond concluded that Joanna's beliefs about the teaching and learning of mathematics were primarily non-traditional—students learning primarily through working with others in solving problems—and her beliefs about mathematics were traditional—mathematics is a collection of facts, rules, and skills that are fixed and predictable—and her enacted teaching practices were primarily traditional—students are passive learners and teacher instructs mostly from a textbook. The author concluded that Joanna's inconsistency between beliefs and practices most likely stemmed from lack of time, concerns over standardized testing, students' behavior, and lack of resources. In addition, the author noted the different influences on teaching practices, though Joanna's instructional practices did not align with her beliefs about the teaching and learning of mathematics, they did align with her beliefs about mathematics content. Though the author cautioned readers to not make claims on this fact because Joanna failed to distinguish which beliefs (mathematics pedagogy or mathematics content) she stated were influential in her enacted instructional practices.

Yurekli et al. (2020) examined the alignment between instructional beliefs concerning "Explicit Attention to Concepts (EAC)" (p. 236)—teachers' ability to make connections between representations, concepts, and solution strategies—and self-reported instructional practices focused on making connections. Participants, 248 Grade 4–8 teachers, were asked to rank, in order of importance, their beliefs concerning practices known to develop students' conceptual understanding of mathematics and to report how often (on a scale from never to almost daily) they implement each practice. The authors found that although teachers reported the importance of students making connections and generalizations based on the connections, teachers reported not using the practice frequently. This finding led the authors to conclude that although teachers believed that it was important to make connections, they found it difficult to implement the practice in their classrooms (p. 242).

Building on previous research findings, Polly et al. (2013) explored the relationships among teachers' beliefs toward mathematics teaching, instructional practices, and student learning outcomes. Using a teachers' belief survey (Swan, 2007), a questionnaire to assess self-reported instructional practices (Swan, 2007), Mathematical Knowledge for Teaching Test (Hill et al., 2005), and end-of-unit assessment for student achievement data, Polly et al. (2013) found that teachers' beliefs aligned with their teaching practices. For example, teachers who believed in transmission-oriented teaching (e.g., teacher-centered, behaviorist) were more likely to use teacher-centered practices. The authors also found a negative association between students gain in mathematics achievement score, as measured by teacher-created, end-of-unit test, and the use of teacher-centered activities ($t(30) = -2.15$, $p = .04$). Lastly, students with teachers who had discovery-oriented teaching beliefs experienced larger gains than students taught by transmission-oriented teachers ($t(30) = -3.44$, $p = .002$). Unlike Raymond (1997) who found a relationship between teachers' beliefs concerning the nature of mathematics and their enacted instructional practices, Polly et al. (2013) found that how a teacher views mathematics as a subject, is not influential on the teachers' instructional practices.

Comparing teachers whose beliefs align with more cognitively based perspective (CB) and less cognitively based perspective (LCB), Peterson et al. (1989) explored the interplay between the beliefs and practices of 39 first grade teachers. Using data from a beliefs survey, the authors determined the teachers' beliefs on cognitively based perspective to learning. The authors conducted interviews to better understand teachers' beliefs on the "conceptions of mathematics, curriculum, the roles of teachers and students, and their goals for instruction in addition and subtraction" (p. 6). In reference to the relationships between beliefs and practices, Peterson et al. found that CB teachers introduced addition and subtraction to their students differently than the LCB teachers. CB teachers introduced the concept by building on student strategies, despite being informal in nature, allowed students to utilize manipulatives to represent quantities within the problem, and viewed the learner as an active participant in their acquisition of mathematical understanding. On the other hand, LCB teachers tended to introduce written symbols and equations to students, using manipulatives to represent the values and symbols, and viewed their role as teacher to be one of organizer and presenter of mathematical knowledge while viewing the students as passive listeners.

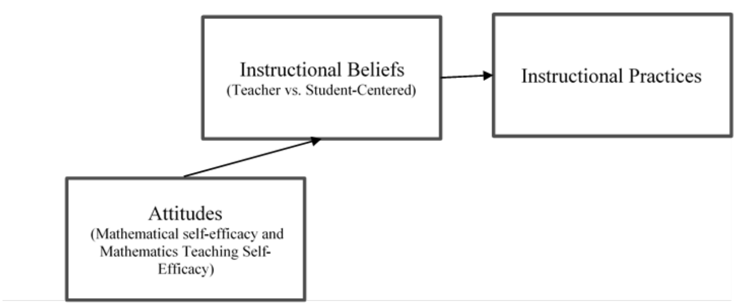
Connections Among Self-Efficacy, Instructional Beliefs and Practices

As previously noted, I based my theoretical understanding of the relationships among beliefs and practices on Wilkins' (2008) theoretical model (see Figure 1). In his model, Wilkins focused on affective factors—beliefs and attitudes about mathematics and mathematics teaching—that influenced teachers' instructional decisions. Because of the effect that beliefs about one's self-efficacies toward mathematics and mathematics teaching have on instructional decisions, I think it is appropriate to consider self-efficacy as part of those teacher attitudes. Thus, I have zoomed in on part of Wilkins model[1] and added detail (see

Figure **6**) to illustrate how I hypothesized that self-efficacies are mediated through instructional beliefs (e.g., toward teacher-centered vs. student-centered practices) to lead to the enacted instructional practices. In addition, the exclusion of content knowledge and teacher backgrounds characteristics from Wilkins' (2008) original model is purposeful as the primary focus of this study involves a more detailed analysis of the relationships between teacher self-efficacies, instructional beliefs, and practices.

Figure 6

Model Relating Teachers' Attitudes, Instructional Beliefs, and Instructional Practices



Note. Adapted from Wilkins, J. L. M. (2008). The relationship among elementary teachers' content knowledge, attitudes, beliefs, and practices. *Journal of Mathematics Teacher Education, 11*, 139–164. (https://doi.org/10.1007/s10857-007-9068-2)

[1] Though Wilkins found a statistical correlation between attitudes and instructional practices that were not mediated through instructional beliefs, I have omitted the link between attitudes and instructional practices in this enlarged section of the model to focus on the part of the model that needs more exploration.

Linking Self-Efficacy to Enacted Practices

Teachers' use of specific instructional practices has been linked to their self-efficacy. Allinder (1994) examined 437 special education teachers' self-efficacy and instructional practices, in which the Teacher Efficacy Scale (Gibson & Dembo, 1984) and the Teacher Characteristics Scale (Fuchs et al., 1992) were used. The Teacher Efficacy Scale (Gibson & Dembo, 1984) was a 30-item survey in which teachers reported their level of agreement on statements such as "when a student did not remember information I gave in a previous lesson, I would know how to increase his/her retention in the next lesson." The Teacher Characteristics Scale (Fuchs et al., 1992) was used to ascertain teachers' self-reported instructional practices including: instructional experimentation, routine, organization and planning, and instructional innovation. Allinder (1994) found that teachers with high TSE were more likely to report being experimental in their choice of instructional practices ($r = .34$, $p < .001$) as well as organized and business-like in their interactions with students ($r = .37$, $p < .001$). Lastly, the author noted that teachers who reported being organized in their instruction were also those who reported being more experimental ($r = .43$, $p < .001$).

Depaepe and König (2018) also found connections between self-efficacy and self-reported instructional practices, and they sought to determine whether general pedagogical knowledge (GPK) was also a related covariate. Depaepe and König (2018) collected survey data from 342 PSTs who had completed a five-month teaching internship using a version of the Teacher Education Development Study in Mathematics (TEDS-M, König et al., 2011). This version of the TEDS-M was used to assess PSTs' GPK with four subscales: classroom structure, motivation and classroom management, student diversity, and classroom assessment. The TEDS-M was a paper-and-pencil open-ended questionnaire in which PSTs analyzed situations and lessons. A revised version of the TSES (Pfitzner-Eden et al., 2014) was used to measure PSTs' self-efficacy for instructional strategies, classroom management, and student engagement. Lastly, instructional practices were self-reported by the PSTs through a survey in which they stated their level of agreement with items focused on cognitive activation of students (e.g., "When working on challenging tasks, I allowed students to apply their own strategies"), classroom management (e.g., "I always knew exactly what happened in the classroom"), and learning support (e.g., "I showed interest in every student's learning"). Interestingly, the authors did not report finding a significant relationship between GPK and self-efficacy, as they had predicted, and there were weak associations between GPK and instructional practices. Similar to Allinder (1994), Depaepe and König (2018) found strong correlations between self-efficacy and instructional practices (varying between $r = .22$ and $r = .35$ for the different subscales, $p < .001$). Depaepe and König concluded that the relationship among the three subscales of the TSES and GPK was insignificant and the assumption that greater knowledge is associated with high self-efficacy (see Fives, 2003) was not true for the PSTs in the study.

2020 Pandemic

As mentioned previously, my study took place during the pandemic of 2020. As I prepared to collect data, I searched out relevant literature concerning teaching during a pandemic. Once I completed data collection and analysis, new relevant literature existed. The following sections focus on literature pre-data collection and post-data collection.

Pre-Data Collection Relevant Literature

The sudden closing of schools and hurried transition to digital learning in March of 2020 due to the spread of COVID-19, put teachers, students, and parents in circumstances that none had experienced before. Some school districts had previously established digital learning days, confident that the students had the resources to learn at home, while others scrambled to determine how to digitally teach students who may lack necessary resources for online learning. As the 2019–2020 school year came to an end, students, parents, and teachers did not know what fall 2020 would be like, but most knew that there was a possibility that digital learning would play a major role in the new school year. Burgess and Sievertsen (2020) stated that the immediate transition to learning and working from home was a "not only massive shock to parents' productivity, but also to children's social life and learning" (para 1). Often parents were left in a position in which they were managing their children as the child navigated through the online lesson platform (Clarkson et al., 2020).

To aid in the transition to online learning, McCarthy and Wolfe (2020) reported on one school that engaged the parents in the student-teacher relationship, making sure the parents had a voice in how online learning would happen. The newfound relationship in the context of online learning resulted in a school wide organization scheme that included class schedules and templates for lessons. Likewise, Vu et al. (2020) described one Kindergarten teacher's experience as she moved from face-to-face instruction to online teaching. Using a blended model, including asynchronous and synchronous components, the teacher continued providing instruction to her students. Throughout the week, students completed activities and worksheets and watched videos. The teacher met with the students once a week, at a consistent designated time, via an online conferencing program (Zoom). During the online meeting, the teacher answered questions and provided additional information to students. Though Vu et al. did not collect data to evaluate the teacher's effectiveness, the teacher reported that the quality of student work was the same during online learning as it had been during in-person learning. Vu et al.'s description of the transition to online learning was similar to stories shared with me by other teachers and parents. I, as a parent and teacher, experienced the transition from face-to-face learning to online learning, and I also perceived that the quality of student work was similar to student work I had observed previously, during in-person classes.

Though there was a quick transition from face-to-face online learning, Wolfe and McCarthy (2020) explained that many of the strategies teachers used in the classroom could be leveraged for an online learning environment with specific adjustments and understandings. Providing teachers in a private K–6 school with professional development workshops based on the Community of Inquiry (COI; Garrison et al., 2000) framework, the authors aided teachers in the transition to online teaching. The COI framework described elements of an educational experience, which included a cognitive presence, social presence, and teacher presence (see Garrison et al., 2000, for detail). The professional development workshop consisted of a two-hour session with teachers and school administrators followed by weekly or bi-weekly meetings with school administrators, who then conducted weekly meetings with teachers (Wolfe & McCarthy, 2020). Throughout the two-hour session, teachers were asked to revisit how they created the elements of the COI framework in their regular classroom and then use the information to adjust their approach for an online platform. Many teachers reported having high anxiety transitioning to online teaching, but after the professional development they felt less anxiety. The authors also noted that teachers who had “strong teaching pedagogy” (p. 146) during face-to-face instruction transitioned to online teaching more quickly.

Post-Data Collection Relevant Literature

Because more literature was being published that focused on elements of my study, the following sections discuss literature pertaining to the pandemics influences on teachers’ self-efficacy and their teaching practices.

Pandemic and Teachers' Self-Efficacy

Weißenfels et al. (2022) used survey data to determine the influence of COVID-19 on teachers' general teacher self-efficacy (GTSE) as it related to use of digital media and attitudes toward e-learning. The authors acquired data prior to the start of the pandemic and compared that data to data gathered after schools began to shut down because of the COVID-19 virus. Using surveys to measure GTSE (TSES; Tschannen-Moran & Woolfolk Hoy, 2001), the authors found that teachers GTSE for digital media increased between the two data points for most teachers. Weißenfels et al. stated that this increase in GTSE could be the result of having mastery experiences (Bandura, 1986) with teaching online and using digital media.

Conversely, Pressley and Ha (2021) and Pellerone (2021), also using the TSES (Tschannen-Moran & Woolfolk Hoy, 2001), found teachers' GTSE decreased during phases of remote teaching. Pressley and Ha (2021) found that GTSE was lower for those who taught primarily online classes during the 2020–2021 school year than those who taught hybrid or all in-person classes. Furthermore, though contradictory to the authors' hypothesis, there was no significant difference between teachers who had received a teacher-of-the-year accolade and those who had not received such recognition. The authors suggested that having prior success in the classroom did not influence teachers' general self-efficacy. Investigating differences among kindergarten, primary, and middle school teachers in Italy, Pellerone (2021) found that only the middle school teachers experienced a decrease in the GTSE beliefs. Pellerone stated that middle school teachers—who displayed greater aptitude for conflict resolution and executive leadership—needed to "cope with the emotions and conflicts activated by daily contact with adolescent problems" (p. 508). Furthermore, Pressley and Ha (2021) noted that compared to previous studies, conducted prior to the pandemic, teachers had lower self-efficacy during fall 2020.

Pandemic and Teachers' Instructional Practices

For many teachers, the pandemic brought on many challenges to their teaching, specifically to their instructional practices as they transitioned to remote teaching (Aldon et al., 2021; Barlovits et al., 2021; Echeverría et al., 2022). Teachers often attempted to employ instructional practices they used in the physical classroom into an online learning environment (Aldon et al., 2021) or planned a student-centered lesson (Echeverría et al., 2022) but failed to achieve what they envisioned. Similarly, in a survey sent to mathematics teachers in Spain and Germany, Barlovits et al. (2021) found that teachers (i.e., the primarily secondary teachers who responded) from both countries struggled to use the teaching practices they previously used in the physical classroom, often resulting in a focus on mathematical content that teachers perceived students could acquire through independent learning.

Though teaching in the remote or online environment was difficult, post-pandemic return to the physical classroom brought about changes to teachers' instructional practices (Barlovits et al., 2021; Martin et al., 2021). Barlovits et al. (2021) reported that about half the teacher participants changed their teaching practices because of the pandemic, while a quarter of the teachers stated they had resumed their pre-pandemic instructional practices. Many of the teachers who changed noted the primary change was the use of digital tools to support their students during lessons. Moreover, Martin et al. (2021) found that teachers reassessed their pre-pandemic teaching, noting a frequent comment made by teachers was that "teaching virtually reminded them that students need and require opportunities to engage in hands-on manipulatives" (p. 346).

Conclusion

The self-efficacy of teachers, qualities of effective teachers, effective teaching practices, and teachers' instructional beliefs are all components of my study. These components function as guides for me as I attempted to determine the possible relationships among teachers' beliefs and practices. Self-efficacy has been shown to influence a person's decision-making process even as it is situation specific. Effective teachers are those who provide rich mathematical learning opportunities, through their choice of effective instructional practices. Gaining a more detailed description of effective teachers' self-efficacy, both mathematical and mathematics teaching, in conjunction with their instructional beliefs and enacted instructional practices, can better inform mathematics educators about factors affecting the teaching and learning of mathematics. Lastly, the 2020 pandemic and the resulting transition to online teaching and learning have put teachers in a position that was jarringly different from the standard school context and complicated in many ways. Because COVID-19 is not likely to be the last pandemic (Future Agenda, 2020; World Health Organization, 2020) or other disruptive event (Arcanjo, 2018; Rigaud et al., 2018) to affect teachers, schools, and students, it is imperative to understand the pandemic's role in teachers' self-efficacy and practices.

CHAPTER III: METHODOLOGY

To explore the relationships among mathematical self-efficacy, mathematics teaching self-efficacy, teachers' instructional beliefs, and teachers' use of effective teaching practices for K–6 mathematics teachers, I employed a case study design. I used surveys, observations, and interviews to describe teachers' self-efficacy, instructional beliefs, and enacted teaching practices. As a result, differing levels of self-efficacies were compared to the use of effective teaching practices by teachers of mathematics. To make comparisons among the cases in my study, I used the theory of self-efficacy (Bandura, 1986) as a guide for my data collection and analysis.

Researcher Positionality

I am a former teacher of mathematics in Grades 4–8. I hold constructivist beliefs about the nature of learning. This means that I believe that if I explain a concept to a student, they will not necessarily understand or make sense of that concept. Instead, the student must struggle with the concept, exploring it from multiple perspectives to construct their own understanding. Likewise, I believe that students need opportunities to engage with mathematical concepts as they relate them to their perceptions of the real world. To obtain this type of engagement, teachers must employ effective teaching practices, such as those discussed earlier in this paper.

My experience as a mathematics teacher provides me with insights into the challenges of helping students learn mathematical ideas, relationships, and representations. During my 15 years of public K–12 education, I became curious about how mathematics could be taught in a way that led to better student understanding and retention.

In my current roles as researcher and mathematics instructional coach, I hear from former colleagues and see firsthand that, even with support, teachers struggle to consistently implement student-centered practices promoted by NCTM (2000, 2014) and Common Core Standards for Mathematics (CCSS-M; NGA & CCSSO, 2010). I am intrigued by the fact that teachers hesitate to give up direct control of all classroom events (i.e., engage in student-centered practices rather than teacher-centered practices), even if they cite the value in having students engage in problem solving and critical thinking. Observing these everyday struggles and apparent inconsistencies has led me to wonder about the role of teacher's self-efficacy.

I recognize that my own mathematical self-efficacy may be relatively low. I vividly recall my own struggles in advanced high school mathematics classes when I believed that the only way to succeed in mathematics courses was to memorize facts and algorithms. From those experiences, I have lingering doubts about my ability to successfully learn the concepts and procedures required in a post-secondary-level mathematics course (e.g., calculus). In contrast, I have a strong mathematics teaching self-efficacy. I believe I am capable of effectively engaging others in mathematical learning in a way that challenges students and helps them make sense of concepts and procedures, as well as successfully apply reasoning and problem-solving skills to novel situations.

Study Design

I used a case study design to develop an "in-depth description and analysis" (Merriam & Tisdell, 2016, p. 37) of participating teachers who were labeled as effective. More specifically, I used a descriptive multi-case study (Yin, 2003). Using this approach allowed me to analyze several sources of data within each case and make comparisons across the two cases (Yin, 2003). A characteristic of multi-case studies is the focus on multiple specific units of analysis (Yin, 2003; i.e., an individual teacher who has been labeled as an effective teacher of mathematics). In addition, by using a descriptive case study approach each portrayal of a teacher includes descriptions of their self-efficacies—mathematical and mathematics teaching—their instructional beliefs, and their use of effective practices in a real-life classroom environment (Merriam & Tisdell, 2016; Yin, 2003).

Participants

To recruit effective teachers of mathematics for my study, I requested recommendations from experts[2] in mathematics education. Through this process a total of six teachers were recommended, four high school level, one middle level, and one primary level. I contacted the building administration for each of the potential participants by email to request verification that the teacher was considered an effective teacher of mathematics and requesting permission to contact the teacher. Of the five administrators contacted, two teachers were from the same district, four provided verification and permission to contact the teachers. The administrator for one of the six teachers agreed the teacher was an effective teacher of mathematics but due to the pandemic they were not allowing any research to occur within the district.

At this point, I contacted the five remaining potential participants requesting their involvement in my research study. All five teachers, three high school, one middle level, and one primary level, agreed to participate in my study. Due to the influence of the 2020 pandemic, I was unable to obtain district-level permission to conduct research for the schools of the three high school teachers, so they were not included in the final study. The remaining two teachers, Kathy, a Kindergarten teacher, and Frances, a 5th and 6th grade mathematics teacher, (pseudonyms) taught in the same school district located in the Midwestern United States and became my two cases for the final study. I then provided Teacher Consent Forms (see Appendix A) for my participants to complete prior to collecting data.

[2] Experts are those who had extensive knowledge and expertise in mathematics education and effective teaching practices, such as experienced mathematics education researchers and teacher educators.

Instruments

Teachers' choice of enacted teaching practices is dependent on several factors (Boaler & Greeno, 2000; Brown, 2009, Gresalfi & Cobb, 2011; Wilkins, 2008). For my study, I focused on the affective factors of mathematical self-efficacy, mathematics teaching self-efficacy, and instructional beliefs. Table 1 lists my research questions—focused on the relationships among affective factors and the connections to the implementation of effective mathematical teaching practices espoused by NCTM (2014)—and which instruments I used to address each question. The key factors of interest for each question are identified in bold to help the reader know that the listed instruments had the potential to address the key factors. Because I employed a descriptive multi-case study design (Yin, 2003), much of my descriptive data stemmed from semi-structured interviews and classroom observations. I used surveys to determine teachers' self-efficacy and instructional beliefs. In subsequent sections, I describe each of the instruments and its purpose in more detail.

Table 1

Data Collection Instrument Per Research Question

Research Question	Data Collection Instrument
1. How are **mathematical self-efficacy** and **mathematics teaching self-efficacy** related in mathematics teachers who have been labeled as effective?	• Mathematics Teaching and Mathematics Self-Efficacy survey (MTMSE; Kahle, 2008) • Pre- and post-lesson interviews • End-of-study interview

Table Continues

Table Continued

2. How do teachers' **instructional beliefs** relate to their **mathematical self-efficacy, mathematics teaching self-efficacy,** and their use of **effective teaching practices**?	• Mathematics Teaching and Mathematics Self-Efficacy survey (MTMSE; Kahle, 2008) • Pre- and post-lesson interviews • End-of-study interview • Classroom observations • Lesson plans and artifacts • Instructional Beliefs survey (Part 5 of MTMSE; Kahle, 2008; O'Hanlon et al., 2015)
3. How did the spring 2020 coronavirus school shutdown, the immediate transition to remote learning, and the atypical fall 2020 semester, influence effective mathematics **teachers' self-efficacies and instructional practices** during the spring 2021 semester?	• Classroom observations • Lesson plans and artifacts • Pre- and post-lesson interviews • End-of-study interviews

Self-Efficacies

To measure both mathematical and mathematics teaching self-efficacy, I used the Mathematics Teaching and Mathematics Self-Efficacy (MTMSE) survey (Kahle, 2008; see Appendices B, C, and D Parts 1, 2, 3, 4). This six-part survey was developed by Kahle from previously published surveys and sources including the Mathematics Self-Efficacy Survey – Revised (MSES-R; Kranzler & Pajares, 1997), the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI; Enochs et al., 2000), and NCTM Process Standards (2000). Kahle (2008) showed that the MTMSE is a reliable instrument with an alpha level of 0.942 and that it produced positive results for both face and content validity. The MTMSE originally intended for teachers who taught Grades 3 through 6, Kahle (personal communication, June 8, 2020) verified that the survey was suitable for elementary and middle school teachers but needed modification to accurately depict the mathematical and mathematics teaching self-efficacies of high school teachers. The original MTMSE contained language that was specific to elementary school teachers, so three versions were created, one for each grade level—elementary, middle school, high school. The MTMSE for middle school teachers was altered slightly for language (e.g., elementary was replaced by middle school) but questions remained the same in regard to content (see Appendix C). To alter the MTMSE for high school teachers (see Appendix D), I referred to the current state standards (CCSS-M, 2010) and determined which content strands high school teachers are expected to teach, then incorporated questions from those strands into the survey. To do so, I used items from SAT practice tests (College Board, 2016). To establish content validity for the MTMSE for high school teachers, the modified survey was sent to experts[3] in high school mathematics teaching and content, including the original author of the MTMSE (Kahle, 2008). Each expert provided feedback concerning the applicability of content within the survey to those who teach high school mathematics and agreed that the survey was appropriate for high school mathematics teachers. In the following sections, I provide more details on the self-efficacy surveys.

Mathematical Self-Efficacy Survey. Mathematical self-efficacy are those beliefs individuals hold concerning their perceived ability to be successful when solving mathematical tasks (Kahle, 2008). I measured mathematical self-efficacy using the MTMSE (Kahle, 2008) survey. Part 1 and Part 3 of the survey focused on teachers' beliefs about their capabilities with regard to a variety of mathematical tasks. In the survey, teachers were asked to rate their level of confidence in their capability to complete each task on a 1 (not confident at all)-to-6 (completely confident) scale. The teachers were not required to complete the tasks. For example, teachers were asked to assess their capability to solve this task: "On a map, 7/8 inch represents 200 miles. How far apart are two towns whose distance apart on the map is 3 1/2 inches?" (The full sets of

[3] Experts were those who had advanced degrees in mathematics education and who had taught or were currently teaching secondary mathematics.

tasks from Part 1 and Part 3 are available in Appendices B, C, & D.) To build a thorough case description of each teacher, I also posed questions in the pre-lesson and end-of-study interviews focused on teachers' beliefs in their capability of solving mathematical tasks related to the content they taught during each lesson. For example, in the pre-lesson interview teachers were asked, "how confident are you with respect to the mathematical content you are teaching today?" and "Do you believe you are a person who is good at math?" in the end-of-study interview.

Mathematics Teaching Self-Efficacy Survey. Mathematics teaching self-efficacy are those beliefs that teachers hold about their capability to teach mathematics (Bandura, 1997; Enochs et al., 2000; Kahle, 2008; Swars, 2005). These beliefs were determined using Part 2 and Part 4 of the MTMSE (Kahle, 2008) survey. The survey was used to determine several aspects of teachers' belief systems in accordance with the teaching of mathematics. In Part 2 of the MTMSE teachers were asked to rate, using a Likert scale of 1 (strongly disagree)–6 (strongly agree), their agreement with statements such as "I will generally teach mathematics ineffectively" or "I will typically be able to answer students' questions" (see Appendices B, C, & D, part 2). In Part 4 of the survey, teachers were asked to rate (on a scale of 1–6, from low to high) how confident they felt about teaching specific content (e.g., fractions, decimals, shapes; see Appendices B, C, & D, part 4). During each pre- and post-lesson interview, I asked teachers how confident they were with teaching the content in the current lesson so that I gathered information on their self-efficacy (see Table 2 and Table 11). In addition, during the end-of-study interview, I referenced back to the survey responses to request more information concerning teachers' experiences with the teaching and learning of mathematics that potentially influenced their mathematics teaching self-efficacy (see Appendix F).

Instructional Practice Beliefs

Instructional beliefs are views teachers hold about the best teaching practices to use when teaching students and are mediated by teachers' attitude about mathematics and mathematics teaching (Wilkins, 2008). Additionally, instructional beliefs often are categorized as either student-centered (e.g., reform oriented, learner centered) or teacher-centered (e.g., traditional; Woolley et al., 2004). To determine whether teachers' instructional beliefs align with student- or teacher-centered instructional practices, I used items selected from O'Hanlon et al.'s (2015) Teaching and Learning Mathematics Beliefs survey and included these items as part 5 (see Appendices B, C, & D) to the Mathematical Self-Efficacy, Mathematics Teaching Self-Efficacy, and Instructional Beliefs Survey for Elementary teachers. In developing the survey, O'Hanlon et al. designed items to reflect NCTM's (2000) process standards recommendations which are consistent with values described in the CCSS-M standards (NGA & CCSSO, 2010). O'Hanlon et al. (2015) developed items that were from different perspectives (personal learning, student learning, and teaching). I selected items that focused on teachers' beliefs on how students learn best (e.g., student learning) and teachers' beliefs about how best to teach mathematics. I chose to omit questions focused on personal learning because those survey items focused more on how one prefers to learn mathematics, which was not a focus of my study. Although O'Hanlon et al. (2015) used a five-point scale, I used a six-point Likert scale, 1 (strongly disagree)–6 (strongly agree), to maintain consistency with other parts of the survey. The beliefs survey was applicable to all K–12 mathematics teachers as the items were designed to elicit teachers' instructional beliefs about the content they teach at their own grade level.

To gather additional information concerning the teachers' instructional beliefs, I posed questions related to teachers' instructional beliefs during the pre- and post-lesson and end-of-study interviews. In the pre-lesson interview, I posed questions that focused on how the teacher would deliver the mathematical content to students during the observations. These questions provided insight into beliefs that might influence how the teachers chose to teach mathematical content (see Table 2). In addition, in the post-lesson interview, I posed questions which focused on reflecting on their teaching and whether what was observed was typical in their classroom (see Table 11). Lastly, during the end-of-study interview, some questions focused specifically on the teacher's instructional beliefs and how they might have changed throughout their teaching careers (see Appendix F).

Enacted Teaching Practices

According to Fennema and Franke (1992), teachers make decisions that affect instruction when they are engaged in lesson planning. During this process, teachers determine what to teach, how to teach the material, how to organize student participation, and what accommodations or adaptations are needed throughout the lesson. To obtain a complete understanding of teachers' enacted practices, I collected lesson plans and materials for each of the observed lessons[4]. Prior to the observations, the teacher and I conducted a short meeting to discuss the lesson plan and the teachers' confidence ratings for the content and the teaching of the content (see Table 2). In addition, I requested information about what adjustments the teachers made due to COVID restrictions and regulations enforced by the district.

[4] Due to COVID restrictions and availability of teachers, I observed Kathy a total of five times and Frances a total of six times.

Table 2

Pre-Lesson Interview Questions

Question	Purpose	Research Question
1. What do you hope the students will understand by the end of the lesson?	Use of goal setting practice	2
2. Give me a brief description of what the lesson will look like? a. Follow up: What will you be doing and what will the students be doing? b. How will the materials (e.g., handouts, examples, technology) be used?	Use of teacher-centered or student-centered practices	2
3. Where did you get the idea for this lesson?	Link to prior experiences or other sources of self-efficacy information	2
4. What made you decide that this would be a good lesson to use?	Link to instructional beliefs	2
5. Would you want to use this lesson on a day that your principal or other district evaluator was observing? Why or why not?	Background context for the cases and link to other sources of self-efficacy information Context for how instructional beliefs fit within school expectations	1 2
6. Have you taught this lesson before or observed another teacher implement this lesson? If so, did you find it was a successful lesson? Why or why not?	Link to prior experiences or other sources of self-efficacy information	2
7. What indicators will help you know whether the lesson is going well?	Use of goal setting practice	2
8. How confident are you with respect to the subject matter you are teaching today? What aspects are you comfortable with and what aspects are you concerned about?	Background context for the cases and link to other sources of mathematical self-efficacy information Triangulation of data	1 2 3

Table Continues

Table Continued

9. How confident are you with respect to your ability to enact this lesson today? What aspects are you comfortable with and what aspects are you concerned about?	Background context for the cases and link to other sources of mathematics teaching self-efficacy information. Triangulation of data	1 2 3
10. How would this lesson be different if you were teaching in the absence of COVID restrictions?	Link to instructional beliefs Self-efficacy related to pandemic	2 3

Implementation of Lesson

I followed Gleason et al.'s (2017) recommendation to conduct multiple observations of each teaching over a period of time so that I could establish a comprehensive view of each teacher's practice. I conducted five observations of Kathy and six observations of Frances within the spring of 2021 (i.e., roughly a 3-month time span). The difference in the number of observations was a result of scheduling limitations. I chose to use the *Mathematics Classroom Observation Protocol for Practices* ($MCOP^2$; Gleason et al., 2015) because of its focus on the student-centered practices in the SMPs (NGA & CCSSO, 2010), even though the $MCOP^2$ is not directly aligned with the eight mathematical teaching practices (MTPs; NCTM, 2014). At the time of my study, there was no instrument available that directly measured NCTM's (2014) teaching practices. In Table 3, I show how the $MCOP^2$ (Gleason et al., 2015) observation guide aligns with teaching implementation of the SMPs (NGA & CCSSO, 2010).

Table 3

Relationship between the MCOP² and NGA & CCSSO's (2010) SMPs

	Standards for Mathematical Practice							
MCOP² item	Make sense of problems and persevere in solving them	Reason abstractly and quantitatively	Construct viable arguments and critique the reasoning of others	Model with mathematics	Use appropriate tools strategically	Attend to precision	Look for and make use of structure	Look for and express regularity in repeated reasoning
1	X						X	X
2	X				X			
3	X							
4	X		X					X
5	X	X	X		X			
6							X	X
7		X		X				
8							X	X
9	X							
10			X			X		
11	X							
12			X					
13			X					
14	X							
15			X					
16	X							

Note. From "Mathematics Classroom Observation Protocol for Practices (MCOP²): A Validation Study," by J. Gleason, S. Livers, and J. Zelkowski, 2017, *Investigations in Mathematics Learning, 9*(3), p. 14 (http://doi.org/10.1080/19477503.2017.1308697). Copyright 2017 by Research Council on Mathematics Learning.

I video and audio recorded each observation, taking necessary precautions to omit students' faces. I completed the MCOP² (Gleason et al., 2015; see Appendix E) during the observation and subsequent viewing of videos. The MCOP² was developed to examine aspects of teacher facilitation and student engagement for the purpose of teaching mathematics for conceptual understanding and was grounded in the *Instruction as Interaction* framework (Cohen et al., 2003). Teacher facilitation refers to the role of the teacher to provide lesson structure and guidance through problem solving and mathematical discourse. Student engagement refers to students fulfilling the role as active learner within the classroom environment (Gleason et al., 2015). Table 4 shows the alignment of each item and whether the description displayed teacher facilitation, student engagement, or both.

Table 4

Item Subscales of the MCOP²

Item	Student Engagement	Teacher Facilitation
1	X	
2	X	
3	X	
4	X	X
5	X	
6		X
7		X
8		X
9		X
10		X
11		X
12	X	
13	X	X
14	X	
15	X	
16		X

Note. From "Mathematics Classroom Observation Protocol for Practices (MCOP²): A Validation Study," by J. Gleason, S. Livers, and J. Zelkowski, 2017, *Investigations in Mathematics Learning, 9*(3), p. 14 (http://doi.org/10.1080/19477503.2017.1308697). Copyright 2017 by Research Council on Mathematics Learning.

My original intent was to use the MCOP² to determine the use of the eight mathematical teaching practices (MTPs; NCTM, 2014) but when I began observing and completing the MCOP², I found instances in which the MCOP² was not catching all components of the MTPs. Furthermore, I found that some MTPs were not being recorded when using the MCOP². Thus, I created a careful alignment between the MCOP^2 and the MTPs based on the classroom observations and from descriptions in *Principles to Actions* (P2A; NCTM, 2014)[5].

To begin my alignment, I read through the MCOP^2 Descriptors Manual (Gleason et al., 2015) and P2A (NCTM, 2014) descriptions of the eight MTPs to verify the alignment among the MCOP^2 items and the MTP descriptions and teacher actions in P2A. I started by documenting examples of teacher actions, key verbs, and descriptions of each of the MTPs. Then, I distilled the descriptions down to several main observable components. I performed the same process using Gleason et al.'s (2015) published descriptors manual, which the authors designed to maintain the reliability of the MCOP^2 among various observers. The descriptors manual contains evidence and support from mathematics education literature the authors used to illustrate each of the 16 observation items, including teacher and student actions. Table 5 shows an example of teacher actions when implementing the MTP, use and connect mathematical representations, as provided by the authors of P2A (NCTM, 2014), and excerpts from the MCOP^2 Descriptors Manual (Gleason et al., 2015). In the table, I have bolded the key words and phrases that supported the alignment between the MTPs and MCOP^2. As seen in Table 5, the bolded terms for both P2A and MCOP^2 focused on teachers showing or providing opportunities for students to make or be introduced to multiple representations of mathematical concepts. Throughout the process of verifying alignment among the statements, I regularly consulted with another expert to maintain content validation.

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[5] After I completed my work, I learned that Zelkowski et al. (2020) published a book chapter describing an "initial crosswalk" (Zelkowski, personal communication, November 28, 2022) between the MCOP^2 and the MTPs. In Appendix G, I provide details concerning the alignment between Zelkowski et al. (2020) and my own work aligning the MCOP^2 to the MTPs (NCTM, 2014).

Table 5

Alignment Among MCOP² General Descriptions and NCTM (2014) Teacher Actions for Use and Connect Mathematical Representations

MCOP² General Descriptions	NCTM (2014) Teacher Action Description
Common for teacher to use **various representations** (models, drawings, graphs, concrete materials, manipulatives, graphing calculators, compass & protractor, i.e., tools for the mathematics classroom) to focus students' thinking on and develop their conceptions of a mathematical concept.	Introducing forms of representations that can be useful to students.
The students manipulated or **generated two or more representations** to represent the same concept, and the **connections across various representations.**	Asking students to make math drawings or use other visual supports to explain and justify their thinking.

After the process of determining alignment between the MTPs and the MCOP², I found the MCOP² was not able to provide the necessary information concerning teachers' enactment of all MTPs. For example, the MCOP² did not contain an item that covered the first NCTM practice, establish mathematics goals to focus learning. According to P2A, when establishing goals that focus learning, teachers should create goals that focus on "the mathematics students are learning as a result of instruction" (NCTM, 2014, p. 16) and know how the current goal fits in with the progression of mathematics learning.

Because of the discrepancies between P2A and MCOP², I returned to the literature to attempt to locate an observation protocol that was better aligned with the eight MTPs in P2A. Some of the observation protocols I considered included: Mathematical Quality of Instruction (Hill, 2014); The Mathematics Scan (Berry et al., 2010); Inside the Classroom (Horizon Research, 2002); and Reformed Teaching Observation Protocol (Sawada & Piburn, 2000). For each protocol, I tried, unsuccessfully, to match vocabulary in descriptions of the MTP to the vocabulary in each of the protocols. Unable to locate an appropriate protocol, I worked to modify the MCOP^2 to align with the MTP. The following paragraphs detail the process I completed to develop the Enhanced-Mathematics Classroom Observation Protocol for Practice (E- MCOP^2).

To begin the process of developing the E- MCOP^2, I returned to the alignment notes I had previously developed when first choosing the MCOP^2 as an observation protocol, as described in the previous section. This process resulted in multiple MCOP^2 items that were well aligned with some the MTPs whereas other MTPs had no corresponding MCOP^2 items. Further, there were some MCOP^2 items that aligned with multiple MTPs (e.g., MCOP^2 item 11 incorporated aspects of both facilitate meaningful mathematical discourse and pose purposeful questioning). Table 6 shows my final alignment between the MTPs (NCTM, 2014) and MCOP^2 item descriptions (Gleason et al., 2015), I included excerpts from both documents to highlight the correspondences. To maintain reliability throughout the process of aligning MTPs with MCOP^2, I asked another mathematics educator to review my descriptions to determine whether they accurately reflect NCTM's language and intent.

Table 6

Alignment Among Descriptors for MCOP² Items and MTPs

Description of MTPs (NCTM, 2014)	MCOP² Item Focus (Gleason et al., 2017)
Establish mathematics goals to focus learning	No aligned items
Implement tasks that promote reasoning and problem solving	
"Motivating students' learning of mathematics through opportunities for exploration and solving problems that build on and extend their current mathematical understanding." (p. 24)	Item 1: The focus of this item is on the role of exploration, investigation, and problem solving in the teaching process. Students should have the opportunity to determine their own solution strategies.
"[Tasks that allow] students to engage in active inquiry and exploration or encourage students to use procedures in ways that are meaningfully connected with concepts or understanding." (p. 19)	Item 8: The focus of this item is the opportunity given to students to contextualize and/or decontextualize mathematical tasks in order to explore and make sense of mathematical structure or to use repeated reasoning to form generalizations.
"Encouraging students to use varied approaches and strategies to make sense of and solve tasks." (p. 24)	Item 9: The focus of this item is for students to have the opportunity to look for multiple paths to a solution or engage with tasks that have multiple solutions.
Use and connect mathematical representations	
"Designing ways to elicit and assess students' abilities to use representations meaningfully to solve problems." (p. 29)	Item 2: This focus of this item is for students to have the opportunity to use various representations to represent their thinking and the development of their mathematical understanding.
Facilitate meaningful mathematical discourse	
"Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations" (p. 35)	Item 13: The focus of this item is that students are expected to effectively communicate (i.e., listen, question, and critique) with peers during mathematical discussions.
"Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches." (p. 35)	Item 15: The focus of this item is that students were involved in the communication of their ideas and are active participants.

Table Continues

Table Continued

Pose purposeful questions	
"Purposeful questions allow teachers to discern what students know and adapt lessons to meet varied levels of understanding, help students make important mathematical connections, and support students in posing their own questions." (pp. 35–36)	Item 11: the focus of this item is that the teacher's talk promotes students to think, reason, argue, and critique. The teacher's questions and statements focus on allowing students to reason and/or discuss the mathematical concept.
Build procedural fluency from conceptual understanding	
"This approach supports students in developing the ability to understand and explain their use of procedures, choose flexibly among methods and strategies to solve contextual and mathematical problems, and produce accurate answers efficiently." (p. 46)	Item 4: The item focuses on the opportunities provided to students to choose the appropriateness of a strategy and consider the efficiency of that strategy. Students were provided time to assess mathematical strategies by comparing and contrasting different strategies, discussing the generalizability of the strategies, or discussing the efficiency of strategies.
"Asking students to discuss and explain why the procedures that they are using work to solve particular problems." (p. 47)	Item 6: The item focuses on students having the opportunity to discuss and understand both the how and why, creating a relational/conceptual understanding.
"Connecting student-generated strategies and methods to more efficient procedures as appropriate." (p. 48)	Item 16: The item focuses on the teacher using student comments and questions to enhance student conceptual understanding of mathematical concepts.
Support productive struggle in learning mathematics	
"Giving student time to struggle with tasks and asking questions that scaffold students' thinking without stepping in to do the work for them." (p. 52)	Item 5: The focus of the item is the teacher providing a space for students to persevere in problem solving.
"Teachers must accept that struggle is important to students' learning of mathematics, convey this message to students, and provide time for them to try to work through their uncertainties." (p. 50)	Item 14: The item focus is on the amount of wait time provided to the students by the teacher. The wait time must be appropriate for the task. In addition, students were required to reason, make sense, and articulate thoughtful responses.

Table Continues

Table Continued

Elicit and use evidence of student thinking	
"Listen to learn whether students are precise in using concept-based language in discussing their reasoning." (p. 53)	Item 10: The focus of this item is the teacher's promotion of precision of mathematical language to better assess student thinking and uses the information to direct or redirect the lesson.

Note. From "Mathematics Classroom Observation Protocol for Practices (MCOP^2): A Validation Study," by J. Gleason, S. Livers, and J. Zelkowski, 2017, *Investigations in Mathematics Learning, 9*(3), (http://doi.org/10.1080/19477503.2017.1308697) and *Principles to Action: Ensuring Mathematical Success for All,* by NCTM, 2014

Because the $MCOP^2$ protocol was unable to completely describe the use of all eight MTPs (NCTM, 2014) and those that did align did not completely encompass all that each MTP entails, I began the process of identifying the gaps. This process involved reading the descriptions and exemplars provided in P2A (NCTM, 2014) and developing a matrix to organize what might be seen in the classroom when each MTP was implemented.

Table 7 shows an example of one MTP in which no $MCOP^2$ items aligned. The first column provides a point in time when the teacher action might occur, the middle column provides the description of the teacher action as described in P2A, and the last column shows a description of an observable action one would see a teacher perform from the E-$MCOP^2$. Once again, I often verified the alignment, descriptions, and where the action might be seen in the classroom with other mathematics education researchers to maintain reliability of the developing framework. For MTPs that aligned with $MCOP^2$ items, I completed a similar process but did not provide new E-$MCOP^2$ items for teacher actions that were previously covered by $MCOP^2$ items. In Table 8, I provide the moment within an observation one might notice the teacher action or the $MCOP^2$ item, support from P2A (NCTM, 2014), and the E-$MCOP^2$ item which describe the observable action one might notice for the MTP, elicit and use evidence of student thinking.

Table 7

Establishing Mathematical Goals to Focus Learning E-MCOP² Items With NCTM Support Descriptions

Data Collection Opportunity	E-MCOP² Teacher Action Description	NCTM (2014) Reference
Pre-lesson interview	Teacher discussed the mathematical goals for the lesson and related those goals to a learning progression.	NCTM referenced Fosnot and Jacob (2010) and Ma (2010), "situating learning goals with the mathematical landscape supports opportunities to build explicit connections so that students see how ideas build on and relate to one another and come to view mathematics as a coherent and connected discipline." (p. 13)
Pre-lesson/ Observation	Teacher related current mathematical topic/concept to an already discussed mathematical topic/concept.	"The goals that guide instruction, however, should not just be a reiteration of a standard statement or clustered but should be more specifically linked to the current classroom curriculum and student learning needs." (pp. 12–13)
Observation	Teacher refocused students to align with mathematical goal or reassessed mathematical goal based on evidence of student thinking noticed during the lesson.	"Teachers need to establish clear and detailed goals that indicate what mathematics students are learning, and they need to use these goals to guide decision making during instruction." (p. 16)
Post-lesson interview	Teacher reflected on the mathematical goal and whether the students were successful. If goal was not successful, teacher explained using evidence of student thinking.	"Teachers need to establish clear and detailed goals that indicate what mathematics students are learning, and they need to use these goals to guide decision making." (p. 16)

Table 8

Elicit and Use Evidence of Student Thinking: E-MCOP² Descriptions, NCTM References, and Data Collection Opportunities

Data Collection Opportunity	E-MCOP² Teacher Action Description	NCTM (2014) Reference
Pre-lesson interview	Teacher planned for times within the lesson in which they planned to stop and assess student understanding.	"Preparation of each lesson needs to include intentional and systematic plans to elicit evidence that will provide 'a constant stream of information about how student learning is evolving toward the desired goal' (Heritage, 2008, p. 6)." (p. 53)
Observation	Teacher prompted students to explain their understanding of a concept as a mode of assessment.	Using high-level tasks paired with requiring students to "explain, represent, and justify mathematical understanding and skills provide stronger evidence of their understanding for ongoing assessment and instructional decisions." (p. 54)
Observation	Teacher listened during group/partner discussions and there is evidence of using gathered student thinking in a whole class discussion.	"It is important to identify and address potential learning gaps and misconceptions when it matters most to students, which is during instruction, before errors or faulty reasoning becomes consolidated and more difficult to remediate." (p. 53)

Table Continues

Table Continued

MCOP² Item 10: The focus of this item is the teacher's promotion of precision of mathematical language to better assess students thinking and uses the information to direct or redirect the lesson.	The teacher "attends to precision" with respect to communication during the lesson. The students also "attend to precision" in communication, or the teacher guides students to modify or adapt non-precise communication to improve precision.	Teachers are, "Interpreting student thinking to assess mathematical understanding, reasoning, and methods" (p. 56). Students are, "Revealing their mathematical understanding, reasoning, and methods in written work and classroom discourse." (p. 56)
Post-lesson interview	Teacher reflected on evidence of student thinking and discussed how the thinking might influence the next lesson or set of lessons.	"Reflecting on evidence of student learning to inform the planning of the next instructional steps." (p. 56)

After I determined how to fill the gaps found during the alignment process between the MTPs and the MCOP² for teacher actions, I created a draft of the E-MCOP². Table 9 displays the MTPs along with the aligned MCOP² items and the number of items added to fill the gaps between the MTPs and the MCOP².

Table 9

Number of Added Items to MCOP² to Form the E-MCOP²

Mathematical Teaching Practice	Aligned MCOP²	Number of Items Added to Fill Alignment Gaps
Establish mathematics goals to focus learning	N/A	4
Implement tasks that promote reasoning and problem solving	Items 1, 8, 9	2
Use and connect mathematical representations	Item 2	0
Facilitate meaningful mathematical discourse	Items 13, 15	0
Pose purposeful questions	Item 11	4

Table Continues

Table Continued

Build procedural fluency from conceptual understanding	Items 4, 6, 16	0
Support productive struggle in learning mathematics	Items 5, 14	3
Elicit and use evidence of student thinking	Item 10	4

To enhance the clarity of the E-MCOP^2 for myself and other potential users, I curated examples from my observation data to illustrate the teacher actions described for each of the MTPs. When I was unable to locate an exemplar in my own data, I searched for examples in mathematics education literature. Often, these examples of MTPs came from Smith et al.'s (2020) *The 5 Practices: Successfully Orchestrating Mathematics Discussions in Your Elementary Classroom.* I chose this resource because Smith et al. specifically referenced the MTPs (NCTM, 2014) in their writing. To add clarity to the examples, I developed descriptions of exactly how each example matched key indicators in the MTP descriptions. Table 10 shows the E-MCOP^2, including the MTP description, an exemplar, and explanation of how the exemplar illustrated the relevant MTP. The purpose of the E-MCOP^2 was to verify the use of the MTPs rather than rating the teacher on some kind of performance scale. Thus, rather than a rating scale, I designed the data-collection instrument to include additional columns for notes. Specifically, I added a column for recording whether the actions associated with the MTP description were observed. Also, I included a column in which the observer would record evidence to support their claim. Keeping with my previous alignment and description practices, I verified the accuracy of my examples, descriptions, and information with others knowledgeable of the MTPs (NCTM, 2014).

Table 10

Elicit and Use Evidence of Student Thinking: E-MCOP² With Exemplars and Justification for Connections to MTP

Data Collection Opportunity or MCOP² Item	MTP or MCOP² Description	Example (*Italicized* from alternate source, bolded background detail)	Connection between example and MTP
Pre-lesson interview	Teacher plans times during the lesson at which to stop and assess student understanding.	*While anticipating student solution strategies, Ms. Stastny stated that she would ask the following questions to assess the student thinking if they used a corresponding solution strategy: a) "What do your diagrams show about how much lasagna Tanesha and David ate?" b) "Who got the most lasagna and how do you know?"* (Smith et al., 2020, p. 62)	*In this example, a teacher described their plan to stop the lesson to pose specific questions (i.e., a) "What do your diagrams show about how much lasagna Tanesha and David ate?" b) "Who got the most lasagna and how do you know?") to assess student understanding in connection with their strategy choices.*
Observation	Teacher prompts students to explain their understanding of a concept.	Students were working in pairs. Teacher asked, "You agreed with the first two strategies, did you get through all the strategies? Were there any that you disagreed with?" Student responds by stating that the last one (incorrect strategy picked by the teacher) was questionable. Teacher asked why the strategy was questionable and recorded the students' explanation on a clipboard. (5_S1 Ob)	In the example, the teacher's questions were used to prompt students to explain their identification of an incorrect solution. This explanation exposed student understanding of the concept underlying the task.

Table Continues

Table Continued

Observation	Teacher notices student thinking, through student verbalization or students' written work that is evidence of understanding, then uses the evidence in a whole class discussion.	**During a whole-class discussion focused on student strategies for solving 7 x 1/3, one strategy that all students agreed with had a solution of 2 1/3 and one that they disagree with has the solution of 7/21.** Teacher asked, "How does 2 1/3 compare with 7/21?" **The teacher posed this question because the teacher noticed that not all students were able to articulate how the fractions were different during pair discussions. Then, whole-class discussion focused on how these two values were different. (5_S1 Ob)**	In the example, the teacher gathered student thinking data (students did not recognize how 2 1/3 compared to 7/21) during paired discussion time to use during whole class (whole class discussion focused on how these two values were different).
MCOP^2 Item 10: The focus of this item is the teacher's promotion of precision of mathematical language to better assess student thinking and uses the information to direct or redirect the lesson.	The teacher "attends to precision" in communication during the lesson. The students also "attend to precision" while communication, or in response to teacher encouragement to modify or adapt imprecise communication.	**During a discussion on students' knowledge of geometric terms associated with triangles, teacher stated,** "…as we are talking today and as we are thinking of things and sharing ideas, we can really be purposeful in using that vocab to describe what we're talking about." **Student stated that lines are congruent, teacher asked,** "Are lines the only thing that can be congruent?" **The purpose of the question was to help students refine their vocabulary and work towards more general description of the terms.** (5_S2 Ob)	In the example, the teacher attended to precision by prompting students to be purposeful in their use of vocabulary and by probing a student's understanding of the term congruent (Are lines the only thing that can be congruent?) so that a broader definition of congruence could be discussed and established.

Table Continues

Table Continued

Post-lesson interview	Teacher reflects on evidence of student thinking and discussed how the thinking might influence the next lesson or set of lessons.	**The teacher was describing the game students played in which students spun a spinner to see the values that would be added to 10 (dime), "**One child noticed that at the bottom of each column in the graph there were the coins from the spinner with the written amount. It really highlighted his observation skills and he indicated that was his strategy for adding money on the spinner. He matched the spinner amount to the picture and number in the graph. Several children began counting on from 10. That isn't always typical behavior from this group. I have really worked hard at modeling counting on strategies using the ten-frame in particular. I try to have students share their strategy for solving a problem with the class. It provides a richness of options for the other students, and it lets me know what their thinking process is so I can target what learning needs they might have." (K_S1 Post-Ob)	In the example, the teacher reflected on evidence that students were using the counting on from ten strategy, which was not typical for the set of students. Teacher noted that the class had been working on counting using a ten-frame (based on the ability for students to subitize). Further, the teacher stated, "and it lets me know what their thinking process is so I can target what learning needs they might have." Both the teacher's reflection on prior goals and their justification for listening to student strategies illustrate that the teacher engages in a cycle of collecting evidence of student thinking and setting goals based on that evidence.

Note. From *The 5 Practices in Practice: Successfully Orchestrating Mathematics Discussions in your Elementary Classroom,* by M. Smith, V. Bill, and M. Gamoran Sherin, 2020.

Post-Lesson Interview. The purpose of the post-lesson interview was to provide teachers an opportunity to reflect on the lesson in a way that allowed them to talk about their instructional beliefs, their self-efficacy, the extent to which they believed students reached the teacher's goals, and their own teaching practices as they perceived them. Table 11 contains the interview questions I posed to the teachers, along with the purpose for asking each one, and anticipated connections to my research questions. Due to COVID concerns and scheduling conflicts, the post-lesson interview differed between the teachers. I was able to meet with Frances immediately following each of her observed classes for a post-lesson, audio-recorded interview. Frances' interviews lasted between 8 and 15 minutes. However, Kathy had a much tighter schedule, and we were not able to complete post-lesson interviews in-person. Instead, I sent Kathy the interview questions in an online survey format for her to complete at her earliest convenience. Kathy consistently returned the survey to me within 24 hours after each observation.

Table 11

Post-Lesson Interview Questions

Question	Purpose	Research Question
1. How do you feel about how the lesson went today? a. What went well in the lesson today? b. What did not go well in the lesson today? c. What might you change the time you teach this lesson?	Link to self-efficacy Link to use of effective teaching practices	2
2. How do you know if most of the students met the learning goal for today's lesson?	Link to effective teaching practices	2

Table Continues

Table Continued

3. Which events occurred today that you would say are pretty typical of your daily classroom routines?	Link to effective teaching practices Instructional beliefs	2
4. Which events occurred today that you would say really showed your beliefs in action, even if you don't use these practices regularly?	Instructional beliefs	2

End-of-Study Interview

I completed observations for both teachers by the end of the school year in the spring 2021. Over the subsequent summer and fall, I analyzed data from the observations, including the pre- and post-lesson interviews, to select video clips to use in the end-of-study interview. The purpose of each video was to serve as the stimulus in a stimulated-recall interview. I chose classroom excerpts that I believed exemplified some of the NCTM's (2014) MTPs. In alignment with a case study approach, I was interested in creating detailed descriptions of teachers' interpretations of their own lived experiences. Thus, sharing those classroom moments by video during the interviews gave teachers the opportunity to describe their teacher actions through their own lenses.

To select the excerpts, I watched each observation, noting excerpts where the teacher employed one or more MTP (NCTM, 2014). From those excerpts, I chose four excerpts to view with Kathy and six excerpts to view with Frances. The difference in the number of excerpts was a result of Frances teaching both 5^{th} and 6^{th} grade versus Kathy who only taught kindergarten.

In addition, I analyzed the surveys, self-efficacies and instructional beliefs, to determine the need for any follow-up or clarification questions in the end-of-study interview. Lastly, I noted any notes in my researcher's journal that needed clarification or more explanation to use during the last interview with the teachers. In spring 2022, I conducted the semi-structured end-of-study interviews, in which the teachers could recount their past experiences concerning the teaching and learning of mathematics along with reflection on the pre-selected video excerpts.

For the end-of-study interview, I adapted Kahle's (2008) interview protocol (see Appendix F). I made modifications because of the difference in grades the teacher taught between Kahle's original study and my own and because I was investigating *enacted* teaching practices rather than teacher-reported teaching practices, as Kahle had done. I asked teachers questions focused on their past experiences with learning mathematics, their beliefs concerning the teaching of mathematics, and their capability to teach mathematics. Some examples are, "Generally, how confident are you when teaching mathematics?" and "What was your experience learning mathematics?" (see Appendix F part 1 for anticipated questions and prompts).

The stimulated recall portion (see Appendix F part 2 for questions) of the interview consisted of viewing portions of each teacher's observations in which the teacher was using a MTP (NCTM, 2014). In selecting these clips in advance of the interview, I verified specific points in which the teacher used one or more of the MTPs. I chose excerpts that contained a variety of MTPs and where the MTPs were more evident. Aligning with the case study design, my goal was to create detailed descriptions of how teachers viewed their own teaching practices and how they viewed the MTPs in action. In addition, it was important to have teachers reflect on their thoughts and beliefs while watching the videos, so that, I would have a clearer picture of experiences that might influence their decision making at the time, therefore obtaining a better understanding of their self-efficacy. Lastly, this process allowed the teachers to review and provide feedback, clarification, and as a way to verify my understanding of their beliefs and practices (i.e., member check; Creswell, 2014).

Prior to watching the video excerpts, I provided the teacher with a brief description of the eight MTPs (NCTM, 2014), provided them with time to read through each of the practice descriptions, and verified that they understood each MTP. Prior to watching each excerpt, I informed the teachers of the MTPs I noticed in the video segment and requested they discuss their thoughts and reaction to the use of the practices. I also requested the teacher to note whether they noticed any other MTPs I had not noted. Teachers were also requested to recall why they chose that particular practice and how confident they felt at that moment in time about their capability to implement the practice skillfully, as well as their expectation for whether the practice would lead to the desired learning outcome for students.

Researcher's Journal

I kept a researcher's journal in which I recorded my thoughts and conjectures throughout the study. The purpose of keeping such a journal was to record any notions I had throughout the process to aid in building each of the teachers' stories. During observations, I used my journal in conjunction with the $MCOP^2$. For example, during an observation, I had the observation protocol visible to note evidence that might have occurred outside the video or audio recordings. This included points within observation in which I thought significant events occurred that would merit rewatching at a later date. At the end of each observation, I recorded my initial thoughts on the general use of student-centered practices and evidence of beliefs I notice in the teacher's communication with their students.

During the interviews, I used my journal to note points of interest that I wanted the teacher to expand on but did not want to interrupt them during their current thought, story, or description of an event. At the conclusion of each interview, I noted any experiences or statements teachers shared that could help me better understand their self-efficacies and instructional beliefs.

2020 Pandemic

To collect information from the teachers concerning teaching during a pandemic, I posed questions to each teacher during the pre-lesson interview and the end-of-study interview (see Table 12). The purpose of these questions was to gather information concerning each teacher's perception of the pandemic's influence on their self-efficacy and enacted teaching practices. In addition to these questions, I recorded any statements teachers provided spontaneously during other parts of the interviews or during the observations. Lastly, I took note of classroom environmental adjustments teachers made in response to COVID restrictions.

Table 12

Questions on Possible Influences on Teachers During the Pandemic

	Question
Pre-Lesson Interview	How would this lesson be different if you were teaching in the absence of COVID restrictions?
	What is your level of confidence this year compared to past years in the teaching of mathematics?
End-of-Study Interview	During the transition to online learning in the spring of 2020, how did you feel about the lessons you were providing for your students?
	Overall, how confident do you feel about the level of understanding your students acquired during the transition to online learning in the spring? In the fall?
	Do you believe that your experience teaching during the pandemic has influenced your ability to teach mathematics effectively?

Analysis

In the following sections, I describe the methods used to analyze each of the instruments described in the previous sections. Because of the low number of participants, no statistical calculations were performed on any data.

Self-Efficacies

In the following sections, I describe the methods used to analyze data from surveys and interviews to determine the self-efficacies of my participants.

Survey. I analyzed the Mathematical Self-Efficacy, Mathematics Teaching Self-Efficacy, and Instructional Beliefs Survey (see Appendices B, C, D Parts 1, 2, 3, 4) to determine whether the teachers' MSE and MTSE were high or low. I used Parts 1 and 3 to determine teachers' MSE and Parts 2 and 4 to determine teachers' MTSE. Because survey questions were both positively and negatively worded, in order to determine consistencies in participant responses, I reversed the Likert scale scores for the negatively worded statements, so that they would be comparable to the corresponding positively worded statements. Then I tallied the number of 4, 5, or 6 rating for each statement. For Parts 1 and 3, I classified teachers as having high MSE if they provided at least 17 responses (out of a total of 31 questions for both parts) as a rating of 4, 5, or 6. For Parts 2 and 4, I classified teachers as having a high MTSE if they provided at least 15 responses (out of 26 total questions) as a rating of 4, 5, or 6. Because of the small sample size, I did not follow the recommendations of Kahle (2008), who performed statistical analysis with a large participant pool. I based the cutoff scores on that teachers would be more likely to choose more statements with ratings of 4, 5, or 6.

Interviews. In addition to the self-efficacy surveys, I culled the transcripts from each pre- and post-lesson interview and the end-of-study interview looking for statements that related to the teachers' self-efficacy. These statements included descriptions of their confidence or capability of carrying out mathematical thinking or computational tasks (e.g., MSE) or the teaching of mathematics (e.g., MTSE). A statement could be as short as a phrase or as long as several sentences as long as they were uttered within the same talk turn and focused on the same idea. If the teacher stated they were confident in their capability to successfully solve a particular task or computation, I recorded the statement as evidence of high MSE. Alternatively, if the teacher spoke negatively about their capability or stated they were not confident in solving a particular task, I recorded the statement as evidence of low MSE. Similarly, if a teacher stated they were capable of teaching a mathematical concept or were confident in their teaching, I labeled the statement as evidence of high MTSE. If the teacher stated they were not confident about their capability to or were incapable of successfully teaching a mathematical concept. The statement was labeled as evidence of low MTSE. Once all statements were identified and categorized, I tallied the number of statements within each of the categories.

Instructional Beliefs

In the following sections, I describe the methods used to analyze data from surveys and interviews to determine the instructional beliefs of my participants.

Survey. I used the Mathematical Self-Efficacy, Mathematics Teaching Self-Efficacy, and Instructional Beliefs Survey (see Appendices B, C, D Part 5) to assess teachers' instructional beliefs. Again, the survey statements were both positively and negatively worded statements. To determine consistencies in participant responses I reversed the Likert scale scores for the negatively worded statements, so that they would be comparable to the corresponding positively worded statements. Then, for each teacher, I tallied the number of responses corresponding to each Likert scale rating. I considered teachers to have instructional beliefs more aligned with student-centered practices if the majority of their responses were in the top half of the Likert scale (scores of 4, 5, or 6). If the majority of the responses were in the bottom half of the Likert scale (scores of 1, 2, or 3), I classified those teachers as having teacher-centered instructional beliefs. This analysis criterion—classifying beliefs according to which end of the scale (low or high) correspond to the majority of ratings—was not consistent with O'Hanlon et al.'s (2015) original recommendation for interpreting results from the instrument due to the small number of participants in the study.

Interviews. In addition to the survey, I examined transcripts from each pre- and post-lesson interview and end-of-study interview for statements concerning teachers' instructional beliefs. I looked for statements in which the teacher described their beliefs regarding how students engage in or learn mathematics. A statement could be as short as a phrase or as long as several sentences as long as they were uttered within the same turn talk and focused on the same idea. If the statement focused on a description of student learning that was consistent with descriptions of teaching practices in P2A (NCTM, 2014) or the SMPs (NGA & CCSSO, 2010), wherein students are viewed as active learners in the building of their mathematical understanding, I recorded the statement as evidence of student-centered instructional beliefs. Conversely, if the teacher described a teacher as the person in the classroom whose role was to disseminate knowledge to students or if the teacher indicated that the students' role was to be a passive recipient of knowledge (Skemp, 1978), I recorded the statement as evidence of teacher-centered instructional beliefs. Once all statements were identified and categorized, I tallied the number of students within each of the categories. If the teacher shared more statements focused on a student-centered belief, they were considered to have student-centered instructional beliefs.

On the other hand, if a greater number of the teacher's statements focused more on the teacher disseminating knowledge to students, they were classified as having teacher-centered beliefs.

Enacted Teaching Practices

In the following sections, I describe the methods used to analyze data from observation protocols to determine the use of effective teaching practices of my participants.

MCOP^2. I classified each teacher's performance on the use of SMPs by their score in the relevant sections of the MCOP^2 (Gleason et al., 2015; see Appendix E). As previously noted, teacher facilitation was defined as "the role of the teacher as one who provides structure for the lesson and guides the problem-solving process and classroom discourse" (Gleason et al., 2015, p. 3) and student engagement refers to students' engagement in the learning process. According to its design, I used the descriptors to rank teachers' practice for each item of performance on a scale of 0–3 (see Appendix E). The scale represents a continuum ranging from 0 (not engaging students in the SMP) to 3 (engaging students in the SMP). Zelkowski and Gleason (2016) designed the MCOP^2 as a protocol for "observing and scoring" (p. 129) PSTs during clinical experiences. In their validation study, Zelkowski and Gleason were working with PSTs, they did not anticipate that an average of 3 was a realistic mean to obtain. Thus, they equated a mean rating score of 1.0 with a failing score in the mid-60s and a mean rating score of 2.5 with a top score of 100 to establish a linear function for converting rating means to their typical grading scale.

I did not follow Zelkowski and Gleason's procedure for a couple of reasons. First, the ratings were discrete and ordinal, so it did not make sense to use a mean. Second, I am working with expert teachers, so, theoretically, any score or sum of scores should be attainable. Instead, I described teachers' practices by individual component of the scale or as a median for each subscale, or with a visual display to show the distribution of their ratings for each subscale (i.e., teacher facilitation and student engagement). I interpreted the median for a subscale using the scale ranging from 0–3, in which the top two levels (i.e., 2 and 3) indicated at least an intentional effort to use the SMPs (NGA & CCSSO, 2010). I inferred that a teacher is likely to use the SMPs (i.e., student-centered practices) on a regular basis if they earned a median rating of 2 for student engagement across all observations and a median rating of 2 for teacher facilitation across all observations.

E-MCOP^2. To determine teachers' use of MTPs (NCTM, 2014), I analyzed the videos using the E-MCOP^2. I focused on instances of MTPs that were not-well documented by the MCOP^2 alone. If I observed a MTP at all during a class observation, I counted that as evidence that the teacher used the practice during the lesson. I did not distinguish between more use or less use of the practice. I recorded the MTP as "highly evident" if more than half of the items within the MTP's category on the E-MCOP^2 were noticed during the observation. If the items in the MTP's category were noticed equaled or were less than half but more than zero items, I recorded the MTP as being "moderately evident." If none of the items within the MTP were noticed, I recorded the MTP as "not evident." I completed this process for each observation. Because my goal was to observe whether or not the MTP was used and not to what degree the teacher used the MTP, I did not collect numerical data for the E-MCOP^2. Thus, my analysis was simply creating a list of which practices were used by each teacher.

2020 Pandemic

To analyze teachers' responses to questions pertaining to the pandemic's influence on the teachers, I examined teachers' responses to questions specific to the pandemic (see Table 12) and transcripts for statements in which the teacher described alterations to their instructional practices or changes in self-efficacy. A statement could be as short as a phrase or as long as several sentences as long as they were uttered within the same talk turn and focused on the same idea. For each statement, I noted whether the teacher focused on changes in instructional practices or detailed an influence on self-efficacy.

CHAPTER IV: FINDINGS

This qualitative case study focused on the stories of two teachers labeled as effective teachers of mathematics. In this chapter, I describe each teacher in terms of the data collected throughout the study as it pertains to each of the beliefs and practices. Using the analyzed data from each teacher, I then revisit my research questions to provide descriptions of relationships among MSE, MTSE, instructional beliefs, and enacted teaching practices.

Before sharing my findings, I provide background information about each of the teachers. In each main findings section, I support my findings with results from individual surveys, observations, and interviews. I draw on these data sources to answer my first two research questions, as each of the questions focuses on the relationships among enacted teaching practices, teachers' instructional beliefs, and teachers' mathematical and mathematics teaching self-efficacies.

When I describe the teachers' use of MTPs (NCTM, 2014) and SMPs (NGA & CCSSO, 2010), I report both my interpretations of what I observed as well as the teachers' interpretations of their actions. The findings related to the teachers' instructional practices were drawn from multiple observations during which I used the MCOP^2 and E-MCOP^2 observation protocols. I also drew on data from the pre- and post-lesson interviews, to inform the reader about the teachers' perspective on their own practices. And I incorporated teachers' reflections on their practices from their end-of-study interview into my analysis of the teachers' practices.

Lastly, I conclude this chapter by reporting on data from interview questions focused on teaching during a pandemic, as I answer my last research question.

Participants

Both participants taught in the same mid-sized school of about 350–400 students in a mid-sized urban area in the midwestern United States. In this school, about 82.5% of the student body identified as white and 75.3% of the teachers identified as white. About 8% of the children were eligible for free and reduced lunch. Both were experienced teachers, yet they faced unprecedented constraints on their teaching, like most teachers in the US, during the time in which data was collected. Despite teaching in the same school, the teaching protocols differed at the two grade levels at which they taught, due to the ages of the children.

Kathy

Kathy was a kindergarten teacher who identified as Hispanic non-white. Kathy had earned her bachelor's and master's degrees in elementary education, with a concentration in early childhood education, and was qualified as a Nationally Board-Certified teacher. During her 30 years as an educator, Kathy had taught preschool, first grade, and sixth grade but stated that she enjoyed teaching kindergarten the most.

Kathy was identified as an effective mathematics teacher by her administrator. In addition, throughout observations and interviews I noticed several qualities of an effective teacher. Kathy engaged her students in research-based practices (Gay, 2012; Liang et al., 2012; Perry, 2007) and used worthwhile tasks with her students (Smith et al., 2020). Lastly, Kathy kept focus on her students' thinking throughout each lesson, verifying understanding through purposeful questions (NCTM, 2014).

Frances

At the time of the study, Frances identified as a white female who taught Grade 5 and 6 mathematics. Frances earned her bachelor's and master's degrees in middle level mathematics education and was enrolled in a mathematics education doctoral program. During her 15 years of teaching, Frances taught mathematics in grades 5–7. She had started her career as a Grade 7 teacher, but at the time of the study was working as a Grades 5 and 6 mathematics teacher.

Frances was recommended to be a participant by trusted mathematics educators and her administration. Through interactions with Frances, I noticed she exhibited qualities of effective teachers noted in previous research. Frances engaged herself in mathematics teaching research (Liang et al., 2012; Wang & Cai, 2007), as she was enrolled in a doctorate program. During observations she focused her teaching on student thinking and problem solving using highly cognitively demanding tasks (Smith et al., 2020). The tasks Frances presented to her students often allowed for exploration to begin then would allow students opportunities to generalize their strategies.

Mathematical Self-Efficacy

In the following section, I describe the results regarding Kathy's and Frances' mathematical self-efficacy. I used two methods for assessing each teacher's MSE. To determine their confidence in their ability to solve mathematical tasks, I used Parts 1 and 3 of the instrument shown in Appendix B for Kathy. I used Parts 1 and 3 of the instrument shown in Appendix C for Frances. The two parts of the surveys differed in that the parallel Part 1 components focused more on formal, school-based tasks, like those one might see in a mathematics textbook, whereas the parallel Part 3 components focused on practical, out-of-school tasks, like those one might encounter in their everyday life. A sample textbook question might be, "On a certain map, 7/8 inch represents 200 miles. How far apart are two towns whose distance apart on the map is 3 1/2 inches?" A sample everyday life questions might involve doubling a recipe that contains fractions.

In addition, I asked questions related to MSE during interviews. To locate statements concerning teachers' MSE, I read through each interview transcript and highlighted statements of interest. These statements often included discussion of the teacher's experiences learning mathematics or comments they made about specific mathematical content. Statements could be as short as a phrase or several sentences in length as long as they focused on the same idea or thought. Kathy's and Frances' surveys indicated that both teachers had high MSEs, but interview data provided evidence that their MSEs varied in response to specific situations.

Kathy's Mathematical Self-Efficacy

The following sections describe data from a survey, interviews, and observations used to determine Kathy's MSE.

Survey

Table 13 shows the distribution of Kathy's MSE survey ratings indicating her confidence in successfully completing each of the task types: formal, school-based tasks, or practical, out-of-school tasks (Resnick, 1987). Because Kathy rated at least 17 of the 31 tasks at a level of 4, 5, or 6, her results indicated that Kathy had a high MSE, without any noticeable distinction based on the task classification type.

Table 13

MSE: Number of Tasks Kathy Assigned to Each Rating Level by Task Type

Ratings	1 (Not confident at all)	2	3	4	5	6 (Completely confident)	Count: 4, 5, or 6 Ratings
Part 1: Formal Tasks	1	0	0	3	6	8	17
Part 2: Practical Tasks	0	2	0	0	4	7	11

Interviews

During interviews, Kathy made comments that illustrated a range of comfort levels that varied by content or task type. Table 14 shows that throughout all interviews, including the pre- and post-lesson and the end-of-study interviews, Kathy made a total of eight statements related to her MSE. From the survey and pre- and post-lesson interviews, Kathy's MSE appeared to be robust. In Kathy's high MSE statements, she would often state that she was "very confident or "comfortable" with the material when responding to pre-lesson interview questions like, "How confident are you with respect to the mathematical content you are teaching today?" However, additional information from the end-of-study interview, made classifying Kathy's MSE less clear.

Table 14

Kathy's Responses Regarding MSE

Statements	High MSE	Low MSE
"I'm comfortable with the concept of place value." (Pre-lesson interview, March 11, 2021)	X	
"I am very confident [adding onto 10 to make teen numbers]." (Pre-lesson interview, March 25, 2021)	X	
"I am comfortable with the subject matter [adding three numbers]." (Pre-lesson interview, April 15, 2021)	X	
"And that's probably in part because that [story problems] was my least favorite." (End-of-study interview, April 21, 2022)		X
"Oh, I don't want to do it. I really didn't want to do it. But I know two things. I knew that math was my weakest subject area." (End-of-study interview, April 21, 2022)		X
"Middle school, I didn't do as well in math and didn't feel as confident. High school, I did really, really well." (End-of-study interview, April 21, 2022)	X	X
"I obviously knew that I was doing and what addition and subtraction and all those things meant and what story problems meant." (End-of-study interview, April 21, 2022)	**X**	
Count	**5**	**3**

During the end-of-study interview, when discussing mathematical concepts that were Kathy's least favorite to teach, she discussed her struggles as a student solving word problems. Kathy's responses indicated she struggled when presented with non-routine problems. For example, she stated, "Just give me the steps. Give me the flow chart, and I can do it, but I don't know what I'm doing." Then, when I posed the question, "Do you believe yourself as being a person who is good at math?" Kathy responded:

> No, I guess is the short answer. I believe that I'm okay at maths…I did well, like, I had peaks. Middle school, I didn't do well in math and didn't feel as confident. High school, I did really well, really well. And that kind of created that "ooh, I like math. I'm interested in this." And then I went to college and jumped into math there, and I did very poorly. I was done with math.

Though Kathy showed general confidence in her ability to solve the mathematical tasks she was teaching during her lessons to kindergartners with her current knowledge base, it became evident in her interviews, that Kathy's MSE varied when discussing some of her past experiences with mathematics. Kathy's lower MSE was only evident in the end-of-study interview in which she was often asked to discuss her past experiences with mathematics. A possible explanation for the difference could relate to the pre- and post-lesson interviews being conducted through an online survey whereas the end-of-study interview was conducted face-to-face. The experience of verbally discussing past experiences versus writing responses to questions could potentially alter the reporting of aspects of Kathy's MSE.

Observations

Though, during interviews, Kathy often stated that she believed that mathematical concepts geared for learners at the kindergarten level are easy for adults, indicating a possible high MSE, there were no instances in which Kathy's actions or words indicated aspects of her MSE within the observation data.

Frances' Mathematical Self-Efficacy

The following sections describe data from the survey, interviews, and observations used to determine Frances' MSE.

Survey

Table 15 shows the distribution of Frances' ratings for each type of task, formal or practical tasks. As shown in Table 15, Frances rated all 31 tasks at levels of 4, 5, or 6. Because the criterion for classifying the respondent as having a high MSE was rating tasks as a 4, 5, or 6 for at least 17 of the 31, the survey results indicated that Frances had a high MSE, without any notable distinction based on task classification.

Table 15

MSE: Distribution of Frances' Task Rating by Task Type

Ratings	1 (Not confident at all)	2	3	4	5	6 (Completely Confident)	Count: 4, 5, or 6 Ratings
Part 1: Formal tasks	0	0	0	0	3	15	18
Part 3: Practical tasks	0	0	0	0	1	12	13

Interviews

During interviews, Frances made comments that illustrated her aptitude for mathematical content and her comfort with not knowing everything. Table 16 shows that throughout all pre- and post-lesson interviews and the end-of-study interview, Frances made a total of six statements related to her MSE. Within the four high MSE statements, Frances would often state that she was "pretty confident" or "does not mind the discomfort" with the material. Frances made these statements when responding to pre-lesson interview questions like, "How confident are you with respect to the mathematical content you are teaching today?" Frances was very vocal in her ability to be "comfortable with the uncomfortable." For example, during a pre-lesson interview concerning a lesson on the hierarchy of quadrilaterals, Frances stated, "If I don't know something, I think I have gotten better at being, like, oh, let's think about that or let's let that simmer." This interaction with Frances provided evidence that she was confident about having a way to navigate situations in which she might not have immediate recognition of or deep knowledge about the mathematical content. This is an example of Frances' high MSE because though she mentions she might now know the content; she is willing to continue to persevere in the task (Hackett & Betz, 1989). In another interview, Frances stated that she was confident in her capability to solve tasks with fractions—showing her high MSE.

Table 16

Frances' Interview Statements Regarding MSE

Statements	High MSE	Low MSE
"Fractions, I am pretty confident with." (Pre-lesson interview Grade 5, March 22, 2021)	X	
"I am pretty confident with shapes." (Pre-lesson interview Grade 5, May 3, 2021)	X	
"I am pretty confident with it [vocabulary associated with quadrilaterals]." (Pre-lesson interview Grade 5, May 5, 2021)	X	
"It has taken awhile, but I feel pretty confident about it [dividing fractions]." (Pre-lesson interview Grade 6, March 23, 2021)	X	
"I struggle to see the big picture when it comes to expressions and equations." (End-of-study interview, May 19, 2022)		X
"I majored in middle level mathematics. I didn't want to teach all the subjects in elementary and I also didn't feel like my strength was secondary [mathematics]." (End-of-study interview, May 19, 2022		X
Count	**4**	**2**

During the end-of-study interview, when describing why she chose middle level mathematics rather than secondary teaching, Frances discussed how she perceived secondary mathematics education for people who "really enjoyed math, like in the sense of, like, they liked algebra, and the algorithm and efficiency." Frances felt differently about her mathematical knowledge, that instead of enjoying the "algorithms and efficiency" she enjoyed "different ways of thinking of things [mathematical strategies to solve tasks]" and appreciated the emphasis on exploring mathematical concepts in the middle grades. As previously stated, I classified statements in which Frances discussed her capability for persevering and confidence in mathematical concepts as indicative of high MSE.

Frances showed her confidence regarding feeling okay with her level of content understanding and knowing that she was capable of learning content that she needed to know. However, Frances did state that she struggled to see the "big picture when it comes to expressions and equations." Frances discussed a lack of confidence in her mathematical understanding of algebraic content. Frances continued to explain that this lack of confidence in her mathematical knowledge leads her to spend more time concerned about the mathematical content while teaching. This was the single instance throughout all of her interviews in which Frances statements may have been interpreted as indicated of a low MSE.

Lastly, Frances exhibited a high MSE when she began discussing what she believed made a person good at mathematics. She discussed her belief that people who are good at math are those who have an "open mind" and are willing to take risks and ask questions. Frances described a situation in a graduate level algebra course in which she experienced moments of confusion and frustration. Yet, she also reported that even though she was confused and frustrated, she was willing to accept that she was not confident with the material and she was willing to ask questions to improve her understanding (Collins, 1984). Frances stated that she believed that being good at mathematics meant being okay with not knowing everything. In this way, she interpreted her own lack of mastery as still indicative of high MSE. During the interview, Frances discussed the algebraic content she encountered throughout the class in which she struggled to understand. Frances referenced the content within the course as “algebra” and did not specify specific algebraic concepts. Initially, it seemed as though Frances’ MSE was not as robust for algebra as it was for fractions. However, during our conversation, Frances stated that although she felt stumped by the class content, she still believed she was capable of completing the task because she was willing to seek assistance and to persevere until she reached a solution.

In summary, Frances’ comments during the interviews provided evidence of confidence in her own definition of being “good at math,” while at the same time admitting that her confidence in her content knowledge varied, depending on the specific content area. In particular, she identified some mixed levels of confidence associated with algebra. Thus, I classified this evidence as indicating that Frances primarily had a high MSE overall, with some inconsistencies in her MSE in some content areas.

Observations

Throughout all observations, Frances often responded to student contributions with confidence, showing no hesitation with respect to content. Thus, there was no evidence during the lessons of any visible self-doubt regarding Frances’ MSE.

Mathematics Teaching Self-Efficacy

In the following section, I describe the results for Kathy’s and Frances’ mathematics teaching self-efficacy. I used two methods for assessing each teacher’s MTSE. I used Parts 2 and

4 of the Mathematical Self-Efficacy, Mathematics Teaching Self-Efficacy, and Instructional Beliefs Surveys to elicit MTSE from the teachers. For Kathy, the survey was tailored for elementary teachers (see Appendix B). For Frances, the survey was tailored for middle school teachers (see Appendix C). As noted by Kahle (2008), Part 2 focused on a teacher's level of agreement with statements centered on the teaching of mathematics (e.g., I understand mathematics concepts well enough to be effective in teaching elementary school mathematics) and Part 4 focused on a teacher's confidence level in their ability to effectively teach students mathematical topics (e.g., number patterns, fractions, shape properties). For Part 2, a rating of 1 indicated the teacher strongly disagreed with the statement while a rating of 6 indicated the teacher strongly agreed with the statement. Part 4 rated the confidence in teaching wherein a rating of 1 indicated "not confident at all" and a rating of 6 "completely confident.".

During interviews, I asked questions related to the teacher's MTSE. To locate statements concerning teachers' MTSE, I read through each interview transcript and highlighted statements of interest. These statements often included discussion where the teacher related their experience with respect to the teaching of mathematics. Statements could be as short as a phrase or several sentences in length as long as they focused on the same idea or thought. The results show that both Kathy and Frances have a high MTSE.

Kathy's Mathematics Teaching Self-Efficacy

The following sections describe data from the survey, interviews and observations used to determine Kathy's MTSE.

Survey

Table 17 shows Kathy's score of 26, rating each statement as a 5 or 6 on all survey items, resulting in a high MTSE.

Table 17

MTSE: Number of Statements Kathy Assigned to Each Rating Level

	1 (low MTSE indicator)	2	3	4	5	6 (high MTSE indicator)	Count: 4, 5, or 6 Ratings
Part 2: Teaching Practices	0	0	0	0	3	10	13
Part 4: Teaching Concepts	0	0	0	0	2	11	13

Interviews

Kathy's statements often focused on past experiences while teaching mathematics or experiences focused on learning how to teach mathematics (see Table 18). Once again, the statements could be short phrases or long statements focused on the same idea or thought. I identified nine statements in which Kathy exhibited evidence of her MTSE.

Table 18

Kathy's Interview Statements Regarding MTSE

Statements	High MTSE	Low MTSE
"I am comfortable with teaching what is next in the learning sequence." (Pre-lesson interview, March 11, 2021)	X	
"I am comfortable with teaching what teen numbers are." (Pre-lesson interview, March 11, 2021)	X	
"I feel very confident in teaching addition and helping children understand this concept." (Pre-lesson interview, March 18, 2021)	X	
"I feel confident teaching the content [placing values on a number line with 0, 5, 10 marked] and organizing the game environment." (Pre-lesson interview, April 1, 2021)	X	
"I am very comfortable with enacting today's lesson." (Pre-lesson interview, April 15, 2021)	X	

Table Continues

Table Continued

"I really struggle with how to introduce it [story problems], how to teach it in a way that was meaningful for the students, how to help them understand conceptually what was going on." (End-of-study interview, April 21, 2022)		X
"But I didn't know how to teach the kids conceptually, I taught them kind of the rules." (End-of-study interview, April 21, 2022)		X
"So, I was not very strong with teaching that [mathematics], I followed the book, I did what the book said. There were some things I skipped in the book." End-of-study interview, April 21, 2022).		X
"I feel very confident as a teacher." (End-of-study interview, April 21, 2022)	X	
"It was very hard for me to teach math [during the pandemic]." (End-of-study interview, April 21, 2022)		X
Count	**6**	**4**

The data from the pre-lesson interviews was consistent with Kathy's survey results; Kathy exhibited a high MTSE. In my pre-lesson interviews, Kathy often related her confidence in her knowledge of the mathematical content she was teaching during the lesson to her ability to teach the content to students. For example, Kathy stated that although she was confident in adding teen numbers, she was apprehensive about using money to teach the concept because she felt the students were coming into the lesson with little background knowledge. For another lesson, also focused on addition, Kathy stated, "I feel very confident in teaching addition and helping children understand this concept." Interestingly, when directly questioned about her confidence in teaching the mathematical content, Kathy often deviated from her ability to teach the content and instead made comments related to "organizing the environment" to provide students opportunities to interact, get along with other students, and develop or reinforce skills.

Though questions in the post-lesson interview were more focused on reflections about the observed lesson rather than direct questions about MTSE, Kathy's responses continued to indicate a high MTSE. Kathy often welcomed and discussed deviations from the planned lesson.

In her interview question responses, there was no evidence that Kathy was bothered by unplanned student responses. For example, during a review lesson about the pan balance, one child questioned whether switching the objects in each pan would produce a different result, this then led students "questioning whether the weight was dependent on the number of items, size of items, or just the actual weight" (post-lesson interview, March 18, 2021). Kathy commented on how many of the students were using imprecise vocabulary, such as "bigger" or "smaller," although she was confident they had mastered the use of "heavier" and "lighter" when referring to weighing items with the pan balance. Kathy's original goal for this lesson was to quickly revisit the pan balance, then move the students into "math workshop," during which they would practice mathematical concepts they had learned throughout the week. Because of the students' questions, the lesson deviated from Kathy's goal and, instead, focused solely on the pan balance and vocabulary concerning weight. In my observation notes, I commented that though this deviation occurred, Kathy showed no indication of anxiety or fear (e.g., wavering of voice, stopping students from asking questions; Usher, 2009). Instead, Kathy stayed very calm and allowed students to bring or suggest different items to test using the pan balance. Kathy's satisfaction with students derailing the lesson was also evident because when Kathy reflected on the lesson during the post-lesson interview, she did not mention the interruption as an event that stood out in her mind.

As happened with Kathy's MSE results, the end-of-study interview uncovered some of Kathy's doubts about her own MTSE. Although Kathy noted, in multiple interviews, that kindergarten mathematical concepts are simple for adults, during the end-of-study interview, Kathy displayed a lack of confidence in her ability to teach mathematics effectively when the content is related to story problems or word problems. For example, in her discussion concerning her least favorite concept to teach, she stated, “I really struggle with how to introduce it [story problems], how to teach it in a way that was meaningful for students. How to help them understand conceptually what was going on.” As previously noted, Kathy seemed to rely on processes and procedures in her learning and doing of mathematics, so with respect to story or word problems, there is a possible relationship between Kathy’s MSE and MTSE.

During the end-of-study interview, Kathy described a transformation she experienced in recent years. As a veteran teacher, Kathy had spent time improving her teaching in the other core subjects she taught (e.g., reading, writing, science) but had only recently decided that it was time to enhance her mathematics teaching. Kathy discussed how she relied on the mathematics textbook and taught in a traditional manner, telling the students what they needed to know and learn. She described that she had not felt confident enough to allow students to explore or in her ability to pose questions that would develop students’ conceptual understanding. Kathy knew that she needed to change her teaching. She stated “I kept trying to make my students believe that I loved math, and I didn’t really love it. But I wanted them to think they were good at math, regardless of anything that was going on.” So, she enrolled in a weeklong professional development (PD) at a local university focused on teaching mathematics effectively. As the week approached, she remembers having second thoughts about attending the PD, stating, “I really didn’t want to do it [the PD], I knew that math was my weakest subject area [teaching].” She knew the facilitators of the PD, felt comfortable learning from them, and knew that she needed to teach mathematics better, so despite her doubts, she attended the PD.

Kathy’s description of what she gathered from the PD, focused on the role of questioning in mathematics teaching, for both the learner and teacher. She recalled that at the conclusion of the PD, the facilitators visited Kathy’s classroom to observe and model different aspects of teaching they had discussed in the PD, including interacting with students, modeling effective questioning and task selection. Kathy, once again, focused much of the description of what she learned from the experience on the act of questioning students about their thinking, potentially showing that questioning might be an aspect of effective teaching that is influenced by a teacher's MTSE.

Observations

Throughout all observations, there was no evidence during the lessons of any visible evidence of Kathy's MTSE.

Frances' Mathematics Teaching Self-Efficacy

The following sections describe data used from the survey, interviews, and observation to determine Frances' MTSE.

Survey

Table 19 shows Frances' score of 26, rating each statement as a 5 or 6 on all survey items. By the criteria I identified earlier, this score indicates that Frances had a high MTSE.

Table 19

MTSE: Distribution of Frances' Statements Ratings by Survey Component

Ratings	1 (low MTSE indicator)	2	3	4	5	6 (high MTSE indicator)	Count: 4, 5, or 6 Ratings
Part 2: Teaching Practices	0	0	0	0	4	9	13
Part 4: Teaching Concepts	0	0	0	0	2	11	13

Note. Part 2: A rating of 1 indicated "strongly disagree" and a rating of 6 indicated "strongly agree." Part 4: A rating of 1 indicated "not confident at all" and a rating of 6 indicated "completely confident."

Interviews

Frances' statements regarding her MTSE often focused on her perceived capability to use students' prior knowledge to engage them in active learning experiences (see Table 20).

Table 20

Frances' Interview Statements Regarding MTSE

Statements	High MTSE	Low MTSE
"I guess I feel comfortable in that even when teaching fractions is not what I thought it was going to be like, I can rebound from it and get some resources and materials instead of walking away and have no clue where to go next." (Pre-lesson interview Grade 5, March 22, 2021)	X	
"I am confident with shapes, but I've never tried it this way [shape sort focused on vocabulary hierarchy]. So, I'm like, 'What's going to happen?' But I think I'm ready to roll with whatever happens." (Pre-lesson interview Grade 5, May 3, 2021)	X	
"I am confident with taking the knowledge students have and doing something with it." (Pre-lesson interview Grade 5, May 5, 2021)	X	
"I've gotten more comfortable with the types of questions they [students] are going to ask and where they're going to get stuck, so that has helped with my confidence. When I first started doing this type of like, here's the situation and let's work through the situation, what are we going to do? What actually are we doing? That was uncomfortable." (Pre-lesson interview Grade 6, March 23, 2021	X	
"I think I'm pretty confident when they're [students] are struggling. And they're like, not seeing stuff [patterns and connections]. Like, I feel I have the questions and the tools or like to get them to start thinking about it [mathematics]." (Pre-lesson interview Grade 6, March 23, 2021)	X	
"I do not necessarily dread but something I feel less confident about, only because I feel like I'm all, 'There's a better way to do it.' Is that introduction to expressions and equations…I'm here and I haven't taken enough time to think about where they [students] should start to learn that stuff and how could I start prepping the language." (End-of-study interview, May 19, 2022)		X
"I think I'm doing fine, like, I think if other teachers came in, they'd be like, 'This is great, like you model language." (End-of-study interview, May 19, 2022)	X	

Table Continues

Table Continued

"I'm just always like, 'How can I make this better? And how can I provide opportunities for kids, so they can see that they can do this too?' Yeah, it's a little bit of confidence too when I think of those things, I think like, you're a secondary major, they would in and be like, 'got it' and I'm not there." (End-of-study interview, May 19, 2022)		X
"I think probably those fraction operations and ratios, I have more room in my brain to think about these more like the teaching side of it. Because I feel like competent about the content, I have either from experience done it enough times, and I kind of know a little bit more about what to anticipate. So, I'm not loaded with that. So, I can think a little bit more about like, my side of things, like, how I'm instructing it and engaging with it with the kids." (End-of-study interview, May 19, 2022)	X	
Count	**7**	**2**

Consistent with her survey responses, Frances' comments during pre-lesson interviews indicated that she had an overall high MTSE. In many of Frances' high MTSE statements, she often focused on facilitating classroom discourse and engaging students in problem solving. When describing a task in a pre-lesson interview, Frances focused on "how they [students] could use their background knowledge for whole number multiplication to multiply whole numbers and fractions." During her description of the lesson goals, Frances discussed the lesson with excitement and even stated that, "fractions are my favorite unit." This lack of stress over the teaching of fraction content, is evidence of Frances high MTSE because she exhibited her confidence in teaching a commonly difficult concept by the lack of signs of stress (i.e., physiological arousal; Bandura, 1997) and instead showing excitement for teaching the concept. Frances also exhibited her capability to teach concepts when she might be unsure about students' level of understanding. In a pre-lesson interview, Frances reported that because of the pandemic and remote learning environment the previous year, she was not sure about the level of student understanding concerning segments and angles in relation to triangles and quadrilaterals. Frances continued by stating she was also unsure about whether students would use vocabulary correctly.

In the lesson, Frances planned time for students to discuss, then publicly share, geometric vocabulary they remembered from previous experiences. Though Frances seemed uncertain about students' prior knowledge and vocabulary related to quadrilaterals, during the interview she was not apprehensive, nor did she show signs of stress or nervousness about the lesson. Because the lack of physiological arousal with signs of stress (i.e., non-wavering voice, relaxed, smiling; Usher, 2009), I interpreted Frances' comments and demeanor as indicating a high MTSE. In other words, even heading into a lesson with a sense of insufficient information about her students' prior knowledge, Frances was comfortable about facing this situation.

The post-lesson interview data also supported Frances' high MTSE. Often, Frances reflected not only on her actions but on the intended goal of the lesson and how well she believed students had achieved the goal by the end of the lesson. In addition, throughout the interviews, Frances spoke in terms of her students being active learners. Because of this use of language, evidence of Frances' MTSE often was in the form of her physiological arousals (i.e., lack of hesitancy, excitement; Usher, 2009). For example, after a lesson focused on dividing fractions, Frances started her response to a question focused on how she thought the lesson went, she stated, "I thought it was super messy." Frances continued her response focused on "embracing the mess" of student learning instead of trying to have a "perfect little lesson." Throughout Frances description of the lesson, her voice was often in an excited tone, along with laughing and animated hand movements. The example showed that Frances was aware of effective teaching practices, as she allowed students to be active learners despite the "mess" that might occur. For this reason, I interpreted these statements as evidence of Frances' high MTSE. In a later interview, Frances mentioned again that sometimes she is "on a roller coaster" because she believes that the student thinking is "all over the place." Frances added that despite the roller coaster of thinking and the lesson appearing messy, she loved that everyone had a different way of thinking even when the variety of ideas appears “hard to manage.” This example further illustrated Frances’ comfort with her lesson design and implementation leading to “messy” interactions. Such messiness makes it difficult for teachers to navigate the variety of student ideas, yet Frances believed that she has the capability to guide the discussion in ways that leads to productive student learning.

During the end-of-study interview, some of Frances’ responses may have indicated that her otherwise robust MTSE may have been less robust in the context of teaching algebra. For example, Frances stated that expressions and equations are a mathematical concept in which she does not feel overly confident in teaching. When describing why she feels this way about teaching equations and expressions, she discussed her inability to take the time to “think about where they [students] should start learning that stuff [equations and expressions].” Frances stated that she “puts a lot of pressure” on herself to “try and get it right.” In addition, Frances recognized that technically her students had been working with “algebraic concepts even before you [students] came to school” and the “concept comes with a lot of baggage.” Frances’ statement seemed to indicate a lack of confidence in her ability to understand where she should start when teaching equations and expressions. Similar to Frances’ pre- and post-lesson interviews, she continued to exhibit her level of MTSE in statements focused on her capability to allow her students to be active learners.

In summary, Frances’ statements focused on her capability and confidence in allowing her students to be active learners despite the “messy” lesson structure, provided evidence of her high MTSE.

Observations

Throughout all observations, there was no evidence during the lessons of any visible evidence of Frances' MTSE.

Research Question One: Connections among MSE and MTSE

In this section, I answer the first research question as it applies to both Kathy and Frances. The research question was, "How are mathematical self-efficacy and mathematics teaching self-efficacy related in mathematics teachers who have been labeled as effective?"

Both teachers were identified as effective teachers by both trusted mathematics educators and their principal. The self-efficacy surveys indicated that both Kathy and Frances had high MSE and MTSE, though Kathy's survey results indicated she had more robust MSE. In addition, both teachers provided evidence of having a high MSE and MTSE throughout all of their interviews.

As for a relationship between MSE and MTSE, Kathy's and Frances' data provided evidence of possible connections among the self-efficacies. Kathy provided insight into a PD experience that not only provided both a vicarious and mastery experience that potentially increased her MTSE. When discussing this experience, Kathy also noted that she would be interested in revisiting mathematics as a learner, which leads me to believe that the experience also influenced her MSE. Likewise, Frances' statements concerning her view of mathematics and mathematics teaching (e.g., importance of perseverance, refining her teaching skills to provide worthwhile experiences) were revealing of the relationship I noticed between her MSE and MTSE. Like Kathy, Frances tended to relate her own mathematical capability (e.g., MSE) to her capability to teach mathematics (e.g., MTSE).

Instructional Beliefs

To determine Kathy and Frances' instructional beliefs, I used data from part 5 of the Mathematical Self-Efficacy, Mathematics Teaching Self-Efficacy, and Instructional Beliefs Survey for Elementary Teachers (see Appendix B) for Kathy and the middle level rendition (see Appendix C) for Frances and interviews. The results showed that both Kathy and Frances' instructional beliefs were primarily student-centered beliefs. Additionally, because beliefs potentially influence the use of specific instructional practices (Wilkins, 2008), I found it difficult to identify whether each teacher's actions should be categorized as an indicator of their instructional beliefs or whether they were an indication of enacted teaching practices. Therefore, I used Kathy's and Frances' comments about their lessons as indicators of their beliefs and their actions as indicators of their enacted teaching practices.

Kathy's Instructional Beliefs

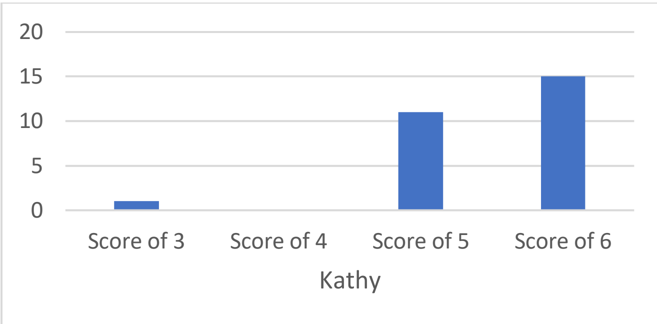
In the following sections, I describe Kathy's instructional beliefs using data from the survey, interviews, and observations.

Survey

The aim of the instructional beliefs survey was to determine whether Kathy's instructional beliefs were primarily student- or teacher-centered. The results of Kathy's instructional beliefs survey are depicted in Figure 7. Because Kathy selected a 4, 5, or 6 for 26 out of the 27 survey items, the survey indicated that her instructional beliefs were primarily student-centered.

Figure 7

Distribution of Scores for Instructional Beliefs Survey – Kathy



Note. One indicated "strongly disagree" and six indicated "strongly agree."

One pair of matched statements indicated a possible discrepancy with the overall finding that Kathy was student-centered. These items focused on student-centered beliefs. One item with which Kathy disagreed (score of 3) was, "Students should understand computational procedures before they spend time practicing them." However, Kathy strongly disagreed with the converse statement, "Students should spend time practicing computational procedures before they are expected to understand those procedures." Though Kathy might have misread the question, the response might also indicate that Kathy's specific instructional beliefs concerning when mathematical practice should be assigned to students might not be as strong as other aspects of her instructional beliefs.

Interviews

During the pre-lesson interview, I attempted to learn more about Kathy's instructional beliefs. Toward this end, I asked Kathy, "What are some of the teacher actions that you plan to use during the lesson?" Kathy often used the words "model" or "demonstrate" to describe teacher actions she planned to use. Often, she would follow up with a statement stating that she would "monitor" the students as they interacted with a game or lesson materials. For example, in the fourth pre-lesson interview, Kathy stated, "I'll demonstrate the game for the whole class with a group of students. I'll put the children into groups to play. While they are playing the game, I will monitor groups and observe what they do." Though Kathy seemed to use language that would be considered primarily teacher-centered (e.g., model, demonstrate), she also seemed to see value in planning time to monitor, or listen to, students (see Smith et al., 2020) which could be considered a student-centered action. In Table 21, I describe more instances in which Kathy discussed actions taken by her or supported by her teaching that focused on student-centered beliefs. In many of Kathy's responses, she focused both on students' need to communicate about mathematics as well as on her concern that students ought to attend to social aspects of working with others. Kathy's emphasis on communication and social interactions indicates an alignment with NCTM's suggestion that effective teachers facilitate discourse among students to build shared understanding.

Table 21

Instructional Action Statement From Interview – Kathy

Instructional Action	Statement
Demonstrate or model an activity or game leading to monitoring students.	"I will demonstrate the game, then monitor students during their partner work." "I'll demonstrate the game for the whole class with a group of students…while they are playing the game, I will monitor groups and observe what they do."
Use student misconceptions as opportunities for learning.	"I can help students work through misconceptions and grow as learners."
Share and compare student strategies	"I hope they will share their strategies with their partners and their partner is paying attention and giving them feedback."
Pose purposeful questions	"I really strive to pose questions to the students. What do you think? How would you solve that? How would you figure that out? Does someone else have another strategy?"

Kathy's student-centered instructional beliefs were also evident in her post-lesson interviews. Specifically, Kathy's reflections focused on student thinking and instances in which students made mathematical connections. For example, in response to the question, "How do you feel the lesson went today?" Kathy responded that she was able to learn a lot from listening to and observing the students. When she described an instance of students exploring different objects using a pan balance, Kathy stated, "I loved that one child was thinking about moving the objects being weighed to different pan balance buckets. This gave me insight into what he was thinking, and, in turn, what others might be thinking." When Kathy stated that she "loved" what had occurred in the lesson, it seemed that this was further indication of her student-centered instructional beliefs. Because the focus of Kathy's comment was on student thinking rather than on what she was thinking or doing during the lesson. Later in that same post-lesson interview, Kathy discussed how she intended to add a ping pong ball (roughly 1.5 in. diameter) and a steel ball (roughly 0.75 in. diameter) to the pan balance station during math workshop to allow students the opportunity to explore that objects with greater volumes are not always heavier than objects with comparatively smaller volumes. This was in response to the misconception about size-weight (i.e., volume-weight) correspondence that Kathy noted had occurred during the lesson.

Later during the end-of-study interview, Kathy gave further evidence of her student-centered instructional beliefs by describing aspects of her teaching that she believed supported students in understanding mathematical concepts. When viewing a video excerpt in which Kathy used the MTP to pose purposeful questions (NCTM, 2014), she noted that, although understanding money was not part of the curriculum, she had decided to launch her lesson by having students sort coins. Kathy stated that coin sorting was a good place to start the lesson because she wanted students to start thinking about the process of sorting. She noted that having the coins sorted would help support student reasoning while they explored the task posed during the lesson. This launch positioned Kathy to pose purposeful questions during the lesson that were designed to highlight the existence of multiple answers or different ways to reason about and solve the task, an element of student-centered beliefs (Smith et al., 2020). For example, as the students were sorting the coins, Kathy students to describe how they were sorting their coins and why they were using that approach. Though many students were sorting the piles of coins into nickels and pennies, Kathy attempted to highlight the different category descriptions that students provided. In response to a student who stated that they sorted the coins by their values Kathy posed questions such as, "How did you know those coins were different values?" and "So, these [pointing to the nickels] are worth 5, while these [pointing to the pennies] are only 1?" To another student, who sorted their coins according to the size, Kathy asked, "What kinds of words did you use for how you sorted your coins?"

Observations

Because of the difficulty determining whether Kathy's actions should be categorized as an indication of her instructional beliefs or whether they were an indication of enacted teaching practices, I did not make assumptions concerning Kathy's instructional beliefs as seen through her practices.

Frances' Instructional Beliefs

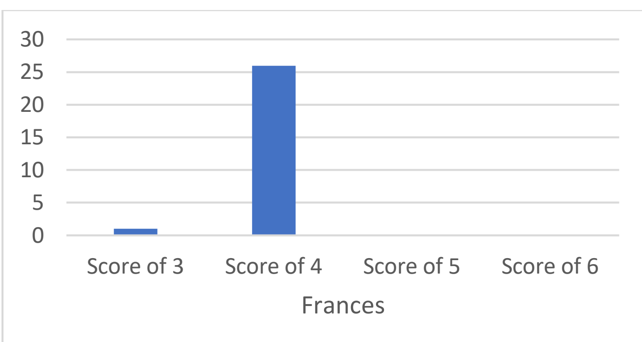
In the following sections, I describe Frances' instructional beliefs using data from the survey, interviews, and observations.

Survey

The aim of the instructional beliefs survey was to determine whether Frances' instructional beliefs were primarily student- or teacher-centered. The results of Frances' instructional beliefs survey are depicted in Figure 8. Because Frances selected a 4, 5, or 6 for 26 out of the 27 survey items, the survey indicated that her instructional beliefs were primarily student-centered. As seen in Figure 8, Frances chose a rating of 4 for all statements except for one. The only statement that focused on student-centered beliefs with which Frances gave a score of 3—disagree—was, "Students should understand computational procedures before they spend time practicing them."

Figure 8

Distribution of Scores for Instructional Beliefs Survey – Frances



Note. One indicated "strongly disagree" and six indicated "strongly agree."

Interviews

During the pre-lesson interview, I attempted to elicit from Frances a verbal description of her instructional beliefs, by asking, "What made you decide this was the best lesson for today?" Though my goal was to encourage Frances to discuss her instructional beliefs during the interviews, I often noticed that Frances was more apt to discuss her values. For clarification, a value can be seen as “beliefs in action” (Bishop, 2005, p. 107) whereas beliefs are a complex system that integrate one’s beliefs concerning intertwined relationships (e.g., instructional beliefs integrate teachers’ beliefs concerning the nature of mathematics and beliefs about the relationship between the teaching and learning of mathematics; O’Hanlon, 2015). Table 22 includes values and examples statements that were suggestive of Frances’ student-centered instructional values. I have recorded and described Frances’ values because these values have the potentially to influence and interact with her beliefs and practices. For example, in Observation 1 (see Table 22), Frances provided students with student generated strategies from a previous homework assignment. In the pre-lesson interview, Frances stated:

> I’m starting today with those student samples. And going, ‘here’s what your classmates did.’ I’ve left some incorrect ones in there…my question is going to be, ‘Let’s look at the strategies, look at the ways of thinking that your classmates had, what do you think about them?’ Kind of open up the discussion, see where that takes us.

During this interview, Frances stated that she believed it was important to “open up the discussion” using student samples because it is “capitalizing on students’ background knowledge to look at a concept.” This was an indication that Frances valued using student-centered practices.

Table 22

Instructional Values Statements From Interviews – Frances

Instructional Value	Statement
Risk taking and making mistakes	"I'm hoping that they just like, take some **risks**, engage. Maybe it's not in the full class, but it's during the time with their partner."
	"Students are welcome to **take risks, make mistakes, and question/be skeptical** of ideas."
Problem solving and making connections	"If I see them also, not only asking questions or trying to **problem solve, like think through this, use a number line or a tool? Are they seeing connections** at all?"
	"Trying to get them to **use what they know** about whole number division to make sense of non-whole number division."
Eliciting and using student thinking	"So, I hope I can, I feel like I can help progress a student with **questioning and like activities and prompting** more if they're consistent versus, like, 'it's magic, I just did it!' And I don't know where it came from. So, I'm going to be looking for a **consistent strategy,** a consistent entry point."
	"I attempted to create a more authentic forum for this **analysis of ways of thinking** and ideas by **pulling in the student work** demonstrating the variety, common strategy, and misconceptions."
	"But I think teaching should be catered towards what your **students are actually doing** and **what they know and where they take you** for the most part, having the plan."
Support student discourse and discussion	"Students are regularly asked to engage in whole-class discussion (reviewing student work, number talk, problem string) and sharing ideas/questions with small group/partner."
	"Some people like posing maybe a problem or question, giving them some time to think and grapple with it on their own. Then getting them together to **share with someone else** and then kind of bring it **back to the whole group**."

Frances continued to support student-centered values when asked, "What indicators will help you know whether the lesson is going well?" Many of Frances' responses focused on language centered around students engaging and taking risks in their strategies and explanations. It is also important to note that Frances focused on students' need to communicate about mathematics, question strategies, and engage in pattern finding. Frances' emphasis on discourse, exploration of mathematical strategies, and taking risks as students engage with a task indicated her alignment with NCTM's (2014) suggestion that effective teachers facilitate discourse among students to build shared understanding.

Frances' student-centered instructional values continued to be evident in her post-lesson interviews, as her reflections focused on student thinking and situations in which students made mathematical connections. For example, in a post-lesson interview for a Grade 6 lesson, Frances stated that she phrased problems focused on making muffins so that they would require students to divide a whole number by a fraction. Frances described how she intentionally set aside class time for students to make connections between the muffin problem and earlier work they had done with division.

During the end-of-study interview, Frances continued to support her student-centered instructional values by focusing on aspects of her teaching that support elicit and using student thinking through engagements in student discourse. When viewing a video excerpt in which Frances used the MTP elicit and use student thinking, she reflected on the fact that the students were engaging in a task that included student-generated strategies (i.e., eliciting student strategies) for dividing a whole number by a fraction. Frances stated that this was a good task to get the students thinking of what strategies other students were using (i.e., supporting student discourse). The task also gave Frances the opportunity to determine whether students were able to identify the incorrect strategy and solution that was included in the task. Frances also noted that this excerpt showed her use of mathematical discourse, because students were conversing with one another for most of the class, discussing and comparing different strategies.

Observations

Because beliefs potentially influence the use of specific instructional practices (Wilkins, 2008), it was difficult to distinguish whether an action was taken by Frances could be categorized as an instructional belief or an enacted teaching practice. For this reason, I did not make assumptions about Frances' instructional beliefs based on her practices. Instead, I report on the enacted instructional practices of both teachers in the following sections.

Enacted Teaching Practices

To determine the teaching practices employed by Kathy and Frances, I used two observation protocols. To record the use of the Standards for Mathematical Practices (SMP; NGA & CCSSO, 2010), I used the MCOP^2 (Gleason et al., 2015). In mapping the SMPs to the MCOP^2, Gleason et al. (2015) did not equally distribute the items to each of the SMP. For example, "model with mathematics" (NGA & CCSSO, 2010) maps to one item on the MCOP^2, while the SMP "make sense of problems and persevere when solving them" (NGA & CCSSO, 2010) maps to nine items on the MCOP^2. For this reason and the purpose of the current study, I focused on the medians of scores of the items that map to the SMP, which allowed me to notice differences in each teachers' uses of the individual SMP. Though the SMPs were not the practices I proposed to observe, the MCOP^2 provided an additional insight into each teacher's instructional practices. Because the MCOP^2 did not align with my proposed verification of the use of the MTPs (NCTM, 2014), I also used the E-MCOP^2, described in previous chapters and Appendix G. The E-MCOP^2 provided an in-depth documentation of Kathy and Frances' use of the MTPs. Because the observations were video recorded, I completed the protocol using the recordings, allowing me to focus on the interactions between teacher and students during the observations versus focusing on completing the protocol and potentially missing support for the SMPs.

Kathy’s Enacted Teaching Practices

In this section, I provide information about each lesson I observed. Table 23 provides the mathematical content Kathy focused on during each lesson. Furthermore, to develop a full description of Kathy’s teaching practices, I provide a brief summary from my observation notes, including the goal, activity or task, and the perceived student engagement (see Table 24). Following the tables, I report Kathy’s $MCOP^2$ and E-$MCOP^2$ observation results.

Table 23

Mathematical Content Focus During Observed Lessons - Kathy

Observation	Mathematical Content Focus
Observation 1	Add some quantity to 10 to make teen numbers.
Observation 2	Add some quantity to 5 and write a number sentence (game).
Observation 3	Use a pan balance to make predictions whether select objects were less than, greater than, or equal to a pound of potatoes.
Observation 4	Place values on a number line that contains the values of 0, 5, and 10 (game).
Observation 5	Add three single-digit values.

Table 24

Lesson Observation Summary – Kathy

Observation	Lesson Goal	Planned Task or Activity	Perceived Student Engagement
1	Recognize teen numbers, know that a teen number of ten and some more, and add a group of ones to a ten to determine the total amount.	Students started by sorting dimes and pennies they brought from home then had a class discussion on how they chose to sort the coins. They proceeded to count by tens and ones using their coins. Students then learned a game in which one player spins the spinner (marked with values 1–6) and adds the value they landed on to ten. At the bottom of their worksheet where the spinner was located, there was a graph in which students recorded the sum. The graph included values of 11–16. The students played the games in pairs following the initial lesson on how to play the game.	High – Students were engaged in sorting their coins. When playing the game, students were engaged in the game but were not engaging with their partner as requested by the teacher.
2	Explore strategies of addition to add sums up to 10.	Students worked in pairs playing a game. The worksheet for the game consisted of an empty graph with ten frames (values 5–10). Students rolled a die (values 0–5) and added the rolled value to 5 and colored in one section corresponding to the sum on their graph. They then described to their partner how they added the two values.	Moderate – Students were engaged with the game but were often not discussing their strategies with a partner.

Table Continues

Table Continued

3	Describe the weight of an object using precise language (e.g., heavier, lighter, equal) as they directly compared the weights of two objects using a pan balance.	Using a class graph, students made predictions of heavier, lighter, equal weights of different objects to a pound of potatoes. As a class they placed each object in a pan balance where the other side contained a pound of potatoes. The class then recorded whether the object was heavier, lighter, or equal to the pound of potatoes.	Moderate – Students who were placing the items in the pan balance or drawing pictures to place on the graph were engaged in the content. Students who were making predictions often disengaged until a new item was presented.
4	When given a number line with values 0, 5, 10 labeled, locate the values 0–10 on the number line. Compare two values from 0–10 by identifying which one is greater than, less than, or equal to the other.	Students were paired to play a game in which they worked to place numbers 0–10 on a number line. Students were given cards that contained two sets of numbered (0–10) cards. The two decks were shuffled together and passed out between the two students. The students also had a spinner labeled with the greater than, less than, and equal to symbols. At the start of the game one student chose a card from their hand and placed it on a number line. The second student spun the spinner and placed a number card from their hand that corresponded with the symbol they spun. For example, if the first student placed a 7 on the number line and the second student spun a greater than symbol on the spinner, then the second student would need to place a number card containing a value greater than 7 on the number line. The game continued until all cards were played.	High – There was some confusion of the game but as students began to recognize the process, there was high engagement with the content.
5	Explore ways to add 3 sets of numbers.	Students worked in pairs using a deck of ten-frame cards ranging from 0–10. One student drew three cards and both students thought and shared how they would add the three values. Each student wrote the three numbers and the sum on a worksheet in the form of an equation.	High – Students engaged in the game and sharing their thinking.

Standards for Mathematical Practice

In determining the use of the SMPs (NGA & CCSSO, 2010), I completed the MCOP^2 observation protocol (Gleason et al., 2015) for each of the five lessons I observed. For teacher facilitation—the role of the teacher to provide lesson structure and guidance through problem solving and mathematical discourse—Kathy scored a median of 2 across all observations. For student engagement—students fulfilling the role as active learner with the classroom environment—Kathy scored a median of 2 across all observations. Table 25 shows Kathy's scores per item and observation sessions and is organized by the items associated with each SMP. I have included the median score for each SMP.

Table 25

Distribution of MCOP^2 Item Scores for Each Observation – Kathy

MCOP^2 Item	Observation 1	Observation 2	Observation 3	Observation 4	Observation 5	Median for SMP
	Make sense of problems and persevere in solving them					**2**
1	2	2	3	3	3	3
2	1	1	2	1	2	1
3	2	3	3	3	3	3
4	2	2	2	2	3	2
5	0	2	2	0	2	2
9	1	1	2	1	2	1
11	2	0	2	2	2	2
14	3	2	2	2	2	2
16	2	1	2	1	1	1
	Reason abstractly and quantitatively					**2**
5	0	2	2	0	2	2
7	2	2	1	2	2	2

Table Continues

Table Continued

	Construct viable arguments and critique the reasoning of others					**2**
4	2	2	2	2	3	2
5	0	2	2	0	2	2
10	2	2	3	2	2	2
12	2	2	3	2	3	2
13	1	2	2	3	3	2
15	1	2	2	2	2	2
	Model with mathematics					**2**
7	2	2	1	2	2	2
	Use appropriate tools strategically					**1.5**
2	1	1	2	1	2	1
5	0	2	2	0	2	2
	Attend to precision					**2**
10	2	2	3	2	2	2
	Look for and make use of structure					**3**
1	2	2	3	3	3	3
6	2	1	2	1	3	2
8	1	3	3	2	3	3
	Look for and express regularity in repeated reasoning					**2.5**
1	2	2	3	3	3	3
4	2	2	2	2	3	2
6	2	1	2	1	3	2
8	1	3	3	2	3	3

Note. A rating of 0 indicated action was not noticed and a rating of 3 indicated action was highly noticed.

As see in Table 25, Kathy's use of "look for and make use of structure" was used more often in the observed lessons than "use appropriate tools strategically." Because most of the medians were greater than a score of 2 across all observations, I determined that Kathy had a skill of providing students with opportunities to engage with the SMPs.

Mathematical Teaching Practices

To determine the use of effective teaching practices in mathematics, I used the E-MCOP^2 to analyze each of the five observations. The purpose of the E-MCOP^2 was to record the visible MTPs (NCTM, 2014) Kathy used during each of the lessons. Overall, Kathy was more likely to facilitate meaningful mathematical discourse than use and connect mathematical representations.

Table 26 shows the extent to which there was evidence of Kathy using MTPs in each of the five observations.

Table 26

Extent of Evidence of MTPs – Kathy

	Observation 1	Observation 2	Observation 3	Observation 4	Observation 5
Establish mathematics goal to focus learning	Highly evident	Moderately evident	Moderately evident	Moderately evident	Not evident
Implement tasks that promote reasoning and problem solving	Highly evident	Highly evident	Highly evident	Moderately evident	Highly evident
Use and connect mathematical representations	Not evident	Not evident	Not evident	Not evident	Highly evident
Facilitate meaningful mathematical discourse	Not evident	Highly evident	Highly evident	Highly evident	Highly evident
Pose purposeful questions	Highly evident	Not evident	Highly evident	Moderately evident	Moderately evident
Build procedural fluency from conceptual understanding	Highly evident	Moderately evident	Highly evident	Moderately evident	Highly evident
Support productive struggle in learning mathematics	Moderately evident	Highly evident	Moderately evident	Highly evident	Highly evident

Table Continues

Table Continued

Elicit and use evidence of student thinking	Moderately evident	Not evident	Moderately evident	Moderately evident	Moderately evident

Note. "Not evident" indicates that there were no descriptors noticed. "Moderately evident" indicates there were up to half, including exactly half of the descriptors noticed. "Highly evident" indicates that there were more than half the descriptor noticed.

In determining the descriptions and alignment between the MCOP^2 and MTPs, I found that some MTPs required more descriptor categories than others. Because of the difference in number of subscales, I determined the extent of use by the number of subscales evident compared to the total number of subscales of the MTP. If none of the descriptors were evident, then I categorized the use of the MTP as "not evident." If up to half, including exactly half, of the descriptors were evident, the MTP was labeled as "moderately evident." Lastly, if there were more than half the descriptors evident, I labeled the MTP as "highly evident." As seen in

Table **26**, Kathy was able to use most MTPs on a regular basis as seen by "moderately evident" and "highly evident" are prominent across all observations. Overall, Kathy used most of the MTP throughout the observations. The only MTP I noticed in only one observation was use and connect mathematical representations. The following section focuses on Frances and her enacted teaching practices.

Frances' Enacted Teaching Practices

In this section, I provide information about each lesson I observed while in Frances' classroom. Table 27 provides the mathematical content Frances focused on during each lesson. Table 28 shows a brief summary from my observation notes, including the goal, activity or task, and the perceived student engagement. Following the tables, I report Frances' $MCOP^2$ and E-$MCOP^2$ observation results.

Table 27

Observed Lessons Mathematical Content Focus – Frances

	Grade 5	Grade 6
Observation 1	Multiply a fraction and a whole number	Divide a fraction and a whole number with context
Observation 2	Identify characteristics of 2D shapes – Triangles	Divide a whole number and fraction with context
Observation 3	Identify characteristics of 2D shapes – Quadrilaterals	Divide fractions and whole numbers without context

Table 28

Lesson Observation Summary – Frances

	Observation	Lesson Goal	Planned Task or Activity	Perceived Student Engagement
Grade 5	1	Use background knowledge of whole number multiplication to multiply fractions.	Students were provided with five student solution strategies for 1/3 x 7. They then worked in pairs to document what they noticed and wondered about each of the strategies. Teacher then conducted a whole class discussion focused on what students noticed and wondered.	High – Most students were actively involved in discussions both with their partners and in the whole class discussion.
	2	Formatively assess students' knowledge of geometric vocabulary concerning triangles and use that vocabulary to begin describing quadrilaterals.	With a partner, students discussed how they categorized different triangles from a previous assignment. The teacher recorded the vocabulary terms students used to describe different types of triangles on the board. Students were then given a polygon shape set (manipulatives) and told to generate different categories for classifying the shapes.	Low – Most students were engaged in both partner and whole class discussion concerning geometric vocabulary. Some students misunderstood the shape sort directions and made categories based on color or overall shape. Other students made one set of groups but then did not explore other ways to group the shapes.
	3	Use vocabulary to determine whether a shape matches a statement and justify the answer.	Students were given a picture of a rectangle, then a series of statements (e.g., "This shape is a quadrilateral" or "This shape is a trapezoid") in which they were to answer true or false and justify.	Moderate – Many students were engaged in recording their responses and justification on their papers. Some were engaged in the class discussion focused on each statement.

Table Continues

Table Continued

Grade 6	1	Divide a whole number by a fraction.	Students were given the problem of "How many 3/4 are in 14?" Students worked individually, then shared their answers with the class. During the whole class discussion, students shared their strategies for determining the answer. Students were given a problem focused on distributing fractional amounts of cheese blocks to make pizzas (e.g., Student had 9 blocks of cheese. How many pizzas can they make if each pizza uses: 1/3 block; 1/4 block; 1/5 block; 2/3 block; 4/3 block).	High – Most students were engaged in determining a solution when working individually. Most students provided strategies or described what they noticed or wondered about other student strategies shared with the class.
	2	Formatively assess students' use of different strategies to divide a fraction by a whole number or a whole number by a fraction.	The teacher provided students with five previously class-generated solution strategies—using a number line, drawing a picture, repeated subtraction—for determining the among of flour needed to bake a given number of muffins. Students worked with partners to discuss what they noticed and wondered about each solution strategy. Partner work was followed by a whole class discussion focused on what students noticed and wondered.	High – Most students were engaged in discussion both with their partner and whole class discussion.
	3	Identify connections among strategies for dividing a whole number by a fraction in context-free tasks.	Teacher provided students individual work time on a problem string (e.g., 5 divided by 1/4; 5 divided by 1/8; 5 divided by 1/16). Students shared their solutions after each problem.	High – Students were actively solving each string and engaged in the whole class discussion focused on strategies students used to solve each problem.

Standards for Mathematical Practices

To determine Frances' use of the SMPs, I completed a MCOP^2 observation protocol (Gleason et al., 2015) for each of the six lessons I observed: three in Grade 5 and three in Grade 6. For teacher facilitation—the role of the teacher to provide lesson structure and guidance through problem solving and mathematical discourse—Frances scored a median of 3 across all observations. For student engagement—students visibly active while completing tasks during class—Frances scored a median of 2.5 across all observations. Though I observed Frances' teaching both in Grade 5 and Grade 6, I chose to combine the overall means for both teacher facilitation and student engagement to display Frances' use of each SMP. Because the content taught to each of the grade levels differed, Grade 5 lessons focused on geometrical shapes and Grade 6 lessons focused on fraction computation, I found it useful to also show Frances' MCOP^2 scores for each of the grades.

Table 29 and Table 30 show Frances' scores per item and observation sessions for both Grade 5 and Grade 6, respectively. The item scores are organized by SMP. I have also included the median score for each of the SMPs.

Table 29

Distribution of MCOP² Item Scores for Each Observation - Frances Grade 5

MCOP² Item	Observation 1	Observation 2	Observation 3	Median for SMP
	Make sense of problems and persevere in solving them			**3**
1	3	1	3	3
2	3	2	2	2
3	3	1	3	3
4	3	2	3	3
5	2	0	1	1
9	3	2	2	2
11	3	2	2	2
14	3	2	3	3
16	3	2	3	3
	Reason abstractly and quantitatively			**0.5**
5	2	0	1	1
7	0	0	2	0
	Construct viable arguments and critique the reasoning of others			**2.5**
4	3	2	3	3
5	2	0	1	2
10	3	3	3	3
12	3	2	2	2
13	3	3	2	3
15	3	2	2	2
	Model with mathematics			**0**
7	0	0	2	0
	Use appropriate tools strategically			**1.5**
2	3	2	2	2
5	2	0	1	1
	Attend to precision			**3**
10	3	3	3	3
	Look for and make use of structure			**3**
1	3	1	3	3
6	3	1	3	3
8	3	3	3	3
	Look for and express regularity in repeated reasoning			**3**
1	3	1	3	3
4	3	2	3	3
6	3	1	3	3
8	3	3	3	3

Note. A rating of 0 indicated that I did not notice the action. A rating of 3 indicated that I noticed the action frequently.

Table 30

Distribution of MCOP² Item Score for Each Observation - Frances Grade 6

MCOP² Item	Observation 1	Observation 2	Observation 3	Median for SMP
	Make sense of problems and persevere in solving them			**3**
1	3	3	2	3
2	2	3	2	2
3	2	3	3	3
4	3	2.5	2	2.5
5	3	2	2	2
9	3	3	2	3
11	2	3	2	2
14	3	2	3	3
16	3	3	3	3
	Reason abstractly and quantitatively			**2.5**
5	3	2	2	2
7	3	2	3	3
	Construct viable arguments and critique the reasoning of others			**2**
4	3	2.5	2	2.5
5	3	2	2	2
10	3	2	2	2
12	2	2	3	2
13	2	3	2	2
15	3	3	3	3
	Model with mathematics			**3**
7	3	2	3	3
	Use appropriate tools strategically			**2**
2	2	3	2	2
5	3	2	2	2
	Attend to precision			**2**
10	3	2	2	2
	Look for and make use of structure			**3**
1	3	3	2	3
6	3	3	3	3
8	3	3	3	3
	Look for and express regularity in repeated reasoning			**3**
1	3	3	2	3
4	3	2.5	2	2.5
6	3	3	3	3
8	2	3	3	3

Note. A rating of 0 indicated that I did not notice the action. A rating of 3 indicated that I noticed the action frequently.

As seen in Table 29, Frances' use of "model with mathematics" was limited, possibly due to the mathematical content. I make this conjecture because in Table 30 shows that I observed "model with mathematics" in all three lessons with the Grade 6 students. It was evident that despite the mathematical content, Frances often engaged students in "making sense of problems and persevering" and "looking for and making use of structure," as seen by both SMP for Grade 5 and Grade 6 had medians of 3. Because most of the medians were greater than a score of 2 across all observations (see Table 31), I concluded that Frances was skilled at providing students with opportunities to engage with the SMPs.

Table 31

Median Scores of SMPs Across all Observation – Frances

	Make sense of problems and persevere in solving them	Reason abstractly and quantitatively	Construct viable arguments and critique the reasoning of others	Model with mathematics	Use appropriate tools strategically	Attend to precision	Look for and make use of structure	Look for and express regularity in repeated reasoning
Median	3	2	3	2	2	3	3	3

Note. A median of 0 indicated no use of SMP and a median of 3 indicated high use of SMP.

Mathematical Teaching Practices

To determine the use of MTPs (NCTM, 2014), I used the E-MCOP^2 to analyze each of the six observations. The purpose of the E-MCOP^2 was to record the visible MTPs the teacher used during each of the lessons. Similar to completing the MCOP^2 to record the use of SMPs, I used video recordings that allowed me to better identify actions and statements indicative of use of each MTP. Overall, Frances showed evidence of using all MTPs throughout the observations. Table 32 shows the extent to which there was evidence of Frances using MTPs in each of the six observations.

Table 32

Extent of Evidence of MTPs – Frances

	Grade 5			Grade 6		
	Observation 1	Observation 2	Observation 3	Observation 1	Observation 2	Observation 3
Establish mathematics goals to focus learning	Highly evident	Highly evident	Moderately evident	Highly evident	Moderately evident	Highly evident
Implement tasks that promote reasoning and problem solving	Highly evident	Highly evident	Highly evident	Highly evident	Highly evident	Highly evident
Use and connect mathematical representations	Highly evident	Highly evident	Highly evident	Highly evident	Highly evident	Highly evident
Facilitate meaningful mathematical discourse	Highly evident	Highly evident	Highly evident	Highly evident	Highly evident	Highly evident
Pose purposeful questions	Highly evident	Highly evident	Highly evident	Highly evident	Highly evident	Highly evident
Build procedural fluency from computational understanding	Highly evident	Highly evident	Highly evident	Highly evident	Highly evident	Highly evident
Support productive struggle in learning mathematics	Highly evident	Moderately evident	Moderately evident	Highly evident	Moderately evident	Highly evident

Table Continues

Table Continued

Elicit and use evidence of student thinking	Highly evident	Highly evident	Highly evident	Highly evident	Highly evident	Moderately evident

Note. "Not evident" indicates that there were no descriptors noticed. "Moderately evident" indicates there were up to half, including exactly half of the descriptors noticed. "Highly evident" indicates that there were more than half the descriptor noticed.

In determining the descriptors and alignment between the MCOP^2 and MTPs, I found that some MTPs required more descriptor categories than others. Table 32 shows that Frances was able to use most of the MTPs on a regular basis as seen by consistent ratings of "moderately evident" and "highly evident" across all observations. Overall, Frances used more of the MTPs throughout the observations than Kathy.

Research Question Two: Effective Teachers' Self-Efficacies, Beliefs, and Effective Practices

To answer my second research question, "How do teachers' instructional beliefs relate to their mathematical self-efficacy, mathematics teaching self-efficacy, and their use of effective teaching practices?" I begin with summaries from both cases to exemplify the possible relationship relating self-efficacy—mathematical and mathematics teaching. I then interweave evidence of each teacher's instructional beliefs with their use of effective teaching practices. After, I report on the relationships I noticed among self-efficacies, instructional beliefs, and enacted teaching practices.

Self-Efficacy

As stated previously, there seems to be some relation between each participant's MSE and MTSE (Kahle, 2008). Both Kathy and Frances' surveys indicated that they had both a high MSE and a high MTSE. However, I question the precision of those results as I consider the interview data. Kathy's interview comments showed that even though her MSE and MTSE were robust, overall, those beliefs may not have been as strong as the survey data indicated. Likewise, survey data indicated that Kathy's MSE was more robust than Frances' MSE, but after considering interview data, Frances' MSE appeared to be more robust than Kathy's MSE.

Instructional Beliefs and Implementation of Effective Mathematics Teaching Practices

According to my data, both Kathy and Frances had student-centered instructional beliefs, meaning that they both believed that students should be actively engaged in their own learning of mathematics. However, because Kathy chose more of the highest ratings (i.e., 5 or 6) for statements on the survey and Frances choose more ratings that was at the low end of the high ratings (i.e., 4), I conjectured that Kathy may have had more robust student-centered beliefs. The interview data, however, showed quite the opposite as each teacher described their upcoming lesson plans. During her interviews Kathy often used more teacher-centered terms (e.g., demonstrate, monitor) whereas Frances' descriptions of her planned lessons indicated that she valued more student-centered practices (i.e., eliciting and using student thinking, valuing risk taking). Thus, it seems that the survey results were not entirely consistent with evidence from the interviews. To be fair, the instruments' designers did not make claims about the survey's ability to identify fine distinctions. They suggested only that ratings could be roughly considered in the high end of the rating scale or the low end of the rating scale. My findings support the limited use of the instrument. Even so, my results show that teachers may respond in ways that overestimate their own beliefs, once those beliefs are closer to enactment. Though there was a difference in which of the highest ratings were chosen by each of the teachers, it is important to note that the instrument and the intent of the study was not to measure to what degree teachers had student- or teacher-centered practices. Instead, the focus was to determine whether the teacher had primarily student- or teacher-centered instructional beliefs.

With respect to the use of SMPs (NGA & CCSSO, 2010) and MTPs (NCTM, 2014), both teachers enacted effective teaching practices. Each teacher addressed some, but not all, effective teaching practices in each individual lesson. However, across all observations both teachers implemented most of the effective teaching practices.

Thus, overall, both Kathy and Frances exhibited student-centered instructional beliefs and implemented effective teaching practices (i.e., student-centered practices), regardless of which lens was used to measure practice.

Relationships Among Self-Efficacy, Instructional Beliefs, and Implementation of MTP

As for the potential relationship among teachers' instructional beliefs, MSE, MTSE, and use of effective teaching practices, given the evidence from these two cases, I believe there is a possible relationship, though more complex than previous research has noted using surveys and self-reported teaching practices (e.g., Allinder, 1994; Depaepe & König, 2018; Kahle, 2008).

Through survey results, I found both teachers exhibited high MSE and MTSE and had student-centered beliefs. Additionally, from observation data, I showed that both teachers engaged students in SMPs (NGA & CCSSO, 2010) and used MTPs (NCTM, 2014) across multiple lessons. But despite gathering data from multiple sources (e.g., surveys, interviews, classroom observations), I found the relationship among MSE, MTSE, instructional beliefs, and enacted teaching practices to be complex. I found it difficult to determine how one factor influenced another or the interplay among the factors. Since both teachers had student-centered instructional beliefs, it was difficult to determine how their self-efficacies contributed to having student-centered instructional beliefs. There is the possibility that both Kathy and Frances chose to continue to use student-centered practices despite teaching during challenging circumstances because they had robust student-centered instructional beliefs. Or it is possible that because of their high self-efficacy, they both felt capable of employing effective teaching practices despite the COVID restrictions.

Teacher Examples of Relationships Among Self-Efficacies, Instructional Beliefs, and Practices

Despite the relationships among self-efficacy, instructional beliefs, and practice being complex, I noted connections among the factors in both Kathy and Frances. I share two examples of the apparent links, one for each teacher.

Kathy. When Kathy often identified kindergarten mathematical concepts as “being easy for adults” showing her confidence in her ability to complete kindergarten leveled mathematical tasks (MSE) and in her ability to monitor mathematics activities and understanding a mathematical concept enough to help students understand it better (MTSE). For example, Kathy described her overall confidence in supporting students in conceptually understanding mathematical concepts and her ability to guide the students through her questioning capability. In Kathy’s end-of-study interview, she stated:

> Understanding, again, some of this, what I might think is simplistic mathematically, how important that is to build foundations to make those conceptual connections. I mean, I just didn't know any of that. And now that I do, I don't overlook those things anymore. Or skip them or whatever. And I would say just kind of that that whole probing and questioning, also…now as a teacher, I try to find it, I always can, something they're doing, right. They're on the path to getting there. And I often point that out. I'm like, “Okay, here's what you've got, right? You figured this part out. But this is what we're

asking. And we need to do more" …Or, "Yes, you started here, but you need to go backwards instead of forward." Those types of things, kind of validating where they are in the process, and where they need to move to.

In this excerpt, Kathy's statements offer insight into her self-efficacies and her instructional beliefs. When she stated that the content was "easy for adults" I interpreted that as indication that Kathy's MSE is high with respect to the content she teaches her kindergarten students. When Kathy noted that it was important to "build foundations to make those conceptual connections," I interpreted that as a statement about her instructional beliefs. When she followed up by stating that she doesn't "overlook those things [concepts] anymore," I classified that as statement about her MTSE because she seemed to be implying that she had some confidence in her ability to enact teaching for conceptual understanding. Kathy exemplified her use of "probing and questioning" with some typical statements that she used with children. Kathy seemed to be using those examples to illustrate her techniques for helping students develop conceptual understandings.

Kathy then exhibited an alignment between her instructional beliefs and the practice of probing and questioning students, during a lesson focused on adding to 10 to make teen numbers. Kathy began the lesson having students sort coins (dimes and pennies). During this lesson, a student stated they sorted "by ones." Kathy followed up with, "explain that to me, show me what you mean?" This example shows the alignment between Kathy's instructional beliefs and practices because her probing questioning was focused on students' conceptual understanding of place value.

Overall, I see several intertwined threads that imply possible connections among her instructional beliefs and practices, and these woven threads appear to be supported by the confidence underlying Kathy's self-efficacies. During the end-of-study interview, Kathy explained that she believed that it was important to "build foundations to make those conceptual connections." And during Kathy's enactment of the lesson, she supported the development of students' conceptual foundations of tens and ones by asking about their thinking as students sorted coins into piles of dimes and pennies. Because Kathy indicated that the content of place value was "easy for adults," thus, implying a level of confidence in her own content knowledge (MSE), it is possible that Kathy's MSE contributed to both her instructional beliefs and her enacted practices. Likewise, Kathy's confidence in her own ability to probe students' conceptual understanding ("I don't overlook those things now") also appeared to link to Kathy's instructional beliefs and practices, at least in her own mind.

Frances. When Frances stated that she was completely confident with fractions (MSE) and in her ability to teach fractions (MTSE), which was evident in both her survey results and interviews. For example, responding to a question during the end-of-study interview, focused on Frances favorite concept to teach, she responded, "I like a good fractions unit." Frances continued by describing her enjoyment of students "putting [fraction] concepts together and pulling in stuff we've known" and her confidence in teaching fractions (MTSE). Frances then went on to describe why she liked teaching fractions which provided evidence of her student-centered instructional beliefs focused on students building conceptual understanding of fractions through exploration using different representational models, which she referred to as "tools."

> Because the tools we start off with, just writing statements for every blank and just making comparisons which they love to do. Then we start using a tape diagram to help us organize and problem solve. Then we use a double number line…We started using squares and circles to represent our ratio, as a comparison. Then that got too

> cumbersome. "We don't want to draw 50 circles, right?" Then we use the tape diagram, but I don't want to draw tons of blocks [as values increase] and then we get to the double number line… And then we go to a ratio table, "Oh, this is the best thing ever." I like seeing them start making connections and that progression…And then you connect it to precents…I feel like it's pretty natural. For most of them to be like, "Oh, I can just use this tools that I use for ratio, because the ratio is a percent."

In the excerpt, Frances described the exploration of different tools (e.g., tape diagram, double number line, area models, ratio table) that she believed support students' conceptual understanding of fractions by leading them through a progression of representations from ratios to percents.

During the first observation for Frances' 5th grade classroom, the lesson focused on the beginning concepts of fraction multiplication, students were given five student-generated strategies (see Figure 9) from a previous assignment. In the pre-observation interview, Frances stated, "I left some incorrect ones [strategies] in that were kind of on point. With just enough incorrect that hopefully, we can elicit some discussion of, 'Why? Maybe that doesn't make sense.'"

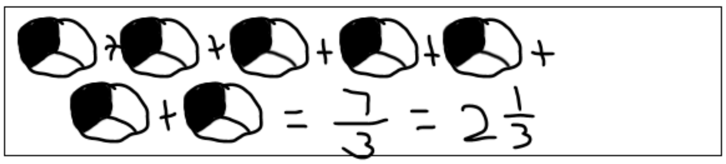
Figure 9

Student Generated Solution Strategies for 1/3 x 7

1/3 + 1/3 + 1/3 + 1/3 + 1/3 + 1/3 + 1/3 = 2 1/3

Way #2 – Groups of: 7 groups of 1/3 or 1/3 groups of 7 (7 groups of 1/3 makes the most sense)

I could do 1/3 x 7 = 7/21. I did this by multiplying the 7 by both numbers.



1/3 + 3/3 + 3/3 + 3/3 + 3/3 + 3/3 + 3/3 + 3/3 = 7 1/3

During the observation, Frances handed out copies of the strategies, assigned students to small groups, and directed students to discuss the different strategies. As part of the discussion, students were to state whether they agreed or disagreed with each strategy and why. After students had an opportunity to review and consider the five strategies, Frances engaged the whole class in a discussion about the strategies, stating,

> Who wants to start us off? This could be “I agree with this statement because…,” “I disagree with this statement…,” “I don't even know what this strategy even means…,” “I wish I could talk with the student who wrote it because I'm not sure….” Um, remember we are not disagreeing with anyone who did it, we might just be disagreeing with their idea [be]cause we don't understand it and that is okay too.

This example shows how Frances' instructional beliefs—students playing a primary role in mathematical discussion and student exploration of mathematical concepts to build conceptual understanding—aligned with her enacted practices, as shown during the observation, as students had ownership over the strategies and discussion.

Overall, I noticed several aspects to Frances' thoughts toward the teaching and learning of fractions that suggest possible connections among her instructional beliefs and practices. These aspects appear to be reinforced by her self-efficacies specific to fractions. During the end-of-study interview, Frances explained the importance of representations ("tools") when helping students build a conceptual understanding of fractions. In addition, during a pre-observation interview, Frances stated, "hopefully, we can elicit some discussion" surrounding students' incorrect strategies. Because Frances indicated that she was confident in her fraction knowledge (MSE) on the survey (see Appendix C) and noting that she "likes a good fractions unit" (MTSE), it is possible that her self-efficacies contribute to her instructional beliefs and enacted practices. Similarly, Frances' confidence in teaching fractions using a progression of representations and strategies to build students' conceptual understanding indicated a link between Frances' instructional beliefs and practices.

2020 Pandemic

Because my study took place during the 2020 pandemic and subsequent school year, I considered the effects of this context on teachers' self-efficacies and instruction. Vaccines were scheduled to be available to teachers in January 2021, however, in practice, many teachers were unable to get the vaccine due to shortages and priority to older individuals and health care workers (State off Illinois, 2023; Stock, 2021). Because vaccines were only available to people aged 16 and older at the end of the 2020–2021 school year, masks and social distancing were the primary methods to prevent the spread of the COVID-19 virus among school-aged children throughout the time I was collecting data. In the following section I describe the results of my analysis related to the effects of the pandemic on each of the teachers. I then answer my third research question, reporting on the possible influences the pandemic had on effective teachers of mathematics.

To collect data pertaining to the possible influences of the 2020 pandemic on enacted teaching practices, I specifically posed, "How would this lesson be different if you were teaching in the absence of COVID restrictions?" during the pre-lesson interview. To address the question, I scoured the teachers' interview transcripts for any statements related to the effects of the pandemic on their self-efficacy or practices (see Table 33). Within Table 33, I have bolded statements that suggested a focus (e.g., change in practices or influence on a belief) and noted the focus.

Table 33

Statements for Teachers Focused on Changes Due to the Pandemic

Teacher	Statement	Further Detail	Focus
Kathy (Pre-lesson interview, March 11, 2021)	"This lesson is right about the time we went remote, so this might be my first time teaching this lesson…Ideally, I would demonstrate the game and then the children would play with partners. **I've tried partner play, but they have difficulty seeing what each other is doing and have difficulty staying focused."**	Kathy described how she did not have previous experience with this lesson due to a quick shift to remote learning the previous year and using a new curriculum. In addition, Kathy noted that because of the shields between children, students struggled to keep focused and see what their partner was doing.	Change in practice
Kathy (pre-lesson interview, March 25, 2021)	"For math workshop, children would **typically share markers and supplies,** there would not be two of everything. They would be **closer together filling in information together**…They **could work with anyone in the room, not just the person across from them.** In fact, we **don't typically sit at tables all day**…there would be tubs on a shelf for students to grab and move all over the room to work at tables or on the floor. **The students would have freedom to change from various workstations when they want."**	Kathy described the changes that were made to their math workshop time due to COVID restrictions that were designed to limit close contact—contact with those infected with COVID-19—with other students.	Change in practice

Table Continues

Table Continued

Kathy (End-of-study interview, April 21, 2022)	“It was **very hard** for me to teach math [during the pandemic].”	Kathy noting her difficulty teaching math while remote teaching and having a newer curriculum.	Influence on MTSE
Kathy (End-of-study interview, April 21, 2022)	“But that **sense of doing math in isolation,** you know, **they didn’t work with partners.** They didn’t work in small groups. All the materials were their own materials. That was really hard. And **I felt that was not best practice,** that was not what we should be doing or wanted to do.”	Kathy noted her conflict between best practices in teaching mathematics and how those were not seen while remote teaching.	Change in practice
Frances (Pre-lesson interview Grade 5, March 22, 2021)	“Probably do a **little more whole group** during COVID restrictions than I normally would do.”	Frances discussed that, though she would like to get students in smaller groups, she perceived that she may have been doing more whole class discussions due to restrictions.	Change in practice
Frances (Pre-lesson interview Grade 5, May 3, 2021)	“I know with last year, you know, in **March, kind of learning, shifting, with the pandemic, to remote [learning]** and then what as a fourth grader, **what did they [the students] get shape wise?”**	Frances discussed that she needed to change this lesson from previous years because she was not confident that the students would have adequate prior knowledge related to shapes and related vocabulary.	Response to possible learning loss
Frances (Pre-lesson interview Grade 6, March 23, 2021)	“I think I did this lesson **so that you talk to someone else and think through it.** So, my plan is to continue [the] course, like I **normally would**…I’m not necessarily doing much different for this lesson because of COVID.”	When Frances’ responded to the pre-lesson question focused on changes made to the lesson due to COVID restrictions, she indicated that she had not changed her implementation significantly.	Choosing not to change practice

Table Continues

Table Continued

Frances (Pre-lesson interview Grade 6, March 30, 2021)	"So, what I've been doing on Fridays, when we're all remote, is **kind of shifting towards another unit, and working on things that are more, I think, conducive to a remote environment."**	Frances stated that she did not find it helpful to have students work on the current unit's material (i.e., dividing fractions and whole numbers) while the class met remotely on Friday's. Instead, Frances focused on other material that was within the Grade 6 standards but was more adapted for effective remote instruction.	Change in practice
Frances (Pre-lesson interview Grade 6, April 6, 2021)	"There are days though, where I'm like, 'wow, I'm how many weeks now into this, like three.' **I think there are things that are worth taking the time on. I don't know. And I would hate to sacrifice that this year, because I felt like pressure to just get it done.** So, on a normal year, I'd see them every day. I I think that I notice a difference, **at least on my end of the consistency from day to day to day, like five days a week, we would have touched on this, maybe not as much in a day,** but like, pretty close, like a lot. And **now I'm only seeing them in-person two days,** so, **I've been trying to break it up a little bit so we're not working on problems for an hour.** But I think consistency would help. I feel a little bit disconnected from it, because I'm only in it [dividing fractions] two days a week with them.	In this statement, Frances commented that she felt concerned that students might be uncomfortable spending so much time discussing one fraction division task during in-person class. She implied that this was her normal way of treating the content, but that students were not used to this process of comparing and connecting strategies because they were only in-person two days per week. On the other hand, Frances stated that she chose to continue her practice facilitating lengthy task discussions for fraction content because the time was well spent. Her concern about the pandemic was that students were experiencing inconsistent classroom norms between in-person and remote instruction.	Response to possible learning loss AND Change in practice

In my researcher's journal, I noted any environmental elements I noticed in the physical classroom. In Table 34, I provide some examples of classroom adaptations I noticed during my observations.

When students returned in fall 2020 (prior to my observations), the district protocols allowed half the students to attend school in-person, wearing masks, while the other half worked remotely. The in-person and remote students switched roles every other day. In addition, the district recommended teachers to place one student per table and the seats were to be at least ten feet apart. Group work was discouraged but if students were to engage in group work, plexiglass shields were to be placed between students. Protocols were updated for spring 2021, when I completed my observations. All students were allowed to attend in person Monday through Thursday, though families could choose to have their children learn remotely. Each Friday was a remote learning day for everyone. During my observations, I noted that Kathy had one student and Frances had three students attending remotely via an online video platform. As shown in Table 34, there was a difference in schedules depending on grade level. In addition, there were updates to group work protocols, allowing teachers to have students engage in group work.

Table 34

Classroom Adaptations as a Result of COVID Restrictions During Spring 2021

Classroom Adaptation	Description
Kindergarten	Throughout the classroom, I noted there were plexiglass dividers between students who sat across from one another.
Kindergarten	Students were in-person Monday through Thursday and asynchronous on Friday.
Grade 5 and 6	Students were in-person two days per week for 90-minute sessions. Friday all students were asynchronous.

Table Continues

Table Continued

All classrooms	Students and teachers wore masks for the entirety of each lesson.
All classrooms	There was at least one but no more than three students attending class through an online platform who were not physically in the class.
All classrooms	Students were spread out, often having one student per table. Students were often staggered in Frances' classroom and in Kathy's students were on either end of each table

As shown in Table 33 and Table 34, there were several changes that both teachers noted of described changes in the classroom environment. Throughout my time in Kathy's classroom students often worked with the same partners—those who sat across from them. In contrast, Frances tended to have different pairings each time I visited. This difference could be partly due to the differing modes of instruction, despite both teachers being in the same district. Kathy led in-person classes every day except Friday, however Frances led in-person classes twice a week per grade level (e.g., Grade 5 on Monday and Wednesday and Grade 6 on Tuesday and Thursday), leading two classes of students each day they attended. It was evident that both Kathy and Frances had to change their teaching practices and classroom routines as a result of the pandemic, often to a point at which they noticed that they were always living up to their own standards for best practice.

Thus, as a result of these restrictions, neither Kathy nor Frances was able to continue their normal classroom structure and routine due to the influence of the 2020 pandemic.

Research Question Three: Effects of the Pandemic on Teachers

For my last research question, I hoped to contribute to the expanding literature focused on the effects on teachers of the 2020 pandemic. In addition, I wanted to learn whether teachers reported that the MSE, MTSE, and enacted teaching practices were affected by virus mitigation procedures in schools, hybrid classes, and remote teaching.

Effects of the Pandemic on Teachers' Self-Efficacy

Because I began collecting data after the pandemic began, I was unable to determine Kathy and Frances' pre-pandemic self-efficacy and therefore do not have evidence of changes in their self-efficacies due to teaching during the pandemic. Though purposefully posing questions throughout the pre- and post-lesson interviews and the end-of-study interview, both teachers described a possible fluctuation in their self-efficacy due to the pandemic. Kathy reported that she found it hard to teach during the pandemic, showing possible evidence of a decrease in Kathy's MTSE. While Frances' self-efficacy seemed to fluctuate due to the uncertainty of knowing how students might respond to a task due to a lack of prior understanding as a result of learning remotely.

Though there was evidence to support possible effects from the pandemic on each teacher's MTSE, I could not find direct evidence that either teacher's MSEs changed as a result of teaching during the 2020 pandemic. Next, I connect my work connecting pandemic effects on teaching practices.

Effects of the Pandemic on Teachers' Instructional Practices

The pandemic effects I have described were particular to the spring of 2021, when I conducted my observations. For most of the time I was collecting data, vaccinations were not yet available. Students and teachers wore masks and were seated in ways that complied with physical distancing requirements of 3 ft between students. Kathy's class used physical, plexiglass dividers between children as well. Although there were grade level distinctions, both teachers used some version of hybrid instruction on Mondays–Thursdays. And, both teachers taught remotely on Fridays. In addition to these visually observable distinctions and delivery mode changes, both teachers reported making changes to classroom procedures as a result of the pandemic.

Kathy reported relying primarily on individual instruction, with some use of pairs for collaboration. Pairs were the only viable collaborative option because the plexiglass dividers hampered communication. Although Frances did not have plexiglass barriers between students, she strategically paired remote students with in-person students who could more efficiently navigate the Zoom breakout rooms. Both of these grouping approaches were distinct from pre-pandemic practices for these teachers. Both teachers reported using more whole-class instruction.

Both Kathy and Frances reported ways in which teaching through a pandemic had altered their teaching practices in a negative way. Kathy reported that she found mathematics hard to teach remotely. Meanwhile, Frances had difficulty knowing where her students' mathematical understanding would be after spending the spring 2020 semester online because students had not been required to submit work between March and the end of the school year. Both teachers indicated that the whole-class, individual, and grouping changes were needed to adapt to the situation, but were not as effective as their preferred, pre-pandemic approaches.

Disruptions to daily standard practice were commonly reported by teachers during the pandemic (Aldon et al., 2021; Barlovits et al., 2021; Echeverría et al., 2022), however I did not find evidence that teaching during a pandemic altered Kathy nor Frances' instructional practices from pre-pandemic times as compared to post-pandemic times. Though I was not in the classroom once all COVID-19 restrictions were lifted—both teachers were still on an altered schedule at the completion of the study—there was evidence that they both were possibly returning to their pre-pandemic routines. For example, by the last observation it was evident that Kathy no longer was concerned about limiting student collaboration to paired table partners when playing mathematical games. Likewise, in a pre-lesson interview in March Frances stated she was engaging students in more whole group discussion. But by the last observation in May, Frances' students spent a majority of class time interacting in small groups.

CHAPTER V: SUMMARY, DISCUSSION, AND IMPLICATIONS

Overview

The purpose of this study was to investigate potential relationships among mathematical and mathematics teaching self-efficacy, instructional beliefs, and enacted teaching practices of teachers who were labeled as effective. During the 2020 pandemic, I recruited two teachers—both identified as effective—to participate in this study. Both participants worked in the same mid-sized urban elementary school in the midwestern United States. Kathy, a Kindergarten teacher, and Frances, who taught Grade 5 and 6 mathematics, completed surveys and participated in interviews focused on classroom observations. At the conclusion of the observations, each participated in a stimulated recall end-of-study interview.

This chapter begins with a summary of my research questions followed by a discussion focused on revisiting Wilkins' (2008) model concerning the relationships between teacher background characteristics and teacher practices. I then discuss various themes as they relate my results with existing literature. Next, I discuss what was learned about effective teachers and their teaching practices as a result of the 2020 pandemic. At the conclusion of this chapter, I discuss the limitations and implications of this study.

Research Questions Revisited

Three research questions were posed during this study. First, how are mathematical self-efficacy and mathematics teaching self-efficacy related in mathematics teachers who have been labeled as effective? Evidence indicated that there was a relationship between Kathy and Frances' MSE and MTSE, as surveys and interview data indicated they both had high MSE and MTSE. Second, how do teachers' instructional beliefs relate to their mathematical self-efficacy, mathematics teaching self-efficacy, and their use of effective teaching practices? The results indicated that there were relationships among these factors but that the relationships among beliefs and practices continues to prove complex. Lastly, how did the spring 2020 coronavirus school shutdown, the immediate transition to remote learning, and the atypical fall 2020 semester, influence effective mathematics teachers' self-efficacies and instructional practices during the spring 2021 semester? I found that the pandemic had a lasting influence on teachers, and they were remarkably resilient despite challenging situations they faced.

Connecting Results to Literature

Throughout reviewing literature as it pertained to my results, I found ten themes. In this section, I use these themes to relate my results to literature.

Connections to Self-Efficacy

Self-Efficacy and Instructional Beliefs. In this first theme, connecting self-efficacy to instructional beliefs, I found both Kathy and Frances to have high MSE and MTSE along with having primarily student-centered (i.e., constructivist) instructional beliefs. These findings align with Bas (2022) and Gürbüztürk and Şad (2009) who both measured the three dimensions of general teaching self-efficacy—student engagement, instructional strategies, and classroom management—in relation to teacher beliefs in a population of student teachers. Furthermore, Çobanoğlu (2011) studied general self-efficacy and teacher beliefs in conjunction with curriculum implementation. These researchers noted a positive correlation between teachers' self-efficacy and their teacher instructional beliefs. Bas (2022) found that teacher beliefs were able to predict self-efficacy while Gürbüztürk and Şad (2009) found a positive correlation between constructivist teaching beliefs and self-efficacy in relation to student engagement—one dimension of the teacher self-efficacy scale (Tschannen-Moran, Woolfolk Hoy, & Hoy, 1998). Interestingly, Gürbüztürk and Şad (2009) found that traditional teaching beliefs (i.e., teacher-centered) were positively correlated with self-efficacy related to classroom management and instructional strategies—two dimension of the teacher self-efficacy scale (Tschannen-Moran, Woolfolk Hoy, & Hoy, 1998). I find this to be interesting as it contrasts with what I noticed during interviews with Kathy and Frances. Though my focus was on situation specific self-efficacies, I found that both teachers focused their discussion of instructional strategies, a dimension of general teaching self-efficacy, on student-centered beliefs, not on teacher-centered beliefs as Gürbüztürk and Şad (2009) had noted that their participating student teachers held more traditional teacher beliefs.

Self-Efficacy and Student-Centered Practices. Allinder (1994) and Depaepe and König (2018) researched the connection between general teaching self-efficacy and self-reported teaching practices. Allinder (1994) found that special education teachers who had high self-efficacies were more experimental with their choice of instructional practices (i.e., reform-oriented, student-centered). Likewise, Depaepe and König (2018) sought to determine relationships among general pedagogical knowledge (GPK), self-efficacy, and self-reported instructional practices of PSTs. The authors determined that though there were weak associations between GPK and self-efficacy and GPK and instructional practices, there was a strong correlation between self-efficacy and instructional practices. My findings are able to support and extend the findings of Allinder (1994) and Depaepe and König (2018) because I found that both Kathy and Frances had high self-efficacies and both engaged students in student-centered practices. Similar to Gibson and Dembo (1984), I was able to show that these relationships were still present in the teachers when an outside observer assesses the use of student-centered practices in mathematics, rather than relying solely on teacher self-reported practices. Though unlike the authors, I was able to extend the literature concerning mathematics specific self-efficacy and enacted instructional practices in effective mathematics teachers.

Self-Efficacy and Engagement With Professional Learning. As noted previously, instructional beliefs are complex in nature as they integrate one's beliefs concerning the nature of mathematics and mathematics teaching (Ernest, 1989a, 1989b). Self-efficacies are also complex in that they stem from experiences and are situation specific (Bandura, 1986, 1997). Bas (2022) noted the complexity of these beliefs in teachers. The author found that student teachers' instructional beliefs and their attitudes toward mathematics (i.e., nature of mathematics) were mediated by teacher's self-efficacy which then predicted their level of motivation to teach. The predictive influence of self-efficacy on motivation to attain a goal echoes Bandura (1997). Likewise, my results are able to support this relationship. Both teachers showed student-centered beliefs and positive attitudes towards the teaching and learning of mathematics. In addition, they both held high MSE and MTSE, therefore supporting their motivation to continue growing as effective mathematics teachers through PD or coursework.

Situation Specific Self-Efficacy. There were several instances where the specificity of both MSE and MTSE were evident for both Kathy and Frances. Each teacher recounted a specific instance or experience in which they felt less efficacious, although their overall MSE was high. For Kathy, she experienced a desire for an algorithm to solve a word problem because she was less confident about her capability to draw upon a conceptual understanding. Similarly, during a college algebra course for teachers, Frances experienced a struggle with the course content, which further illustrated the situation-specific nature of her MSE. However, when Frances described her hesitancy toward teaching expressions and equations, she qualified her comments about her confidence by stating that she imagined that there existed a more efficient method to teach the concept than she could envision in the moment. This qualification indicated that perhaps Frances felt capable, just not fully prepared in the moment. Bandura (1986) noted that self-efficacies are situation specific and can differ in *level* and *generality*. Although both teachers experienced situations at a specific level—limited to a single task or situation—in which they might have felt less efficacious, their general feelings of capability concerning mathematical tasks remained high.

Further, according to survey results, Kathy's MTSE was more robust than Frances' MTSE. Yet, though through interviews, I was able to document instances where Frances seemed to be more robust than Kathy in her MSTE. Thus, by reporting both survey results, to measure generality MSE, and interview findings, which provided insight in more specific self-efficacies, provided an opportunity to investigate self-efficacy comprehensively. This observation echoes Glackin's & Hohenstein's (2018) call for using qualitative means to identify nuances in teachers' self-efficacy. Without conducting interviews, I might have missed the opportunity to document the full picture of the teachers' self-efficacy.

Sources of Self-Efficacy. My interviews showed that, for my participants, self-efficacies are not static. They change based on one's experiences and one's interpretation of those experiences (Bandura, 1986, 1997; Hackett & Betz, 1989; Tschannen-Moran et al., 1998; Wilkins, 2008). Both Kathy and Frances recounted experiences that may have negatively influenced their self-efficacy. But when reflecting on those experiences both teachers found ways to think positively about the outcomes of the experiences or their potential to be effective despite those challenges. Recall that experiences during which one successfully completes a task or has a positive outcome are categorized as mastery experiences (Bandura, 1997). Kahle (2008) postulated that an increase in MSE through mastery experiences, could, in turn, influence one's

MTSE. Though I was unable to find evidence to support Kahle's postulation, I did find that mastery experiences did appear to influence the teachers' MTSE. For example, Kathy, through a PD experience, received training focused on posing purposeful questioning during mathematics instruction. This PD experience allowed Kathy to experience successful outcomes in terms of questioning students which increased her MTSE related to questioning (Conroy et al., 2019). For Frances, she noted feeling more confident after she had persevered through a difficult graduate level algebra course. So, these mastery experiences seemed to support each teacher's self-efficacy. Additionally, vicarious experiences (Bandura, 1997) potentially influenced Kathy's self-efficacy, specifically her MTSE. Kathy described her experience watching (i.e., vicariously experiencing) the PD facilitator posing questions to students. When Kathy described these events, she stated that the experience of seeing the facilitators successfully employ the questioning strategies gave Kathy the confidence to pose more purposeful questions to her own students.

Relationship Between MSE and MTSE

Having both survey and interview data allowed me to investigate the relationship between teachers' MSE and MTSE. Through interviews, I found that both teachers recounted experiences in which their self-efficacy as learners of mathematics (e.g., MSE) influenced their beliefs about their capability to teach mathematics (e.g., MTSE). Both teachers reported negative experiences as a learner, for Kathy it was with solving word problems and Frances struggled with an algebra for teachers course. Both teachers exhibited with high MSE and MTSE using survey and interview data. These experiences mirror the results from Coppola et al.'s (2003) study. Coppola et al. found that PSTs used their past experiences with learning mathematics—crucial element in developing MSE—to form their beliefs on teaching mathematics (e.g., MTSE). In other words, those who had positive experiences with mathematics had a positive outlook on their capabilities in teaching mathematics, though the inverse was not always true. Those who had negative experiences with mathematics learning did not consistently have a negative perspective on teaching mathematics.

The results I have summarized echo Zuya et al. (2016) and Kahle's (2008) finding that there was a relationship between teachers' MSE and MTSE. The authors used survey results to determine a significant relationship between MSE and MTSE. Kahle (2008) found that if a teacher had a low MSE then they were more likely to have a low MTSE. Likewise, a teacher with a high MSE was more likely to have a high MTSE. In addition, Kahle noted that out of the 75 teachers in her study, her data showed that only one teacher had both a low MSE and a high MTSE and no teachers exhibited the converse—a high MSE with a low MTSE. Although my findings are consistent with Kahle's findings, my results—incorporating observation and interview data—offer more insight into the complexity of teachers' self-efficacy (Glackin & Hohenstein, 2018).

Instructional Beliefs and Practice

Torff (2006) investigated expert secondary teachers, representing a variety of subject areas, and their beliefs about using critical thinking activities—which aligns with student-centered practices allowing for student exploration of skills and concepts—with high and low advantaged learners. In their study, Torff found that expert teachers, who showed stronger support for using critical thinking activities, were more likely to engage both high- and low-advantaged learners in high critical thinking activities. Because critical thinking activities are consistent with tasks in student-centered classrooms, I considered whether Kathy and Frances behaved similarly to Torff's participants because despite having diverse learners, they continued to use worthwhile tasks and student-centered practices.

Some (Gay, 2012; Perry, 2007) researchers have examined the relationship between instructional beliefs and practices for effective teachers of mathematics. These authors found that teachers' instructional beliefs, which were often student-centered, were often aligned with the teachers' instructional practices. Kathy's and Frances' data echoed these findings about effective teachers. However, when other researchers (Raymond, 1997; Yurekli et al., 2020) have looked at the relationship between beliefs and practices without focusing specifically on those who were recognized as being effective, these authors have noted inconsistencies between the beliefs and practices. Specifically, some teachers report holding student-centered beliefs but, in practice, these teachers enact teacher-centered practices. Raymond (1997) ascribed the apparent conflict to teachers feeling that they had insufficient time for planning and implementing student-centered practices, worries about state testing, or uncooperative behavior from students. The participants in Yurekli et al. (2020) noted that they found it difficult to implement student-centered practices. My results contradict these author's conclusions, as both teachers' instructional beliefs and enacted teaching practices aligned with student-centered pedagogy. Perhaps this indicates that effective teachers find ways to overcome the challenges reported by a more general teaching population.

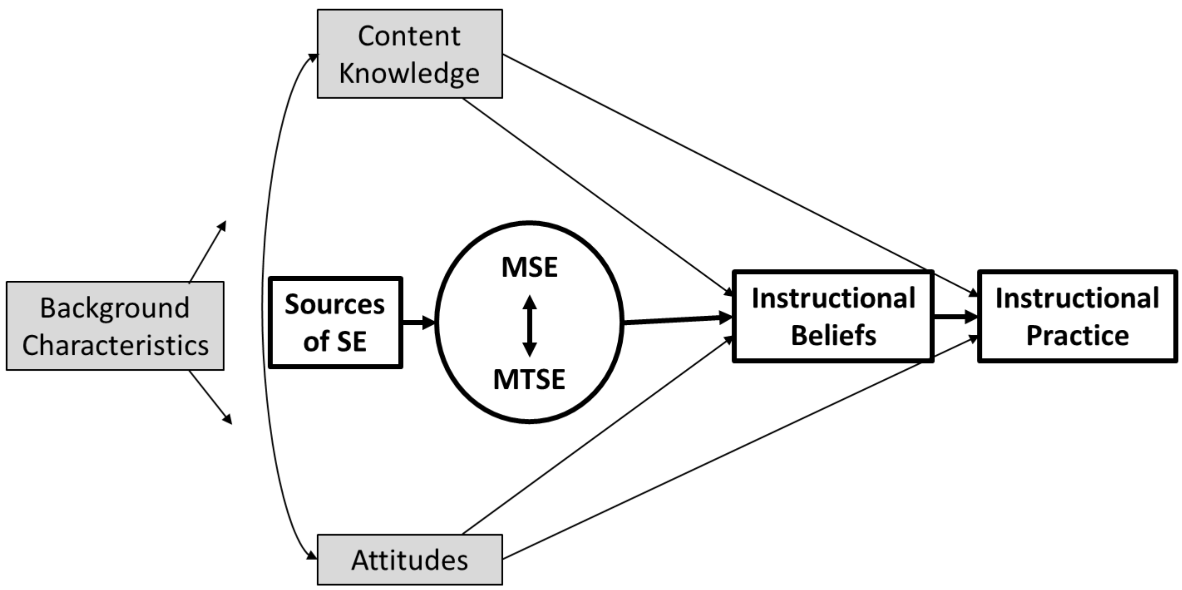
Revisiting Wilkins' Model

In examining connections between my findings and the literature, I revisited Wilkins' (2008) theoretical model (see Figure 1) which showed how various teacher attributes (i.e., background characteristics, content knowledge, instructional beliefs, and attitudes towards mathematics and mathematics teaching) interact with one another and mediate instructional practices. Based on his empirical results, Wilkins theorized that teachers' instructional beliefs are influenced, in part, by teachers' affective factors, including their attitudes towards mathematics and mathematics instruction. Self-efficacies—also affective factors—have separately been shown to be related to instructional decisions (Allinder, 1994; Conroy et al., 2019; Depaepe & König, 2018; Gibson & Dembo, 1984). Furthermore, Ernest (1989a, 1989b) discussed how beliefs concerning the nature of mathematics and beliefs about the teaching and learning of mathematics can both influence one's instructional beliefs as well as how these instructional beliefs are pivotal in teachers' decision on enacted teaching practices. For these reasons, I considered how self-efficacies might fit into Wilkins' model.

Based on findings from other research, I added detail to a portion of Wilkins' model. I shared this model in Chapter 2 (see Figure 6) to illustrate my hypothesis that self-efficacies are encompassed by the larger family of attitudes that Wilkins (2008) stated were comprised of attitudes toward mathematics and mathematics teaching. In this way, like attitudes in Wilkins original model, I hypothesized that self-efficacies are mediated by instructional beliefs to, in turn, influence the enactment of teaching practices. Here, in Figure 10, I share a revised portion of Wilkins' model, based on my findings in this study. My revised model shows that self-efficacies are complex affective factors that are distinct from other attitudes. These factors are distinct because there is a dynamic relationship among the influences (i.e., personal, behavioral, and environmental) that build one's self-efficacy (Bandura, 1977, 1986; Pajares & Usher, 2008). Bandura (1977) referred to this dynamic and influential relationship as triadic reciprocal determinism. Though personal experiences might influence one's attitudes towards mathematics and mathematics teaching, one does not often reflect on their attitudes even when they do reflect on their behavior (Bandura, 1977, 1986). This act of self-reflection and the dynamic relationship among influences on one's behaviors distinguishes self-efficacy from other teacher attitudes. The revised Wilkins' model I propose reflects my insights into the roles that MSE and MTSE play in the complex relationship among instructional beliefs and enacted teaching practices.

Figure 10

Proposed Model Relating Teacher' MSE, MTSE, Instructional Beliefs, and Enacted Teaching Practices



Note. This model was adapted from Wilkins, J. L. M. (2008). The relationship among elementary teachers' content knowledge, attitudes, beliefs, and practices. *Journal of Mathematics Teacher Education, 11*, 139–164. (https://doi.org/10.1007/s10857-007-9068-2). Components of the diagram in bold reflect evidence found in the current study. Factors that I did not investigate but that were part of Wilkins (2008) model include text in shaded boxes and the non-bold arrows that illustrate relationships.

In Figure 10, I illustrate the connection among the factors I examined in this study. Consistent with prior research (Bandura, 1986, 1997; Hackett & Betz, 1989; Kahle, 2008; Tschannen-Moran et al., 1998), I found compelling evidence in my participating teachers' descriptions of their experiences that Bandura's (1986) sources of self-efficacy (specifically, mastery and vicarious experiences) affected their MSE and MTSE. My adaptations to Wilkins' model also reflect my conclusions that teachers' situation-specific self-efficacies of MSE and MTSE are related. However, contrary to Kahle's (2008) conjecture that MSE directly influences one's MTSE, I found a counterexample within Frances' data. Frances struggled with her confidence in a college algebra course for teachers but still believed she was capable of designing a classroom experience around similar content. For this reason, my arrow between MSE and MTSE is bidirectional, indicating that either self-efficacy may influence the other, or that each self-efficacy may influence instructional beliefs independently. Finally, because my findings re-confirmed Wilkins' conclusion that instructional beliefs are associated with enacted instructional practices, that relationship arrow is indicated in bold.

Although I did not seek to investigate connections between self-efficacies and attitudes, I surmise that there may be a complex relationship between attitudes and self-efficacies. While espousing their instructional beliefs, the teachers made statements that may indicate that attitudes about mathematics and mathematics teaching may be linked to both self-efficacies and instructional beliefs. Others (Ernest, 1989a, 1989b; Kahle, 2008) have found that a person's attitudes about mathematics may influence the development of one's MTSE. Yet it is also possible that some attitudes and beliefs are not mediated through one's MSE. Because I did not explicitly investigate teachers' attitudes or examine links between attitudes and other aspects of

Wilkins' (2008) model, I did not include relationship arrows between self-efficacies and attitudes, nor revise Wilkins' original relationship arrows.

Teachers' Belief and Practice Changes During and After the Pandemic

Though I was unable to measure teachers' pre-pandemic beliefs and practices, I was able to note experiences they shared which provided some insight into possible changes in their beliefs and practices. Pressley and Ha (2021) and Pellerone (2021) measured general teaching self-efficacy, noting a decrease in teachers' self-efficacy as a result of teaching during a pandemic. In responding to interview questions, Kathy expressed concern about the pandemic's effects on her students because she felt as though she was limited to using individual instruction versus group work. Kathy commented that the missed group interactions were important for kindergarten students because, during group interactions, the students learned social skills in conjunction with academic content. Likewise, Frances discussed her lack of confidence in knowing the students' mathematical understanding from the previous year. Frances' limited view of students' prior knowledge made her less confident in her ability to predict how students might engage with a particular task. These situations possibly reduced each teachers' confidence in teaching mathematics which might indicate a negative impact on their MTSE. However, despite these challenges there was no evidence that either teacher's self-efficacies were significantly influenced by the changes in practice.

Both teachers in this study altered their mathematics teaching in some way. Kathy found it difficult to modify her mathematics workshop which provided students with exploration opportunities pre-pandemic. While Frances felt the need to oscillate between different mathematical concepts depending on whether the students were learning remotely or in-person on a given school day. This phenomenon was similar to one reported by Aldon et al. (2021) and

Barlovits et al. (2021), who both reported that during the pandemic, teachers found it difficult to teach some concepts remotely. As a result, teachers chose to focus on more procedural mathematical content (Aldon et al., 2021) or to target specific mathematical concepts which were more appropriate for independent learning (Barlovits et al., 2021).

Unlike previous research (Barlovits et al., 2021; Martin et al., 2021), I did not find evidence that teaching during the pandemic altered Kathy nor Frances' instructional practices from pre-pandemic times as compared to post-pandemic times. Though I was not in the classroom once all COVID-19 restrictions were lifted—both teachers were still on an altered schedule at the completion of the study—there was evidence that they both were possibly returning to their pre-pandemic routines. By the conclusion of my time observing each of the teachers, the most prominent COVID restriction still enforced was the wearing of masks. Both teachers had returned to flexible grouping of students and used fewer whole-group discussions. It is possible that Kathy's and Frances' relatively quick return to pre-pandemic practices was due to their knowledge that they were recognized as effective teachers of mathematics. Perhaps, they did not feel the need to reassess and change their practices (Martin et al., 2021) nor to focus on integrating more technological tools into their teaching (Barlovits et al., 2021), phenomena that have been documented among a convenience sample of Grades 1–9 teachers surveyed in the US (Martin et al., 2021) and mostly secondary mathematics teachers surveyed in Germany and Spain (Barlovits et al., 2021), respectively.

Effective Mathematics Teachers

I chose effective teachers of mathematics as participants because having these types of teachers in the classroom increases the likelihood that students engage in effective teaching practices. Studies have shown that effective teachers engage students in student-centered practices (Gay, 2012; Perry, 2007), involve themselves in mathematics teaching research to better implement research-based practices (Liang et al., 2012; Wang & Cai, 2007) and often possess mathematical knowledge for teaching (Cai & Wang, 2010; Gay, 2012; Liang et al., 2012; Perry, 2007; Wang & Cai, 2007). Kathy and Frances embodied many of these qualities and characteristics. In addition, I was able to document how these effective teachers navigated teaching during difficult times while still adhering to effective teaching practices—and their student-centered instructional beliefs—during a variety of teaching modalities.

Limitations

Though I was able to provide some insights into the relationships among MSE, MTSE, instructional beliefs, and use of effective teaching practices, SMPs (NGA & CCSSO, 2010) and MTPs (NCTM, 2014), for two effective mathematics teachers, my study was limited by several factors.

First, my study was limited by the small number of willing or available participants. The 2020 school shutdowns—as a consequence of COVID-19 and subsequent restrictions—caused several administrators to limit the number of people visiting their schools. This made it difficult for an outsider like me to gain access to classrooms, even when teachers were willing to invite me in. In addition, it is possible that teachers may have declined to participate in my study because I was focused on the use of effective practices and teachers recognized that they were not enacting those practices under the existing conditions.

Second, only recruiting two teachers limited the potential richness of data I could have collected to pinpoint nuances in the relationship among MSE, MTSE, instructional beliefs and enacted teaching practices. Because both teachers taught within the same mid-sized urban K–8 school, I was not able to observe teachers in a broader range of school environments (e.g., high school, rural). Also, by being a novice researcher and having a limited number of participants, I potentially missed out on aspects of their self-efficacies, instructional beliefs, and enacted practices, that could provide insight on possible relationships among the factors.

I recognize that my interpretation of events was limited by the questions I chose to pose to the teachers. Though each teacher was generous with their time, more specific follow-up questions may have enabled me to develop more detailed descriptions of each case including building a stronger argument for the connections among MSE, MTSE, instructional beliefs, and practices. Perhaps if I had asked teachers to reflect on their own self-efficacies and instructional beliefs, I could have painted a more nuanced picture of the ways that affective factors influence teaching practices. Likewise, had I asked teachers to reflect on the interaction between MSE and MTSE or the links between these self-efficacies and other attitudes, I may have been able to collect more robust evidence to support my hypothesized relationship indicators (or additional connection) in Wilkins' model.

Lastly, my validation and reliability procedures were limited. The E-MCOP^2 instrument I created has the potential to document teachers' use of NCTM's (2014) MTPs. In my exploratory study, I engaged in a limited, informal validation process with an experienced mathematics education researcher. Specifically, this researcher compared my NCTM practice descriptions to NCTM's (2014) text, suggested category refinements, and commented on whether my practice exemplars drawn from the data were aligned with my revised E-MCOP^2 criteria. Although helpful, this form of assistance did not constitute a robust validation procedure like the one completed by Gleason et al. (2017). Likewise, my classification of the qualitative interview data did not undergo a robust code-checking procedure.

Implications

Teachers who employ effective practices have been shown to positively influence student outcomes (Fennema et al., 1996; Jong et al., 2010; Kane et al., 2011). Through my study, I produced a more detailed understanding of the potential connections among teachers' MSE, MTSE, instructional beliefs, and enacted teaching practices. Having effective teachers in the classroom is the goal of any educational organization, which is why I was motivated to conduct this case study focusing on traits of these effective teachers. In doing so, I hope my research highlights some ways that research can contribute to practice.

Research

My revisitation of Wilkins' (2008) theoretical model and the roles of self-efficacy, experiences, beliefs, and behaviors contributes to the vast research on the self-efficacy of teachers (e.g., Ashton, 1984; Holzberger et al., 2013; Pajares & Miller, 1994). This revised Wilkins' (2008) model may help understand the links and intricacies among various affective factors, including self-efficacy and teaching practices.

There are a variety of tools for measuring effectiveness of mathematics teachers (e.g., Berry et al., 2010; Gleason et al., 2017; Hill 2014; Sawada & Piburn, 2000), many measuring the use of student-centered practices. Because my study focused on NCTM's (2014) MTPs, I was unable to find an observational tool to document these specific practices. This provided me an opportunity to build a tool focused on NCTM's MTPs. The E-MCOP^2 may afford future researchers the opportunity to document the use of MTP in a variety of mathematics classrooms.

Practice

Although I worked with a small group of teachers, I was able to model how these factors work together in teachers who were identified as effective. My findings may lead to a better understanding of teachers who use effective teaching practices. I hope this added insight can allow mathematics teacher educators, professional development coordinators, and school leadership to better support teachers who aspire to teach in ways that are more closely aligned with NCTM's (2014) description of effective teaching practices. From these effective teachers, I learned that PD experiences that include model teaching and observations, such that might occur during academic coaching, can be context for vicarious and mastery experience that not only influence self-efficacy but also, perhaps practices.

The findings of my study have implications for both preservice and in-service teachers. First, those who educate preservice teachers could be intentional about providing positive mastery and vicarious experiences (Bandura, 1997) within the context of the course materials and teaching experiences to enhance PSTs' MSE and MTSE. For example, teacher educators could provide students with opportunities to engage in the MTPs (NCTM, 2014) as learners (i.e., vicarious experience), then provide teaching opportunities—small group teaching in a classroom or teaching rehearsals with other PSTs acting as students—wherein the PSTs could possibly have mastery experiences using the MTPs. These experiences with the MTPs could provide the PSTs with opportunities to increase their MSE and MTSE or influence their own instructional beliefs.

For in-service teachers, engaging in worthwhile PD that intentionally focuses on the SMPs (NGA & CCSSO, 2010) or the MTPs (NCTM, 2014) may also provide experiences—mastery or vicarious—that may influence teachers' MSE, MTSE, instructional beliefs, and teaching practices. Kathy's experience with PD focused on questioning is a good example of how PD can spark a change in MTSE and, in turn, influence instructional practices. Providing teachers with model examples of the practices in action or coaching opportunities is another implication of my findings, Kathy experienced firsthand what the practice of pose purposeful questions (NCTM, 2014) looked like with her students. Kathy was then able to learn from that experience and alter her beliefs and practice.

Researchers who have investigated teachers who have been labeled as effective have often focused on teachers' mathematical knowledge, course work, or experience (Ball et al., 2005; Ball et al., 2008; Shulman, 1986; Wilkins, 2008). Even though some researchers have focused on affective factors of teachers (Ashton, 1984; Clark & Peterson, 1986; Hackett & Betz, 1989; Tschannen-Moran et al., 1998), there is limited in-depth evidence related to how affective factors influence teachers' choices of instructional practices. As a result of creating those rich descriptions of teachers' self-efficacies, instructional beliefs, and enacted teaching practices, I was able to contribute to the knowledge base of effective mathematics teachers. Both teachers displayed high self-efficacy, student-centered instructional beliefs, and engaged students in effective teaching practices. But each teacher had different experiences that led them to becoming effective teachers of mathematics. These differences allow for an opportunity to dig deeper into what types of experiences may support the development of effective mathematics teachers.

My focus on an affective factor (i.e., self-efficacy) widens the developing definition of who effective teachers are. Although some researchers have found that affective factors such as self-efficacy, mathematics anxiety, and instructional beliefs may have a greater influence on students' perception of quality (Holzberger et al., 2013; Howard & Whitaker, 2011; Perera & John, 2020; Torff & Sessions, 2005), teachers' choice of instructional practices (Allinder, 1994; Raymond, 1997; Wilkins, 2008), and student learning (Azkiyah, 2017; Peterson et al., 1989), many of the conclusions were made using student or teacher self-reported data. My study differed in that I was able to generate firsthand accounts (Merriam & Tisdell, 2016) of enacted teaching practices, enabling a more in-depth description and understanding of teachers' self-efficacies, instructional beliefs, and enacted teaching practices.

Lastly, despite the difficult circumstances of the pandemic, these effective teachers felt the pressure, yet still did a valiant job of prioritizing their instructional values to the extent possible. This was different from other teachers who were studied because despite situations which challenged their beliefs, both teachers had high MSE and MTSE. Furthermore, the 2020 pandemic is not likely to be the last disruptive event (Arcanjo, 2018; Future Agenda, 2020; Rigaud et al., 2018; World Health Organization, 2020) to affect teachers, schools, and students. Having documented a part of these teachers' stories and their response provides insight into how effective teachers were able to navigate these difficult situations.

Directions for Future Research

Though I was able to provide further insight into the relationships among MSE, MTSE, instructional beliefs, and enacted teaching practices, there are still many unanswered questions. For example, it is possible that the relationships I described among MSE, MTSE, instructional beliefs, and enacted practices for the elementary-level teachers I observed, were not typical for other teachers who also exclusively teach mathematics, particularly those who teach at the middle or high school level. Thus, more research is needed that includes a variety of teachers from different grade levels (e.g., middle and high school teachers) and different types of schools (e.g., rural or suburban).

In addition, my focus was on teachers who were identified as effective teachers of mathematics. Extending the pool of participants to any teacher of mathematics could provide more insight into the relationships among beliefs and practices. Both Kathy and Frances appeared to have high MSE and MTSE which potentially resulted in the increased use of the

MTPs (NCTM, 2014) and engagement of students in the SMPs (NGA & CCSSO, 2010). Would other teachers who enacted student-centered practices, as noted by an outside observer, also have a high MSE and a high MTSE? As stated previously, the relationships among MSE, MTSE, instructional beliefs, and enacted teaching practices are complex, more research if necessary to determine all the interactions and the potential causal influences among these factors in teachers of mathematics.

Furthermore, through my research concerning Bandura's (1986) social cognitive theory, I noted the triadic reciprocal determinism: the dynamic interplay of personal, behavioral, and environmental influences. Though the triadic reciprocal determinism was not a primary focus of my study, it is essential to note the importance of this interplay when assessing self-efficacy (Woodcock & Tournaki, 2023). For example, though the pandemic could be considered an environmental influence on the teachers' self-efficacy, more research is necessary to investigate the interplay of that environmental factor on specific behavioral and personal influences, since self-efficacy are situation specific.

Moreover, through this study I was able to document a PD experience noted by Kathy which altered her MSE, MTSE, and instructional practices. Future research is needed to design ways to use vicarious and mastery experiences so that teachers, pre-service and in-service, can encounter situations that could lead to worthwhile changes in their self-efficacy, instructional beliefs, and practices.

Lastly, these effective mathematics teachers gave insight among factors and nuances which fit into Wilkins' (2008) theoretical model. I proposed a modified version of Wilkins' model, though this model was based on those who were similarly classified as effective. Future research could consider the proposed modifications to Wilkins' model with a greater variety of teachers.

Closing Thoughts

At the inception of this study, I was hopeful to recruit many teachers so that I could create a deeper understanding of the relationships among self-efficacies, beliefs, and practices, but the influence of the 2020 pandemic required that I reassess the scope of my intended goal. Though I was only able to recruit two effective teachers of mathematics, I believe that the descriptions and relationships I generated provides another step in the long journey to fully understanding the complexity of beliefs—self-efficacy and instructional—and enacted practices. Through this process, I realized that the stories of these teachers brought significance to the influence the 2020 pandemic had on them as effective teachers of mathematics and hope that there are lessons to be learned from the data I collected. At the conclusion of this study, I am left with some unanswered questions concerning the self-efficacies, beliefs, and practices of teachers of mathematics and I look forward to continuing the journey toward finding more answers to those questions.

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APPENDIX A: TEACHER CONSENT FORM

You are being asked to participate in a research study conducted by Amy Roehrig, Instructional Assistant Professor and doctoral student in the Mathematics Department at Illinois State University and Dr. Tami Martin, Professor. The purpose of this study is to describe the relationships between teachers' self-efficacy concerning mathematics, instructional beliefs, and their enacted use of effective teaching practices.

Why are you being asked?
You have been asked to participate because you have been identified as a mathematics teacher who uses effective teaching practices in mathematics. Your participation in this study is voluntary. You may withdraw from the study at any time and there are no penalties for choosing not to participate or withdrawing at any time.

What would you do?
If you choose to take part in this research study, you will be asked to complete an online survey containing about 75 questions. This survey will take approximately 20 minutes. Following the completion of the survey, you and I will schedule approximately 5–10 observations of your in-person or remote synchronous teaching. The length of the observation is determined by the length of time you spend teaching mathematics on the scheduled day. I will conduct a pre- and post-observation interview with each observation, each of which will take approximately 10–15 minutes. All observations are video recorded, and students' confidentiality will be maintained. To protect students in the online environment, I ask that names of students appearing on screen be altered to include at most their first name and last initial. At the conclusion of the observations, an interview will occur which will take approximately an hour.

Are any risks expected?
There is a potential that your voice and face may be recognizable during future conference presentations or professional development, therefore causing a risk of loss confidentiality. Should a loss of confidentiality occur there could be the risk to your reputation or employability despite the fact that the focus of this study is on exemplary teachers.

Will your information be protected?
We will use all reasonable efforts to keep any provided personal information confidential unless you provide permission to use the videos in educational settings, such as professional conferences or in classes for preservice teachers. All participants will be identified using pseudonyms and the location of their school nor district will not be identified. Information obtained from this study will be used as part of a dissertation study and subsequent journal articles.

Could your responses be used for other research?
In the following sections, you will also be asked to indicate whether it is okay to use excerpts of your classroom observation video educational settings. If you prefer not to have the videos shown in educational settings, please use the attached consent form and refrain from initialing the relevant box. After your data has been deidentified, your data may be used in other research projects.

Who will benefit from this study?
There are no direct benefits to participants. However, you may benefit from additional reflection on your teaching. Your participation will help us gain insight into the relationships among self-efficacy specific to mathematics, instructional beliefs, and effective teaching practices.

Whom do you contact if you have any questions?
If you have any questions about the research or wish to withdraw from the study, contact Amy Roehrig can be contacted at 309–824–8549 or aroehr@ilstu.edu.

If you have any questions about your rights as a participant, or if you feel you have been placed at risk, contact the Illinois State University Research Ethics & Compliance Office at (309) 438-5527 or IRB@ilstu.edu.

Sincerely,

Amy E. Roehrig
Doctoral Student & Instructional Assistant Professor
Department of Mathematics
Illinois State University
Normal, IL 61790-4520
(309) 824–8549

Tami S. Martin
Professor
Department of Mathematics
Illinois State University
Normal, IL 61790-4520
(309) 438–7864

I consent to present in the project described above. I have the right to full information about the project, and that all information will be used confidentially. I give my permission for my classroom and teaching to be observed, audiotaped, and videotaped by the project staff and complete associated surveys.

I do not consent, I do not wish to participate

I *also agree* to allow video excerpts from the recorded lessons to be shown at academic conferences, classes, and for other educational purposes. Every attempt will be made to exclude identifying information, such as my name or school.

I agree that members of the research team may show my videos at academic conferences, classes, and other educational experiences.

I DO NOT agree that members of the research team may show my videos at academic conferences, classes, and other educational experiences.

Your name:

E-mail address:

Telephone:

Date:

APPENDIX B: ELEMENTARY TEACHER MATHEMATICS SELF-EFFICACY, MATHEMATICS TEACHING SELF-EFFICACY, AND INSTRUCTIONAL BELIEFS SURVEY

Mathematics Teaching and Mathematics Self-Efficacy (MTMSE) Survey (Kahle, 2008). Part 5 adapted from Teaching and Learning Mathematics Beliefs survey (O'Hanlon et al., 2015)

This survey will take approximately fifteen minutes to complete. Your opinions are very important to me. Thank you in advance for your participation in this study.

DIRECTIONS

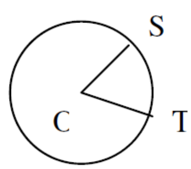
Part 1: Suppose that you were asked the following math questions in a multiple-choice form. Please indicate how confident you are that you would give the correct answer to each question *without using a calculator.*

PLEASE DO NOT ATTEMPT TO SOLVE THESE PROBLEMS.

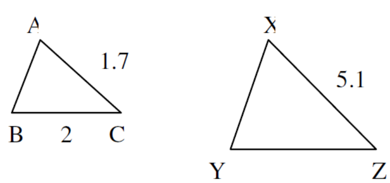
1	2	3	4	5	6
Not confident at all					Completely confident

1. In a certain triangle, the shortest side is 6 inches. The longest side is twice as long as the shortest side, and the third side is 3.4 inches less than the longest side. What is the sum of the three sides? 1 2 3 4 5 6
2. ABOUT how many times larger than 614,360 is 30,688,000? 1 2 3 4 5 6
3. There are three numbers. The second is twice the first and the first is one-third of the other number. Their sum is 48. Find the largest number. 1 2 3 4 5 6
4. Five points are on a line. T is next to G. K is next to H. C is next to T. H is next to G. Determine the positions of the points along the line. 1 2 3 4 5 6
5. If $y = 9 + x/5$, find x when $y = 10$. 1 2 3 4 5 6
6. A baseball player got two hits for three times as bat. This could be represented by 2/3. Which decimal would most closely represent this? 1 2 3 4 5 6
7. If P = M + N, then which of the following would be true? 1 2 3 4 5 6
    a. N = P – M
    b. P – N = M
    c. N + M = P
    d. All of the above
8. Find the measure of the angle that the hands of a clock form at 8 o'clock. 1 2 3 4 5 6

9. Bridget buys a packet containing 9-cent and 13-cent stamps for \$2.65. If there are 25 stamps in the packet, how many are 13-cent stamps? 1 2 3 4 5 6
10. On a certain map, 7/8 inch represents 200 miles. How far apart are two towns whose distance apart on the map if 3 ½ inches? 1 2 3 4 5 6
11. Fred's bill for some household supplies was \$13.64. If he paid for the items with a \$20 bill, how much change should he receive? 1 2 3 4 5 6
12. Some people suggest that the following formula be used to determine the average weight for boys between the ages of 1 and 7: W = 17 + 5A, where W is the weight in pounds and A is the boy's age in years. According to this formula, for each year older a boy gets, should his weight become more or less, and by how much? 1 2 3 4 5 6
13. Five spelling tests are to be given to Mary's class. Each test has a value of 25 points. Mary's average for the first four tests is 15. What is the highest possible average she can have on all five tests? 1 2 3 4 5 6
14. $3\frac{4}{5} - \frac{1}{2} =$ ________ 1 2 3 4 5 6
15. In an auditorium, the chairs are usually arranged so that there are x rows and y seats in each row. For a popular speaker, an extra row is added, and an extra seat is added to every row. Thus, there are $x + 1$ rows and $y + 1$ seats in each row. Write a mathematical expression to show how many people the new arrangement will hold. 1 2 3 4 5 6
16. A Ferris wheel measures 80 feet in circumference. The distance on the circle between two of the seats S and T, is 10 feet. See the figure below. Find the measure in degrees of the central angle SCT whose rays support the two seats. 1 2 3 4 5 6



17. Write an expression for "six less than twice 4 5/6? 1 2 3 4 5 6
18. The two trianlges shown below are similar. Thus, the corresponding sides are proportional, and AC/BC = XZ/YZ. If AC = 1.7, BC = 2, and XZ = 5.1, find YZ. 1 2 3 4 5 6



Part 2: Directions: Please use the following scale to answer each question.

1	2	3	4	5	6
Strongly Disagree	Moderately Disagree	Disagree	Agree	Moderately Agree	Strongly Agree

1. I will continually find better ways to teach mathematics.	1	2	3	4	5	6
2. Even if I try very hard, I will not teach mathematics as well as I teach most subjects.	1	2	3	4	5	6
3. I know how to teach mathematics concepts effectively.	1	2	3	4	5	6
4. I will not be very effective in monitoring mathematics activities.	1	2	3	4	5	6
5. I will generally teach mathematics ineffectively.	1	2	3	4	5	6
6. I understand mathematics concepts well enough to be effective in teaching elementary mathematics.	1	2	3	4	5	6
7. I will find it difficult to use manipulatives to explain to students why mathematics works.	1	2	3	4	5	6
8. I will typically be able to answer students' questions.	1	2	3	4	5	6
9. I wonder if I have the necessary skills to teach mathematics.	1	2	3	4	5	6
10. Given a choice, I will not invite the principal to evaluate my mathematics teaching.	1	2	3	4	5	6
11. When a student has difficulty understanding a mathematics concept, I will usually be at a loss as to how to help the student understand it better.	1	2	3	4	5	6
12. When teaching mathematics, I will usually welcome student questions.	1	2	3	4	5	6
13. I do not know what to do to turn students on to mathematics.	1	2	3	4	5	6

Part 3: Directions: How much confidence do you have that you are able to successfully perform each of the following tasks? Please use the following scale:

1	2	3	4	5	6
Not confident at all					Completely confident

1. Add two large numbers in your head.	1	2	3	4	5	6
2. Multiply quantities in a recipe to feed a larger group.	1	2	3	4	5	6
3. Doubling a recipe that contains fractions.	1	2	3	4	5	6
4. Figure out how long it will take to travel from City A to City B driving x mph.	1	2	3	4	5	6
5. Understand a graph accompanying an article on business profits.	1	2	3	4	5	6
6. Figure out how much you would save if there were a 15% markdown on an item you wish to buy.	1	2	3	4	5	6
7. Estimate your grocery bill in your head as you pick up items.	1	2	3	4	5	6

8. Figure out which of two summer jobs is the better offer: one with a higher salary but no benefits, the other with a lower salary plus room, board, and travel expenses.	1 2 3 4 5 6
9. Figure out the tip on your part of a dinner bill.	1 2 3 4 5 6
10. Figure out how much lumber you need to buy in order to build a set of bookshelves.	1 2 3 4 5 6
11. Measure your height in centimeters.	1 2 3 4 5 6
12. Determine how many boxes of a certain size will fit into a closet.	1 2 3 4 5 6
13. Explain your chances of flipping tails on both of two coins.	1 2 3 4 5 6

Part 4: Directions: Please rate the following mathematics topics according to how confident you would be teaching students each topic. Please use the following scale:

1	2	3	4	5	6
Not confident at all					Completely confident

1. Average, Mean, Median, & Mode	1 2 3 4 5 6
2. Multiplication	1 2 3 4 5 6
3. Number Patterns	1 2 3 4 5 6
4. Shape Properties	1 2 3 4 5 6
5. Fractions	1 2 3 4 5 6
6. U.S. Customary Measurement System (e.g., feet, pounds, gallons)	1 2 3 4 5 6
7. Probability	1 2 3 4 5 6
8. Decimals	1 2 3 4 5 6
9. Order of Operations	1 2 3 4 5 6
10. Metric System (e.g., meters, liters, grams)	1 2 3 4 5 6
11. Division	1 2 3 4 5 6
12. Perimeter & Area	1 2 3 4 5 6
13. Tables & Graphs	1 2 3 4 5 6

Part 5: Directions: Please rate the following statements to how much you agree or disagree. Please use the following scale:

1	2	3	4	5	6
Strongly Disagree	Moderately Disagree	Disagree	Agree	Moderately Agree	Strongly Agree

1. It is important for a student to discover mathematics.	1 2 3 4 5 6
2. Students should spend time practicing computational procedures before they are expected to understand those procedures.	1 2 3 4 5 6
3. The teacher should demonstrate how to solve mathematics problems before the students are allowed to solve problems.	1 2 3 4 5 6
4. Students should understand computational procedures before they spend time practicing them.	1 2 3 4 5 6

5. During class discussion, the teacher should be the authority in terms of whether a student's mathematical conjecture or justification is correct.	1	2	3	4	5	6
6. During class discussions, students should play a role in determining whether mathematical justifications are valid.	1	2	3	4	5	6
7. Teachers should encourage students to invent ways to solve mathematical problems.	1	2	3	4	5	6
8. Students should understand how mathematics ideas interconnect and build on one another to produce a coherent whole.	1	2	3	4	5	6
9. Students should engage in problem solving before they master computational procedures and basic concepts.	1	2	3	4	5	6
10. Students should be given the opportunities to learn mathematics by developing and investigating mathematical conjectures, arguments, and proofs.	1	2	3	4	5	6
11. Allowing students to discuss mathematics with a partner, or in a group, is an important instructional strategy.	1	2	3	4	5	6
12. Struggling with mathematical concepts is detrimental to developing understanding.	1	2	3	4	5	6
13. During instruction, teachers should teach each mathematical idea separately rather than emphasizing the interconnections among ideas.	1	2	3	4	5	6
14. Group discussions often lead to tangents, or incorrect mathematics, and should be limited in their use.	1	2	3	4	5	6
15. Students learn mathematics best from teachers' demonstrations and explanations.	1	2	3	4	5	6
16. Students learn mathematics by studying mathematical arguments and proofs presented by the teacher or shown in the textbook.	1	2	3	4	5	6
17. It is not necessary for students to understand the interconnections among mathematical ideas as long as they have some understanding of the individual topics.	1	2	3	4	5	6
18. Students should not attempt problem solving until the understand basic concepts.	1	2	3	4	5	6
19. The best way to teach problem solving is to focus on one type of mathematics problem at a time.	1	2	3	4	5	6
20. Teachers should provide opportunities for students to critique mathematical arguments and discuss their own conjectures.	1	2	3	4	5	6
21. The teacher should provide verification for mathematical arguments rather than expecting students to do so.	1	2	3	4	5	6
22. During instruction, teachers should emphasize the interconnections among mathematical ideas.	1	2	3	4	5	6
23. Struggling with mathematical concepts is beneficial to developing understanding.	1	2	3	4	5	6
24. If a teacher encounters a student who is struggling with a problem, they should ask a question that will help the student find a solution but should not directly give the answer.	1	2	3	4	5	6

25. If a student is having difficulties solving a problem, then the teacher should tell the student how to solve it. 1 2 3 4 5 6
26. My job as a teacher is to make mathematics interesting and engaging. 1 2 3 4 5 6
27. The structure of mathematics as a subject is more important in making instructional decisions than the natural development of students' ideas. 1 2 3 4 5 6

Part 6: Demographic Questions

1. Which type of school do you work in? (Circle one on each line)
   a. Urban   Suburban   Rural
   b. Parochial   Public   Private   Charter
2. Is teaching your first career?   Yes   No
3. What is your gender?   Male   Female   Prefer not to answer
4. Are you a parent?   Yes   No
5. What is your race? (optional)
   African American   Asian   Hispanic
   White   Mixed   Other ___________________
6. What is your highest level of degree earned?
   Associates   Bachelor's   Master's   Doctorate
7. What was your major in college? _____________________
8. How many years have you been teaching?
   (0–2) (3–5) (6–10) (11–15) (16–20) (21–30) (30+)
9. What type of teaching certificate/license do you hold? Circle all that apply.
   a. Type: Teaching license   Provisional certificate   Permanent Certificate
   b. Grades: PreK K 1 2 3 4 5 6 7 8 9 10 11 12
   c. Other: Math Specialist   Middle grade validation   Math concentration
      Other: __________________________
10. Circle all the subjects that you teach.
    Language Arts   Mathematics   Reading   Science   Social Studies
    Other: _____________________
11. What subjects are you most confident teaching in an elementary school?
    Language Arts   Mathematics   Reading   Science   Social Studies
12. What subject are you least confident teaching in an elementary school?
    Language Arts   Mathematics   Reading   Science   Social Studies

Please keep in mind that all answers will be kept strictly confidential. Please provide the following information:

Name: Grade:
School: School District:

Thank you for taking the time to participate in my study!

Survey Number

APPENDIX C: MIDDLE SCHOOL TEACHER MATHEMATICS SELF-EFFICACY, MATHEMATICS TEACHING SELF-EFFICACY, AND INSTRUCTIONAL BELIEFS SURVEY

Mathematics Teaching and Mathematics Self-Efficacy (MTMSE) Survey (Kahle, 2008). Part 5 adapted from Teaching and Learning Mathematics Beliefs survey (O'Hanlon et al., 2015)

This survey will take approximately fifteen minutes to complete. Your opinions are very important to me. Thank you in advance for your participation in this study.

DIRECTIONS

Part 1: Suppose that you were asked the following math questions in a multiple-choice form. Please indicate how confident you are that you would give the correct answer to each question *without using a calculator.*

PLEASE DO NOT ATTEMPT TO SOLVE THESE PROBLEMS.

1	2	3	4	5	6
Not confident at all					Completely confident

1. In a certain triangle, the shortest side is 6 inches. The longest side is twice as long as the shortest side, and the third side is 3.4 inches less than the longest side. What is the sum of the three sides? 1 2 3 4 5 6
2. ABOUT how many times larger than 614,360 is 30,688,000? 1 2 3 4 5 6
3. There are three numbers. The second is twice the first and the first is one-third of the other number. Their sum is 48. Find the largest number. 1 2 3 4 5 6
4. Five points are on a line. T is next to G. K is next to H. C is next to T. H is next to G. Determine the positions of the points along the line. 1 2 3 4 5 6
5. If $y = 9 + x/5$, find x when $y = 10$. 1 2 3 4 5 6
6. A baseball player got two hits for three times as bat. This could be represented by 2/3. Which decimal would most closely represent this? 1 2 3 4 5 6
7. If P = M + N, then which of the following would be true? 1 2 3 4 5 6
    a. N = P – M
    b. P – N = M
    c. N + M = P
    d. All of the above
8. Find the measure of the angle that the hands of a clock form at 8 o'clock. 1 2 3 4 5 6

9. Bridget buys a packet containing 9-cent and 13-cent stamps for \$2.65. If there are 25 stamps in the packet, how many are 13-cent stamps? 1 2 3 4 5 6

10. On a certain map, 7/8 inch represents 200 miles. How far apart are two towns whose distance apart on the map if 3 ½ inches? 1 2 3 4 5 6

11. Fred's bill for some household supplies was \$13.64. If he paid for the items with a \$20 bill, how much change should he receive? 1 2 3 4 5 6

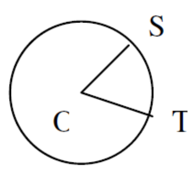
12. Some people suggest that the following formula be used to determine the average weight for boys between the ages of 1 and 7: W = 17 + 5A, where W is the weight in pounds and A is the boy's age in years. According to this formula, for each year older a boy gets, should his weight become more or less, and by how much? 1 2 3 4 5 6

13. Five spelling tests are to be given to Mary's class. Each test has a value of 25 points. Mary's average for the first four tests is 15. What is the highest possible average she can have on all five tests? 1 2 3 4 5 6

14. $3\frac{4}{5} - \frac{1}{2} =$ ________ 1 2 3 4 5 6

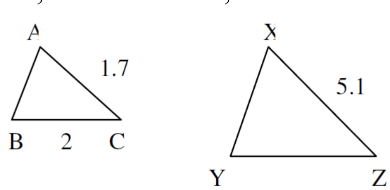
15. In an auditorium, the chairs are usually arranged so that there are x rows and y seats in each row. For a popular speaker, an extra row is added, and an extra seat is added to every row. Thus, there are $x + 1$ rows and $y + 1$ seats in each row. Write a mathematical expression to show how many people the new arrangement will hold. 1 2 3 4 5 6

16. A Ferris wheel measures 80 feet in circumference. The distance on the circle between two of the seats S and T, is 10 feet. See the figure below. Find the measure in degrees of the central angle SCT whose rays support the two seats. 1 2 3 4 5 6



17. Write an expression for "six less than twice 4 5/6? 1 2 3 4 5 6

18. The two trianlges shown below are similar. Thus, the corresponding sides are proportional, and AC/BC = XZ/YZ. If AC = 1.7, BC = 2, and XZ = 5.1, find YZ. 1 2 3 4 5 6



Part 2: Directions: Please use the following scale to answer each question.

1	2	3	4	5	6
Strongly Disagree	Moderately Disagree	Disagree	Agree	Moderately Agree	Strongly Agree

1. I will continually find better ways to teach mathematics.	1	2	3	4	5	6
2. Even if I try very hard, I will not teach mathematics as well as I teach most subjects.	1	2	3	4	5	6
3. I know how to teach mathematics concepts effectively.	1	2	3	4	5	6
4. I will not be very effective in monitoring mathematics activities.	1	2	3	4	5	6
5. I will generally teach mathematics ineffectively.	1	2	3	4	5	6
6. I understand mathematics concepts well enough to be effective in teaching middle school mathematics.	1	2	3	4	5	6
7. I will find it difficult to use manipulatives to explain to students why mathematics works.	1	2	3	4	5	6
8. I will typically be able to answer students' questions.	1	2	3	4	5	6
9. I wonder if I have the necessary skills to teach mathematics.	1	2	3	4	5	6
10. Given a choice, I will not invite the principal to evaluate my mathematics teaching.	1	2	3	4	5	6
11. When a student has difficulty understanding a mathematics concept, I will usually be at a loss as to how to help the student understand it better.	1	2	3	4	5	6
12. When teaching mathematics, I will usually welcome student questions.	1	2	3	4	5	6
13. I do not know what to do to turn students on to mathematics.	1	2	3	4	5	6

Part 3: Directions: How much confidence do you have that you are able to successfully perform each of the following tasks? Please use the following scale:

1	2	3	4	5	6
Not confident at all					Completely confident

1. Add two large numbers in your head.	1	2	3	4	5	6
2. Multiply quantities in a recipe to feed a larger group.	1	2	3	4	5	6
3. Doubling a recipe that contains fractions.	1	2	3	4	5	6
4. Figure out how long it will take to travel from City A to City B driving x mph.	1	2	3	4	5	6
5. Understand a graph accompanying an article on business profits.	1	2	3	4	5	6
6. Figure out how much you would save if there were a 15% markdown on an item you wish to buy.	1	2	3	4	5	6
7. Estimate your grocery bill in your head as you pick up items.	1	2	3	4	5	6

8. Figure out which of two summer jobs is the better offer: one with a higher salary but no benefits, the other with a lower salary plus room, board, and travel expenses. 1 2 3 4 5 6
9. Figure out the tip on your part of a dinner bill. 1 2 3 4 5 6
10. Figure out how much lumber you need to buy in order to build a set of bookshelves. 1 2 3 4 5 6
11. Measure your height in centimeters. 1 2 3 4 5 6
12. Determine how many boxes of a certain size will fit into a closet. 1 2 3 4 5 6
13. Explain your chances of flipping tails on both of two coins. 1 2 3 4 5 6

Part 4: Directions: Please rate the following mathematics topics according to how confident you would be teaching students each topic. Please use the following scale:

1	2	3	4	5	6
Not confident at all					Completely confident

1. Average, Mean, Median, & Mode 1 2 3 4 5 6
2. Multiplication 1 2 3 4 5 6
3. Number Patterns 1 2 3 4 5 6
4. Shape Properties 1 2 3 4 5 6
5. Fractions 1 2 3 4 5 6
6. U.S. Customary Measurement System (e.g., feet, pounds, gallons) 1 2 3 4 5 6
7. Probability 1 2 3 4 5 6
8. Decimals 1 2 3 4 5 6
9. Order of Operations 1 2 3 4 5 6
10. Metric System (e.g., meters, liters, grams) 1 2 3 4 5 6
11. Division 1 2 3 4 5 6
12. Perimeter & Area 1 2 3 4 5 6
13. Tables & Graphs 1 2 3 4 5 6

Part 5: Directions: Please rate the following statements to how much you agree or disagree. Please use the following scale:

1	2	3	4	5	6
Strongly Disagree	Moderately Disagree	Disagree	Agree	Moderately Agree	Strongly Agree

1. It is important for a student to discover mathematics. 1 2 3 4 5 6
2. Students should spend time practicing computational procedures before they are expected to understand those procedures. 1 2 3 4 5 6
3. The teacher should demonstrate how to solve mathematics problems before the students are allowed to solve problems. 1 2 3 4 5 6
4. Students should understand computational procedures before they spend time practicing them. 1 2 3 4 5 6

5. During class discussion, the teacher should be the authority in terms of whether a student's mathematical conjecture or justification is correct.	1	2	3	4	5	6
6. During class discussions, students should play a role in determining whether mathematical justifications are valid.	1	2	3	4	5	6
7. Teachers should encourage students to invent ways to solve mathematical problems.	1	2	3	4	5	6
8. Students should understand how mathematics ideas interconnect and build on one another to produce a coherent whole.	1	2	3	4	5	6
9. Students should engage in problem solving before they master computational procedures and basic concepts.	1	2	3	4	5	6
10. Students should be given the opportunities to learn mathematics by developing and investigating mathematical conjectures, arguments, and proofs.	1	2	3	4	5	6
11. Allowing students to discuss mathematics with a partner, or in a group, is an important instructional strategy.	1	2	3	4	5	6
12. Struggling with mathematical concepts is detrimental to developing understanding.	1	2	3	4	5	6
13. During instruction, teachers should teach each mathematical idea separately rather than emphasizing the interconnections among ideas.	1	2	3	4	5	6
14. Group discussions often lead to tangents, or incorrect mathematics, and should be limited in their use.	1	2	3	4	5	6
15. Students learn mathematics best from teachers' demonstrations and explanations.	1	2	3	4	5	6
16. Students learn mathematics by studying mathematical arguments and proofs presented by the teacher or shown in the textbook.	1	2	3	4	5	6
17. It is not necessary for students to understand the interconnections among mathematical ideas as long as they have some understanding of the individual topics.	1	2	3	4	5	6
18. Students should not attempt problem solving until the understand basic concepts.	1	2	3	4	5	6
19. The best way to teach problem solving is to focus on one type of mathematics problem at a time.	1	2	3	4	5	6
20. Teachers should provide opportunities for students to critique mathematical arguments and discuss their own conjectures.	1	2	3	4	5	6
21. The teacher should provide verification for mathematical arguments rather than expecting students to do so.	1	2	3	4	5	6
22. During instruction, teachers should emphasize the interconnections among mathematical ideas.	1	2	3	4	5	6
23. Struggling with mathematical concepts is beneficial to developing understanding.	1	2	3	4	5	6
24. If a teacher encounters a student who is struggling with a problem, they should ask a question that will help the student find a solution but should not directly give the answer.	1	2	3	4	5	6

25. If a student is having difficulties solving a problem, then the teacher should tell the student how to solve it. 1 2 3 4 5 6
26. My job as a teacher is to make mathematics interesting and engaging. 1 2 3 4 5 6
27. The structure of mathematics as a subject is more important in making instructional decisions than the natural development of students' ideas. 1 2 3 4 5 6

Part 6: Demographic Questions

1. Which type of school do you work in? (Circle one on each line)
   a. Urban Suburban Rural
   b. Parochial Public Private Charter
2. Is teaching your first career? Yes No
3. What is your gender? Male Female Prefer not to answer
4. Are you a parent? Yes No
5. What is your race? (optional)
   African American Asian Hispanic
   White Mixed Other ___________________
6. What is your highest level of degree earned?
   Associates Bachelor's Master's Doctorate
7. What was your major in college? _____________________
8. How many years have you been teaching?
   (0–2) (3–5) (6–10) (11–15) (16–20) (21–30) (30+)
9. What type of teaching certificate/license do you hold? Circle all that apply.
   a. Type: Teaching license Provisional certificate Permanent Certificate
   b. Grades: PreK K 1 2 3 4 5 6 7 8
      9 10 11 12
   c. Other: Math Specialist Middle grade validation Math concentration
      Other: __________________________
10. Circle all the subjects that you teach.
    Language Arts Mathematics Reading Science Social Studies
    Other: _____________________
11. What subjects are you most confident teaching in an elementary school?
    Language Arts Mathematics Reading Science Social Studies
12. What subject are you least confident teaching in an elementary school?
    Language Arts Mathematics Reading Science Social Studies

Please keep in mind that all answers will be kept strictly confidential. Please provide the following information:

Name: Grade:
School: School District:

Thank you for taking the time to participate in my study!

Survey Number

APPENDIX D: HIGH SCHOOL TEACHER MATHEMATICS SELF-EFFICACY, MATHEMATICS TEACHING SELF-EFFICACY, AND INSTRUCTIONAL BELIEFS SURVEY

Mathematics Teaching and Mathematics Self-Efficacy (MTMSE) Survey (Kahle, 2008). Part 5 adapted from Teaching and Learning Mathematics Beliefs survey (O'Hanlon et al., 2015)

This survey will take approximately fifteen minutes to complete. Your opinions are very important to me. Thank you in advance for your participation in this study.

DIRECTIONS

Part 1: Suppose that you were asked the following math questions in a multiple-choice form. Please indicate how confident you are that you would give the correct answer to each question *without using a calculator.*

PLEASE DO NOT ATTEMPT TO SOLVE THESE PROBLEMS.

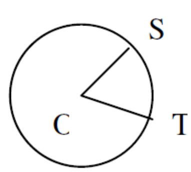
1	2	3	4	5	6
Not confident at all					Completely confident

1. If $\frac{x-1}{3} = k$ and $k = 3$, what is the value of *x*? 1 2 3 4 5 6
2. On Saturday afternoon, Armand sent *m* text messages each hour for 5 hours, and Tyrone sent *p* text messages each hour for 4 hours. Write an expression that represents the total number of messages sent by Armand and Tyrone on Saturday afternoon. 1 2 3 4 5 6
3. A line in the *xy-plane* passes through the origin and has a slope of 1/7. Find a point that lies on the line. 1 2 3 4 5 6
4. If $(ax + 2)(bx + 7) = 15x^2 + cx + 14$ for all values of *x,* and $a + b = 8$, what are the two possible values for *c*? 1 2 3 4 5 6
5. The table below shows the distribution of age and gender for 25 people who entered a contest. If the contest winner will be selected at random, what is the probability that the winner will be either female under the age of 40 or male age 40 or older? 1 2 3 4 5 6

	Age		Total
Gender	Under 40	40 or older	
Male	12	2	14
Female	8	3	11
Total	20	5	25

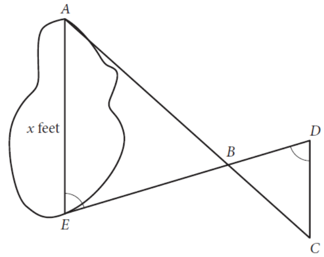
6. A food truck sells salads for $6.50 each and drinks for $2.00 each. The food truck's revenue from selling a total of 209 salads and drinks in one day was $836.50. How many salads were sold that day? 1 2 3 4 5 6

7. Find an equation of a circle in the *xy*-plane with center (0,4) and a radius with endpoint (4/3, 5). 1 2 3 4 5 6
8. Katrina is a botanist studying the production of pears by two types of pear trees. She noticed that Type A trees produced 20 percent more pears than Type B trees did. Based on Katrina's observation, if the Type A trees produced 144 pears, how many pears did the Type B trees produce? 1 2 3 4 5 6
9. For the polynomial *p(x)*, the value of *p(3)* is -2. What can you say about the degree of *p(x)*? 1 2 3 4 5 6
10. The equation $h = -4.9t^2 + 25t$ expresses the approximate height, *h*, in meters, of a ball *t* seconds after it is launched vertically upward from the ground with an initial velocity of 25 meters per second. After approximately how many seconds will the ball hit the ground? 1 2 3 4 5 6
11. For what value of *x* is the function $h(x) = \frac{1}{(x-5)^3+4(x-5)+4}$ undefined? 1 2 3 4 5 6
12. Wyatt can husk at least 12 dozen ears of corn per hour and at most 18 dozen ears of corn per hour. Based on this information, what is the possible amount of time, in hours, that it could take Wyatt to husk 72 dozen ears of corn 1 2 3 4 5 6
13. A dairy farmer uses a storage silo in the shape of a right circular cylinder. If the volume of the silo is 72π cubic yards and the height of the silo is 8 yards, what is the diameter of the base of the cylinder in yards? 1 2 3 4 5 6
14. In a right triangle on angle measures $x°$, where $\sin x° = 4/5$. What is $\cos(90° - x°)$? 1 2 3 4 5 6
15. The posted weight limit for a covered wooden bridge in Pennsylvania is 6000 pounds. A delivery truck that is carrying *x* identical boxes each weighing 14 pounds will pass over the bridge. If the combined weight of the empty delivery truck and its driver is 4500 pounds, what is the maximum possible value for *x* that will keep the combined weight of the truck, driver, and boxes below the bridge's posted weight limit? 1 2 3 4 5 6
16. A Ferris wheel measures 80 feet in circumference. The distance on the circle between two of the seats S and T, is 10 feet. See the figure below. Find the measure in degrees of the central angle SCT whose rays support the two seats. 1 2 3 4 5 6

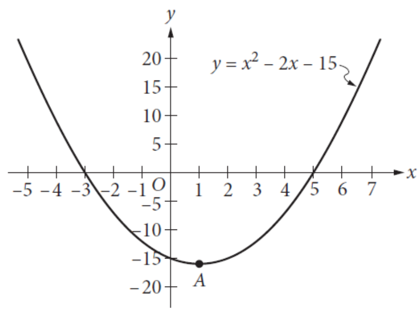


17. A summer camp counselor wants to find a length, *x*, in feet, across a lake as represented in the sketch below. The lengths represented by *AB, EB, BD, and CD* on the sketch were determined to by 1800 feet, 1400, feet, 700 feet, and 800 feet, 1 2 3 4 5 6

respectively. Segments AC and DE intersect at B, and $\angle AEB$ and $\angle CDB$ have the same measure. What is the value of x?



18. Which of the following is an equivalent form of the equation of the graph shown in the xy-plane, from which the coordinates of vertex A can be identified as constants in the equation? 1 2 3 4 5 6



Part 2: Directions: Please use the following scale to answer each question.

1	2	3	4	5	6
Strongly Disagree	Moderately Disagree	Disagree	Agree	Moderately Agree	Strongly Agree

1. I will continually find better ways to teach mathematics. 1 2 3 4 5 6
2. Even if I try very hard, I will not teach mathematics as well as I teach most subjects. 1 2 3 4 5 6
3. I know how to teach mathematics concepts effectively. 1 2 3 4 5 6
4. I will not be very effective in monitoring mathematics activities. 1 2 3 4 5 6
5. I will generally teach mathematics ineffectively. 1 2 3 4 5 6
6. I understand mathematics concepts well enough to be effective in teaching high school mathematics. 1 2 3 4 5 6
7. I will find it difficult to use manipulatives to explain to students why mathematics works. 1 2 3 4 5 6
8. I will typically be able to answer students' questions. 1 2 3 4 5 6
9. I wonder if I have the necessary skills to teach mathematics. 1 2 3 4 5 6
10. Given a choice, I will not invite the principal to evaluate my mathematics teaching. 1 2 3 4 5 6
11. When a student has difficulty understanding a mathematics concept, I will usually be at a loss as to how to help the student understand it better. 1 2 3 4 5 6

12. When teaching mathematics, I will usually welcome student questions. 1 2 3 4 5 6
13. I do not know what to do to turn students on to mathematics. 1 2 3 4 5 6

Part 3: Directions: How much confidence do you have that you are able to successfully perform each of the following tasks? Please use the following scale:

1	2	3	4	5	6
Not confident at all					Completely confident

1. Add two large numbers in your head. 1 2 3 4 5 6
2. Multiply quantities in a recipe to feed a larger group. 1 2 3 4 5 6
3. Doubling a recipe that contains fractions. 1 2 3 4 5 6
4. Figure out how long it will take to travel from City A to City B driving x mph. 1 2 3 4 5 6
5. Understand a graph accompanying an article on business profits. 1 2 3 4 5 6
6. Figure out how much you would save if there were a 15% markdown on an item you wish to buy. 1 2 3 4 5 6
7. Estimate your grocery bill in your head as you pick up items. 1 2 3 4 5 6
8. Figure out which of two summer jobs is the better offer: one with a higher salary but no benefits, the other with a lower salary plus room, board, and travel expenses. 1 2 3 4 5 6
9. Figure out the tip on your part of a dinner bill. 1 2 3 4 5 6
10. Figure out how much lumber you need to buy in order to build a set of bookshelves. 1 2 3 4 5 6
11. Measure your height in centimeters. 1 2 3 4 5 6
12. Determine how many boxes of a certain size will fit into a closet. 1 2 3 4 5 6
13. Explain your chances of flipping tails on both of two coins. 1 2 3 4 5 6

Part 4: Directions: Please rate the following mathematics topics according to how confident you would be teaching students each topic. Please use the following scale:

1	2	3	4	5	6
Not confident at all					Completely confident

1. Ratios and Proportions 1 2 3 4 5 6
2. Quadratic Functions 1 2 3 4 5 6
3. Conditional Probability 1 2 3 4 5 6
4. Geometric Properties and Relationships 1 2 3 4 5 6
5. Fractions 1 2 3 4 5 6
6. Polynomials 1 2 3 4 5 6
7. Complex Number Systems 1 2 3 4 5 6

8. Geometric Measurement and Dimensions	1	2	3	4	5	6
9. Trigonometric Functions	1	2	3	4	5	6
10. Vectors & Matrix Quantities	1	2	3	4	5	6
11. Interpreting Categorical & Quantitative Data	1	2	3	4	5	6
12. Geometric Congruence	1	2	3	4	5	6
13. Reasoning with Equations & Inequalities	1	2	3	4	5	6

Part 5: Directions: Please rate the following statements to how much you agree or disagree. Please use the following scale:

1	2	3	4	5	6
Strongly Disagree	Moderately Disagree	Disagree	Agree	Moderately Agree	Strongly Agree

1. It is important for a student to discover mathematics.	1	2	3	4	5	6
2. Students should spend time practicing computational procedures before they are expected to understand those procedures.	1	2	3	4	5	6
3. The teacher should demonstrate how to solve mathematics problems before the students are allowed to solve problems.	1	2	3	4	5	6
4. Students should understand computational procedures before they spend time practicing them.	1	2	3	4	5	6
5. During class discussion, the teacher should be the authority in terms of whether a student's mathematical conjecture or justification is correct.	1	2	3	4	5	6
6. During class discussions, students should play a role in determining whether mathematical justifications are valid.	1	2	3	4	5	6
7. Teachers should encourage students to invent ways to solve mathematical problems.	1	2	3	4	5	6
8. Students should understand how mathematics ideas interconnect and build on one another to produce a coherent whole.	1	2	3	4	5	6
9. Students should engage in problem solving before they master computational procedures and basic concepts.	1	2	3	4	5	6
10. Students should be given the opportunities to learn mathematics by developing and investigating mathematical conjectures, arguments, and proofs.	1	2	3	4	5	6
11. Allowing students to discuss mathematics with a partner, or in a group, is an important instructional strategy.	1	2	3	4	5	6
12. Struggling with mathematical concepts is detrimental to developing understanding.	1	2	3	4	5	6
13. During instruction, teachers should teach each mathematical idea separately rather than emphasizing the interconnections among ideas.	1	2	3	4	5	6
14. Group discussions often lead to tangents, or incorrect mathematics, and should be limited in their use.	1	2	3	4	5	6
15. Students learn mathematics best from teachers' demonstrations and explanations.	1	2	3	4	5	6

16. Students learn mathematics by studying mathematical arguments and proofs presented by the teacher or shown in the textbook.	1	2	3	4	5	6
17. It is not necessary for students to understand the interconnections among mathematical ideas as long as they have some understanding of the individual topics.	1	2	3	4	5	6
18. Students should not attempt problem solving until the understand basic concepts.	1	2	3	4	5	6
19. The best way to teach problem solving is to focus on one type of mathematics problem at a time.	1	2	3	4	5	6
20. Teachers should provide opportunities for students to critique mathematical arguments and discuss their own conjectures.	1	2	3	4	5	6
21. The teacher should provide verification for mathematical arguments rather than expecting students to do so.	1	2	3	4	5	6
22. During instruction, teachers should emphasize the interconnections among mathematical ideas.	1	2	3	4	5	6
23. Struggling with mathematical concepts is beneficial to developing understanding.	1	2	3	4	5	6
24. If a teacher encounters a student who is struggling with a problem, they should ask a question that will help the student find a solution but should not directly give the answer.	1	2	3	4	5	6
25. If a student is having difficulties solving a problem, then the teacher should tell the student how to solve it.	1	2	3	4	5	6
26. My job as a teacher is to make mathematics interesting and engaging.	1	2	3	4	5	6
27. The structure of mathematics as a subject is more important in making instructional decisions than the natural development of students' ideas.	1	2	3	4	5	6

Part 6: Demographic Questions

1. Which type of school do you work in? (Circle one on each line)
   a. Urban   Suburban   Rural
   b. Parochial   Public   Private   Charter
2. Is teaching your first career?   Yes   No
3. What is your gender?   Male   Female   Prefer not to answer
4. Are you a parent?   Yes   No
5. What is your race? (optional)
   African American   Asian   Hispanic
   White   Mixed   Other ___________________
6. What is your highest level of degree earned?
   Associates   Bachelor's   Master's   Doctorate
7. What was your major in college? _____________________
8. How many years have you been teaching?
   (0–2) (3–5) (6–10) (11–15) (16–20) (21–30) (30+)
9. What type of teaching certificate/license do you hold? Circle all that apply.
   a. Type: Teaching license   Provisional certificate   Permanent Certificate

b. Grades: PreK K 1 2 3 4 5 6 7 8
9 10 11 12

c. Other: Math Specialist Middle grade validation Math concentration
Other: ___________________________

10. Circle all the subjects that you teach.
    Language Arts Mathematics Reading Science Social Studies
    Other: ______________________
11. What subjects are you most confident teaching in an elementary school?
    Language Arts Mathematics Reading Science Social Studies
12. What subject are you least confident teaching in an elementary school?
    Language Arts Mathematics Reading Science Social Studies

Please keep in mind that all answers will be kept strictly confidential. Please provide the following information:

Name: Grade:
School: School District:

Thank you for taking the time to participate in my study!

Survey Number

APPENDIX E: MATHEMATICS CLASSROOM OBSERVATION PROTOCOL FOR PRACTICES (MCOP^2)

(Gleason et al., 2015)

1. Students engaged in exploration/investigation/problem solving.				
SE		**Description**		**Comments**
3		Students regularly engaged in exploration, investigation, or problem solving. Over the course of the lesson, the majority of the students engaged in exploration/investigation/problem solving.		
2		Students sometimes engaged in exploration, investigation, or problem solving. Several students engaged in problem solving, but not the majority of the class.		
1		Students seldom engaged in exploration, investigation, or problem solving. This tended to be limited to one or a few students engaged in problem solving while other students watched but did not actively participate.		
0		Students did not engage in exploration, investigation, or problem solving. There were either no instances of investigation or problem solving, or the instances were carried out by the teacher without active participation by any students.		
2. Students used a variety of means (models, drawings, graphs, concrete materials, manipulatives, etc.) to represent concepts.				
SE		**Description**		**Comments**
3		The students manipulated or generated two or more representations to represent the same concept, and the connections across the various representations, relationships of the representations to the underlying concept, and applicability or the efficiency of the representations were explicitly discussed by the teacher or students, as appropriate.		
2		The students manipulated or generated two or more representations to represent the same concept, but the connections across the various representations, relationships of the representations to the underlying concept, and applicability or the efficiency of the representations were **not** explicitly discussed by the teacher or students.		
1		The students manipulated or generated one representation of a concept.		
0		There were either no representations included in the lesson, or representations were included but were exclusively manipulated and used by the teacher. If the		

		students only watched the teacher manipulate the representation and did not interact with a representation themselves, it should be scored a 0.	
3. Students were engaged in mathematical activities.			
SE		**Description**	**Comments**
3		Most of the students spend two-thirds or more of the lesson engaged in mathematical activity at the appropriate level for the class. It does not matter if it is one prolonged activity or several shorter activities. (Note that listening and taking notes does not qualify as a mathematical activity unless the students are filling in the notes and interacting with the lesson mathematically.)	
2		Most of the students spend more than one-quarter but less than two-thirds of the lesson engaged in appropriate level mathematical activity. It does not matter if it is one prolonged activity or several shorter activities.	
1		Most of the students spend less than one-quarter of the lesson engaged in appropriate level mathematical activity. There is at least one instance of students' mathematical engagement.	
0		Most of the students are not engaged in appropriate level mathematical activity. This could be because they are never asked to engage in any activity and spend the lesson listening to the teacher and/or copying notes, or it could be because the activity they are engaged in is not mathematical – such as a coloring activity.	
4. Students critically assessed mathematical strategies.			
SE	**TF**	**Description**	**Comments**
3	**3**	More than half of the students critically assessed mathematical strategies. This could have happened in a variety of scenarios, including in the context of partner work, small group work, or a student making a comment during direct instruction or individually to the teacher.	
2	**2**	At least two but less than half of the students critically assessed mathematical strategies. This could have happened in a variety of scenarios, including in the context of partner work, small group work, or a student making a comment during direct instruction or individually to the teacher.	
1	**1**	An individual student critically assessed mathematical strategies. This could have happened in a variety of scenarios, including in the context of partner work, small group work, or a student making a comment	

		during direct instruction or individually to the teacher. The critical assessment was limited to one student.	
0	**0**	Students did not critically assess mathematical strategies. This could happen for one of three reasons: 1) No strategies were used during the lesson; 2) Strategies were used but were not discussed critically. For example, the strategy may have been discussed in terms of how it was used on the specific problem, but its use was not discussed more generally; 3) Strategies were discussed critically by the teacher but this amounted to the teacher telling the students about the strategy(ies), and students did not actively participate.	
5. Students persevered in problem solving.			
SE		**Description**	**Comments**
3		Students exhibited a strong amount of perseverance in problem solving. The majority of students looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary. When confronted with an obstacle (such as how to begin or what to do next), the majority of students continued to use resources (physical tools as well as mental reasoning) to continue to work on the problem.	
2		Students exhibited some perseverance in problem solving. Half of students looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary. When confronted with an obstacle (such as how to begin or what to do next), half of students continued to use resources (physical tools as well as mental reasoning) to continue to work on the problem.	
1		Students exhibited minimal perseverance in problem solving. At least one student but less than half of students looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary. When confronted with an obstacle (such as how to begin or what to do next), at least one student but less than half of students continued to use resources (physical tools as well as mental reasoning) to continue to work on the problem. There must be a roadblock to score above a 0.	
0		Students did not persevere in problem solving. This could be because there was no student problem solving in the lesson, or because when presented with a problem solving situation no students persevered. That is to say, all students either could not figure out how to get started	

TF	Description	Comments
	on a problem, or when they confronted an obstacle in their strategy, they stopped working.	
6.	**The lesson involved fundamental concepts of the subject to promote relational/conceptual understanding.**	
TF	**Description**	**Comments**
3	The lesson includes fundamental concepts or critical areas of the course, as described by the appropriate standards, and the teacher/lesson uses these concepts to build relational/conceptual understanding of the students with a focus on the "why" behind any procedures included.	
2	The lesson includes fundamental concepts or critical areas of the course, as described by the appropriate standards, but the teacher/lesson misses several opportunities to use these concepts to build relational/conceptual understanding of the students with a focus on the "why" behind any procedures included.	
1	The lesson mentions some fundamental concepts of mathematics but does not use these concepts to develop the relational/conceptual understanding of the students. For example, in a lesson on the slope of the line, the teacher mentions that it is related to ratios, but does not help the students to understand how it is related and how that can help them to better understand the concept of slope.	
0	The lesson consists of several mathematical problems with no guidance to make connections with any of the fundamental mathematical concepts. This usually occurs with a teacher focusing on procedure of solving certain types of problems without the students understanding the "why" behind the procedures.	
7.	**The lesson promoted modeling with mathematics.**	
TF	**Description**	**Comments**
3	Modeling (using a mathematical model to describe a real-world situation) is an integral component of the lesson with students engaged in the modeling cycle (as described in the Common Core State Standards).	
2	Modeling is a major component, but the modeling has been turned into a procedure (i.e. a group of word problems that all follow the same form and the teacher has guided the students to find the key pieces of information and how to plug them into a procedure.); or modeling is not a major component, but the students engage in a modeling activity that fits within the corresponding standard of mathematical practice.	

1	The teacher describes some type of mathematical model to describe real-world situations, but the students do not engage in activities related to using mathematical models.	
0	The lesson does not include any modeling with mathematics.	

8. The lesson provided opportunities to examine mathematical structure. (symbolic notation, patterns, generalizations, conjectures, etc.)

TF	Description	Comments
3	The students have a sufficient amount of time and opportunity to look for and make use of mathematical structure or patterns.	
2	Students are given some time to examine mathematical structure but are not allowed adequate time or are given too much scaffolding so that they cannot fully understand the generalization.	
1	Students are shown generalizations involving mathematical structure but have little opportunity to discover these generalizations themselves or adequate time to understand the generalization.	
0	Students are given no opportunities to explore or understand the mathematical structure of a situation.	

9. The lesson included tasks that have multiple paths to a solution or multiple solutions.

TF	Description	Comments
3	A lesson which includes several tasks throughout; or a single task that takes up a large portion of the lesson; with multiple solutions and/or multiple paths to a solution and which increases the cognitive level of the task for different students.	
2	Multiple solutions and/or multiple paths to a solution are a significant part of the lesson, but are not the primary focus, or are not explicitly encouraged; **or** more than one task has multiple solutions and/or multiple paths to a solution that are explicitly encouraged.	
1	Multiple solutions and/or multiple paths minimally occur and are not explicitly encouraged; **or** a single task has multiple solutions and/or multiple paths to a solution that are explicitly encouraged.	
0	A lesson which focuses on a single procedure to solve certain types of problems and/or strongly discourages students from trying different techniques.	

10. The lesson promoted precision of mathematical language.

TF	Description	Comments
3	The teacher "attends to precision" in regard to communication during the lesson. The students also "attend to precision" in communication, or the teacher	

SE	TF	Description	Comments
		guides students to modify or adapt non-precise communication to improve precision.	
	2	The teachers "attends to precision" in all communication during the lesson, but the students are not always required to also do so.	
	1	The teacher makes a few incorrect statements or is sloppy about mathematical language, but generally uses correct mathematical terms.	
	0	The teacher makes repeated incorrect statements or incorrect names for mathematical objects instead of their accepted mathematical names.	
11. The teacher's talk encouraged student thinking.			
	TF	**Description**	**Comments**
	3	The teacher's talk focused on **high levels** of mathematical thinking. The teacher may ask lower level questions within the lesson, but this is not the focus of the practice. There are three possibilities for high levels of thinking: analysis, synthesis, and evaluation. **Analysis**: examines/ interprets the pattern, order or relationship of the mathematics; parts of the form of thinking. **Synthesis**: requires original, creative thinking. **Evaluation**: makes a judgment of good or bad, right or wrong, according to the standards he/she values.	
	2	The teacher's talk focused on **mid-levels** of mathematical thinking. **Interpretation**: discovers relationships among facts, generalizations, definitions, values and skills. **Application**: requires identification and selection and use of appropriate generalizations and skills	
	1	Teacher talk consists of **"lower order"** knowledge based questions and responses focusing on recall of facts. **Memory**: recalls or memorizes information. **Translation**: changes information into a different symbolic form or situation.	
	0	Any questions/ responses of the teacher related to mathematical ideas were rhetorical in that there was no expectation of a response from the students.	
12. There were a high proportion of students talking related to mathematics.			
SE		**Description**	**Comments**
3		More than three quarters of the students were talking related to the mathematics of the lesson at some point during the lesson.	
2		More than half, but less than three quarters of the students were talking related to the mathematics of the lesson at some point during the lesson.	

SE	TF	Description	Comments
1		Less than half of the students were talking related to the mathematics of the lesson.	
0		No students talked related to the mathematics of the lesson.	
13. There was a climate of respect for what others had to say.			
SE	**TF**	**Description**	**Comments**
3	**3**	**Many** students are sharing, questioning, and commenting during the lesson, including their struggles. Students are also listening (active), clarifying, and recognizing the ideas of others.	
2	**2**	The environment is such that **some** students are sharing, questioning, and commenting during the lesson, including their struggles. Most students listen.	
1	**1**	Only a **few** share as called on by the teacher. The climate supports those who understand or who behave appropriately. **Or** Some students are sharing, questioning, or commenting during the lesson, but most students are actively listening to the communication.	
0	**0**	No students shared ideas.	
14. In general, the teacher provided wait-time.			
SE		**Description**	**Comments**
3		The teacher **frequently** provided an ample amount of "think time" for the depth and complexity of a task or question posed by either the teacher or a student.	
2		The teacher **sometimes** provided an ample amount of "think time" for the depth and complexity of a task or question posed by either the teacher or a student.	
1		The teacher **rarely** provided an ample amount of "think time" for the depth and complexity of a task or question posed by either the teacher or a student.	
0		The teacher **never** provided an ample amount of "think time" for the depth and complexity of a task or question posed by either the teacher or a student.	
15. Students were involved in the communication of their ideas to others (peer-to-peer).			
SE		**Description**	**Comments**
3		**Considerable** time **(more than half)** was spent with peer to peer dialog (pairs, groups, whole class) related to the communication of ideas, strategies and solution.	
2		**Some** class time **(less than half, but more than just a few minutes)** was devoted to peer to peer (pairs, groups, whole class) conversations related to the mathematics.	
1		The lesson was primarily teacher directed and little opportunities were available for peer to peer (pairs, groups, whole class) conversations. A few instances	

	TF	Description	Comments
		developed where this occurred during the lesson but only lasted less than 5 minutes.	
0		No peer to peer (pairs, groups, whole class) conversations occurred during the lesson.	
16. The teacher uses student questions/comments to enhance conceptual mathematical understanding.			
	TF	**Description**	**Comments**
	3	The teacher **frequently** uses student questions/ comments to coach students, to facilitate conceptual understanding, and boost the conversation. The teacher sequences the student responses that will be displayed in an intentional order, and/or connects different students' responses to key mathematical ideas.	
	2	The teacher **sometimes** uses student questions/ comments to enhance conceptual understanding.	
	1	The teacher **rarely** uses student questions/ comments to enhance conceptual mathematical understanding. The focus is more on procedural knowledge of the task verses conceptual knowledge of the content.	
	0	The teacher **never** uses student questions/ comments to enhance conceptual mathematical understanding.	
Additional Notes: Live or Video, # of Students, Grade Level, topic/subject, date, other demographics, school, etc.			

APPENDIX F: END-OF-STUDY INTERVIEW PROTOCOL

(Adapted from Kahle, 2008)

Part 1: I will ask you some questions about your survey results and overall experience teaching and learning mathematics.

1. Why did you choose to teach ________ grade? (Insert the current grade)
2. What subjects/concepts do you teach?
3. What is your least favorite mathematical concept to teach? Why?
4. What is your favorite mathematical concept to teach? Why?
5. Do you think your confidence level is different when teaching your least favorite mathematical concept versus your favorite mathematical concept? Why?
6. Do you believe yourself to be a person that is good at mathematics?
7. What was your experience with the learning of mathematics?
8. Describe a time in your learning of mathematics that you felt the most confident. Least confident.
9. Has your district provided PD on mathematics teaching methods? If so, have you attended?
    a. If participant attended – What takeaways did you get from attending? Did it change your teaching methods?
    b. If participant did not attend – Why did you not attend the PD?
10. How long have you been teaching mathematics?
    a. How has your teaching methods changed over those years?
11. *Using the survey answers, ask the participant to* _____________________ Describe a lesson which introduces (most confident mathematics teaching topic).
12. *Using the survey answers, ask the participant to* _____________________ Describe a lesson which introduces (least confident mathematics teaching topic).

For questions 8& 9 use a sampling of the following probing questions:

Survey Response	Follow-up question
Teaching Goal: Explaining "why" or explaining "how".	When explaining why a math procedure works, how much can students understand?
Algorithms: memorize steps or discover steps	How does a student come to understand the steps of an algorithm?
Number of Skills: Teaching sequential isolated skills or mixing math concepts.	Should skills be taught in isolation or mixed?
Calculators: for problem solving or for computations	What is the role of calculators in your classroom?
Wrong Answers: Should be corrected or should lead to discussion.	What do you do when a student gives a wrong answer?

Survey Response	Follow-up question
Goal for Students: Understanding or speed and accuracy	How important is developing students' speed and accuracy of getting answers?
Focus: Concept development or skill drill	What is the most important thing for students to learn in mathematics?
Solution Process: One right way or many right ways	How important is it to learn a solution process for a particular type of problem?
Problem Solving or Solving Word Problems	What types of problem solving do students experience in your class?
Questioning: Justify reasons or recite facts	What types of questions are important to ask in your mathematics class?
Manipulatives: to explore or to models	How and when do your students use manipulatives?
Role of Teacher: Teacher demonstrates or teacher facilitates	What is the role of the teacher in your math class?

Part 2: I will show you some video from the observations. For each of the excerpts, I will ask the following questions.

1. What effective practice did you use in this video clip?
2. What led you to choose that practice?
3. How confident do you feel integrating these practices in your teaching?
4. Do you usually plan to use the effective practices? If not, how do you know you are integrating them in your teaching?

Part 3: The following questions will help me to better understand how you have modified your instruction during the 2020 Pandemic.

1. What is your level of confidence this year compared to past years in the teaching of mathematics?
2. During the transition to online learning in the spring of 2020, how did you feel about the lessons you were providing for your students?
3. Overall, how confident do you feel about the level of understanding your students acquired during the transition to online learning in the spring? In the fall?
4. Do you believe that your experience teaching during the pandemic has influenced your ability to teach mathematics effectively?

APPENDIX G: ALIGNMENT OF MCOP^2 AND E-MCOP^2

After I had completed my data analysis using the E-MCOP^2 framework, I spoke with one of the original authors of the MCOP^2 (personal communication, November 19, 2022). In this communication, the author alerted me to a document in which he and others had attempted to create an alignment between the MCOP^2 and NCTM's (2014) MTPs (Zelkowski et al., 2020). Following this communication, I returned to the E-MCOP^2 to verify my alignment between NCTM's (2014) MTPs and the observation frameworks (e.g., E-MCOP^2 and MCOP^2; see Table 35). The first column of Table 35 contains the item number from the MCOP^2 (see Appendix E; Gleason et al., 2015). The second column is split for the purpose of showing my initial thoughts on the alignment between the eight MTPs—numbered 1-8 (NCTM, 2014)—the 16 MCOP^2 items and my final determination of the alignment between MCOP^2 item and MTP. As noted previously, my goal was to align a single MTP with each MCOP^2 item to provide clarity in the E-MCOP^2. The final column of Table 35, includes Zelkowski et al. (2020) alignment between the MCOP^2 items and the eight MTP, allowing for multiple MTPs to align with the same MCOP^2 item.

Table 35

Comparison of E-MCOP^2 Items to Zelkowski et al. (2020) Item List

Item from MCOP^2	E-MCOP^2		Zelkowski et al. (2020)
	Initial Thoughts	MTP	MTP
1	2	2	1, 2
2	3	3	3
3	1, 2	-	2
4	3, 4	6	4
5	7	7	7
6	1, 6	6	6
7	**3**	**-**	**2**

8	**3**	**2**	**1**
9	2	2	2
10	4, 5, 8	8	4
11	4, 5, 8	5	5, 8
12	4	-	4
13	4	4	4
14	**7**	**7**	**8**
15	4	4	4
16	4, 5, 6, 8	6	4, 5, 8

Note. Includes "Engaging mentor teachers with teacher candidates during methods courses in clinical settings," by J. Zelkowski, J. Yow, M. Ellis, & P. Waller, 2020, in W. G. Martin, B R. Lawler, A. E. Lischka, & W. M. Smith (Eds.), *The mathematics teacher education partnership: The power of a networked improvement community to transform secondary mathematics teacher preparation*. (pp. 211–234). AMTE.

After I inspected the alignment between my thoughts on MCOP^2 items matched with MTPs (see Table 35, third column) and Zelkowski et al.'s (2020) MCOP^2 items matched with MTPs (see Table 35, fourth column), I revisited items in which there was no agreement among my original thoughts and the E-MCOP^2 and Zelkowski et al.'s (2020) alignment. As shown in Table 35, there was no agree between the authors and me for Items 7, 8, and 14 of the MCOP^2 (bolded for emphasis in Table 35). In Table 36, I show the Item descriptions and the MTP aligned with each item in both the E-MCOP^2 and the MCOP^2. In the following paragraphs, I describe the process of revisiting each of the items in relation to the context of the MTP, E-MCOP^2, and the MCOP^2.

Table 36

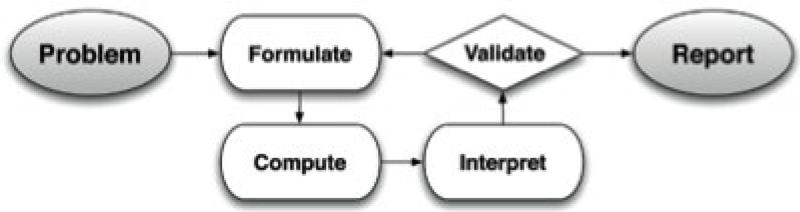
MCOP^2 Item Description and MTP Discrepancies

MCOP^2 Item	E-MCOP^2 MTP	Zelkowski et al. (2020) MTP
Item 7 The lesson promoted modeling with mathematics.	None	Implement tasks that promote reasoning and problem solving.
Item 8 The lesson provided opportunities to examine mathematical structure. (Symbolic notation, patterns, generalizations, conjectures, etc.)	Implement tasks that promote reasoning and problem solving.	Establish mathematics goals to focus learning.
Item 14 In general, the teacher provided wait-time.	Support productive struggle in learning mathematics.	Elicit and use evidence of student thinking.

Item 7 involves students "modeling (using a mathematical model to describe a real-world situation) …with students engaged in the modeling cycle." (Gleason et al., 2015, p. 11). Figure 11 shows the modeling cycle, as described in the Common Core State Standards. In the Modeling Cycle, students are working through a cyclic process of problem solving—formulate, compute, interpret, and validate—until they report a solution. In P2A, NCTM (2014) described implement tasks that promote reasoning and problem solving as engaging "students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies" (p. 17). Because of the focus in the NCTM statement on "reasoning and problem solving" and the Modeling Cycle is also focused on a problem-solving process rather than on using models to represent real-world situations, as indicated in Item 7 of the MCOP^2, I chose to not use Item 7 to represent the implementing tasks MTP.

Figure 11

Modeling Cycle



Note. From "Common Core State Standards for Mathematics," by NGA & CCSSO, 2010, p. 72 (https://www.isbe.net/Documents/math-standards.pdf).

Zelkowski et al. (2020) found that Item 8 aligned with the MTP focused on establishing mathematics goals to focus learning. Though a teacher could write a mathematics goal which focuses on students generalizing and using mathematical structure to solve tasks, that might not always be the case. For the E-MCOP^2, I aligned Item 8 with the MTP focused on teachers implementing tasks in which students are engaging in "solving and discussing takes that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies" (NCTM, 2014, p. 10). In the description of each item, Gleason et al. (2015) described Item 8 in terms of students exploring and making use of mathematical structure or "to use repeated reasoning to generalize" (p. 12). Although the language differs slightly, students who are reasoning mathematically through tasks are, in essence, exploring and using the mathematical structure.

Lastly, Item 14 was aligned by Zelkowski et al. (2020) with the MTP in which teachers are to elicit and use evidence of student thinking. Whereas I aligned Item 14 with MTP to support productive struggle in learning mathematics. In P2A, NCTM (2014) described teacher actions while engaging in each teaching practice and stated, “giving students time to struggle with tasks,” (p. 52) as an action for supporting students in productive struggle. I believe this aligns with Item 14, which focuses on the teacher providing wait time so that students can provide responses that are based in reasoning and making sense of their thinking. Though Item 14 does focus on providing the time for students to organize their thinking, the actions from the teacher in P2A (NCTM, 2014) are not specific that wait time is an action that would yield using the MTP focused on eliciting and using evidence of student thinking.