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Gribov horizon beyond the Landau gauge

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ABSTRACT

Gribov and Zwanziger proposed a modification of Yang–Mills theory in order to cure the Gribov copy problem. We employ field-dependent BRST transformations to generalize the Gribov–Zwanziger model from the Landau gauge to general R_ξ gauges. The Gribov horizon functional is presented in explicit form, in both the non-local and local variants. Finally, we show how to reach any given gauge from the Landau one.

1. Introduction and summary

It is long known that the covariant quantization of Yang–Mills theory is beset by the Gribov problem: the existence of infinitely many discrete gauge copies even after gauge fixing [1]. A natural remedy, suppressing the field integration outside the Gribov horizon, is accomplished by adding to the action a Gribov horizon functional [1–5]. The latter, however, is not BRST invariant and usually chosen in the Landau gauge. For a better understanding of its effect on the gauge variance of Green's functions, a knowledge of the horizon functional in other gauges is desirable [6].

Recently, we have discovered an explicit way to change the gauge in Faddeev–Popov quantization by effecting a suitable field-dependent BRST transformation [7].[1] Here, we utilize this strategy to define horizon functionals for the non-local and local forms of the Gribov–Zwanziger model in any R_ξ gauge. At the end of the Letter, we present the horizon functional in an arbitrary gauge.

2. Yang–Mills theory with Gribov horizon

Yang–Mills theory with gauge group SU(n) in d spacetime dimensions features gauge potentials $A^a_\mu(x)$ with $a = 1, \dots, n^2-1$ and $\mu = 0, 1, \dots, d-1$. The classical action has the standard form

$$S_0(A) = -\frac{1}{4}\int d^dx\, F^a_{\mu\nu} F^{\mu\nu a} \quad \text{with}$$
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu, \tag{2.1}$$

where f^{abc} denote the (totally antisymmetric) structure constants of the Lie algebra $su(n)$. The action (2.1) is invariant under the gauge transformations

$$\delta A^a_\mu = D^{ab}_\mu \xi^b \quad \text{with } D^{ab}_\mu = \delta^{ab}\partial_\mu + f^{acb} A^c_\mu. \tag{2.2}$$

The BRST formulation of the quantum theory extends the field content to

$$\{\phi^A\} = \{A^a_\mu, B^a, C^a, \bar{C}^a\} \tag{2.3}$$

by adding the Nakanishi–Lautrup auxiliary fields as well as the Faddeev–Popov ghost and antighost fields, in the order above. The Grassmann parities ε and ghost numbers gh are

$$\varepsilon(C^a) = \varepsilon(\bar{C}^a) = 1, \qquad \varepsilon(A^a_\mu) = \varepsilon(B^a) = 0,$$
$$gh(A^a_\mu) = gh(B^a) = 0, \qquad gh(C^a) = -gh(\bar{C}^a) = 1. \tag{2.4}$$

In DeWitt notation [9], the quantum action à la Faddeev and Popov [10] takes the form

$$S(\phi) = S_0(A) + \bar{C}^a K^{ab}(A) C^b + \chi^a(A) B^a, \tag{2.5}$$

with the Faddeev–Popov operator

$$K^{ab}(A) = \frac{\delta\chi^a(A)}{\delta A^c_\mu} D^{cb}_\mu = \partial^\mu D^{ab}_\mu = \delta^{ab}\partial^\mu\partial_\mu + f^{acb} A^c_\mu \partial^\mu \tag{2.6}$$

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[1] An analogous formula had been derived differently in [8].

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for the gauge-fixing functions χ^a of the Landau gauge,

$$\chi^a(A) = \partial^\mu A^a_\mu. \tag{2.7}$$

The action (2.5) is invariant under the BRST transformation [11,12]

$$\delta_\lambda A^a_\mu = D^{ab}_\mu C^b \lambda, \qquad \delta_\lambda \bar{C}^a = B^a \lambda, \qquad \delta_\lambda B^a = 0,$$
$$\delta_\lambda C^a = \frac{1}{2} f^{abc} C^b C^c \lambda \tag{2.8}$$

where λ is an odd constant Grassmann parameter. Introducing the Slavnov variation sX of any functional $X(\phi)$ via

$$\delta_\lambda X(\phi) = \big(sX(\phi)\big)\lambda \quad \text{so that } sX(\phi) = \frac{\delta X(\phi)}{\delta \phi^A} R^A(\phi) \tag{2.9}$$

with the notation

$$\big\{R^A(\phi)\big\} = \left\{D^{ab}_\mu C^b, 0, \frac{1}{2} f^{abc} C^b C^c, B^a\right\} \quad \text{and}$$
$$\varepsilon\big(R^A(\phi)\big) = \varepsilon_A + 1, \tag{2.10}$$

the action (2.5) can be written in the compact form

$$S(\phi) = S_0(A) + s\psi(\phi), \tag{2.11}$$

where $\psi(\phi)$ denotes the associated fermionic gauge-fixing functional (in the Landau gauge),

$$\psi(\phi) = \bar{C}^a \chi^a(A) = \bar{C}^a \partial^\mu A^a_\mu. \tag{2.12}$$

The Gribov horizon [1] in the Landau gauge can be taken into account by adding to the action (2.11) the non-local horizon functional

$$M(A) = \gamma^2 f^{abc} A^b_\mu \big(K^{-1}\big)^{ad} f^{dec} A^{e\mu} + \gamma^2 d\big(n^2 - 1\big), \tag{2.13}$$

where K^{-1} inverts the (matrix-valued) Faddeev–Popov operator $K^{ab}(A)$ of (2.6) and $\gamma \in \mathbb{R}$ is the so-called thermodynamic or Gribov parameter [2,3]. The effective action of the Gribov–Zwanziger model,

$$S_M(\phi) = S(\phi) + M(A) = S_0(A) + s\psi(\phi) + M(A), \tag{2.14}$$

is not BRST invariant because

$$sM(A,C) = \gamma^2 f^{abc} f^{cde} \big[2 D^{bq}_\mu C^q \big(K^{-1}\big)^{ad} - f^{mpn} A^b_\mu \big(K^{-1}\big)^{am} K^{pq} C^q \big(K^{-1}\big)^{nd}\big] A^{e\mu} \neq 0. \tag{2.15}$$

In [6], we have investigated the resulting gauge dependence of the vacuum functional, assuming the existence of a horizon functional beyond the Landau gauge. With the help of recent results [7] (see also [8]), we now verify this assumption and propose an explicit form for such a functional in general R_ξ gauges.

3. Gribov horizon in R_ξ gauges

The vacuum functional for the Gribov–Zwanziger model is given by a functional integral,

$$Z = \int \mathcal{D}\phi \exp\left\{\frac{i}{\hbar}\big(S_0(A) + s\psi(\phi) + M(A)\big)\right\}. \tag{3.1}$$

Let us perform a change of variables which amounts to a particular field-dependent BRST transformation,

$$\phi^A \longmapsto \phi^A + \big(s\phi^A\big)\Lambda_\xi(\phi) \qquad \text{with}$$
$$\Lambda_\xi(\phi) = \bar{C}^a B^a \big(B^2\big)^{-1}\left(\exp\left\{\frac{\xi}{2i\hbar} B^2\right\} - 1\right), \tag{3.2}$$

where $B^2 = B^a B^a$. Taking into account the Jacobian and using $\ln(1 + s\Lambda_\xi) = \frac{\xi}{2i\hbar} B^2$, the vacuum functional then reads [7]

$$Z = \int \mathcal{D}\phi \exp\left\{\frac{i}{\hbar}\big(S_0(A) + s\psi_\xi(\phi) + M_\xi(\phi)\big)\right\}, \tag{3.3}$$

with a shifted fermionic gauge-fixing functional and a modified horizon functional,

$$\psi_\xi(\phi) = \bar{C}^a\left(\partial^\mu A^a_\mu + \frac{\xi}{2} B^a\right) \quad \text{and}$$
$$M_\xi(\phi) = M(A) + \big(sM(A,C)\big)\Lambda_\xi(\phi), \tag{3.4}$$

respectively. The explicit expression for $sM(A,C)$ is given in (2.15).

We have moved away from the Landau gauge and reached a general R_ξ gauge. Therefore, we propose

$$\begin{aligned} M_\xi(\phi) = {} & \gamma^2 f^{abc} A^b_\mu \big(K^{-1}\big)^{ad} f^{dec} A^{e\mu} + \gamma^2 d\big(n^2 - 1\big) \\ & + \gamma^2 f^{abc} f^{cde}\big[2 D^{bq}_\mu C^q \big(K^{-1}\big)^{ad} \\ & - f^{mpn} A^b_\mu \big(K^{-1}\big)^{am} K^{pq} C^q \big(K^{-1}\big)^{nd}\big] \\ & \times A^{e\mu} \bar{C}^\ell B^\ell \big(B^2\big)^{-1}\big(e^{\frac{\xi}{2i\hbar}B^2} - 1\big) \end{aligned} \tag{3.5}$$

as the explicit form for the horizon functional in a general R_ξ gauge. Under further BRST transformations, its Slavnov variation is

$$sM_\xi = sM(A,C)\big[1 - s\Lambda_\xi(\phi)\big]. \tag{3.6}$$

In linear approximation in ξ we have $\Lambda_\xi(\phi) = \frac{\xi}{2i\hbar} \bar{C}^a B^a$ and get

$$\begin{aligned} M_\xi = M(A) + \frac{\xi \gamma^2}{2i\hbar} f^{abc} f^{cde} \big[2 D^{bq}_\mu C^q \big(K^{-1}\big)^{ad} \\ - f^{mpn} A^b_\mu \big(K^{-1}\big)^{am} K^{pq} C^q \big(K^{-1}\big)^{nd}\big] A^{e\mu} \bar{C}^\ell B^\ell \end{aligned} \tag{3.7}$$

still depending on all field variables. For $\xi = 0$, it smoothly reduces to the Landau-gauge functional, $M_0 = M(A)$. It is important to note that our extension (3.5) of the Gribov–Zwanziger model is done in such a way as to render its vacuum functional gauge invariant. Indeed, since an infinitesimal change of gauge $\delta\psi = i\hbar \Lambda_\xi$ (in linear approximation) is merely a field redefinition in the path integral, we have that

$$\langle \delta M(\phi)\rangle + \langle s\delta\psi(\phi)\rangle = 0 \tag{3.8}$$

for expectations values $\langle \cdots \rangle$ in the Gribov–Zwanziger model defined by (3.1).

4. Gribov–Zwanziger action

Originally, the Gribov–Zwanziger model was presented in the non-local form (2.13) and (2.14) [1,2]. Later, the non-locality was 'resolved' by adding auxiliary field variables [3–5]. The resulting local action is referred to as the Gribov–Zwanziger action and takes the form (for details, see [13])

$$S_{GZ}(\Phi) = S_0(A) + s\psi(\phi) + S_\gamma(A, \varphi, \bar{\varphi}, \omega, \bar{\omega}) \tag{4.1}$$

where

$$\begin{aligned} S_\gamma = {} & \bar{\varphi}^{ac}_\mu K^{ab} \varphi^{\mu bc} - \bar{\omega}^{ac}_\mu K^{ab} \omega^{\mu bc} + 2i\gamma f^{abc} A^b_\mu \big(\varphi^{\mu ac} + \bar{\varphi}^{\mu ac}\big) \\ & + \gamma^2 d\big(n^2 - 1\big) \end{aligned} \tag{4.2}$$

represents the horizon functional written in local form for the Landau gauge. The set of fields has been further enlarged to

$$\big\{\Phi^{\mathcal{A}}\big\} = \big\{\phi^A, \varphi^{ac}_\mu, \bar{\varphi}^{ac}_\mu, \omega^{ac}_\mu, \bar{\omega}^{ac}_\mu\big\}. \tag{4.3}$$

The fields φ^{ac}_μ and $\bar\varphi^{ac}_\mu$ are commuting while ω^{ac}_μ and $\bar\omega^{ac}_\mu$ are anticommuting. The additional fields form BRST doublets [14],

$$\delta_\lambda\varphi^{ac}_\mu=\omega^{ac}_\mu\lambda,\qquad \delta_\lambda\bar\varphi^{ac}_\mu=0,$$
$$\delta_\lambda\omega^{ac}_\mu=0,\qquad \delta_\lambda\bar\omega^{ac}_\mu=-\bar\varphi^{ac}_\mu\lambda. \tag{4.4}$$

The local horizon functional S_γ is not BRST invariant,

$$sS_\gamma=f^{adb}\big[\bar\varphi^{ac}_\mu K^{de}C^e\varphi^{\mu bc}+\bar\omega^{ac}_\mu K^{de}C^e\omega^{\mu bc}+2\mathrm{i}\gamma\big(D^{de}_\mu C^e\big(\varphi^{\mu ab}+\bar\varphi^{\mu ab}\big)+A^d_\mu\omega^{\mu ab}\big)\big]\neq 0. \tag{4.5}$$

Note that in case $\gamma=0$ the action (4.2) is reduced to

$$S_{\gamma=0}=\bar\varphi^{ac}_\mu K^{ab}\varphi^{\mu bc}-\bar\omega^{ac}_\mu K^{ab}\omega^{\mu bc}. \tag{4.6}$$

Then, in the vacuum functional, integration over $\bar\varphi$ and φ yields $(\det K)^{-1}$, while integration over $\bar\omega$ and ω reproduces $\det K$, so that in the configuration space $\{\phi\}$ we recover the Yang–Mills vacuum functional.

Like in the previous section, we may move to a general R_ξ gauge by performing the specific field-dependent BRST transformation (3.2) in the vacuum functional integral of the Gribov–Zwanziger model based on the local action (4.1). As a result, the action gets modified,

$$S_{GZ}(\Phi)\longmapsto S_0(A)+s\psi_\xi(\phi)+S_{\gamma\xi}(\Phi) \tag{4.7}$$

where

$$\psi_\xi(\phi)=\bar C^a\left(\partial^\mu A^a_\mu+\frac{\xi}{2}B^a\right)\quad\text{and}$$
$$S_{\gamma\xi}(\Phi)=S_\gamma(A,\varphi,\bar\varphi,\omega,\bar\omega)+\big(sS_\gamma(A,C,\varphi,\bar\varphi,\omega,\bar\omega)\big)\Lambda_\xi(\phi). \tag{4.8}$$

We propose this $S_{\gamma\xi}$ together with (3.2), (4.2) and (4.5) as the proper extension of the local horizon functional to a general R_ξ gauge. Its Slavnov variation reads

$$sS_{\gamma\xi}=sS_\gamma(A,C,\varphi,\bar\varphi,\omega,\bar\omega)\big[1-s\Lambda_\xi(\phi)\big]. \tag{4.9}$$

In the limit $\gamma\to 0$ we expect the action (4.7) again to produce the standard Yang–Mills theory. Putting $\gamma=0$ in (4.8) we get

$$S_{\gamma=0,\xi}=\bar\varphi^{ac}_\mu D^{ab}_\xi\varphi^{\mu bc}-\bar\omega^{ac}_\mu D^{ab}_\xi\omega^{\mu bc}\quad\text{where}$$
$$D^{ab}_\xi=K^{ab}+f^{adb}K^{de}C^e\Lambda_\xi(\phi). \tag{4.10}$$

Like at ξ=0 before, integrating out all auxiliary fields indeed leads back to the Yang–Mills vacuum functional.

With this information, we may revisit the gauge dependence of Green's functions proposed in [6], by taking into account the gauge variation of source terms to be added in the path integral.

5. Horizon functional in an arbitrary gauge

Although the R_ξ gauges were easy to reach, they are not the only ones accessible by our method. In fact, [7] provides a general formula for connecting any two gauges in terms of their fermionic gauge-fixing functionals ψ: to get from a reference gauge ψ_0 to a desired gauge ψ, change the variables inside the generating functional $Z(J)$ by a BRST transformation with a field-dependent parameter

$$\Lambda_\psi(\phi)=(\psi-\psi_0)\big(s(\psi-\psi_0)\big)^{-1}\left(\exp\left\{\frac{1}{\mathrm{i}\hbar}s(\psi-\psi_0)\right\}-1\right)=\frac{1}{\mathrm{i}\hbar}(\psi-\psi_0)\sum_{n=0}^{\infty}\frac{1}{(n+1)!}\left(\frac{1}{\mathrm{i}\hbar}s(\psi-\psi_0)\right)^n. \tag{5.1}$$

The corresponding change of the horizon functional reads

$$M_\psi(\phi)-M_0(\phi)=\big(sM_0(\phi)\big)\Lambda_\psi(\phi). \tag{5.2}$$

The Gribov–Zwanziger model can now be studied explicitly in an arbitrary gauge.

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