Data-driven distributed MPC of dynamically coupled linear systems

Matthias Köhler * **Julian Berberich** * **Matthias A. Müller** **
Frank Allgöwer *

Matthias Köhler (née Hirche), Julian Berberich and Frank Allgöwer are with the University of Stuttgart, Institute for Systems Theory and Automatic Control, Germany {koehler, berberich, allgower}@ist.uni-stuttgart.de
** *Matthias A. Müller is with the Leibniz University Hannover, Institute of Automatic Control, Germany* mueller@irt.uni-hannover.de

Abstract: In this paper, we present a data-driven distributed model predictive control (MPC) scheme to stabilise the origin of dynamically coupled discrete-time linear systems subject to decoupled input constraints. The local optimisation problems solved by the subsystems rely on a distributed adaptation of the Fundamental Lemma by Willems et al., allowing to parametrise system trajectories using only measured input-output data without explicit model knowledge. For the local predictions, the subsystems rely on communicated assumed trajectories of neighbours. Each subsystem guarantees a small deviation from these trajectories via a consistency constraint. We provide a theoretical analysis of the resulting non-iterative distributed MPC scheme, including proofs of recursive feasibility and (practical) stability. Finally, the approach is successfully applied to a numerical example.

Keywords: Data-based control, predictive control, distributed control, large-scale systems, linear systems.

1. INTRODUCTION

Model predictive control (MPC) is a modern control method with a well-researched theoretical foundation, see e.g. (Rawlings et al., 2020; Grüne and Pannek, 2017). Advantages of MPC are the possibility to explicitly consider constraints on the controlled system and the incorporation of a performance objective. If the system to be controlled is large-scale or consists of many interconnected but otherwise discernible subsystems, a central controller might not be desirable or even computationally infeasible. This is alleviated by distributed MPC (DMPC) where a local MPC is designed for each subsystem and the respective control input is computed based on locally available information. Various different setups for DMPC exist which are suitable for different classes of systems, interconnection structures, and communication topologies using an iterative or non-iterative scheme. See e.g. the surveys (Christofides et al., 2013; Müller and Allgöwer, 2017) or the collection (Maestre and Negenborn, 2014) for a selection and classification of various schemes.

Typically, a model derived from first principles is used in MPC to predict the behaviour of the controlled system. In practice, however, it might be difficult to derive such a model in detail, but obtaining input-output data is comparatively simple. Recently, for this reason, there has been a push to use only input-output data and no explicit model knowledge to design direct data-driven controllers. Many results, including our approach, rely

on Willems' Fundamental Lemma (Willems et al., 2005), which states that all trajectories of a linear time-varying (LTI) system can be constructed from one persistently exciting trajectory.

Based on this, MPC schemes can be designed using only input-output data and no explicit model knowledge (Yang and Li, 2015; Coulson et al., 2019), even admitting closed-loop guarantees on stability and robustness (Berberich et al., 2021a). Further references may be found in the survey (Markovsky and Dörfler, 2021). Recently, extensions to data-driven distributed MPC for dynamically coupled systems with stability guarantees (Allibhoy and Cortés, 2021; Alonso et al., 2021) have been made, which require state measurements and state coupling. Both employ iterative distributed optimisation, requiring a multitude of communication at each time step.

In this paper, we propose a non-iterative data-driven distributed MPC (D3MPC) scheme for dynamically coupled LTI systems. The local MPC optimisation problems are based on only local input-output data and are solved in parallel assuming that the neighbours follow a previously communicated trajectory. Therefore, the scheme scales well with the total number of subsystems. In comparison to the data-driven distributed MPC schemes referenced above, communication is kept to a minimum since in each time step only one trajectory needs to be shared, and a non-iterative parallel implementation is possible, at the price of conservativeness. We show that the proposed scheme practically asymptotically stabilises the origin of the system while meeting input constraints. The main tool is a consistency constraint in the optimisation problems, ensuring that the subsystems' deviation from their communicated trajectories is limited. The idea is based on (Dunbar, 2007) which uses a similar consistency constraint in a model-based setting for continuous-time systems in a stabilising dual-mode DMPC scheme.

2. PRELIMINARIES

We denote the natural numbers containing 0 with $\mathbb{N}_0$. We write $A \succ 0$ ($A \succeq 0$) if $A = A^\top$ is positive (semi-)definite. The smallest eigenvalue of a matrix $A = A^\top$ is denoted by $\lambda_{\min}(A)$. Given another matrix $B = B^\top$, $\lambda_{\min}(A, B) = \min\{\lambda_{\min}(A), \lambda_{\min}(B)\}$. For a set of vectors $v_1, \ldots, v_n$ we denote the stacked vector $v = \begin{bmatrix} v_1^\top & \ldots & v_n^\top \end{bmatrix}^\top = \mathrm{col}_{i=1}^n(v_i)$. For a sequence $\{x_k\}_{k=0}^{N-1}$, we define the Hankel matrix

$$H_L(x) = \begin{bmatrix} x_0 & x_1 & \ldots & x_{N-L} \\ x_1 & x_2 & \ldots & x_{N-L+1} \\ \vdots & \ddots & \ddots & \vdots \\ x_{L-1} & x_L & \ldots & x_{N-1} \end{bmatrix}.$$

Moreover, we introduce the notation $x_{[a,b]} = \mathrm{col}_{k=a}^b(x_k)$. For a (block-)diagonal matrix with n (block-)diagonal elements A_i we write $\mathrm{diag}_{i=1}^n(A_i)$. The set of integers in the interval $[a, b]$ with $a \le b$ is denoted by $\mathbb{I}_{[a,b]}$. We make use of class-$\mathcal{K}$, -$\mathcal{K}_\infty$ and -$\mathcal{K}\mathcal{L}$ comparison functions and refer to (Kellett, 2014) for their definition. The cardinality of a set $\mathcal{S}$ is denoted by $|\mathcal{S}|$. With $\star$ we denote parts that can be inferred by symmetry.

Definition 1. We say that a sequence $\{x_k\}_{k=0}^{N-1}$ with $x_k \in \mathbb{R}^n$ is persistently exciting of order L if $\mathrm{rank}(H_L(x)) = nL$.

3. DATA-DRIVEN DISTRIBUTED MPC

3.1 Distributed system representation

Our objective is to stabilise a group of $M \ge 2$ dynamically coupled subsystems about the origin. The dynamic coupling between the subsystems is given by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, \ldots, M\}$ is the set of nodes corresponding to subsystems and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of all directed edges between nodes in the graph. If the output y^j of a subsystem j appears in the dynamic equation of subsystem i, and $i \ne j$, then subsystem j is neighbour of subsystem i and part of the set of neighbours $\mathcal{N}_i = \{j \in \mathcal{V} \mid j \text{ is a neighbour of } i\}$. Then, $\mathcal{E} = \{(j, i) \in \mathcal{V} \times \mathcal{V} \mid j \in \mathcal{N}_i\}$. It is assumed that communication along the directed edges of $\mathcal{G}$ is possible.

We consider as input-output dynamics for subsystem $i \in \mathcal{V}$:

$$x_{t+1}^i = A_{ii} x_t^i + B_{ii} u_t^i + \sum_{j \in \mathcal{N}_i} B_{ij} y_t^j \tag{1a}$$

$$y_t^i = C_{ii} x_t^i + D_{ii} u_t^i \tag{1b}$$

for $i = 1, \ldots, M$, where $x_t^i \in \mathbb{R}^n$ is the state, $u_t^i \in \mathbb{R}^m$ is the input, and $y_t^i \in \mathbb{R}^p$ is the output of system i, all at time $t \in \mathbb{N}_0$. The input is subject to constraints $u_t^i \in \mathcal{U}_i \subset \mathbb{R}^m$ with compact $\mathcal{U}_i$. We abbreviate $y^{-i} = \mathrm{col}_{j \in \mathcal{N}_i}(y^j)$. The dimensions of every subsystems' input and output are assumed to be the same, for notational simplicity and without loss of generality.

For each individual subsystem i, the matrices A_{ii}, B_{ii}, B_{ij} with $j \in \mathcal{N}_i$, C_{ii}, and D_{ii} are *unknown*, but a data set $\mathcal{D}_i := \{\{u_k^{i,\mathrm{d}}\}_{k=0}^{N-1}, \{y_k^{i,\mathrm{d}}\}_{k=0}^{N-1}, \{y_k^{-i,\mathrm{d}}\}_{k=0}^{N-1}\}$ that satisfies the dynamics (1) is available.

Assumption 2. (1) For each $i \in \mathcal{V}$, the sequence $\begin{bmatrix} u_{[0,N-1]}^{i,\mathrm{d}} \\ y_{[0,N-1]}^{-i,\mathrm{d}} \end{bmatrix}$

is persistently exciting of order $L + n$.

(2) (A, C) is observable, where $C = \mathrm{diag}_{i=1}^M(C_{ii})$ and

$$A = \begin{bmatrix} A_{11} & \ldots & A_{1M}s \\ \vdots & \ddots & \vdots \\ A_{M1} & \ldots & A_{MM} \end{bmatrix}$$

with $A_{ij} = 0$ if $j \notin \mathcal{N}_i$ and $A_{ij} = B_{ij} C_{jj}$ if $j \in \mathcal{N}_i$.

(3) For each $i \in \mathcal{V}$, the pair (A_{ii}, B_i) is controllable, where $B_i = \begin{bmatrix} B_{ii} & B_{ij_1} & \ldots & B_{ij_{|\mathcal{N}_i|}} \end{bmatrix}$.

The following lemma is a consequence of Willems' Fundamental Lemma (Willems et al., 2005) with the neighbours' outputs interpreted as additional inputs. It allows the characterisation of any input-output trajectory of (1) based on suitable local data $\mathcal{D}_i$ and will be the basis of our D3MPC scheme.

Lemma 3. Suppose Assumption 2 holds. Then, for any $i \in \mathcal{V}$, $\{u_k^i, y_k^i\}_{k=0}^{L-1}$, $\{y_k^{-i}\}_{k=0}^{L-2}$ is a trajectory of (1) if and only if there exists $\alpha^i \in \mathbb{R}^{N-L+1}$ such that

$$\begin{bmatrix} H_L(u_{[0,N-1]}^{i,\mathrm{d}}) \\ H_{L-1}(y_{[0,N-2]}^{-i,\mathrm{d}}) \\ H_L(y_{[0,N-1]}^{i,\mathrm{d}}) \end{bmatrix} \alpha^i = \begin{bmatrix} u_{[0,L-1]}^i \\ y_{[0,L-2]}^{-i} \\ y_{[0,L-1]}^i \end{bmatrix}. \tag{2}$$

We only have access to input-output measurements, and not to the minimal state. Hence, it is useful to define the extended state

$$\xi_t^i = \begin{bmatrix} u_{[t-n,t-1]}^i \\ y_{[t-n,t-1]}^{-i} \\ y_{[t-n,t-1]}^i \end{bmatrix}. \tag{3}$$

With suitable matrices $\tilde{A}_{ii}$, $\tilde{B}_{ii}$, $\tilde{B}_{-i}$, $\tilde{C}_{ii}$ and $\tilde{D}_{ii}$, the system

$$\xi_{t+1}^i = \tilde{A}_{ii} \xi_t^i + \tilde{B}_{ii} u_t^i + \tilde{B}_{-i} y_t^{-i}, \tag{4a}$$

$$y_t^i = \tilde{C}_{ii} \xi_t^i + \tilde{D}_{ii} u_t^i, \tag{4b}$$

then has the same input-output behaviour as (1), and there exists T_{x^i} such that $x_t^i = T_{x^i} \xi_t^i$, see, e.g. (Goodwin and Sin, 2014), (Koch et al., 2021, Lemma 2). In particular, (4a) looks like (6) with unknown $G_{i,n}, \ldots, G_{i,1}, S_{i,n}, \ldots, S_{i,1}, F_{i,n}, \ldots, F_{i,1}$ (cf. (Berberich et al., 2021b)). Note that (2) contains only $y_{[0,L-2]}^{-i}$ since there is no feed-through from y^{-i} to y^i in (1b), and thus y_{L-1}^{-i} has no effect on y_{L-1}^i. However, we included y_{t-1}^{-i} in (3) since then $\tilde{B}_{-i}$ in (4a) is known. We exploit this in the data-driven design of terminal ingredients in Section 3.3. Define also $T_{y^i} = [0 \ldots 0\, I]$ with $y_t^i = T_{y^i} \xi_{t+1}^i$. Similarly, a global extended state system

$$\xi_{t+1} = \tilde{A} \xi_t + \tilde{B} u_t \tag{5a}$$

$$y_t = \tilde{C} \xi_t + \tilde{D} u_t, \tag{5b}$$

can be defined with $\xi_t = \mathrm{col}_{i=1}^M(\xi_t^i)$, $u_t = \mathrm{col}_{i=1}^M(u_t^i)$ and $y_t = \mathrm{col}_{i=1}^M(y_t^i)$ and suitable matrices $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ and $\tilde{D}$.

3.2 Data-driven distributed MPC optimisation problem

Each subsystem locally solves at time step t the following optimisation problem given the measurements $\{u_k, y_k\}_{k=t-n}^{t-1}$ and communicated trajectories $y_{[-n+1,L]}^{-i,*}(t-1)$ of the neighbours.

$$\min_{\alpha^i(t)} \sum_{k=0}^{L-1} \|y_k^i(t)\|_{Q_i}^2 + \|u_k^i(t)\|_{R_i}^2 + \|\xi_L^i(t)\|_{P_i}^2 \tag{7a}$$

subject to

$$\begin{bmatrix} H_{L+n}(u_{[0,N-1]}^{i,\mathrm{d}}) \\ H_{L+n-1}(y_{[0,N-2]}^{-i,\mathrm{d}}) \\ H_{L+n}(y_{[0,N-1]}^{i,\mathrm{d}}) \end{bmatrix} \alpha^i(t) = \begin{bmatrix} u_{[-n,L-1]}^i(t) \\ y_{[-n+1,L-1]}^{-i,*}(t-1) \\ y_{[-n,L-1]}^i(t) \end{bmatrix} \tag{7b}$$

$$\begin{bmatrix} u_{[-n,-1]}^i(t) \\ y_{[-n,-1]}^i(t) \end{bmatrix} = \begin{bmatrix} u_{[t-n,t-1]}^i \\ y_{[t-n,t-1]}^i \end{bmatrix} \tag{7c}$$

$$u_k^i(t) \in \mathcal{U}_i, \ k \in \mathbb{I}_{0:L-1}, \tag{7d}$$

$$
\xi^i_{t+1} =
\begin{bmatrix}
u^i_{t-n+1} \\
\vdots \\
u^i_t \\
\hline
y^{-i}_{t-n+1} \\
\vdots \\
y^{-i}_t \\
\hline
y^i_{t-n+1} \\
\vdots \\
y^i_t
\end{bmatrix}
=
\left[
\begin{array}{cccc|cccc|cccc}
0 & I & \dots & 0 & 0 & \dots & \dots & 0 & 0 & \dots & \dots & 0 \\
\vdots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots \\
0 & \dots & \dots & I & 0 & \dots & \dots & 0 & 0 & \dots & \dots & 0 \\
0 & \dots & \dots & 0 & 0 & \dots & \dots & 0 & 0 & \dots & \dots & 0 \\
\hline
0 & \dots & \dots & 0 & 0 & I & \dots & 0 & 0 & \dots & \dots & 0 \\
\vdots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots \\
0 & \dots & \dots & 0 & 0 & \dots & \dots & I & 0 & \dots & \dots & 0 \\
0 & \dots & \dots & 0 & 0 & \dots & \dots & 0 & 0 & \dots & \dots & 0 \\
\hline
0 & \dots & \dots & 0 & 0 & \dots & \dots & 0 & 0 & I & \dots & 0 \\
\vdots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots \\
0 & \dots & \dots & 0 & 0 & \dots & \dots & 0 & 0 & \dots & \dots & I \\
G^i_n & \dots & \dots & G^i_1 & S^i_n & \dots & \dots & S^i_1 & F^i_n & \dots & \dots & F^i_1
\end{array}
\right]
\xi^i_t
+
\begin{bmatrix}
0 \\
\vdots \\
I \\
\hline
0 \\
\vdots \\
0 \\
\hline
0 \\
\vdots \\
D_{ii}
\end{bmatrix}
u^i_t
+
\begin{bmatrix}
0 \\
\vdots \\
0 \\
\hline
0 \\
\vdots \\
I \\
\hline
0 \\
\vdots \\
0
\end{bmatrix}
y^{-i}_t
\qquad (6)
$$

$$\xi^i_L(t) \in \mathcal{X}^{\mathrm f}_i(\theta_i \epsilon_i), \tag{7e}$$

$$\|u^i_k(t) - u^{i,*}_{k+1}(t-1)\|^2 \le \|\hat u^i_k(t) - \hat u^{i,*}_{k+1}(t-1)\|^2 + \Omega_i,$$
$$k \in \mathbb{I}_{[0,L-1]}, \tag{7f}$$

$$\|y^i_k(t) - y^{i,*}_{k+1}(t-1)\|^2 \le \|\hat y^i_k(t) - \hat y^{i,*}_{k+1}(t-1)\|^2 + \Omega_i,$$
$$k \in \mathbb{I}_{[0,L-1]}, \tag{7g}$$

$$\xi^i_L(t) = \begin{bmatrix} u^i_{[L-n,L-1]}(t) \\ y^{-i,*}_{[L-n+1,L]}(t-1) \\ y^i_{[L-n,L-1]}(t) \end{bmatrix}, \tag{7h}$$

with a constant $\Omega_i \ge 0$, and momentarily defined trajectories $\hat u^i_{[0,L-1]}(t)$ and $\hat y^i_{[0,L-1]}(t)$ as well as a terminal cost matrix P_i and set $\mathcal{X}^{\mathrm f}_i$. The optimal solution of (7) is denoted by $\alpha^{i,*}(t)$ with corresponding optimal input sequence $u^{i,*}_{[-n,L-1]}(t)$ and predicted output sequence $y^{i,*}_{[-n,L-1]}(t)$. Constraint (7b) is based on Lemma 3 and used to predict the input-output behaviour of the system over the prediction horizon L, whereas (7c) fixes the initial condition. The predicted extended state (7h) is confined to a terminal set through (7e) which is tightened using a factor $\theta_i > 0$ to provide recursive feasibility. Each subsystem assumes that the outputs of their neighbours follow a communicated trajectory (cf. (7b)). The neighbours, however, will in general deviate from this trajectory. To ensure recursive feasibility despite this discrepancy, the consistency constraints (7f) and (7g) are included. It forces the predicted input and output to stay as close to what has been communicated as the in 3.4 defined feasible trajectories $\hat u^i_{[0,L-1]}(t)$ and $\hat y^i_{[0,L-1]}(t)$ can. If $\mathcal{U}_i$ and $\mathcal{X}^{\mathrm f}_i(\theta_i \epsilon_i)$ are ellipsoidal or polytopic, (7) is a convex quadratically constraint quadratic program, which can be solved efficiently.

3.3 Terminal ingredients

Assumption 4. For every subsystem i, there exist $P_i \succ 0$, $\mu_i, \eta_i, \epsilon_i > 0$, $\theta_i \in (0,1)$ and a terminal controller K_i such that

$$\|\xi^{i,+}\|^2_{P_i} - \|\xi^i\|^2_{P_i} \le -\eta_i \|\xi^i\|^2 - \|y^i\|^2_{Q_i} - \|\kappa_i(\xi^i)\|^2_{R_i} + \mu_i \tag{8}$$

$$\kappa_i(\xi^i) \in \mathcal{U}_i \tag{9}$$

$$\|\xi^{i,+}\|^2_{P_i} \le \theta_i \epsilon_i \tag{10}$$

if $\xi^i \in \mathcal{X}^{\mathrm f}_i(\epsilon_i) = \{\xi^i \mid \|\xi^i\|^2_{P_i} \le \epsilon_i\}$ for all $i \in \mathcal{V}$ and where $\xi^{i,+} = \tilde A_{ii}\xi^i + \sum_{j \in \mathcal{N}_i} \tilde A_{ij} y^j + \tilde B_{ii}\kappa_i(\xi^i)$.

This assumption implies that inside $X^{\mathrm f}_i(\epsilon_i)$ every subsystem can be driven into an invariant set strictly inside $X^{\mathrm f}_i(\epsilon_i)$ despite the influence of the neighbours' outputs. Implicitly, the dynamic coupling needs to be sufficiently weak. If μ_i is sufficiently small, it can be shown that (8) implies (10) if $\xi^i \in \mathcal{X}^{\mathrm f}_i(\epsilon_i)$ for all $i \in \mathcal{V}$.

We comment on how the method in (Berberich et al., 2021b) could be adapted to compute distributed terminal ingredients as in Assumption 4 from input-output data, if the coupling is sufficiently weak. A general data-driven method to compute these terminal ingredients is left open for future research. We rewrite (4) into

$$\begin{bmatrix} \xi^i_{t+1} \\ z^i_t \end{bmatrix} = \left[\begin{array}{c|ccc} \bar A_i & B_{u^i} & B_{y^{-i}} & B_{w^i} \\ \hline \begin{bmatrix} I \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ I \end{bmatrix} & 0 & 0 \end{array} \right] \begin{bmatrix} \xi^i_t \\ u^i_t \\ y^{-i}_t \\ w^i_t \end{bmatrix},$$

where $w^i_t = \Delta_i z^i_t$ contains all unknown elements in (4), i.e. $\Delta_i = [G_{i,n}, \dots, G_{i,1}, S_{i,n}, \dots, S_{i,1}, F_{i,n}, \dots, F_{i,1}, D_{ii}]$. Define the matrices

$$\Xi_i = \begin{bmatrix} \xi^{\mathrm d,i}_n & \xi^{\mathrm d,i}_{n+1} & \cdots & \xi^{\mathrm d,i}_{N-1} \end{bmatrix}, \ \Xi^+_i = \begin{bmatrix} \xi^{\mathrm d,i}_{n+1} & \xi^{\mathrm d,i}_{n+2} & \cdots & \xi^{\mathrm d,i}_N \end{bmatrix}$$

$$U_i = \begin{bmatrix} u^{\mathrm d,i}_n & u^{\mathrm d,i}_{n+1} & \cdots & u^{\mathrm d,i}_{N-1} \end{bmatrix}, \ Y_{-i} = \begin{bmatrix} y^{\mathrm d,-i}_n & y^{\mathrm d,-i}_{n+1} & \cdots & y^{\mathrm d,-i}_{N-1} \end{bmatrix},$$

and $Z_i = \begin{bmatrix} \Xi^\top_i & U^\top_i \end{bmatrix}^\top$ with the extended state $\xi^{\mathrm d,i}_t$ based on the available input-output data $\mathcal{D}_i$. Further, define $M_i = \Xi^+_i - \bar A_i \Xi_i - B_{u^i} U_i - B_{y^{-i}} Y_{-i}$ and the matrix

$$\bar P^{w^i}_{\Delta_i} = [\star]^\top \begin{bmatrix} -Z_i Z^\top_i & Z_i M^\top_i B_{w^i} \\ B^\top_{w^i} M_i Z^\top_i & -B^\top_{w^i} M_i M^\top_i B_{w^i} \end{bmatrix} \begin{bmatrix} 0 & I \\ B^\top_{w^i} & 0 \end{bmatrix}.$$

Lemma 5. (cf. (Berberich et al., 2021b, Proposition 10)).
Factorise $T^\top_{y^i} Q_i T_{y^i} = Q^\top_{i,\mathrm r} Q_{i,\mathrm r}$ and $R_i = R^\top_{i,\mathrm r} R_{i,\mathrm r}$. Suppose there exist $\mathcal{X}_i \succ 0$, $\Gamma_i \succ 0$, $\mathcal{M}_i$, $\tau_i \ge 0$, $\gamma_i > 0$ such that $\mathrm{trace}(\Gamma_i) < \gamma^2_i$, $\begin{bmatrix} \Gamma_i & I \\ I & \mathcal{X} \end{bmatrix} \succ 0$, and

$$\begin{bmatrix} \tau \bar P^{w^i}_{\Delta_i} - \begin{bmatrix} \mathcal{X}_i & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} \bar A^i \mathcal{X}_i + B_{u^i}\mathcal{M}_i \\ \mathcal{X}_i \\ \mathcal{M}_i \end{bmatrix} & 0 \\ \star & -\mathcal{X}_i & \begin{bmatrix} Q_{i,\mathrm r}\mathcal{X}_i \\ R_{i,\mathrm r}\mathcal{M}_i \end{bmatrix}^\top \\ \star & \star & -I \end{bmatrix} \prec 0.$$

Define $P_i = \mathcal{X}^{-1}_i - T^\top_{y^i} Q_i T_{y^i}$ and $K_i = \mathcal{M}_i \mathcal{X}^{-i}_i$. Then, there exists $\bar\eta_i > 0$ such that

$$\|\bar\xi^{i,+}\|^2_{P_i} - \|\xi^i\|^2_{P_i} \le -\bar\eta_i \|\xi^i\|^2_{P_i} - \|y^i\|^2_{Q_i} - \|\kappa_i(\xi^i)\|^2_{R_i} \tag{12}$$

with $\bar\xi^{i,+} = (\tilde A_{ii} + \tilde B_{ii} K_i)\xi^i$ and $\kappa_i(\xi^i) = K_i \xi^i$.

This result is a slight extension of (Berberich et al., 2021b, Proposition 10) and we refer to (Berberich et al., 2021b) for the

proof. The existence of the additional parameter $\bar{\eta}_i$ is guaranteed by strictness of the LMIs in (Berberich et al., 2021b, Proposition 10). Note that (12) is similar to (8), except that no coupling is considered, cf. $\bar{\xi}^{i,+}$ in (12) to $\xi^{i,+}$ in (8). Thus, the neglected dynamic coupling needs to be sufficiently weak for (12) to imply (8). Furthermore, if a bound on the neighbours' outputs is known, they can be interpreted as bounded noise and it is straightforward to adapt the method in (Berberich et al., 2021b) to design a robust feedback law, which would alleviate this issue. Feasibility of the conditions in Lemma 5 requires a potentially restrictive condition on the dimensions of subsystem (1), as further discussed in (Berberich et al., 2021b).

3.4 Distributed data-driven MPC scheme

The following assumption bypasses the difficult task of constructing an initially feasible candidate in (7).

Assumption 6. At time $t = 0$, for all subsystems $i \in \mathcal{V}$, there exists $\hat{\alpha}^i(0)$ with corresponding $\hat{y}^i_{[-n,L-1]}(0)$ such that it is a feasible candidate in (7) for $y^{-i,*}_{[-n+1,L]}(-1) = \hat{y}^{-i}_{[-n,L-1]}(0)$. In addition, each subsystem knows $\hat{\alpha}^i(0)$.

Since the prediction relies on communicated trajectories of the neighbours, the first communicated output trajectory is $\hat{y}^i_{[-n,L-1]}(0)$ from Assumption 6.

After solving (7) at $t - 1$, the predicted output trajectory of subsystem i, $y^{i,*}_{[0,L-1]}(t - 1)$, is available for communication after a one-step extension as described momentarily. However, in general, it does not correspond to a feasible candidate at time t because of (7b), which now depends on the updated $y^{-i,*}_{[-n+1,L]}(t-1)$, whereas $y^{i,*}_{[0,L-1]}(t-1)$ was computed based on $y^{-i,*}_{[-n+1,L]}(t-2)$. Instead, a feasible candidate at time $t \in \mathbb{N}$ can be constructed using

$$\hat{u}^i_{[-n,L-2]}(t) = u^{i,*}_{[-n+1,L-1]}(t-1), \tag{13a}$$

$$\hat{u}^i_{L-1}(t) = \kappa_i(\hat{\xi}^i_{L-1}(t)), \tag{13b}$$

$$\hat{\xi}^i_{L-1}(t) = \begin{bmatrix} \hat{u}^i_{[L-n-1,L-2]}(t) \\ y^{-i,*}_{[L-n,L-1]}(t-1) \\ \hat{y}^i_{[L-n-1,L-2]}(t) \end{bmatrix}, \tag{13c}$$

as will be shown below. The corresponding candidate $\hat{\alpha}^i(t)$ and output trajectory $\hat{y}^i_{[-n,L-1]}(t)$ are computed with Algorithm 1 based on the updated neighbours' trajectories $y^{-i,*}_{[-n+1,L]}(t-1)$ and output measurement $y^i_{[t-n,t-1]} = y^{i,*}_{[-n+1,0]}(t-1)$.

The D3MPC scheme is stated in Algorithm 2. Note that each subsystem can solve its MPC problem in parallel in Step 1) of Algorithm 2. Hence, the complexity of the scheme increases only with the number of neighbours each subsystem has, but not with the total number of subsystems. In addition, since communication is necessary only once in each time step, the communication overhead is kept to a minimum. In Step 4) of Algorithm 2, we compute an extension of the optimal output trajectory by one step, which is denoted by $y^{i,*}_L(t)$ with a slight abuse of notation.

4. CLOSED-LOOP GUARANTEES

In this section, we prove that the origin is practically asymptotically stabilised if Algorithm 2 is applied to the system (5). An

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Algorithm 1. **Data-driven simulation (subsystem i)** (cf. (Markovsky and Rapisarda, 2008))

Input:
- Data $\mathcal{D}_i$, where $\begin{bmatrix} u^{\mathrm{d}}_{[0,N-1]} \\ y^{\mathrm{d},-i}_{[0,N-2]} \end{bmatrix}$ is persistently exciting of order $L + 2n$ and (A_{ii}, B_i) is controllable.
- Initial condition $\{u^i_k, y^i_k, y^{-i}_k\}^{-1}_{k=-n}$.
- New input and neighbours' output data $\{u^i_k\}^{L-1}_{k=0}$, $\{y^{-i}_k\}^{L-2}_{k=0}$.

Procedure:
(1) Compute α^i satisfying

$$\begin{bmatrix} H_{L+n}(u^{\mathrm{d},i}_{[0,N-1]}) \\ H_{L+n-1}(y^{\mathrm{d},-i}_{[0,N-2]}) \\ H_n(y^{\mathrm{d},i}_{[0,N-L-1]}) \end{bmatrix} \alpha^i = \begin{bmatrix} u^i_{[-n,L-1]} \\ y^{-i}_{[-n,L-2]} \\ y^i_{[-n,-1]} \end{bmatrix} \tag{14}$$

(2) Compute $y^i_{[0,L-1]} = H_L(y^{\mathrm{d},i}_{[n,N-1]})\alpha^i$.

Output: Resulting simulated output trajectory $\{y_k\}^{L-1}_{k=0}$.

Algorithm 2. **Data-driven distributed MPC scheme**

Input for all $i \in \mathcal{V}$:
- Data $\mathcal{D}_i$, where $\begin{bmatrix} u^{\mathrm{d}} \\ y^{\mathrm{d},-i} \end{bmatrix}$ is persistently exciting of order $L + 2n$ and (A_{ii}, B_i) is controllable.
- Initial measurement $\{u^i_k, y^i_k\}^{-1}_{k=-n}$.
- Initially communicated trajectories $y^{-i,*}_{[-n+1,L]}(-1)$.

Procedure: For all $i \in \mathcal{V}$:
(1) Solve the local MPC problem (7).
(2) Apply $u_t = u^{i,*}_0(t)$ and measure y^i_t.
(3) Compute $u^{i,*}_L(t) = \kappa_i(\xi^{i,*}_L(t))$. Then, compute $y^{i,*}_L(t)$ using Algorithm 1 with initial condition $u^{i,*}_{[-n,-1]}(t)$, $y^{-i,*}_{[-n+1,0]}(t-1), y^{i,*}_{[-n,-1]}(t)$ and new input and neighbours' output trajectories $u^{i,*}_{[0,L]}(t), y^{-i,*}_{[1,L]}(t-1)$.
(4) Send $y^{i,*}_{[-n+1,L]}(t)$; receive $y^{-i,*}_{[-n+1,L]}(t)$ from neighbours.
(5) Set $t = t + 1$.
(6) Compute $\hat{u}^i_{[-n,L-1]}(t)$ according to (13) as well as $\hat{y}^i_{[-n,L-1]}(t)$ using Algorithm 1 with initial condition $\hat{u}^i_{[-n,-1]}(t), y^{-i,*}_{[-n+1,0]}(t-1), y^{i,*}_{[-n,-1]}(t)$ and new input and neighbours' output trajectories $\hat{u}^i_{[0,L-1]}(t)$, $y^{-i,*}_{[1,L-1]}(t-1)$.

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important requirement to this end is that in each time step each subsystem is able to solve the MPC optimisation problem (7), i.e. (7) is recursively feasible, which in particular implies that the input constraints are not violated.

4.1 Recursive feasibility

The following assumption is crucial to prove recursive feasibility of (7) and captures the central idea of the scheme. If all subsystems stay close to their communicated trajectory, the unexpected influence on neighbours is sufficiently bounded.

Assumption 7. There exist $\sigma'_i, \tilde{\sigma}_i \in \mathcal{K}$ such that $\|\hat{\xi}^i_{L-1}(t + 1) - \xi^{i,*}_L(t)\|_{P_i} \leq \tilde{\sigma}_i(\Omega_i)$ and $\|\hat{y}^i_k(t + 1) - y^{i,*}_{k+1}(t)\|_{Q_i} \leq \sigma'_i(\Omega_i)$ for all $t \in \mathbb{N}_0$ and $k \in \mathbb{I}_{[0,L-2]}$ if Algorithm 2

is used, with $\hat{\xi}^i_{L-1}(t+1)$ as in (13c) and where $\xi^{i,*}_L(t) = \left[u^{i,*}_{[L-n,L-1]}(t)^\top\ y^{-i,*}_{[L-n+1,L]}(t-1)^\top\ y^{i,*}_{[L-n,L-1]}(t)^\top \right]^\top$.

As stated in (Dunbar, 2007), it is to be expected that this requires sufficiently weak dynamic coupling. Although we conjecture that Assumption 7 can be shown to hold (cf. (Dunbar, 2007, Lemma 3)), it is not straightforward to compute $\tilde{\sigma}_i$ and σ'_i only based on the available input-output data. This is beyond the scope of this paper, yet we want to highlight two possibilities. It may be possible to extract this information from the Hankel matrix of each subsystem in Lemma 3, since it contains the dynamic coupling. Alternatively, one may be able to use bounds on the norms of A_{ii} and B_{ij}, $j \in \mathcal{N}_i$, in (1) to construct $\tilde{\sigma}_i$ and σ'_i. Using a similar approach as in (Wildhagen et al., 2022), the latter can be estimated using input-output data.

We now show that $\hat{u}_{[0,L-1]}(t)$ from (13) leads to a feasible candidate in (7) and the MPC problem is recursively feasible.

Theorem 8. Let Assumptions 2–7 hold. In particular, from Assumption 6, let the MPC problem (7) be feasible for all $i \in \mathcal{V}$ at time $t = 0$. Then, (7) is also feasible for all $i \in \mathcal{V}$ and all $t \in \mathbb{N}$, if $\tilde{\sigma}_i(\Omega_i) \leq (1 - \sqrt{\theta_i})\sqrt{\epsilon_i}$ and $\theta_i \geq 1 - \eta_i \lambda_{\max}(P_i)^{-1}$ for all $i \in \mathcal{V}$ with ϵ_i, η_i, and P_i from Assumption 4.

Proof. Let $i \in \mathcal{V}$. From Assumption 6, a solution $\alpha^{i,*}(0)$ with corresponding $u^{i,*}_{[-n,L-1]}(0)$ and $y^{i,*}_{[-n,L-1]}(0)$ to (7) exists. Assume for induction that (7) is feasible at time t. Consider the candidate input trajectory $\hat{u}^i_{[-n,L-1]}(t+1)$ from (13) together with the updated output data of the neighbours $y^{-i,*}_{[-n+1,L]}(t)$ which, if applied in (1a), yields $\hat{y}^i_{[-n,L-1]}(t+1)$. By Lemma 3, there exists a corresponding $\hat{\alpha}^i(t+1)$ such that (7b) is satisfied. By definition of $\hat{u}^i_{[-n,L-1]}(t+1)$ and $\hat{y}^i_{[-n,L-1]}(t+1)$ the constraint (7c) holds. From feasibility of $u^{i,*}_{[-n,L-1]}(t)$ it follows that $\hat{u}^i_{[-n,L-2]}(t+1) \in \mathcal{U}_i$. Clearly (7f) and (7g) hold. It remains to be shown that (7d) for $k = L-1$ and (7e) are satisfied. Feasibility of (7) at time t and Assumption 7 yield $\|\hat{\xi}^i_{L-1}(t+1)\|_{P_i} \leq \|\xi^{i,*}_L(t)\|_{P_i} + \|\hat{\xi}^i_{L-1}(t+1) - \xi^{i,*}_L(t)\|_{P_i} \leq \sqrt{\theta_i \epsilon_i} + \tilde{\sigma}_i(\Omega_i) \leq \sqrt{\epsilon_i}$, since $\tilde{\sigma}_i(\Omega_i) \leq (1 - \sqrt{\theta_i})\sqrt{\epsilon_i}$ with $\theta_i \in (0,1)$. Hence, $\hat{\xi}^i_{L-1}(t+1) \in \mathcal{X}^{\mathrm{f}}_i(\epsilon_i)$ and by Assumption 4, (7d) is satisfied for $k = L-1$ and for all $i \in \mathcal{V}$. In addition, from (10) in Assumption 4, $\hat{\xi}^i_L(t+1) \in \mathcal{X}^{\mathrm{f}}_i(\theta_i \epsilon_i)$ for all $i \in \mathcal{V}$. □

4.2 Practical stability

We define $V^*_t = \sum_{i=1}^M \sum_{k=0}^{L-1} (\|y^{i,*}_k(t)\|^2_{Q_i} + \|u^{i,*}_k(t)\|^2_{R_i} + \|\xi^{i,*}_L(t)\|^2_{P_i})$ and assume the following.

Assumption 9. There exists $c_{\mathrm{ub}} > 0$ such that $V^*_t \leq c_{\mathrm{ub}}\|\xi_t\|^2$ holds for all $t \in \mathbb{N}_0$.

This assumption can be shown to hold if, e.g. $\mathcal{U}_i$ are compact polytopes for all $i \in \mathcal{V}$ and the cost of the initially feasible candidate from Assumption 6 admits a quadratic upper bound.

The following theorem establishes practical stability of the origin of the global extended system (5). Hence, also of the closed-loop state, since $x^i_t = T_{x^i}\xi^i_t$ for all $i \in \mathcal{V}$. It shows a trade-off between giving each subsystem a larger margin of freedom (Ω_i larger), compared to tight stabilisation of the origin (Ω_i smaller).

Theorem 10. Let Assumptions 2–7 hold. If the D3MPC scheme in Algorithm 2 is applied, then the origin of the resulting closed-

loop system is practically stable. That is, there exist $\beta \in \mathcal{KL}$, $\delta_1, \delta_2 \in \mathcal{K}_\infty$ and $\Omega_{\max}$ such that for all $\Omega \leq \Omega_{\max}$

$$\|\xi_t\| \leq \beta(\|\xi_0\|, t) + \delta_1(\Omega) + \delta_2(\mu)$$

holds for the closed-loop solution of (5), where $\Omega = \max_i \Omega_i$ and $\mu = \sum_{i=1}^m \mu_i$.

Proof. For brevity, we write $\hat{\cdot} := \hat{\cdot}(t+1)$, e.g. $\hat{\xi}^i_L := \hat{\xi}^i_L(t+1)$, and $\cdot^* := \cdot^*(t)$, e.g. $y^{i,*}_{k+1} := y^{i,*}_{k+1}(t)$. Since $\hat{\alpha}^i(t+1)$ with $\hat{u}^i_{[-n,L-1]}(t+1)$ and $\hat{y}^i_{[-n,L-1]}(t+1)$ is a feasible choice in (7) at time $t+1$ (see Theorem 8) and $\hat{u}^i_k(t+1) = u^{i,*}_{k+1}(t)$ for $k \in \mathbb{I}_{[0,L-2]}$, $V^*_{t+1} \leq \sum_{i=1}^M (\sum_{k=0}^{L-1} \|\hat{y}^i_k\|^2_{Q_i} + \|\hat{u}^i_k\|^2_{R_i} + \|\hat{\xi}^i_L\|^2_{P_i}) = V^*_t + \sum_{i=1}^M (\sum_{k=0}^{L-2} \|\hat{y}^i_k\|^2_{Q_i} - \|y^{i,*}_{k+1}\|^2_{Q_i}) + \sum_{i=0}^M (\|\hat{y}^i_{L-1}\|^2_{Q_i} + \|\hat{u}^i_{L-1}\|^2_{R_i} + \|\hat{\xi}^i_L\|^2_{P_i} - \|\hat{\xi}^i_{L-1}\|^2_{P_i} + \|\hat{\xi}^i_{L-1}\|^2_{P_i} - \|\xi^{i,*}_L\|^2_{P_i} - \|y^i_t\|^2_{Q_i} - \|u^i_t\|^2_{R_i})$. Then, since $\hat{\xi}^i_{L-1}(t+1) \in \mathcal{X}^{\mathrm{f}}_i(\epsilon_i)$ for all $i \in \mathcal{V}$ as shown in the proof of Theorem 8, and by definition of $\hat{u}^i_{L-1}(t+1)$ in (13b), from Assumption 4 it follows that $V^*_{t+1} - V^*_t \leq -\|y_t\|^2_Q - \|u_t\|^2_R + \sum_{i=1}^M (\sum_{k=0}^{L-2} \|\hat{y}^i_k\|^2_{Q_i} - \|y^{i,*}_{k+1}\|^2_{Q_i}) + \sum_{i=0}^M (\|\hat{\xi}^i_{L-1}\|^2_{P_i} - \|\xi^{i,*}_L\|^2_{P_i} + \mu_i)$, with $Q = \mathrm{diag}_{i\in\mathcal{V}}(Q_i)$ and $R = \mathrm{diag}_{i\in\mathcal{V}}(R_i)$. Note that for vectors a,b, $\|a\|^2 - \|b\|^2 \leq \|a-b\|^2 + 2\|b\|\|a-b\|$. Hence, with $\|y^{i,*}_{k+1}(t)\|^2_{Q_i} \leq V^*_t$ for all $k \in \mathbb{I}_{[0:L-2]}$, and $\|\xi^{i,*}_L(t)\|^2_{P_i} \leq V^*_t$ for all $i \in \mathcal{V}$, $V^*_{t+1} - V^*_t \leq -\|y_t\|^2_Q - \|u_t\|^2_R + \mu + \sum_{i=1}^M (\sum_{k=0}^{L-2} \|\hat{y}^i_k - y^{i,*}_{k+1}\|^2_{Q_i} + 2\sqrt{V^*_t}\|\hat{y}^i_k - y^{i,*}_{k+1}\|_{Q_i}) + \sum_{i=0}^M (\|\hat{\xi}^i_{L-1} - \xi^{i,*}_L\|^2_{P_i} + 2\sqrt{V^*_t}\|\hat{\xi}^i_{L-1} - \xi^{i,*}_L\|_{P_i})$. From Assumption 7, and with $\sqrt{V^*_t} \leq V^*_t + 1$

$$V^*_{t+1} - V^*_t \leq \sum_{i=1}^M ((L-1)(\bar{\sigma}(\sigma'_i(\Omega_i)) + 2V^*_t \sqrt{\sigma'_i(\Omega_i)})) + \mu$$
$$- \|y_t\|^2_Q - \|u_t\|^2_R + \sum_{i=1}^M \bar{\sigma}(\tilde{\sigma}_i(\Omega_i)) + 2V^*_t \sqrt{\tilde{\sigma}_i(\Omega_i)} \quad (15)$$

with $\bar{\sigma}(r) = r + 2\sqrt{r}$. As in the proof of (Berberich et al., 2021b, Theorem 8), since (A, C) is observable by Assumption 2, $(\tilde{A}, \tilde{C})$ is detectable and (5) admits an input-output-to-state stability Lyapunov function $W(\xi) = \|\xi\|^2_{P_W}$ satisfying

$$W(\tilde{A}\xi + \tilde{B}u) - W(\xi) \leq -\frac{1}{2}\|\xi\|^2 + c_1\|y\|^2_2 + c_2\|u\|^2_2, \quad (16)$$

with $c_1, c_2 > 0$ and $P_W \succ 0$, for all $u \in \mathbb{R}^m$, $\xi \in \mathbb{R}^{n_\xi}$ and $y = \tilde{C}\xi + \tilde{D}u$ (cf. (Cai and Teel, 2008)). Consider now $\mathcal{W}_t = \gamma W(\xi_t) + V^*_t$ with $\gamma = \frac{\lambda_{\min}(Q,R)}{\max(c_1,c_2)}$. Clearly, $\gamma\|\xi_t\|^2_{P_W} \leq \mathcal{W}_t \leq c_{\mathrm{ub}}\|\xi_t\|^2 + \gamma\|\xi_t\|^2_{P_W}$, where the lower bound follows from $V^*_t \geq 0$ and the upper bound from Assumption 9. Hence, from Assumption 9 and combining (15) with (16), $\mathcal{W}_{t+1} - \mathcal{W}_t \leq -\frac{\gamma}{2}\|\xi_t\|^2 + \sum_{i=1}^M ((L-1)\bar{\sigma}(\sigma'_i(\Omega_i)) + \bar{\sigma}(\tilde{\sigma}_i(\Omega_i))) + 2c_{\mathrm{ub}}\|\xi_t\|^2 \sum_{i=1}^M ((L-1)\sqrt{\sigma'_i(\Omega_i)} + \sqrt{\tilde{\sigma}_i(\Omega_i)}) + \mu$. Thus, there exist Ω_i sufficiently small such that $\mathcal{W}_{t+1} - \mathcal{W}_t \leq -\tilde{\gamma}\|\xi_t\|^2 + \sum_{i=1}^M (L-1)\bar{\sigma}(\sigma'_i(\Omega_i)) + \bar{\sigma}(\tilde{\sigma}_i(\Omega_i)) + \mu$ for some $\tilde{\gamma} > 0$. Hence, there also exists $\sigma \in \mathcal{K}_\infty$ such that $\mathcal{W}_{t+1} - \mathcal{W}_t \leq -\tilde{\gamma}\|\xi_t\|^2 + \sigma(\Omega) + \mu$, and the claim follows (Grüne and Stieler, 2014, Theorem 2.4). □

5. NUMERICAL EXAMPLE

We consider the same example as in (Alonso et al., 2021): a system comprising a chain of 64 subsystems with dynamics
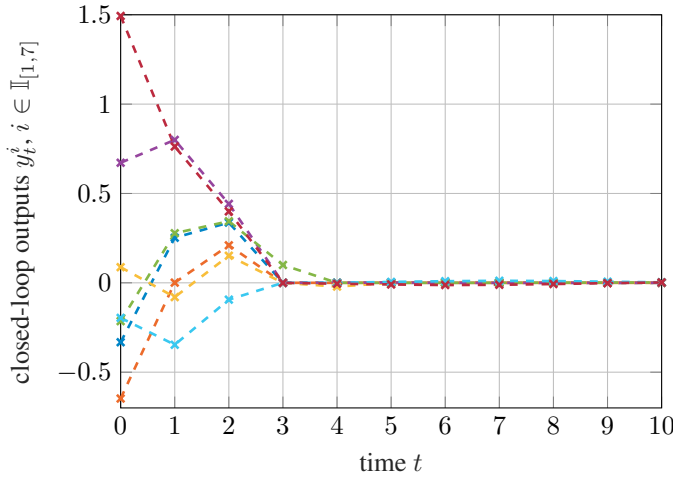
Fig. 1. Evolution of the closed-loop outputs of (17) for $i \in \mathbb{I}_{[1,7]}$.

$$x_{t+1}^i = \begin{bmatrix} 1 & 0.2 \\ -\frac{k_i}{5m_i} & 1 - \frac{d_i}{5m_i} \end{bmatrix} x_t^i + \begin{bmatrix} 0 \\ u_t^i \end{bmatrix} + \sum_{j \in \mathcal{N}_i} \begin{bmatrix} 0 \\ k_{ij} y_t^j \end{bmatrix}, \quad (17a)$$

$$y_t^i = \begin{bmatrix} \frac{0.2}{m_i} & 0 \end{bmatrix} x_t^i. \quad (17b)$$

The parameters are set to $m_i = 1$, $d_i = 0.75$, $k_{ij} = 1.25$ and $k_i = \sum_{j \in \mathcal{N}_i} k_{ij}$ for all $i \in \mathcal{V}$ and $j \in \mathcal{N}_i$. We consider input-output data of each subsystem of length $N = 100$. We choose $L = 5$, $Q_i = R_i = I$, $U_i = [-2, 2]$ and design terminal ingredients as discussed with $\epsilon_i = 10^{-5}$ for all $i \in \mathcal{V}$. For the consistency constraint, we choose $\Omega_i = 0.01$ for all $i \in \mathcal{V}$. We use a suboptimal feasible trajectory as the initially feasible trajectory. The LMIs in Lemma 5 were solved using YALMIP (Löfberg, 2004) and MOSEK (MOSEK ApS, 2020), whereas MOSEK was used for (7). We plot the closed-loop output evolution of subsystems $i \in \mathbb{I}_{[1,7]}$ in Figure 1, displaying (practical) convergence, as expected from Theorem 10.

6. CONCLUSION

We have proposed a direct data-driven distributed MPC scheme for a group of dynamically coupled LTI systems. Each local MPC optimisation problem uses only past measured input-output data for the prediction, without any prior system identification step. We showed that if the dynamic coupling is sufficiently weak, the data-driven distributed MPC scheme is recursively feasible and practically stabilises the origin of the global system. The main mechanism are so-called consistency constraints, i.e. keeping close to a previously communicated trajectory, based on the model-based approach in (Dunbar, 2007). This enables a non-iterative parallel distributed MPC scheme with minimal communication. The complexity of the scheme does not increase with the total number of subsystems, but only with the number of neighbours of each subsystem. Future research will investigate deriving suitable bounds for the consistency based on input-output data, as well as data-driven design of terminal ingredients that take the dynamic coupling into account.

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