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ON THE MONOPHONIC AND MONOPHONIC DOMINATION POLYNOMIAL OF A GRAPH

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ABSTRACT. A set S of vertices of a graph G is a *monophonic set* of G if each vertex u of G lies on an $u-v$ monophonic path in G for some $u, v \in S$. $M \subseteq V(G)$ is said to be a monophonic dominating set if it is both a monophonic set and a dominating set. Let $M(G,i)$ be the family of monophonic sets of a graph G with cardinality i and let $m(G,i) = |M(G,i)|$. Then the monophonic polynomial $M(G,x)$ of G is defined as $M(G,x) = \sum_{i=m(G)}^{n} m(G,i)x^i$, where $m(G)$ is the monophonic number of G. In this article, we have introduced monophonic domination polynomial of a graph. We have computed the monophonic and monophonic domination polynomials of some specific graphs. In addition, monophonic and monophonic domination polynomial of the corona product of two graphs is derived.

Keywords: Monophonic set, Monophonic Dominating set, Monophonic polynomial, Monophonic domination polynomial, Corona product.

AMS Subject Classification: 05C12, 05C69.

1. INTRODUCTION

The graph G considered in this paper is finite, simple, undirected and connected with vertex set $V(G)$ and edge set $E(G)$ respectively. The order and size of G is denoted by n and m respectively. [4, 5] is referred for basic graph theoretic definitions. The *distance* $d(u,v)$ between two vertices u and v is the length of a shortest $u-v$ path in G. The *neighborhood* of a vertex v denoted by $N(v)$ is the set of all vertices adjacent to v. For any subset S of $V(G)$, the *induced subgraph* $\langle S \rangle$ is the maximal subgraph of G with the vertex set S. A vertex v is said to be an *extreme vertex* if the subgraph $\langle N(v) \rangle$ is complete. The join of graphs G and H, denoted by $G+H$, is the graph with vertex set

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$V(G+H) = V(G) \cup V(H)$ and edge set $E(G+H) = E(G) \cup E(H) \cup \{uv : u \in V(G)$ *and* $v \in V(H)\}$. Two vertices (x_1, y_1) and (x_2, y_2) in the cartesian product $G\Box H$ is adjacent if and only if either $x_1 = x_2$ and y_1 is adjacent to y_2 or $y_1 = y_2$ and x_1 is adjacent to x_2. $F \subseteq V(G)$ is said to be a dominating set if every vertex in $V(G) - F$ is adjacent to at least one vertex in F. The least order of the dominating sets of G is said to be the domination number of G and is denoted by $\gamma(G)$. The monophonic number of a graph was studied by Pelayo et al. in [6, 10]. Any chordless path connecting the vertices u and v is called a $u - v$ m-path. The monophonic closure of a subset S of $V(G)$ is given by $J_G[S] = \bigcup\limits_{u,v\in S} J_G[u, v]$, where $J_G[u, v]$ is the set containing u and v and all vertices lying on some $u - v$ m-path. If $J_G[S] = V(G)$, then S is said to be a monophonic set in G. A monophonic set in G of least order is called a minimum monophonic set of G. The order of a minimum monophonic set in G is called the monophonic number of G and is denoted by $m(G)$. Monophonic domination number of a graph is studied in [7, 11]. $M \subseteq V(G)$ is said to be a monophonic dominating set if it is both a monophonic set and a dominating set. The minimum cardinality of a monophonic dominating set of G is the monophonic domination number and is denoted by $\gamma_m(G)$. The domination polynomial of a graph was introduced by Saeid Alikhani et al. in [3] and was later studied in [1, 2]. The monophonic polynomial of a graph was introduced and studied in [8].

In the third section monophonic polynomial of some graphs are derived. In the fourth section we have defined monophonic domination polynomial of a graph and some results corresponding to it are studied. In the last section monophonic and monophonic domination polynomial of the corona product of two graphs are studied.

2. Preliminary Results

Theorem 2.1. [9] *Let G be a graph of order n. Then*
$$m(G + K_p) = \begin{cases} n+p & if \quad G = K_n \\ m(G) & if \quad G \neq K_n \end{cases}$$

Theorem 2.2. [12] *Let G be a connected graph with cutvertex v and let S be a monophonic set of G. Then every component of $G - v$ contains an element of S.*

Theorem 2.3. [12] *No cutvertex of a connected graph G belongs to any minimum monophonic set of G.*

3. Some results on the monophonic polynomial

Theorem 3.1. *Let T be a tree with s pendant vertices, Then $M(T, x) = x^s(1 + x)^{n-s}$.*

Proof. Let s be the number of pendant vertices in the tree T. Since T is a tree, $m(G) = s$. The set of all pendant vertices is the unique monophonic set of T. Hence the number of monophonic sets of cardinality s is 1. For a monophonic set of cardinality $s + y$ $(1 \leq y \leq n - s)$, we have $\binom{n-s}{y}$ choices.

Hence,
$$\begin{aligned} M(T, x) &= \sum_{i=s}^{n} m(t, i)x^i \\ &= x^s + \binom{n-s}{1}x^{s+1} + \binom{n-s}{2}x^{s+2} + \cdots + \binom{n-s}{n-s-1}x^{n-1} + x^n \\ &= x^s(1 + x)^{n-s}. \end{aligned}$$

□

Theorem 3.2. *For the cycle C_n $(n > 3)$, $M(C_n, x) = (1+x)^n - (1 + nx + nx^2)$.*

Proof. For a monophonic set of cardinality two, there are n ways to select the first vertex and $n-3$ ways to select the second vertex. Hence there are $\frac{n(n-3)}{2}$ choices to select two vertices. Hence $m(C_p, 2) = \frac{n(n-3)}{2}$. For the monophonic sets of cardinality $k > 2$, we have $\binom{n}{k}$ choices to select k vertices from a set of n vertices. Hence $m(C_n, k) = \binom{n}{k}$.

$$\begin{aligned} M(C_n, x) &= \frac{n(n-3)}{2}x^2 + \binom{n}{3}x^3 + \cdots + \binom{n}{n-1}x^{n-1} + x^n \\ &= (1+x)^n - 1 - nx + \left[\frac{n(n-3)}{2} - \binom{n}{2}\right]x^2 \\ &= (1+x)^n - (1 + nx + nx^2). \end{aligned}$$

□

Theorem 3.3. *Let G be a connected graph of order n, Then*
$$M(G + K_p, x) = \begin{cases} x^{n+p} & when \quad G = K_n \\ M(G, x)(1+x)^p & when \quad G \neq K_n \end{cases}.$$

Proof. When G = K_n, $G + K_p \cong K_{n+p}$. Hence $M(G + K_p, x) = x^{n+p}$. Next, let us consider the case when $G \neq K_n$. The monophonic set of G will be the monophonic set of $G + K_p$, hence the monophonic polynomial of $G + K_p$ is $M(G, x)(1 + x)^p$.

□

Theorem 3.4. *For each path $P_n, n \geq 3$ and the complete graph K_2,*
$M(P_n \Box K_2, x) = [x(x+2)]^2 (1+x)^{2n-4}$.

Proof. Let P_n be a path on n vertices and K_2 be the complete graph on two vertices. Let $A_1, A_2, \cdots, A_n$ be the n copies of K_2 in $P_n \Box K_2$. It can be verified that $m(P_n \Box K_2) = 2$. A monophonic set of cardinality 2 is selected by choosing a vertex from A_1 and a vertex from A_n. Hence $m(P_n \Box K_2, 2) = 4$. The monophonic set of cardinality 3 is selected either by choosing one vertex from A_1 and one vertex from A_2 or by choosing one vertex from A_1 and two vertex from A_2 or by choosing one vertex from A_2 and two vertex from A_1. Thus, $m(P_n \Box K_2, 3) = 4(2n-3)$. The monophonic set of cardinality $k+2, (2 \leq k \leq 2n-3)$ is obtained either by choosing one vertex from A_1 and A_n or by choosing two vertices from A_1 and one vertex from A_n or by choosing two vertices from A_n and one vertex from A_1 or by choosing two vertices from both A_1 and A_n. Hence the number of monophonic sets of cardinality $k+2, (2 \leq k \leq 2n-3)$ is $4\binom{2n-3}{k} + \binom{2n-4}{k-2}$. Also, $m(P_n \Box K_2, 2n) = 1$. Therefore,

$$\begin{aligned} M(P_n \Box K_2, x) &= 4x^2 + 4(2n-3)x^3 + \sum_{k=2}^{2n-3}\left[4\binom{2n-3}{k} + \binom{2n-4}{k-2}\right]x^{k+2} + x^{2n} \\ &= [x(x+2)]^2 (1+x)^{2n-4}. \end{aligned}$$

□

Lemma 3.1. *For each star graph $K_{1,n}$ and the complete graph K_2, $m(K_{1,n} \Box K_2) = n$.*

Proof. Let $\{v_1, v_2, \cdots, v_n, v_{n+1}\}$ and $\{u_1, u_2\}$ be the vertices of $K_{1,n}$ and K_2 respectively and $deg(v_1) = n$. Let $A_1, A_2, \cdots, A_n, A_{n+1}$ be the $n+1$ copies of K_2 in $K_{1,n} \Box K_2$. The vertices in the n copies of K_2 is adjacent to at least one vertex of A_1. Let $w_{ij} = (v_i, u_j)$. $J = \{w_{2j}, w_{3j}, \cdots, w_{(n+1)j} / j = 1 \text{ or } 2\}$ is a monophonic set of $K_{1,n} \Box K_2$. Hence $m(K_{1,n} \Box K_2) \leq$

n. Suppose $m(K_{1,n}\square K_2) < n$. Then there exists a monophonic set F of $K_{1,n}\square K_2$ which does not contain any vertex from k^{th} copy of K_2. Then the vertices $\{w_{k1}, w_{k2}\}$ will not be contained in the monophonic path joining vertices of F, which is a contradiction. Hence $m(K_{1,n}\square K_2) = n$. □

Theorem 3.5. *For each star graph $K_{1,n}$ and the complete graph K_2,* $M(K_{1,n}\square K_2, x) = [x(2+x)]^n\,(x+1)^2$.

Proof. Let $A_1, A_2, \cdots, A_n, A_{n+1}$ be the $n+1$ copies of K_2 in $K_{1,n}\square K_2$. For a monophonic set of cardinality n we have to choose at least one vertex from the n copies $A_2, \cdots, A_n, A_{n+1}$. Therefore, we have 2^n monophonic sets of cardinality n. For a monophonic set of cardinality $n+1$ we can select the vertices in two ways, the monophonic set may contain no vertices of A_1 or it may contain a vertex of A_1. Hence $m(K_{1,n}\square K_2, n+1) = n.2^{n-1} + 2^{n+1} = (n+4)2^{n-1}$. For a monophonic set of cardinality $n+k$, $2 \le k \le n$ the vertices can be selected in the following three ways; it may contain no vertices of A_1 or one vertex of A_1 or two vertices of A_1. By considering the above three ways we get $m(K_{1,n}\square K_2, n+k) = \binom{n}{k}2^{n-k} + \left[\binom{n}{k-1} + \binom{n}{k-2}\right]2^{n-k+2}$. Also, it is found that $m(K_{1,n}\square K_2, 2n+1) = 2n+2$ and $m(K_{1,n}\square K_2, 2n+2) = 1$. Thus,

$$\begin{aligned}
M(K_{1,n}\square K_2, x) &= 2^n x^n + (n+4)2^{n-1}x^{n+1} \\
&\quad + \sum_{k=2}^{n}\left\{\binom{n}{k}2^{n-k} + \left[\binom{n}{k-1} + \binom{n}{k-2}\right]2^{n-k+2}\right\}x^{n+k} \\
&\quad + (2n+2)x^{2n+1} + x^{2n+2} \\
&= 2^n x^n + (n+4)2^{n-1}x^{n+1} + \sum_{k=2}^{n}\left[\binom{n}{k}2^{n-k} + \binom{n+1}{k-1}2^{n-k+2}\right]x^{n+k} \\
&\quad + (2n+2)x^{2n+1} + x^{2n+2} \\
&= x^{n+1}\left[2^{n+1} + \sum_{k=2}^{n}\binom{n+1}{k-1}2^{n-k+2}x^{k-1} + 2(n+1)x^n + x^{n+1}\right] \\
&\quad + x^n\sum_{k=0}^{n}\binom{n}{k}2^{n-k}x^k \\
&= x^{n+1}(2+x)^{n+1} + x^n(2+x)^n \\
&= [x(2+x)]^n\,(x+1)^2.
\end{aligned}$$

□

4. Monophonic Domination Polynomial

Definition 4.1. *Let $MD(G,i)$ be the collection of monophonic dominating sets of a graph G with cardinality i and $md(G,i) = |MD(G,i)|$. The Monophonic Domination polynomial $MD(G,x)$ is defined as*

$$MD(G,x) = \sum_{i=\gamma_m(G)}^{n} md(G,i)x^i.$$

Example 4.1. *For the graph G given in Figure 1, $M = \{u_2, u_5, u_6\}$ is the unique minimal monophonic dominating set of G. Hence $md(G,3) = 1$. $\{u_1, u_2, u_5, u_6\}, \{u_2, u_3, u_5, u_6\}$, $\{u_2, u_4, u_5, u_6\}$ are the monophonic dominating sets of cardinality 4 and $md(G,4) = 3$.*

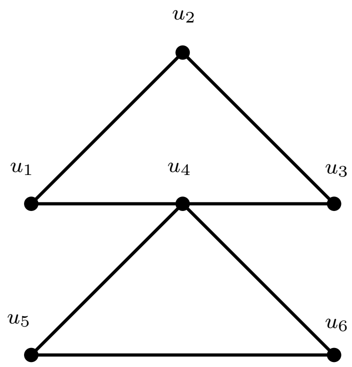

FIGURE 1. A graph G with $m(G) = \gamma_m(G) = 3$.

$\{u_1, u_2, u_3, u_5, u_6\}$, $\{u_2, u_3, u_4, u_5, u_6\}$, $\{u_1, u_2, u_4, u_5, u_6\}$ are the monophonic dominating sets of G with cardinality 5 and therefore $md(G, 5) = 3$. The monophonic domination polynomial of G is, $MD(G, x) = x^3 + 3x^4 + 3x^5 + x^6$.

Theorem 4.1. *Let G be a connected graph of order n. Then*

1. *$md(G, i) = 0$ if and only if $i < \gamma_m(G)$ or $i > n$.*
2. *$MD(G, x)$ is a monic polynomial of degree n.*
3. *Degree of $MD(G, x)$ will be greater than or equal to two always.*
4. *Zero is a root of $MD(G, x)$ with multiplicity $\gamma_m(G)$.*

Proof. (1) Let us assume that $md(G, i) = 0$ (i.e., G has no monophonic dominating set of cardinality i). This condition holds if and only if $i < \gamma_m(G)$ or $i > n$.
(2) The set of all vertices of G will be the unique monophonic dominating set of cardinality n. Hence the coefficient of the highest power of $MD(G, x)$ is one. Thus, $MD(G, x)$ is a monic polynomial of degree n.
(3) Since $\gamma_m(G) \geq 2$, it is clear that the degree of $MD(G, x)$ will be greater than or equal to two.
(4) By the definition of monophonic domination polynomial, it is clear that zero is a root of $MD(G, x)$ with multiplicity $\gamma_m(G)$. □

Theorem 4.2. *For a connected graph G of order n, $MD(G, x) = x^n$ if and only if G is complete.*

Proof. Assume $MD(G, x) = x^n$. Then $md(G, i) = 0$ for $i < n$ and $md(G, n) = 1$. This implies that the set of all vertices of G is the minimum monophonic dominating set of G and hence G is complete. The proof of the converse part is obvious. □

Theorem 4.3. *The monophonic domination polynomial of the complete bipartite graph $K_{n,m}$,*

$$MD(K_{n,m}, x) = \begin{cases} x^2 & if \quad n = m = 1 \\ x^n + x^{n+1} & if \quad n \geq 2, m = 1 \\ x^n(1+x)^m & if \quad n < m \leq 4 \\ [(1+x)^n - 1 - nx]\,[(1+x)^m - 1 - mx] & if \quad n > m > 4 \end{cases}.$$

Proof. Let $A = \{u_1, u_2, \cdots, u_n\}$ and $B = \{v_1, v_2, \cdots, v_m\}$ be the partition of vertices of $K_{n,m}$ respectively.

Case (i) : $n = m = 1$. The monophonic domination number of $K_{1,1}$ is two. Hence the monophonic domination polynomial $MD(K_{1,1}) = x^2$.
Case (ii) : $n \geq 2$, $m = 1$. The monophonic domination number of $K_{n,1}$ is n. The monophonic domination polynomial $MD(K_{n,1}, x) = x^n + x^{n+1}$.
Case (iii): $n, m \geq 2$.
Subcase (i): $n < m \leq 4$. In this case $\gamma_m(K_{n,m}) = n$. There exists a unique monophonic dominating set of cardinality n. For a monophonic dominating set of cardinality $n + k$ ($k \geq 1$), we have $\binom{m}{k}$ choices.

$$\begin{aligned} MD(K_{n,m}, x) &= x^n + \binom{m}{1}x^{n+1} + \binom{m}{2}x^{n+2} + \cdots + x^{n+m} \\ &= x^n(1 + x)^m. \end{aligned}$$

Subcase (ii): $n > m > 4$. For a monophonic dominating set of cardinality 4, we have $\binom{n}{2}\binom{m}{2}$ choices. For a monophonic dominating set of cardinality $k > 4$ we have $\binom{n}{k-2}\binom{m}{2} + \binom{n}{2}\binom{m}{k-2}$ choices. Hence

$$\begin{aligned} MD(K_{n,m}, x) &= \left[\binom{n}{2}x^2 + \binom{n}{3}x^3 + \cdots + \binom{n}{m}x^m + \cdots + \binom{n}{n-1}x^{n-1} + x^n\right] \\ &\times \left[\binom{m}{2}x^2 + \binom{m}{3}x^3 + \cdots + \binom{m}{m-1}x^{m-1} + x^m\right] \\ &= \left[(1 + x)^n - 1 - nx\right]\left[(1 + x)^m - 1 - mx\right]. \end{aligned}$$

□

Theorem 4.4. *If $G = K_n - \{e\}$, then $MD(G, x) = x^2(1 + x)^{n-2}$.*

Proof. Let u and v be the non adjacent vertices of G. $\{u, v\}$ will be the unique minimal monophonic dominating set of G. Hence $md(G, 2) = 1$. For a monophonic set of cardinality $k > 2$, we have $\binom{n-2}{k-2}$ ways to select k vertices. Therefore, $md(G, k) = \binom{n-2}{k-2}$.

$$\begin{aligned} MD(G, x) &= x^2 + \binom{n-2}{1}x^3 + \binom{n-2}{2}x^4 + \cdots + \binom{n-2}{n-3}x^{n-1} + x^n \\ &= x^2(1 + x)^{n-2}. \end{aligned}$$

□

Theorem 4.5. *Let G be a connected graph of order n, Then*
$$MD(G + K_p, x) = \begin{cases} x^{n+p} & \textit{when} \quad G = K_n \\ MD(G, x)(1 + x)^p & \textit{when} \quad G \neq K_n \end{cases}.$$

Proof. **Case 1: G** $= K_n$. In this case $G + K_p = K_{n+p}$. The set of all vertices of K_{n+p} will be the unique monophonic dominating set of $G + K_p$. The monophonic domination polynomial, $MD(G + K_p, x) = x^{n+p}$.
Case 2: $G \neq K_n$. The monophonic dominating set of G will be a monophonic dominating set of $G+K_p$. The monophonic dominating set of cardinality $\gamma_m(G)+k$, where $\gamma_m(G)+k < n$ is obtained either by selecting $\gamma_m(G) + k$ elements from G or by selecting $\gamma_m(G) + l(l < k)$ vertices from G and the remaining $k - l$ vertices from K_p. Hence the monophonic domination polynomial, $MD(G + K_p, x) = MD(G, x)(1 + x)^p$.

□

5. Corona product of two graphs G and H

The *corona product* $G \circ H$ of two graphs G (with p_1 vertices) and H (with p_2 vertices) is defined as the graph obtained by taking one copy of G and p_1 copies of H and then joining the i^{th} vertex of G to every vertex in the i^{th} copy of H.

Lemma 5.1. *Let G and H be connected graphs of order p_1 and p_2, respectively. Then, $m(G \circ H) = m(H) \times p_1$.*

Proof. Let $V(G) = \{v_1, v_2, \cdots, v_{p_1}\}$ be the vertices of G and $\{u_{i1}, u_{i2}, \cdots, u_{ip_2}\}$, $1 \leq i \leq p_1$ be the set of vertices in the i^{th} copy of H in $G \circ H$. Since every vertex of G is a cut vertex of $G \circ H$, the minimum monophonic set contains at least one vertex from each component of $V(G \circ H) - V(G)$. Hence the minimum monophonic set of $G \circ H$ contains at least one vertex from each copy of H in $G \circ H$. Suppose the set F contains exactly one vertex from each copy of H in $G \circ H$. Then $J_{G \circ H}[F]$ will not contain all the vertices of H. Hence F can't be a monophonic set. Let $K = \bigcup_{i=1}^{p_1} S_i$, where S_i is the minimum monophonic set of the i^{th} copy of H. Also, $J_{G \circ H}[S_i] = H_i$, for all $1 \leq i \leq p_1$. Hence $J_{G \circ H}[K] = V(G \circ H)$. Therefore, K is a minimum monophonic set of $G \circ H$ and $m(G \circ H) = m(H) \times p_1$. □

Lemma 5.2. *Let G and H be connected graphs of order p_1 and p_2, respectively. Then, $\gamma_m(G \circ H) = \gamma_m(H) \times p_1$.*

Proof. Let $V(G) = \{v_1, v_2, \cdots, v_{p_1}\}$ be the vertices of G and $\{u_{i1}, u_{i2}, \cdots, u_{ip_2}\}$, $1 \leq i \leq p_1$ be the set of vertices in the i^{th} copy of H in $G \circ H$. Since every vertex of G is a cut vertex of $G \circ H$, every minimum monophonic dominating set contains at least one vertex from each component of $V(G \circ H) - V(G)$. Hence the minimum monophonic dominating set of $G \circ H$ contains at least one vertex from each copy of H in $G \circ H$. Suppose the set F contains exactly one vertex from each copy of H in $G \circ H$. Then $J_{G \circ H}[F]$ will not contain all the vertices of H. Hence F can't be a monophonic dominating set. Let $K = \bigcup_{i=1}^{p_1} S_i$, where S_i is the minimum monophonic dominating set of the i^{th} copy of H. Also, $J_{G \circ H}[S_i] = H_i$, for all $1 \leq i \leq p_1$. Hence $J_{G \circ H}[K] = V(G \circ H)$ and all the vertices of $G \circ H$ is dominated by the vertices of K. Therefore, K is a minimum monophonic dominating set of $G \circ H$ and $\gamma_m(G \circ H) = \gamma_m(H) \times p_1$. □

Theorem 5.1. *Let G and H be connected graphs of order p_1 and p_2, respectively. Then, $M(G \circ H, x) = [M(H, x)]^{p_1}(1 + x)^{p_1}$.*

Proof. The minimum monophonic set of $G \circ H$ contains the minimum monophonic set of H from all p_1 copies in $G \circ H$. Any monophonic set of $G \circ H$ with cardinality $p_1 m(H) + k$ can be obtained by selecting the $p_1 m(H) + k$ vertices from the p_1 copies of H or by selecting the $p_1 m(H) + l, (l < k)$ vertices from the p_1 copies of H and the remaining $k - l$ vertices from G. Hence the monophonic polynomial of $G \circ H$ is given by, $M(G \circ H, x) = [M(H, x)]^{p_1}(1 + x)^{p_1}$.

□

Theorem 5.2. *Let G and H be connected graphs of order p_1 and p_2, respectively. Then, $MD(G \circ H, x) = [MD(H, x)]^{p_1}(1 + x)^{p_1}$.*

Proof. The minimum monophonic dominating set of H from the p_1 copies of H in $G \circ H$ forms the minimum monophonic dominating set of $G \circ H$. Any monophonic dominating set of $G \circ H$ with cardinality $p_1 \gamma_m(H) + k$ can be obtained by selecting the $p_1 \gamma_m(H) + k$ vertices from the p_1 copies of H or by selecting the $p_1 \gamma_m(H) + l, (l < k)$ vertices from the

p_1 copies of H and the remaining $k-l$ vertices from G. Hence the monophonic dominating polynomial of $G \circ H$ is given by, $MD(G \circ H, x) = [MD(H, x)]^{p_1}(1 + x)^{p_1}$.

□

6. Conclusions

This paper provides a brief overview of monophonic and monophonic domination polynomials of graphs. Our results about monophonic polynomials cover the cartesian product of some families of graphs, while we can extend the results by finding out the monophonic polynomial of the cartesian product of any two conneccted graphs G and H.

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