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A practical approach to adaptive sliding mode control

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Abstract: This paper is concerned with the development of a practical approach to the design of adaptive sliding mode controllers. The objective is to define an adaption control law that presents some desired advantages such as non overestimation of the disturbance input, cancellation of the chattering phenomenon, zero overshooting responses, avoid control saturation and simplicity of algorithm tuning. In this practical approach it is provided a solution that uses both, adaptive sliding surfaces and adaptive control gains so the proposed controller is able to manage input disturbances with bounded derivatives.

Keywords: adaptive sliding mode; adaptive sliding surfaces; chattering avoidance ; zero overshoot response

1. INTRODUCTION

Sliding mode control (SMC) has been proposed as a control strategy in a vast collection of control problems because of its robustness properties with respect to model uncertainties, as in can be seen in [1], [2], [3], [4], [5], [6], [7], [8] and [9]. Usually conventional sliding mode controllers use a design procedure with conservative upper bound on the disturbance input to guarantee that sliding will take place. This methodology creates a trade-off with respect to the chattering phenomenon that could causes an impracticable control law design from a practical perspective.

The main contribution of this paper is to purpose a new adaptive SMC scheme that uses an approximation of the sign function to achieve a total cancellation of the chattering (if some conditions about the sampling time are ensured), integrates the reaching and the sliding mode phases in a unique continuous control law that do not overestimate the disturbance bounds and it can be easily configured by means of time domain parameters. The proposed procedure is presented with a practical implementation approach with the following objectives:

- Create responses with zero overshoot, zero steady-state error, robustness and high stability margin with respect to external disturbances.
- The design has to be easy to be tuned, that is, it must be defined with a small number of parameters that could be configured with a time domain perspective (settling time).
- The control input definition must be a continuous function but taking into account that it will be implemented on a computer with finite sampling time an numerical precision in order to avoid chattering.
- The design has to avoid the control saturation by means of a low/high gain profile that bounds the control output at largest sliding distances.

The structure of the present paper is organized as follows. In Section 2. we define the problem statement and the control design objectives. After that in Section 3. we introduce the procedure to compute the adaptive SMC control law and in Section 4. we propose an adaptive sliding mode surface definition. Next in Section 5. we present some numerical simulations of the proposed control algorithm to conclude in Section 6. with final conclusions and ideas for future developments.

2. PROBLEM STATEMENT

Consider a class of *n-th* order uncertain siso nonlinear systems described in the form

$$\begin{aligned} \dot{x}_i(t) &= x_{i+1}(t), \text{ i=1,2,...,}n\text{-1} \\ \dot{x}_n(t) &= f_x(x) + g_x(x)u(t) + d_x(x,t) \end{aligned} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the system measurable state, $u(t) \in \mathbb{R}$ is the control input, $f_x(x) \in \mathbb{R}$ and $g_x(x) \in \mathbb{R}$ are known nonlinear functions and $d_x(x,t) \in \mathbb{R}$ is an unknown term that includes external disturbances and system model uncertainties. The problem is to design a SMC such that the system state converges to zero and remains at zero despite the presence of the unknown term $d_x(x,t)$ by means of a control methodology with two design phases:

- Definition of the control input that leads to reach the sliding surface and next it keeps the trajectory on this manifold.

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- Definition of the sliding surface manifold, such that the system dynamics at this surface have a desired performance and stability properties.

One of the most popular existing sliding mode algorithms is the adaptive super-twisting controller (ASTC) (see [10] page 42) where it is assumed that a sliding variable $\sigma(t) \in \mathbb{R}$ (the distance to the sliding manifold) has been defined such that

$$\dot{\sigma}(t) = u(t) + d_\sigma(x,t) \tag{2}$$

with unknown term $d_\sigma(x,t)$ acting as a disturbance/uncertainty term. The choice of the following control action based on ASTC

$$u(t) = -k_p(t)|\sigma(t)|^{\frac{1}{2}}\,\mathrm{sign}(\sigma(t)) - \int k_i(t)\,\mathrm{sign}(\sigma(t))dt \tag{3}$$

leads to a sliding dynamics in the form

$$\dot{\sigma}(t) + k_p(t)|\sigma(t)|^{\frac{1}{2}}\,\mathrm{sign}(\sigma(t)) + \int k_i(t)\,\mathrm{sign}(\sigma(t))dt - d_\sigma(x,t) = 0 \tag{4}$$

where the controller computes the adaptive gains $k_p(t)$ and $k_i(t)$ with the goal of keep stability and performance without chattering. ASTC creates a PI controller with respect to the sign$(\sigma(t))$ variable, so a generic formulation of the PI control structure could be written in terms of $\sigma(t)$ as follows

$$\dot{\sigma}(t) + \alpha(t)\sigma(t) + \int \eta(t)\sigma(t) - d_\sigma(x,t) = 0 \tag{5}$$

Because $\alpha(t)\sigma(t) = \alpha(t)|\sigma(t)|sign(\sigma(t))$ implies that a linear function over the variable $\sigma(t)$ could be seen as an adaptive function over the variable $sign(\sigma(t))$ where $\alpha(t)$ and $|\sigma(t)|$ modulate the adaptation of the gain. Choosing $\alpha(t) = \frac{k_p(t)}{\sqrt{\sigma(t)}}$ and $\eta(t) = \frac{k_i(t)}{|\sigma(t)|}$ the ASTC formulation could be replicated using the variable $\sigma(t)$ instead of $sign(\sigma(t))$. It is interesting to emphasize that in this formulation the adaptation gains $\alpha(t)$ and $\eta(t)$ tend to infinity if $k_p(t) > \sigma(t)$ and $k_i(t) > \sigma(t)$ as $\sigma(t) \to 0$.

From a practical perspective (that is, taking account of numerical precision, finite sampling time, noise, ...) the real objective of the SMC is to keep the distance to the sliding manifold inside a bounded region defined by a small distance $\varepsilon > 0$. This implies that there is no a significant difference between a terminal sliding mode and a linear sliding mode with asymptotic approximation, because a linear solution ($\sigma(t) = \sigma(0)e^{-\lambda t}$) can be designed to create a terminal time convergence to a bounded region around the sliding surface (using $\sigma(0)e^{-\lambda T} < \varepsilon$ with T a finite time objective).

3. ADAPTIVE SMC CONTROL FUNCTION DESIGN PROCEDURE

3.1. Adaptive SMC control function design.

Let's assume that a sliding mode surface function has been defined such that the sliding variable dynamics are as follows

$$\dot{\sigma}(t) = f_\sigma(x) + g_\sigma(x)u(t) + d_\sigma(x,t) \tag{6}$$

where $\sigma(x) \in \mathbb{R}$ is the sliding variable and $f_\sigma(x) \in \mathbb{R}$, $g_\sigma(x) \in \mathbb{R}$ and $d_\sigma(x,t) \in \mathbb{R}$ depends respectively on $f_x(x)$, $g_x(x)$, $d_x(x,t)$ and the geometric definition of the sliding surface function. The following assumptions are introduced now

Assumption 1: The uncertain/disturbance term $d_\sigma(x,t)$ satisfies.

$$|\dot{d}_\sigma(x,t)| < \dot{D}_\sigma, \forall t \geq 0 \tag{7}$$

Let's note that we don't introduce a condition of the type $|d_\sigma(x,t)| < D_\sigma$ or even the assumption that any bound D_σ exists for $|d_\sigma(x,t)|$.

Assumption 2: $g_\sigma(x)$ is well defined, that is, $g_\sigma(x)$ is a continuous invertible function with $|g_\sigma(x)| \neq 0\ \forall x$. *The singular control problem is out of the scope of this work.*

In [11] we have presented a SMC design procedure that uses a nominal plant to formulate the solution as a tracking control problem. In this work we use the same concept but the nominal system used for the tracking is created using the trajectory of a nominal sliding variable $s(t) \in \mathbb{R}$ with dynamics given as

$$\dot{s}(t) = -\alpha s(t) \tag{8}$$

with $s(0) = \sigma(0)$ and $\alpha > 0\ \forall t$.

Let's also define the sliding error variable $e_\sigma(t) \in \mathbb{R}$ as the difference between the nominal and the control objective sliding variables as

$$e_\sigma(t) = s(t) - \sigma(t) \tag{9}$$

and the function $\phi(t) \in \mathbb{R}$ as

$$\phi(t) = \kappa e_\sigma(t) + z(t) \tag{10}$$

where

$$\dot{z}(t) = \delta e_\sigma(t) + \beta(t)\tanh(\lambda e_\sigma(t)) \tag{11}$$

with $z(0) = 0$. If the control function $u(t)$ is defined as

$$u(t) = -\frac{1}{g_\sigma(x)}(f_\sigma(x) + \alpha\sigma(t) - \phi(t)) \tag{12}$$

it implies that

$$\dot{\sigma}(t) = -\alpha\sigma(t) + \phi(t) + d_\sigma(x,t) \tag{13}$$

Derivation of the $e_\sigma(t)$ leads to

$$\dot{e}_\sigma(t) = -\alpha e_\sigma(t) - \phi(t) - d_\sigma(x,t) \tag{14}$$

$$\ddot{e}_\sigma(t) = -\alpha\dot{e}_\sigma(t) - \dot{\phi}(t)(e) - \dot{d}_\sigma(x,t) \quad (15)$$

such that applying (15), (10) and (11) it is obtained

$$\ddot{e}_\sigma(t) + (\alpha+\kappa)\dot{e}_\sigma(t) + \delta e_\sigma(t) + \beta(t)\tanh(\lambda e_\sigma(t)) + \dot{d}_\sigma(x,t) = 0 \quad (16)$$

The design of the parameters that determines the dynamics of the error variable $e_\sigma(t)$ could be developed assuming that the disturbance has been perfectly compensated ($\beta(t)\tanh(\lambda e_\sigma(t)) + \dot{d}_\sigma(x,t) = 0$) such that

$$\ddot{e}_\sigma(t) + (\alpha+\kappa)\dot{e}_\sigma(t) + \delta e_\sigma(t) = 0 \quad (17)$$

If we set

$$\delta = \frac{(\alpha+\kappa)^2}{4.0} \quad (18)$$

the dynamics of (16) defines an stable under-damped response (double real root solution). On the other hand if the assumption of perfect compensation of the perturbation it is not achieved ($\beta(t)\tanh(\lambda e_\sigma(t)) + \dot{d}_\sigma(x,t) \neq 0$), the condition

$$\beta(t) \geq \frac{\dot{D}_\sigma}{\tanh(\lambda e_\sigma(t))} \quad (19)$$

ensures the stability properties of (16) with the worst value assumed in the disturbance term. If the disturbance is overestimated it implies that $|\beta(t)\tanh(\lambda e_\sigma(t))| > |\dot{d}_\sigma(x,t)|$, such that the under-damped solution of the error dynamics changes and creates a complex pair of poles which generates oscillations when the trajectory cross through the surface $e_\sigma(t) = 0$. In order to mitigate the oscillating behavior we introduce an adaptive algorithm for $\beta(t)$ by means of the definition of the set

$$\Omega_{e_\beta} = \{e_\sigma(t) \mid |e_\sigma(t)| > e_\beta(t)\} \text{ with } e_\beta(t) > 0 \; \forall t \quad (20)$$

The choice

$$\beta(t) = \frac{\dot{D}_\sigma}{\tanh(\lambda e_\beta(t))} \quad (21)$$

implies that

$$\beta(t) > \frac{\dot{D}_\sigma}{\tanh(\lambda e_\sigma(t))} \forall e_\sigma(t) \in \Omega_{e_\beta} \quad (22)$$

such that the stability of the response in Ω_{e_β} could be guaranteed. Thus we are constructing an algorithm that uses an adaptive bounded region around the sliding mode manifold that minimizes its size at the same time that the disturbance term is being compensated by the adaptive control term $\beta(t)\tanh(\lambda e_\sigma(t))$.

In order to choose a function that determines the value of $e_\beta(t)$ we consider the following aspects:

- The function must have a profile such that it provides a larger (lower) value of $e_\beta(t)$ when $|\sigma(t)|$ is large (small), which implies an adaptive gain $\beta(t)$ with the desired low to high gain profile in other to allow a fast and accuracy disturbance compensation that avoids control saturation.
- The dynamics of $e_\beta(t)$ has to be designed such that it avoids the introduction of impulsive behaviors in the parameter value with respect to the system dynamics, therefore its velocity of change must be similar to the system controlled dynamics.

Based on these desired properties in this work the function that determines the value of $e_\beta(t)$ is chosen as

$$e_\beta(t) = (e_\beta^{max} - e_\beta^{min})(1.0 - e^{(-\nu|\sigma(t)|)}) + e_\beta^{min} \quad (23)$$

where e_β^{min} (e_β^{max}) is the minimum (maximum) value allowed of $e_\beta(t)$. We have that

$$\dot{e}_\beta(t) = \nu sign(\sigma(t))\dot{\sigma}(t)(e_\beta^{max} - e_\beta^{min})(1.0 - e^{(-\nu|\sigma(t)|)}) \quad (24)$$

and therefore $e_\beta(t) \to e_\beta^{min}$ if $\sigma(t)\dot{\sigma}(t) < 0$ and $e_\beta(t) \to e_\beta^{max}$ if $\sigma(t)\dot{\sigma}(t) > 0$ which is coherent with the desired properties. Let's note that ν determines the velocity of change of $e_\beta(t)$ in the same way (exponential decay) that a eigenvalue of a stable first order linear system determines the velocity of convergence of its solution. A deepest research of the functions that can be used to define the value of $e_\beta(t)$ will be developed in future works, but it is out of the scope of the present paper.

3.2. Adaptive SMC chattering analysis.

The chattering phenomenon caused by the use of a switched sign function in the conventional sliding modes designs has motivated an active research in order to minimize this problem. Several techniques has been purposed in the literature in order to minimize the chattering, such as higher order sliding modes in [12], [13], [14], switched equivalent control filtering schemes in [15], disturbance estimation in [16], continuous sign function approximations in [17], [11] and adaptive sliding mode gains in [18], [19], [20], [21], [22].

In order to analyze the chattering effect we first consider the following assumptions

Assumption 3: $f_x(x)$,$g_x(x)$ and $d_x(x,t)$ in (1) are smooth vector fields.

Assumption 4: The sliding surface manifold is a smooth continuous function.

Assumptions 3 and 4 implies that control law (12) is defined as a continuous smooth function which implies that theoretically (assuming infinitesimal sampling time) chattering effect has been canceled. However, from a practical point of view we need to consider that the control system is implemented (simulated) in a digital computer with finite sampling time, so we introduce the following assumption

Assumption 5: The control system sampling time is τ and it is simulated with Euler's method.

In order to avoid chattering we impose a condition that delimits the value of $|\dot{\sigma}(t)|$ when $|\sigma(t)|$ is small. If we consider the case $|\sigma(t)| = e_\beta^{min}$ (let's note that that $s(t) \to 0$ implies that $|\sigma(t)| \to |e(t)|$) this condition is given as

$$|\dot{\sigma}(t)| \leq \frac{2e_\beta^{min}}{\tau} \tag{25}$$

such that $|\sigma(t)|$ could not leave a boundary layer of size e_β^{min} in one cycle of the sampling time when $|\sigma(t)| = e_\beta^{min}$.

Equation (21) establishes the condition that ensure the stability properties for error dynamics (16), but an overestimation of the disturbance could cause an excessive oscillating behavior which, combined with the bounded sampling time, could generate the chattering effect. Let's note that condition (21) implies that $\beta(t)\tanh(\lambda e_\sigma(t)) + \dot{d}_\sigma(x,t) = \chi_{e_\sigma}(t)e_\sigma(t)$ with $\chi_{e_\sigma}(t) > 0$ for all the instants where $e_\sigma(t)$ belongs to Ω_{e_β}. In this case the dynamic equation (16) can be rewritten as

$$\ddot{e}_\sigma(t) + (\alpha+\kappa)\dot{e}_\sigma(t) + (\delta + \chi_{e_\sigma}(t))e_\sigma(t) = 0 \tag{26}$$

and thus the roots of the perturbed solution are given as

$$e_{\sigma_{1,2}} = \frac{\alpha+\kappa}{2} \pm \sqrt{\chi_{e_\sigma}(t)}i \tag{27}$$

To avoid the disturbance overestimation that could cause chattering it is introduced a condition in the form

$$\beta(t)|\tanh(\lambda e_\sigma(t))| + |\dot{d}_\sigma(x,t)| < \chi_{e_{max}}|e_\sigma(t)| \tag{28}$$

with $\chi_{e_{max}} > \chi_{e_\sigma}(t) > 0\ \forall t \geq 0$. This condition provides an upper bound on the perturbation generated at the error dynamics with respect to the solution with perfect disturbance cancellation.

It is possible to relate the sampling time τ with the bounded natural frequency $\sqrt{\chi_{e_{max}}}$ (in $\frac{rad}{s}$) by checking that it does not exceed the limit imposed by the Nyquist-Shannon sampling theorem, that is

$$\sqrt{\chi_{e_{max}}} \leq \frac{\pi}{\tau} \tag{29}$$

To estimate a bound for e_β^{min} we assume the case where $|e_\sigma(t)| = e_\beta(t) = e_\beta^{min}$ so from (21), (28) and (29) it is obtained

$$e_\beta^{min} \geq \frac{2.0\tau^2\dot{D}}{\pi^2} \tag{30}$$

which imposes a minimum bound in the size of the boundary layer of the sliding variable with respect to the sampling time and the frequency bandwidth of the external disturbance term $d_\sigma(x,t)$. In order to apply a security margin to avoid the undesired chattering phenomenon in this work we choose the value of e_β^{min} as

$$e_\beta^{min} = \frac{4.0\tau^2\dot{D}}{\pi^2} \tag{31}$$

From (31), (21) and applying the approximation $\tanh(\lambda e_\sigma(t)) \approx \lambda e_\sigma(t)$ when $|e_\sigma(t)| = e_\beta^{min}$ we have that the designed value of $\beta(t)$ must fulfill the inequality

$$\beta(t) < \frac{\pi^2}{4.0\lambda\tau^2} \tag{32}$$

in order to ensure the chattering cancellation.

Because $|e_\sigma(t)| \to 0$ implies that $\tanh(\lambda e_\sigma(t)) \to \lambda e_\sigma(t)$, condition (28) can be approximated when $|e_\sigma(t)| << 1$ as

$$\beta(t)\lambda|e_\sigma(t))| + |\dot{d}_\sigma(x,t)| < \chi_{e_{max}}|e_\sigma(t)| \tag{33}$$

which can be rewritten as

$$(\beta(t)\lambda - \frac{\pi^2}{\tau^2})|e_\sigma(t))| + |\dot{d}_\sigma(x,t)| < 0 \tag{34}$$

A necessary (but not sufficient) condition to fulfill this inequality is

$$\beta(t)\lambda < \frac{\pi^2}{\tau^2} \tag{35}$$

On the other side the function $\tanh(\lambda e_\sigma(t))$ is used as a continuous approximation for the sign function such that

$$\lim_{\lambda\to\infty} \tanh(\lambda e_\sigma(t)) = \text{sign}(e_\sigma(t)) \tag{36}$$

but taking into account that the system is executed with a finite sampling time τ implies that the condition

$$\lambda < \frac{1}{|\dot{e}_\sigma(t)|\tau} \tag{37}$$

it is needed to be satisfied when $e_\sigma(t) \to 0$ in order to obtain a different behavior between a pure switching function and the proposed continuous approximation. Because the designed control law implies that $|\dot{e}_\sigma(t)| \to 0$ when $|e_\sigma(t)| \to 0$ it is possible to consider that a high robustness condition that generates a good smooth approximation of the sign function taking account of the finite sampling time is given as

$$\lambda = \frac{1.0}{\tau} \tag{38}$$

which also implies from (35) that

$$\beta(t) < \frac{\pi^2}{\tau} \tag{39}$$

such that the expression of $\beta(t)$ is compatible with the bound of $\beta(t)$ expressed in (32) if the value of λ is given as in (38).

3.3. Summary of the adaptive SMC algorithm.

The proposed adaptive control function can be summarized as follows

1. Compute the value of α using the settling time for the nominal system (t_σ) as

$$\alpha = \frac{-\log(\varepsilon)}{t_\sigma}$$

so the nominal system runs from an initial condition $s(0) = \sigma(0)$ to a value bounded by ε in a time t_σ.

2. Choose the settling time for the error dynamics (t_{e_σ}) such that the trajectory arrives at $|e_\sigma(t)| < \varepsilon$ before the nominal sliding dynamics achieves the condition $s(t) \approx 0$ ($t_{e_\sigma} << t_\sigma$) and compute γ as

$$\gamma = \frac{-\log(\varepsilon)}{t_{e_\sigma}} = \alpha + \kappa$$

so the error proportional gain in (10) is obtained as

$$\kappa = \gamma - \alpha$$

3. Set the value of δ from (18).
4. Set the value of λ from (38).
5. Set the parameter values related to $e_\beta(t)$ as follows
   - e_β^{min} is obtained from (31)
   - e_β^{max} is chosen equal to the size of the objective convergence region, that is $e_\beta^{max} = \varepsilon$
   - ν is obtained such that $e_\beta(0) \approx e_\beta^{max}$, that is

$$\nu = -log(\frac{\varepsilon}{|\sigma(0)|}) \tag{40}$$

4. ADAPTIVE SMC SLIDING SURFACE DESIGN PROCEDURE

In the previous section we have presented an algorithm to compute the control function once that the sliding surface has been defined. In this section we provide the definition of the dynamics that govern the system when its trajectory flows on the sliding surface manifold.

In the available literature there have been purposed several type of sliding manifolds definitions as functions of the system states beyond the classical fixed parameters linear functions. Some proposals of this advanced manifolds include recursive nonlinear sliding manifolds as in [16] and [23], non linear full order dynamics as in [18], [12] and [15] or adaptive damping parameters with linear functions as in [11].

In this work we use adaptive damping parameters related to a time variable linear sliding manifold definition motivated by the following arguments:

- It can provide the desired low/high gain profile with respect to the distance of the system states to its desired zero values, such that it avoids saturation when the distance is large allowing higher gains at small distances in order to achieve smaller steady state errors.
- The selection of the adaptive parameters values could be developed such that the sliding manifold defines stable dynamics with time variable under dumped characteristics which helps to achieve the desired properties of zero overshooting of the system states.
- It allows to develop a simple procedure to determine the values of the adaptive parameters at each instant, for example, based on classical linear root locus of frequency based design methods.
- As in the case of bounded region adaption law proposed in (21) the calculation of the sliding manifold parameters must not introduce dynamics that could be considered *impulsive* from the point of view of the system dynamics.

Following these ideas let's define the sliding surface function as

$$\sigma(t) = c^T(t)x(t) \tag{41}$$

where $c^T(t) = [c_1(t), c_2(t), \ldots, c_n(t)]$ are the adaptive surface parameters chosen to be positive such that the polynomial $c_n(t)\mu^{n-1} + c_{n-1}(t)\mu^{n-2} + \ldots c_1(t)$ is Hurwitz at every instant. Derivation of (41) leads to

$$\dot{\sigma}(t) = \dot{c}^T(t)x(t) + c^T(t)\dot{x}(t)$$

such that in (6) it is obtained

$$\begin{aligned} f_\sigma(x) &= \dot{c}^T(t)x(t) + c^T(t)\begin{bmatrix} x_2(t) \\ x_3(t) \\ \ldots \\ f_x(x) \end{bmatrix} \\ g_\sigma(x) &= c_n(t)g_x(x) \\ d_\sigma(x,t) &= c_n(t)d_x(x,t) \end{aligned} \tag{42}$$

At this point it is needed to consider that the value of $\sigma(t)$ could not be used to compute $c(t)$. Nevertheless, as we have introduced in [11], it is possible to use the value of the nominal sliding variable at the previous instant (that is, $s(t-\tau)$) as an approximation without affecting the overall system performance assuming that $\sigma(t) \to s(t)$ much faster that $\sigma(t) \to 0$.

Therefore, based on these arguments, we choose a parameter adaption law based in the same form that (21), such that we first compute a time performance scaling factor $c_*(t)$ and then calculate (using one of the mentioned classic linear techniques) the values of $[c_1(t), c_2(t), \ldots, c_n(t)]$ according to $c_*(t)$ in order to achieve the desired under damped solution. In this work we choose the scaling factor adaption law in the form

$$c_*(t) = (c_*^{max} - c_*^{min})(e^{(-\nu|s(t-\tau)|)}) + c_*^{min} \quad (43)$$

and then compute $c_1(t), c_2(t), \ldots, c_n(t)$ using this scale factor as a time domain design requirement. Let's note that, as in (21), the parameter ν is used as the velocity adaption regulator that avoids to introduce undesired impulsive behaviors.

5. NUMERICAL SIMULATIONS

In order to check the performance of the proposed adaptive SMC design we consider first, second and third order examples and compare the results with previous works published in [12], [15], [18] and [28].

5.1. First order case.

In this first order example we check the results with the example proposed in [18] and [28] (with the algorithms configured to achieve a similar settling time) so we consider the case of the SMC control where the system dynamics are given as

$$\dot{x}(t) = u(t) + d_x(x,t) \quad (44)$$

with $\sigma(t) = x(t)$ ($c(t) = [1.0]$ in this case), initial condition $x(0) = 0.0$ and $d_x(x,t) = cos(t)$. The simulation is executed with sampling time $\tau = 0.2ms$, $\varepsilon = 1.0e-3$ and $\dot{D} = 1.0$. Following the algorithm summary we have

1. Set $t_\sigma = 1.0s$ for a value of $\sigma(0) = 0.01$ (we consider this value because in the proposed algorithm it is assumed that the sliding variable starts with non zero initial condition in order to compute the controller parameters) which implies that $\alpha = 2.302$
2. The settling time for the error dynamics is obtained by selecting $t_{e_\sigma} = 0.1 t_\sigma$ which implies that $\kappa = 20.723$.
3. Using α and κ it is computed $\delta = 132.547$
4. From the value of τ we have $\lambda = 5.0e3$.
5. The parameter values related to adaptive choice of $e_\beta(t)$ are obtained as
   - $e_\beta^{min} = 1.62e-8$
   - $e_\beta^{max} = 1.0e-3$
   - $\nu = 690.775$

Figure 1 shows the evolution of the sliding variable and the control. We can observe that the proposed solution achieves a better convergence of the sliding variable than the benchmark solutions. Let's note that control time evolution graphic shows (with transparency to facilitate visualization) the high frequency switched control inputs obtained at the benchmark solutions but it also represents the filtering of these signals that cancels the switching behavior. Compared with computed control law obtained with the proposed algorithm we can see that it avoids the peak in the control input and totally avoids the chattering.

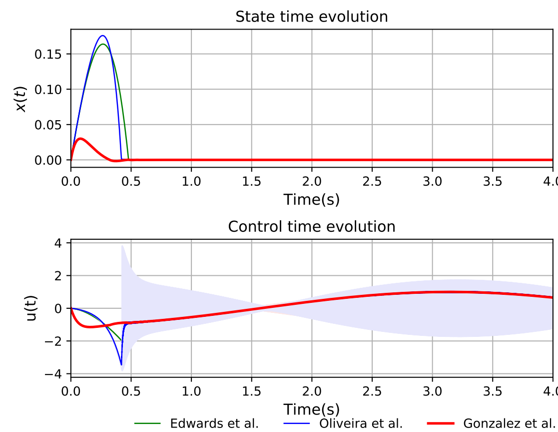

Fig. 1. First order system. State and control evolution.

Figure 2 show the evolution of the input disturbance compared with the adaptive gains of the benchmark solutions and the function $\phi(t)$ which, as previously commented, it has been designed to compensate the disturbance. This figure shows that the performance of the solution with respect to the overestimation of the disturbance is faster and better which implies that a better control solution can be computed to solve the proposed problem.

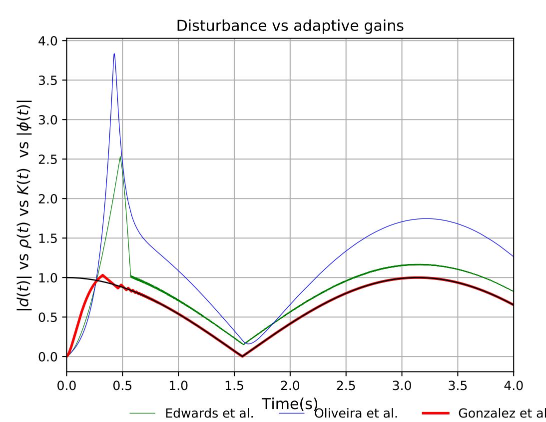

Fig. 2. First order system. Disturbance and adaptive gains evolution.

5.2. Second order case.

In this case we consider the second order example proposed in [15], so we have the following dynamic system

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = u(t) + 0.1sin(20.0t)$$

with initial conditions $x_1(0) = 1.0$ and $x_2(0) = 0.0$. The simulation runs with $\tau = 1.0$ ms, $\varepsilon = 1.0e-3$ and $\dot{D} = 2.0$, so following the algorithm summary we have

1. Set $t_\sigma = 1.0s$ (because the controllers proposed in [12] and [15] achieves an approximate settling time of one second) so $\alpha = 6.91$
2. The settling time for the error dynamics is obtained by selecting $t_{e_\sigma} = 0.1t_\sigma$ which implies that $\kappa = 62.17$.
3. Using α and κ it is computed $\delta = 1192.92$
4. From the value of τ we have $\lambda = 1.0e3$.
5. The parameter values related to adaptive choice of $e_\beta(t)$ are obtained as
   - $e_\beta^{min} = 8.1e-7$
   - $e_\beta^{max} = 1.0e-3$
   - $\nu = 6.91$

In order to define the sliding surface manifold in this example we choose $c_2(t) = 1.0$ and $c_1(t)$ with the general adaption law proposed in (43), that is

$$c_1(t) = (c_1^{max} - c_1^{min})(e^{(-\nu|s(t-\tau)|)}) + c_1^{min}$$

Therefore the sliding surface is defined as

$$\sigma(t) = c_1(t)x_1(t) + x_2(t)$$

which implies that the dynamics $\dot{x}_1(t) = -c_1(t)x_1(t)$ are obtained at the sliding manifold when $\sigma(t) = 0$. To finalize the sliding surface definition we choose

$$c_1^{min} = 1.0$$
$$c_1^{max} = 5.0c_1^{min}$$

so when the condition $\sigma(t) = 0$ is established the system dynamics are defined as $\dot{x}_1(t) = -c_1^{max}x_1(t)$.

Figure 3 shows the time evolution of the system states and the control law obtained in this example. We can observe that the proposed solution achieves a better performance with zero overshooting in both states employing a control law without chattering and with smaller absolute values than the benchmark solutions.

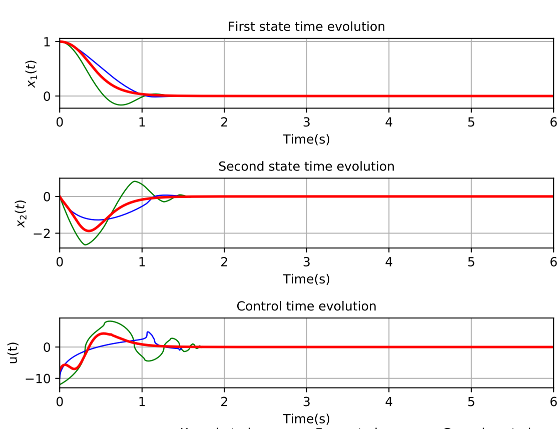

Fig. 3. Second order system. States and control evolution.

Figure 4 shows a detailed view of the system states and control input where it is clear that the chattering behavior at the steady state has been canceled in the proposed method compared with the benchmark selected solutions.

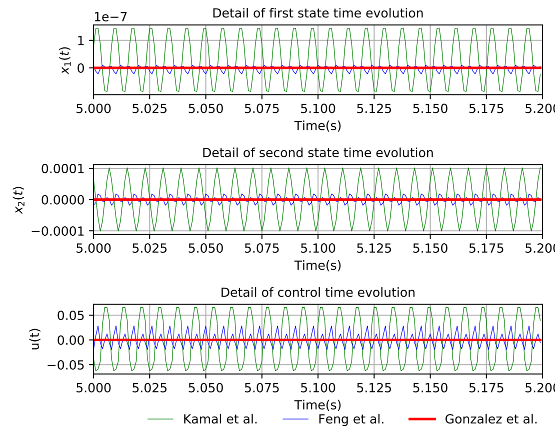

Fig. 4. Second order system. Detail of states and control evolution.

If the limit condition obtained in (30) is used without the anti-chattering security margin (setting $e_\beta^{min} = 4.05e-7$) chattering appears because of finite sampling time as it is clearly shown in Figure 5. This result validates, for the second order plant, the limit obtained for the bounded region around the sliding manifold as a function of the disturbance bandwidth and the simulation sampling time before the chattering phenomenon appears.

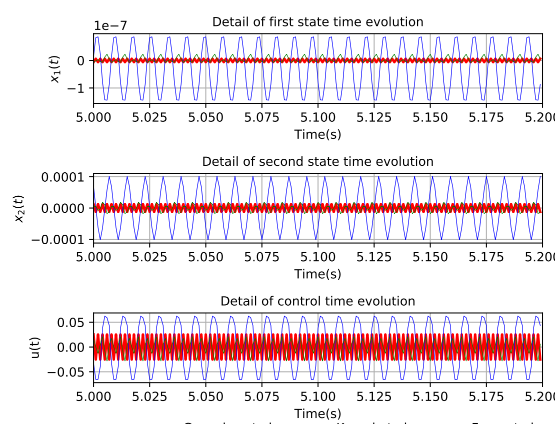

Fig. 5. Second order system. Detail of states and control evolution with chattering.

5.3. Third order case.

In this case we consider the third order example proposed in [12], so we have the following dynamic system

$$\dot{x}_1(t) = x_2(t) \quad (45)$$
$$\dot{x}_1(t) = x_3(t) \quad (46)$$

$$\dot{x}_3(t) = u(t) + 2.0 + sin(20.0\ \mathrm{t}) \tag{47}$$

with initial conditions $x_1(0) = 1.0$, $x_2(0) = 0.5$ and $x_3(0) = 0.0$. The simulation runs with $\tau = 1.0$ ms, $\varepsilon = 1.0e-3$ and $\dot{D} = 20.0$, so we have that

1. Set $t_\sigma = 3.5s$ so $\alpha = 8.70$. Le't note that in this case we adjust the values given in [12] multiplying the proposed gains with a fixed factor of value 2.5 in order to achieve an approximate settling time of one second.

2. The settling time for the error dynamics is obtained by selecting $t_{e_\sigma} = 0.1t_\sigma$ which implies that $\kappa = 78.29$.

3. Using α and κ it is computed $\delta = 1892.04$

4. From the value of τ we have $\lambda = 1.0e3$.

5. The parameter values related to adaptive choice of $e_\beta(t)$ are obtained as

   - $e_\beta^{min} = 1.21e-6$
   - $e_\beta^{max} = 1.0e-3$
   - $\nu = 1.15$

In this case the sliding surface is defined as

$$\sigma(t) = c_1(t)x_1(t) + c_2(t)x_2(t) + x_3(t) \tag{48}$$

where the value of $c_2(t)$ is obtained as

$$c_2(t) = (c_2^{max} - c_2^{min})(e^{(-\nu|s(t-\tau)|)}) + c_2^{min} \tag{49}$$

with

$$c_2^{min} = 1.5 \tag{50}$$
$$c_2^{max} = 5.0c_2^{min} \tag{51}$$

and the value of $c_1(t)$ is obtained in order to generate a double root solution in the sliding surface induced dynamics, that is

$$c_1(t) = \frac{c_2^2(t)}{4.0} \tag{52}$$

such that the sliding surface dynamics are in the form

$$x_3(t) + c_2(t)x_2(t) + \frac{c_2^2(t)}{4.0}x_1(t) = 0 \tag{53}$$

when $\sigma(t) = 0$.

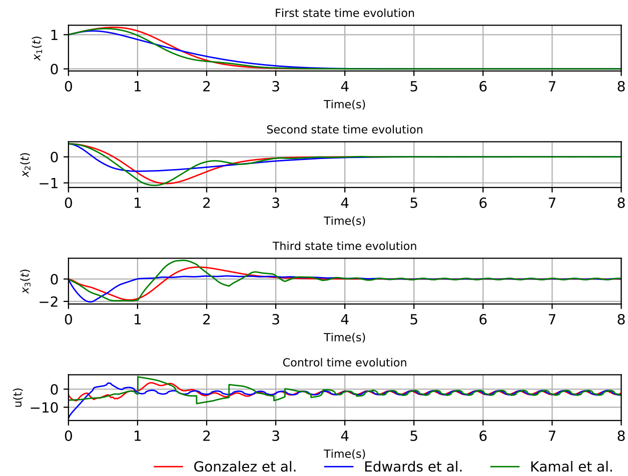

Fig. 6. Third order system. States and control evolution.

Figure 6 shows the results obtained at this third order example, where we can observe that the proposed algorithm is able to achieve the desired performance by keeping the low/high gain profile. This h implies that at the initial state (when it is assumed that the trajectory is far from the sliding manifold) the control input is lower and it is continuously adapted as far as the trajectory converges to the sliding manifold. When the sliding condition is being achieved the algorithm is able to establish robust dynamics without the presence of chattering as in can be observed in Figure 7, where the high gain value and the continuous control definition that are used at the sliding mode allows the system to generate a smaller steady state error.

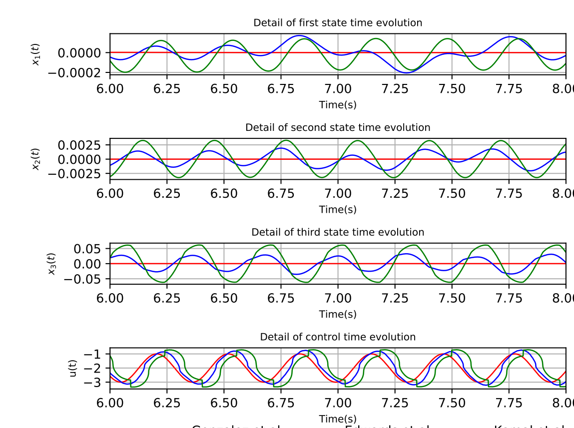

Fig. 7. Third order system. Detail of state and control evolution.

6. CONCLUSIONS AND FUTURE DEVELOPMENT

In this paper a chattering free adaptive sliding mode control solution has been proposed based on the application of nonlinear adaptive gains that are obtained by means of the definition of an adaptive size sliding manifold bounded region and the use of time varying linear sliding surfaces. The design of the SMC control laws creates non-overshooting responses such that the algorithm computes its parameters based on the initial conditions, the desired settling time, the bandwidth of the perturbation term and the sampling time used.

In future works we will consider to extend the analysis of the proposed methodology to new control problems such as the use of hybrid control systems with discrete dynamics related to the initialization (resetting) of the nominal sliding variable in order to compensate impulsive disturbances, application of the framework to solve problems with unmatched disturbances or consider the formulation to be applied on under-actuated plants by means of a backstepping procedure.

7. ACKNOWLEDGEMENT

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