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Abstract

Infrastructure development projects are overwhelmingly managed through engineering-procurement-construction (EPC) contracts, which allow a project end user to shift all project risks to a contractor. Accordingly, the International Federation of Consulting Engineers recommended a contract template based on a lump-sum contract between the end user and main contractor. However, EPC projects often suffer from quality issues due to moral hazard, which is aggravated by the involvement of subcontractors hired by the main contractor to perform parts of the project. Besides, costly rework is frequently needed to achieve the contractually mandated quality. When the main contractor must share some of the subcontractor's rework cost, an externality might arise. In this study, we model an EPC contract with three parties: the end user, the main contractor, and a representative subcontractor. We compare the end user's cost and quality trade-off for the recommended lump-sum contract and the proposed incentive contracts. We find that the lump-sum contract can achieve the first-best trade-off under limited circumstances, whereas the appropriate incentive contract can do so for a wider range of circumstances. Rework cost sharing can cause under- or over-investment in efforts and reduce system welfare compared to the first-best outcome by weakening the incentive contract's ability to overcome moral hazard. However, for a subcontractor with limited liability, rework cost sharing can improve project outcomes by allowing the main contractor to reduce the subcontractor's liability exposure.

Keywords: incentive contract, moral hazard, project management, rework, subcontracting

1. Introduction

Under an engineering-procurement-construction (EPC) contract, an end user engages a main contractor to execute the entire life cycle of the project, from design to procurement and construction. This contracting form is popular with end users, because all responsibility for project management and cost and risk control is placed in the hands of an experienced main contractor (Ishii et al., 2014). The main contractor, in turn, may engage subcontractors to perform a significant part of the work. Thus, the main contractor cedes some control over project quality to subcontractors, who are typically smaller local construction experts with whom the main contractor does not maintain a regular relationship (Steinberg, 2016).

Such EPC contracts are widely used in infrastructure development projects, such as construction of water processing plants, electricity power plants, steel plants, highways, and railways

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(Kynge and Peel, 2017). EPC contracts account for 45% of all construction projects in the energy sector (KPMG, 2015). Further, the top 30 EPC firms in the oil and gas industry in 2014 earned a combined revenue of over $300 million (Statista, 2015). Despite end users' expectation that EPC contracts provide better project control, an industry survey has found that 71% of EPC projects in the oil and gas industry fail in one or more areas of quality, budget, or time control, increasing to 90% for public sector projects (KPMG, 2015). If the project quality falls short of the minimum requirement stated in the contract, the contractor needs to perform rework to achieve the quality specifications. Rework in EPC projects can be extremely costly, such that the contractor makes minimal profit from the project or even suffers a loss. In one noteworthy instance, CITIC, the contractor for a Brazilian coke plant EPC project, was forced to pay EUR 100 million for quality and delay problems, for a contract worth EUR 270 million (Anderlini and Bryant, 2011). Industry estimations of rework costs as a proportion of total project cost range from 5% to more than 20%(Barber et al., 2000; Jason et al., 2012; Love et al., 2010). The rework cost due to failed quality includes both the direct cost of labor, material, and equipment, and the indirect cost related to field supervision, project management, and site safety. The indirect cost is significant and is typically found to be 70% to 80% of the direct cost (Jason et al., 2012), but can climb to up to five times the direct cost (Love, 2002). Therefore, quality management is of paramount importance for the main contractor in EPC.

Project stakeholder behavioral issues have been identified as one of the biggest threats to quality in EPC contracts (Rudolf and Spinler, 2018). Moral hazard occurs when contractors compromise on quality efforts to save costs, which reduces the end user's benefit from the project. Unfortunately, quality efforts are not verifiable and thus, cannot be contracted on directly. Thus, the end user and main contractor contract on verifiable quality instead. However, quality outcomes depend not only on effort levels, but also on external uncertainties, such as weather or environmental and health and safety regulations.

We model a scenario where an end user drafts an EPC contract to outsource a project to a main contractor (she), who in turn outsources part of the work to a subcontractor (he). The contractors exert efforts that stochastically increase the quality outcome of their part of the project work. If the quality fails to meet the end user's minimum requirement, the responsible contractor(s) must undertake costly rework. We consider both direct and indirect rework costs, such that the main contractor may have to bear a portion of the subcontractor's rework cost. To account for the small size of the subcontractor , we compare two types of subcontractors: with and without limited liability. Our model explores the impact of contractual payment terms on project performance and cost in a setting with nested contractual relationships. We start with the lump-sum contract structure recommended in the International Federation of Consulting Engineers (FIDIC) Silver Book, which is an "*agreed amount stated in the contract agreement for the design, execution and completion of the works and the remedying of any defects*" (FIDIC, 1999). Such a lump-sum contract protects the end user from all risk and guarantees a minimum acceptable quality, but cannot overcome the moral hazard problem in contractual relationships.

We investigate the ability of incentive contracts (e.g., (Dawande et al., 2019)) overcome moral hazard and improve project quality while considering the resulting cost–risk profile to the end user.

Our first contribution is that we model nested contractual relationships, in which an end user contracts with a main contractor, who in turn contracts with a subcontractor. While issues of moral hazard in contracts have been widely studied, our setting allows us to explore how moral hazard concerns propagate through the contractual chain and the joint impact on the final project performance and cost. The model and its assumptions are deeply grounded in the specific setting of EPC, which has received little attention in the management literature thus far.

The results of our analysis form our second contribution to theory and practice. We study the friction created by double contracting and by the sharing of the rework cost. We find that incentive contracts cause the main contractor's and the subcontractor's efforts to become complements, even though quality is additive and substitutable. Contrary to expectations, decentralized execution may lead to over-investment in quality efforts compared to the centralized case. We also find a threshold for the main contractor's share of the rework cost above which she is unable to induce the subcontractor to exert first-best effort levels despite of an incentive payment. Therefore, ideally, all or at least most of the rework cost should be borne by the party responsible for the lapse in quality. This recommendation needs to be qualified in the case of a subcontractor with limited liability, in which a shared rework cost can be second-best optimal, as it reduces a subcontractor's risk exposure and thus, also the cost of contracting for the main contractor and end user.

Third, our project quality and cost analysis offers a clear overview of the implications for end users when they choose a lump-sum or incentive EPC contract through a comparison with the centralized first-best outcome. Using the centralized first-best project execution, we determine the optimal cost for every possible level of quality; this yields the efficient frontier, that is, the first-best quality–cost trade-off. Intuitively, in the face of moral hazard and externalities, fixed-fee contracts cannot achieve the efficient frontier, although they reduce the cost risk to zero. Fortunately, if the externality due to the rework cost sharing is low, it is sufficient for the main contractor to introduce an incentive contract; meanwhile, the end user maintains a fixed-fee contract to achieve the efficient frontier—although only the lowest quality can be reached. Compared to the centralized first-best outcome, the end user reaches the same quality level as under centralized execution, but there is no cost risk, thereby achieving the purpose of the Silver Book contract. However, the end user would have to offer an incentive contract if they want to achieve a higher quality than the minimum on the efficient frontier. In this way, when externalities are low and the subcontractor has unlimited liability, the end user may be able to induce the efficient frontier. Interestingly, we show that, unlike the cost risk under centralized planning, the cost variability of an incentive contract is positively correlated with the quality variability; this means that the end user pays a larger cost only for higher realized project quality.

This is unlike the centralized planner's case, in which the cost variability is inversely related to the quality variability. This feature of the cost risk arguably makes it more palatable to the end user. Therefore, EPC contracts should not be regarded as a way to reduce cost risk to zero, but rather as a way to align payments to quality. Finally, while a higher externality prevents the end user from reaching the first-best efficient frontier, the qualitative insights on the different contracts' features and the cost and quality variability trade-off remain the same.

The rest of the paper is organized as follows. We review the related literature in the next section. Section 3 describes the model setting, the first-best case, and the fixed-fee benchmark. Sections 4 and 5 consider two combinations of a fixed fee and incentive payment, respectively. Section 6 examines the impact of moral hazard and contracting choices on effort, quality, and cost in the unlimited liability case of the subcontractor. Section 7 analyzes the limited liability case of the subcontractor. Finally, we present our conclusions in Section 8.

2. Related Literature

To clarify our contribution to the literature, we review studies related to our research on incentive contracts in EPC project management. We add to two relevant streams of literature: moral hazard and incentive mechanism design in project management and supply chain management.

The moral hazard problem is recognized as an important issue in project management and has been widely studied. Atkinson et al. (2006) report that moral hazard occurs in project management because the parties involved in the project are not motivated to work for the best interest of the project owner. Schieg (2008) claims that in construction projects, moral hazard occurs when contractors cannot be completely supervised, and suggests contractual incentives as a way to alleviate the moral hazard problem. Owusu-Manu et al. (2018) empirically study the moral hazard problem in public–private partnership projects. They find that the main causes of moral hazard are lack of knowledge about project conditions and unverifiable effort levels. Consequently, contractors have an incentive to minimize costs, which is not in the best interests of the public welfare. Given the increasing role of EPC in international construction projects and particularly oil and gas processing facilities, Berends (2007) aims to identify contracting strategies for EPC contracts that can help mitigate the effect of moral hazard. The researcher qualitatively studies three commonly used contract structures: a fixed-price/lump-sum contract, cost-plus incentive payment contract, and cost-plus fixed-fee contract. The author recommends an incentive payment based on project quality to improve outcomes. Our study clarifies this recommendation of the quality-based incentive contract by building a quantitative model.

Incentive mechanism design is extensively examined in project management. Bayiz and Corbett (2005) design a duration-based linear incentive contract to solve the moral hazard problem in a project consisting of two sub-projects. The researchers find that the incentive contract can achieve a shorter expected project duration and higher profit than a fixed-price contract. Shi et al. (2021) consider construction projects in which both the owner and the contractor exert unverifiable and complementary efforts affecting the overrun risk and cost. They design a risk-sharing incentive contract in which both parties share the overrun cost, so that they exert high effort to increase the project performance. Zeng et al. (2019) analyze incentive mechanism for supplier development in mega construction projects, and compare the effects on the quality improvement of the supplier of cost-sharing contract and purchase-price contract, as well as whether the owner or the contractor decides the quality level. They find that the situation when the owner determines the quality level of the supplier, as well as the cost-sharing contract where both parties share the cost of quality improvement, lead to a higher degree of quality improvement than when the supplier decides the quality level and the purchase-price contract. Our study extends these studies by further considering a subcontractor's involvement in the project and its effect on the project executions.

Moral hazard is also studied extensively in supply chain management. The literature commonly models a supply chain in which the upstream and downstream parties exert sequential efforts to improve the product quality. The literature proposes the use of performance-based incentives or penalty contracts to introduce an explicit cost of failure, by withholding a reward or imposing a penalty, respectively.

Several studies in the literature analyze whether penalty contracts shifting the cost of failure lapses to the supplier can coordinate the supply chain. Baiman et al. (2000) consider a supply chain in which the supplier's efforts increase product quality, while the buyer can choose to exert quality appraisal efforts. Their results highlight the importance of verifiability: If neither the effort decision nor the appraisal outcome are contractible, a penalty contract can never achieve the first-best outcome. Baiman et al. (2001) further consider a coproduction setting, in which the efforts of both parties affect the product quality. They discuss different product architectures, with and without separable quality. They show that achieving the first-best outcome requires the cause of product defects to be accurately allocated so that the party responsible for the failure pays the penalty. Balachandran and Radhakrishnan (2005) propose a warranty/penalty contract to improve the supply chain performance when a supplier and a buyer co-produce a product. When both parties' efforts are unverifiable, if the supplier is penalized for external or market product failure, regardless of the cause of the failure, the first-best quality cannot be achieved. However, a penalty based on a supplier component's failure to pass the buyer's inspection can achieve the first-best outcome. Chen et al. (2022) also design an inspection-based warranty contract in a supply chain where both a brand owner and a supplier sequentially exert quality effort on a product. They find that the owner can achieve first-best profit through an optimally designed warranty payment even though both parties' efforts are unverifiable and the owner bears the whole failure cost. These studies consider product failure costs borne by a single party. Our context differs from these studies in that the main contractor bears a portion of the rework cost, namely, the cost of failure, caused by the subcontractor's failure; this introduces an extra externality to the moral hazard problem.

Thus, we review the following research that considers shared failure costs. Lee et al.

(2013) propose a quality-compensation contract in which the manufacturer shares the external failure cost with the retailer. The researchers find that it can coordinate the supply chain and achieve the first-best outcome, unlike buyback and revenue-sharing contracts that allow only the manufacturer to make quality-enhancing investments. Lee and Li (2018) expand the model to a setting with binary quality outcomes in which both the buyer and supplier can exert quality-enhancing efforts. To incentivize the supplier to exert effort, the buyer designs a contract with a fixed payment for a high-quality product and a sharing rate for failure costs. The optimal failure cost-sharing rate allows the buyer to achieve the first-best outcome. In Chao et al. (2009), product quality is the sum of the quality of both supply chain partners' efforts. Product failure in the market incurs a recall cost. Regardless of the verifiability of the effort levels, the first-best outcome can be achieved under a cost-sharing contract where the failure cost is shared by the two parties if failure occurs before a threshold time; otherwise, the cost is borne by the responsible party, as identified by a root cause analysis. Zhu et al. (2007) consider a supply chain in which the buyer and manufacturer can invest in quality-improvement efforts. Both parties minimize their annual costs, including a warranty-related cost, payable for non-conforming products. To encourage the appropriate level of quality investment by the manufacturer, the buyer's contract specifies an optimal sharing rate for the warranty cost and the buyer's commitment to quality-enhancing efforts. As these studies show, a shared failure cost, when appropriately allocated, can improve the performance of or even coordinate the supply chain. Our setting differs from the settings in the abovementioned studies of failure-cost sharing in that we consider the nested contracting of three parties in EPC projects. Dong et al. (2016) consider a three-party setting in which a retailer buys from a manufacturer who sources a component from a supplier. The manufacturer and supplier can invest in quality improvement efforts. Given a contract with differentiated prices for high and low quality, Dong et al. (2016) show when the quality failure should be determined through inspection or external failure. We use inspection to identify quality failure and force rework, the cost of which is shared between the contractors. Further, we focus on optimizing the contract structure and the impact on the quality–cost trade-off.

Our study contributes to the literature by analyzing the effectiveness of incentive contracts under a *shared rework cost* in a *nested contractual relationship*—from the end user to the main contractor to the subcontractor—in a setting with simultaneous quality efforts. We provide management suggestions for the end user to obtain the best quality–cost trade-off. Furthermore, we show that limited liability at the subcontractor level can make rework cost sharing attractive.

3. Model Description and Benchmarks

We model an EPC contract with three parties: the end user, the main contractor, and a representative subcontractor. The end user commissions a large-scale engineering project, such as a dam or power plant, and contracts directly with the main contractor (she). The main contractor in turn contracts with a subcontractor (he) for a portion of the work. The main contractor and subcontractor independently exert effort on their portion of the project.

The project quality is affected by the contractors' levels of effort and is subject to uncertainty. The end user cares about the quality of the project, which can be measured objectively. We define the quality of the work undertaken by the main contractor and subcontractor as q_m and q_s, respectively. Further, we assume that the project quality is the sum of the two individual qualities, that is, $q_0 = q_m + q_s$ (see e.g. Chao et al. (2009), Chen et al. (2021), Hu and Wang (2021)).

3.1. Model Framework

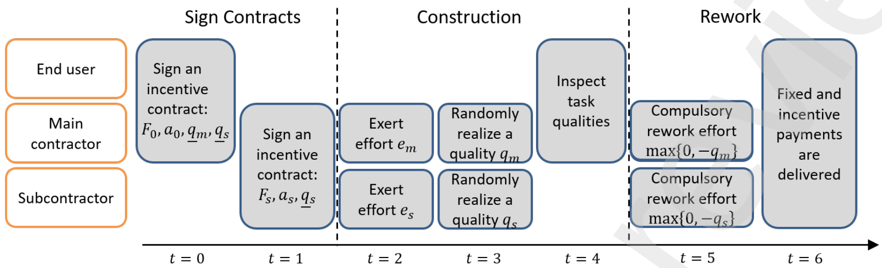


Figure 1: Sequence of events in engineering-procurement-construction

We explain the sequence of events based on the timeline in Figure 1. First, the end user signs an EPC contract with the main contractor at $t = 0$. The contract specifies a fixed fee F_0; the two minimum requirements for the main contractor's and subcontractor's quality, $\underline{q}_m$ and $\underline{q}_s$, respectively, which are standardized to zero; and a variable payment term a_0 rewarding total project quality outcome above the minimum requirement, that is, $\max\{q_m+q_s, 0\}$. Thus, the end user sets a hard constraint for individual quality, which cannot fall below the individual minimum requirement, and offers soft encouragement for total quality through incentive payment. The main contractor accepts the contract as long as it satisfies her minimum reservation utility.

At $t = 1$, the main contractor contracts with the subcontractor for a portion of the work and offers him a contract specifying a fixed fee F_s and a variable component a_s proportional to the quality above the minimum quality level $\underline{q}_s$. The subcontractor accepts the contract if it satisfies his individual rationality (IR) constraint. We assume a subcontractor with unlimited liability first (we discuss limited liability in Section 7).

Next, at $t = 2$, the main contractor and subcontractor set their effort levels e_m and e_s with costs ce_m and ce_s, respectively. The quality outcomes of the main contractor's and subcontractor's effort are subject to a negative random shock $\epsilon_i \sim U[0, 1]$. Their resulting individual quality is $q_i = e_i - \epsilon_i, i \in \{m, s\}$, respectively, and is realized at $t = 3$. The end user, and the main contractor, or a jointly appointed third-party inspector, verifies the individual qualities at $t = 4$. If either quality (q_m, q_s) is below the minimum required quality $(\underline{q}_m, \underline{q}_s)$, the contractor responsible is contractually obliged to perform rework at a unit cost r in $t = 5$. When rework is undertaken by the subcontractor, the rework effort cost is shared between the subcontractor and main contractor, in the proportions of $b \in [0, 1]$ and $1 - b$, respectively. This cost sharing reflects the fact that when the subcontractor is requested to undertake rework, the main contractor may need to assist with the actual rework and/or monitor the subcontractor's effort to avoid further quality problems. Thus, shared rework creates a negative externality for the main contractor, the severity of which is measured by $1-b$. The rework effort is guaranteed to achieve the required minimum quality $(\underline{q}_m, \underline{q}_s)$. At $t=6$, payments are made from the end user to the main contractor and from the main contractor to the subcontractor, including the fixed and incentive payments. The fixed payments F_0 and F_s are paid as long as the minimum quality requirements are achieved upon initial inspection or after rework; meanwhile, the incentive payments are based on the qualities observed during inspection at $t=4$ before rework. The main contractor's incentive payment is $a_0 \max\{q_m+q_s, 0\}$, and the subcontractor's is $a_s \max\{q_s, 0\}$.

Our model includes two payment terms, both of which are made after project completion. This is a simplification from the real-world scenario wherein payments may be made in several installments upon completion of various project milestones. As these intermediate milestone payments are conditional on a successful project review, our abstract model captures the relevant key features of EPC projects while remaining tractable.

We summarize the notation in Table 1. The end user has two decision variables: fixed fee F_0 and incentive unit pay a_0. The main contractor has three decision variables: fixed fee F_s, incentive unit pay a_s, and her own effort level $e_m \in [0,1]$. The subcontractor only determines his effort level $e_s \in [0,1]$. The effort costs are linear in effort. We assume that the unit effort cost c is the same for both the main contractor and subcontractor.

Decision Variables		Parameters	
F_0	main contractor's fixed fee	c	effort cost
a_0	main contractor's unit incentive payment	ϵ_i	random quality shock, $i \in \{m,s\}, \epsilon_i \sim U[0,1]$
F_s	subcontractor's fixed fee	r	rework cost, $r \geq c$
a_s	subcontractor's unit incentive payment	b	subcontractor's share of rework cost
e_i	effort level, $i \in \{m,s\}, e_i \in [0,1]$	q_i	quality level, $i \in \{0,m,s\}$.

Table 1: Notation

The unit rework cost r is the same for the main contractor and subcontractor, with $r \geq c$. This ensures that rework is more expensive than ex-ante work, which prevents the main contractor and subcontractor from shirking at time $t=2$ with the expectation of completing the work at a lower cost at time $t=5$. Finally, a share of the rework cost borne by the subcontractor (resp., main contractor), that is, parameter b (resp., $1-b$) is assumed to be exogenous. The case of an endogenous sharing parameter is analyzed in Section 7.

The contracting model outlined above differs from the moral hazard problem in the existing literature, because it captures the following important features of EPC. First, the model has a nested structure, where the subcontractor's work is contracted for by the main contractor who in turn contracts with the end user. Second, the main contractor may bear a part of the rework cost arising from the subcontractor's quality lapse.

As the end user cares about the trade-off between cost and quality, the following is a natural question: What is the minimum expected cost needed for any given expected project quality? We find the efficient frontier that characterizes the cost–quality trade-off that can be achieved by the end user to use as a benchmark to the EPC contracts.

3.2. First-Best Effort and Efficient Frontier

To obtain the efficient frontier, the central planner chooses the effort levels of the contractors to minimize the end user's costs consisting of the effort and rework costs for a given expected project quality $\tilde{q} \geq 0$. Note that, as quality is stochastic, the central planner can consider only expected quality and not realized quality. We find the efficient frontier by solving the end user's optimization problem for each quality level $0 \leq \tilde{q} \leq 1$.

$$\begin{aligned} \min_{0 \leq e_m, e_s \leq 1} \quad & \mathrm{E}[C] = \mathrm{E}_{\epsilon_m, \epsilon_s}\left[c(e_m + e_s) + r\left((\epsilon_m - e_m)^+ + (\epsilon_s - e_s)^+\right)\right], \\ \text{s.t.} \quad & \mathrm{E}[q] = \mathrm{E}_{\epsilon_m, \epsilon_s}\left[(e_m - \epsilon_m)^+ + (e_s - \epsilon_s)^+\right] \geq \tilde{q}, \end{aligned}$$

where $x^+ \equiv \max(x, 0)$. In the objective function, $(\epsilon_m - e_m)^+$ represents the main contractor's quality shortfall that necessitates rework, and $(\epsilon_s - e_s)^+$ represents the subcontractor's quality shortfall. The following lemma presents the optimal effort levels, expected cost, and expected quality as functions of $\tilde{q}$.

Lemma 3.1 (Efficient Frontier). *Under centralized planning, the main contractor and subcontractor exert symmetric efforts* $e_m = e_s = \max\left\{\frac{r-c}{r}, \sqrt{\tilde{q}}\right\}$.

The efficient frontier is characterized by

$$\begin{aligned} \mathrm{E}[q]^{EF} &= \max\left\{\left(\frac{r-c}{r}\right)^2, \tilde{q}\right\}, \\ \mathrm{E}[C]^{EF} &= 2c\sqrt{\mathrm{E}[q]^{EF}} + r\left(1 - \sqrt{\mathrm{E}[q]^{EF}}\right)^2. \end{aligned}$$

All proofs are in the Appendix. Figure 2 plots the efficient frontier illustrating the first-best cost–quality performance trade-off.

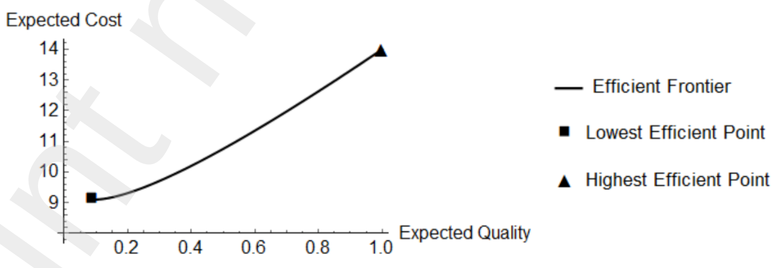


Figure 2: Efficient frontier: Quality–cost trade-off (for $c = 7, r = 10$)

As rework is automatically performed if the quality at $t = 3$ is negative, the lowest efficient expected quality is strictly positive ($\mathrm{E}[q]^{EF} \geq \left(\frac{r-c}{r}\right)^2$), even though the minimum quality requirement is standardized to zero. At the lowest efficient point, the expected quality balances the cost of effort with the cost of mandatory rework. Achieving a lower level of expected quality would increase the total cost, as the decrease in effort cost would be more than offset by the higher rework cost. Thus, any expected quality below this lowest quality is not a part of the efficient frontier.

In practice, the end user cannot set the main contractor's and subcontractor's effort levels directly; rather, the contractors set the effort levels that maximize their own profit given the contract terms offered by the end user and main contractor. We investigate three contract combinations, defined by the type of contract offered from the end user to the main contractor and from the main contractor to the subcontractor. First, we study the case in which both the end user and main contractor offer fixed-fee contracts, as suggested in the FIDIC Silver Book (FF contract). We will later examine contracts with incentive payments.

3.3. Fixed-Fee Contracts: FF Contract Combination

In the fixed-fee contract combinations, the end user and main contractor both employ a lump-sum contract, as described in the FIDIC Silver Book for EPC projects. There are no incentive payments, and all contract payments, from the end user to the main contractor and from the main contractor to the subcontractor, are in the form of a fixed fee payable upon achieving the minimum required quality.

As this is a sequential principal–agent game, we solve the game by backward induction. We start with the subcontractor's effort decision, then consider the main contractor's joint decision on her effort level and contract terms to the subcontractor, and lastly determine the end user's optimal fixed fee to the main contractor.

Contractors' Efforts and Contracting Problems: The subcontractor chooses his effort level to maximize his own profit, consisting of the fixed fee offered by the main contractor minus his effort cost and his share of the expected rework cost.

$$\max_{0\le e_s\le 1} \Omega_s^F(e_s) = \mathrm{E}_{\epsilon_s}\left[F_s - ce_s - br(\epsilon_s - e_s)^+\right]. \tag{1}$$

The main contractor makes her effort decision and designs the contract to maximize her expected profit taking into account the subcontractor's incentive compatibility (IC) and individual rationality (IR) constraints. The main contractor's profit expression consists of revenue from the end user minus her own effort cost and rework cost, minus her payment to the subcontractor and her share in the subcontractor's rework cost.

$$\begin{aligned} \max_{F_s\ge 0, 0\le e_m\le 1} \quad \Omega_m^{FF}(F_s, e_m) \quad &= \mathrm{E}_{\epsilon_m,\epsilon_s}\left[F_0 - ce_m - r(\epsilon_m - e_m)^+ - F_s - (1-b)r(\epsilon_s - e_s^*)^+\right], && (2)\\ \text{s.t.} \quad & \text{IC: } e_s^* = \arg\max_{0\le e_s\le 1} \Omega_s^F(e_s), && (3)\\ & \text{IR: } \Omega_s^F(e_s^*) \ge 0. && (4) \end{aligned}$$

The IC constraint ensures that the subcontractor chooses his effort to maximize his own profit, and the IR constraint guarantees that the subcontractor's reservation utility (normalized to zero) is met. The optimal efforts are characterized below.

Lemma 3.2. *The main contractor sets an effort level $e_m^{FF} = \frac{r-c}{r}$, while the subcontractor's optimal effort level is $e_s^{FF} = \frac{(br-c)^+}{br}$. The fixed fee $F_s^{FF} = ce_s^{FF} + \frac{br}{2}(e_s^{FF}-1)^2$ is set to ensure that the subcontractor's IR constraint is binding.*

Lemma 3.2 provides the main contractor's optimal fixed fee and effort level and the subcontractor's optimal effort level. In the FF contract combination, the main contractor's effort balances her own work and rework cost; meanwhile, the subcontractor balances his work and *perceived* rework cost, that is, his share of the rework cost br. Hence, the subcontractor never exerts the same effort as the main contractor unless the main contractor does not share the rework cost ($b = 1$) or the effort cost c is zero. In fact, the subcontractor shirks altogether when $c > br$: because a fixed payment term does not encourage effort, the only incentive to exert effort comes from the desire to avoid the rework cost. This incentive disappears when the subcontractor's perceived rework cost is lower than the cost of effort. Note that the fixed fee cannot affect the effort, yet the project quality never falls below the minimum quality requirement because of the compulsory rework.

End User's Contracting Decision: The end user wants a project that meets the minimum quality requirement at minimal cost. The end user minimizes the expected cost, that is, the fixed fee F_0, while accounting for the main contractor's IC and IR constraints.

$$\min_{F_0 \geq 0} C = F_0, \tag{5}$$

$$\text{s.t.} \quad \text{IC: } \{F_s^*, e_m^*\} = \underset{F_s \geq 0, 0 \leq e_m \leq 1}{\arg\max} \; \Omega_m^{FF}(F_s, e_m), \tag{6}$$

$$\text{IR: } \Omega_m^{FF}(F_s^*, e_m^*) \geq 0, \tag{7}$$

The IC constraint ensures that the main contractor's effort and fixed-fee decisions maximize her profit, and the IR constraint ensures the main contractor's participation. Thus, the end user pays the optimal fixed fee that incurs expected cost C^{FF} and achieves expected quality $\mathrm{E}[q]^{FF}$:

$$C^{FF} = c\left(\frac{r-c}{r} + \frac{(br-c)^+}{br}\right) + \frac{r}{2}\left[\left(\frac{c}{r}\right)^2 + \left(\frac{(br-c)^+}{br} - 1\right)^2\right], \tag{8}$$

$$\mathrm{E}[q]^{FF} = \frac{1}{2}\left(\frac{r-c}{r}\right)^2 + \frac{1}{2}\left(\frac{(br-c)^+}{br}\right)^2. \tag{9}$$

Unsurprisingly, the presence of moral hazard implies that when both the end user and main contractor offer fixed-fee contracts, the end user achieves a lower expected quality than the lowest quality on the efficient frontier ($\mathrm{E}[q]^{FF} < \min \mathrm{E}[q]^{EF}$) and yet pays a higher cost. This is illustrated in Figure 3, which displays the FF contract combination and the efficient frontier on the same graph. Thus, the end user seems remarkably ill-served by the fixed-fee contract structure recommended for EPC contracts.

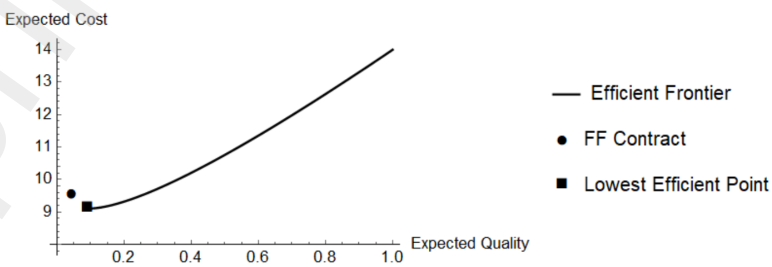


Figure 3: Fixed- fee contract combination: End user's cost and quality (for $c = 7, b = 2/3, r = 10$)

Whether incentive contracts can improve on the performance of traditional EPC contracts, and by how much, is an unanswered question; we explore this in the next two sections. An incentive contract consists of a fixed fee and an incentive payment linked to the quality outcome. First, we consider the case in which the end user continues to offer a fixed-fee contract, while the main contractor switches to an incentive contract—the fixed-incentive contract combination (FI contract). Thereafter, we investigate the incentive-incentive contract combination in which both the end user and main contractor offer an incentive contract (II contract).

4. Fixed-Incentive Contract Combination

In this section, we consider the optimal project execution and the end user's expected cost and quality under the FI contract combination. As in the FF contract combination, we solve the problem backwards, from the subcontractor's decision to the main contractor's problem, and finally to the end user's problem.

4.1. Subcontractor's Problem

In the FI contract combination, the subcontractor chooses his effort level to optimize his expected profit, which consists of the revenue from the main contractor—fixed fee F_s and incentive payment a_s—minus his effort cost and his share of the rework cost. Thus, unlike in the fixed-fee benchmark, the subcontractor's revenue—and not only his cost—depends on his effort.

$$\max_{0 \le e_s \le 1} \quad \Omega_s^I(e_s) \quad = \mathrm{E}_{\epsilon_s}\left[F_s + a_s(e_s - \epsilon_s)^+ - ce_s - br(\epsilon_s - e_s)^+\right].$$

Lemma 4.1. *The subcontractor's optimal effort level is given by the following function:*

$$e_s^* = \begin{cases} 1, & \text{if } a_s \ge \max\{c, 2c - br\}; \\ \frac{br-c}{br-a_s}, & \text{if } a_s \le c \le br; \\ 0 & \text{if } a_s \le 2c - br \text{ and } c \ge br. \end{cases} \tag{10}$$

We find that the subcontractor's optimal effort balances the expected reward a_s with the cost of effort c and his perceived rework cost br. Higher effort is costlier, but increases the expected incentive payment and reduces the expected rework cost, and vice versa. Figure 4 shows the subcontractor's optimal effort level as a function of the main contractor's unit incentive payment a_s and the effort cost c (for a given rework cost $r = 10$ and sharing parameter $b = 2/3$).

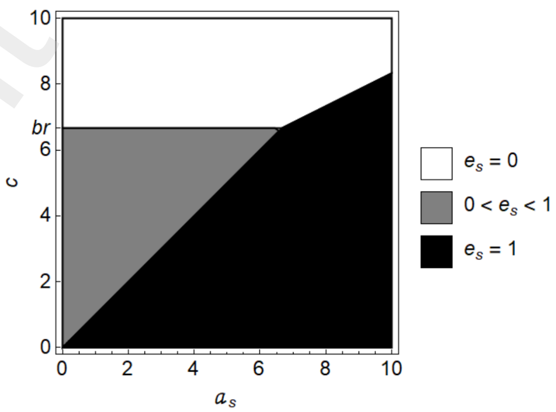


Figure 4: Subcontractor 's optimal effort level (for $b = 2/3, r = 10$)

Corner solutions occur when either no effort or maximum effort is optimal. Taking the case of maximum effort, it is clearly only optimal if the incentive payment is large enough, or

$a_s \geq \max\{c, 2c - br\}$. The case of no effort occurs only if the rework cost—as perceived by the subcontractor—is lower than the effort cost ($br \leq c$). Furthermore, interior efforts (i.e., $0 < e_s < 1$) cannot be achieved when the effort cost is higher than the perceived rework cost. Either the incentive fee is large enough to induce maximum effort by the subcontractor, or low enough for them to make no effort at all. If the effort cost is less than the perceived rework cost, an interior solution is optimal whenever $a_s \leq c \leq br$. The subcontractor chooses an effort level that optimally balances the rework cost and the reward of incentive payment with the cost of effort. This effort level is always higher than the effort level the subcontractor would exert under a fixed-fee contract.

4.2. Main Contractor's Problem

The main contractor sets the subcontractor's contract terms, fixed fee F_s, and incentive payment a_s, as well as her own effort level e_m to maximize her expected profit.

$$\max_{F_s, a_s \geq 0; 0 \leq e_m \leq 1} \Omega_m^{FI}(F_s, a_s, e_m) = \mathrm{E}_{\epsilon_m, \epsilon_s} \begin{bmatrix} F_0 - ce_m - r(\epsilon_m - e_m)^+ \\ -(F_s + a_s(e_s^* - \epsilon_s)^+) - (1-b)r(\epsilon_s - e_s^*)^+ \end{bmatrix}, \quad (11)$$

$$\text{s.t.} \quad \text{IC: } e_s^* = \arg\max_{0 \leq e_s \leq 1} \Omega_s^I(e_s), \quad (12)$$

$$\text{IR: } \Omega_s^I(e_s^*) \geq 0. \quad (13)$$

The optimal decisions of the main contractor are as follows.

Lemma 4.2. *The main contractor sets her effort level $e_m^{FI} = \frac{r-c}{r}$ and the optimal incentive payment a_s^{FI} and induced effort level e_s^{FI} as follows:*

$$a_s^{FI} = \begin{cases} \frac{(1-b)rc}{r-c}, & \text{if } c \leq br; \\ 2c - br, & \text{if } br < c \leq \max\{br, \frac{r}{2}\}; \\ 0, & \text{if } c > \max\{br, \frac{r}{2}\}, \end{cases}$$

$$e_s^{FI} = \begin{cases} \frac{r-c}{r}, & \text{if } c \leq br; \\ 1, & \text{if } br < c \leq \max\{br, \frac{r}{2}\}; \\ 0, & \text{if } c > \max\{br, \frac{r}{2}\}. \end{cases}$$

The fixed fee $F_s^{FI} = ce_s^{FI} - \frac{a_s^{FI}}{2}(e_s^{FI})^2 + \frac{br}{2}(e_s^{FI} - 1)^2$ makes the subcontractor's IR constraint binding.

Lemma 4.2 provides the main contractor's optimal contract terms and effort level, as well as the subcontractor's induced effort level. The main contractor's effort level is identical under the FI and FF contract combinations, because the end user does not use an incentive payment in either contract type. Nevertheless, the main contractor may prefer to offer a strictly positive incentive a_s to the subcontractor, despite receiving a fixed fee from the end user. The main contractor uses the incentive payment to induce the subcontractor to exert higher effort, which reduces her expected share of the rework cost arising from the subcontractor's quality lapses. Whenever the effort cost is less than the perceived rework cost, the main contractor is able to induce the subcontractor to exert the first-best efficient effort. However, if the effort cost is larger than the perceived rework cost, the main contractor must choose between inducing the subcontractor to exert either no effort or maximum effort. In that case, the main contractor chooses to incentivize maximum effort if the cost of effort is less than half the rework cost, and zero effort otherwise. In summary, the main contractor's effort always equals the effort under the lowest efficient point in Lemma 3.1; meanwhile, the subcontractor's effort may be the same as, higher than, or lower than the effort at the lowest efficient point, depending on the effort cost and sharing parameter (see Figure 5).

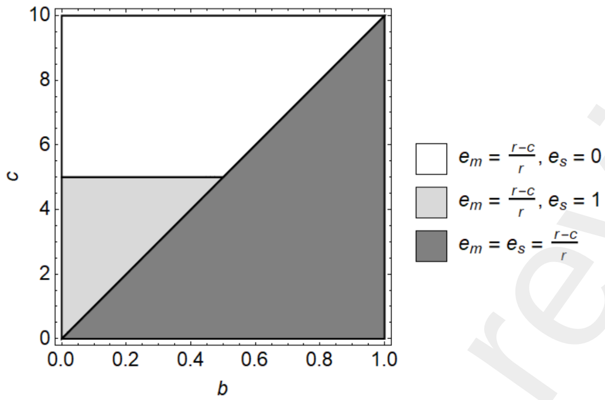


Figure 5: Fixed -incentive contract combination: Optimal effort levels

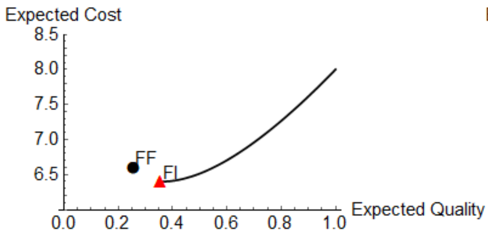
4.3. End User's Contract Decision

The end user's problem under the FI contract combination is similar to that under the FF contract combination in Subsection 3.3, with modifications to the main contractor's revenue expression in the IC and IR constraints to account for the incentive contract to the subcontractor. Solving the end user's optimization problem, we obtain the end user's optimal cost and optimal expected quality as follows:

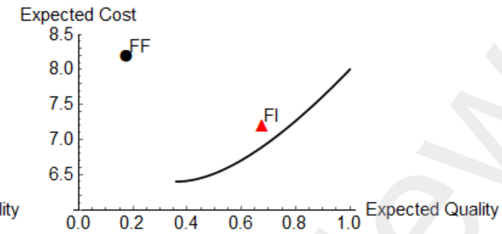
$$C^{FI} = \begin{cases} \frac{c(2r-c)}{r}, & \text{if } c \leq br; \\ \frac{c(2r-\frac{c}{2})}{r}, & \text{if } br < c \leq \max\{br, \frac{r}{2}\}; \\ c + \frac{r^2-c^2}{2r}, & \text{if } c > \max\{br, \frac{r}{2}\}, \end{cases} \tag{14}$$

$$\text{E}[q]^{FI} = \begin{cases} (\frac{r-c}{r})^2, & \text{if } c \leq br; \\ \frac{1}{2}(\frac{r-c}{r})^2 + \frac{1}{2}, & \text{if } br < c \leq \max\{br, \frac{r}{2}\}; \\ \frac{1}{2}(\frac{r-c}{r})^2, & \text{if } c > \max\{br, \frac{r}{2}\}. \end{cases} \tag{15}$$

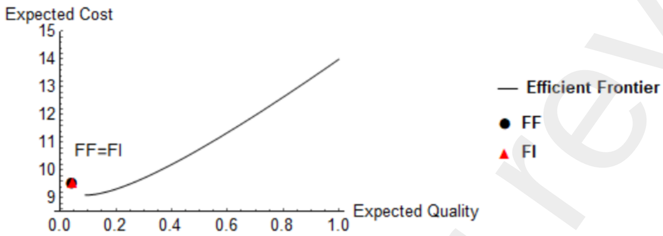
Comparing the project's cost and expected quality under the FI contract combination (Equations (14) and (15)) and the FF benchmark (Equations (8) and (9)), the end user achieves a weakly higher expected quality at a weakly lower cost under the FI contract combination. The performance improvement in the FF contract combination depends on the project parameters. The lowest efficient point can be achieved whenever the subcontractor bears a large enough share of the rework cost, that is, whenever the effort cost is lower than the perceived rework cost (see Figure 6(a)). If the perceived rework cost is lower than the effort cost, the end user may still achieve an improvement in both quality and cost compared to the FF contract combination if the cost of effort is less than half the rework cost. However, the project execution does not lie on the efficient frontier (Figure 6(b)). Finally, when the cost of effort exceeds the perceived rework cost and half the rework cost, the FI contract combination is equivalent to the FF contract combination and does not lie on the efficient frontier (Figure 6(c)).



(a) Case I: $c \leq br$ $(c = 4, b = 2/3)$



(b) Case II: $br < c \leq \max\{br, \frac{r}{2}\}$ $(c = 4, b = 1/3)$



(c) Case III: $c > \max\{br, \frac{r}{2}\}$ $(c = 7, b = 2/3)$

Figure 6: Fixed-incentive contract combination: End user's cost and expected quality

5. Incentive Contracts: II Contract Combination

In the II contract combination, both the end user *and* main contractor offer incentive contracts. Using backward induction, we show how the main contractor's and subcontractor's effort choices and the main contractor's optimal contract terms depend on the end user's contract terms. We exclude the subcontractor's problem, as it is the same as in Subsection 4.1. Finally, we determine the end user's range of optimal fixed and incentive payments for varying expected quality levels.

5.1. Main Contractor's Effort and Contracting Decision

The main contractor maximizes her expected profit, while considering the subcontractor's IC and IR constraints. The main contractor's profit consists of the fixed fee F_0 and the incentive payment a_0 linked to the realization of the project's quality. The main contractor's optimization problem becomes

$$\max_{F_s, a_s \geq 0; 0 \leq e_m \leq 1} \Omega_m^{II}(F_s, a_s, e_m) = \mathrm{E}_{\epsilon_m, \epsilon_s} \left[\begin{array}{l} F_0 + a_0(e_m + e_s^* - \epsilon_s - \epsilon_m)^+ - ce_m - r(\epsilon_m - e_m)^+ \\ -(F_s + a_s(e_s^* - \epsilon_s)^+) - (1-b)r(\epsilon_s - e_s^*)^+ \end{array} \right] \quad (16)$$

$$\text{s.t.} \quad \text{IC: } e_s^* = \arg\max_{0 \leq e_s \leq 1} \Omega_s^I(e_s), \quad (17)$$

$$\text{IR: } \Omega_s^I(e_s^*) \geq 0. \quad (18)$$

The main contractor's optimal decisions and the subcontractor's induced effort level are characterized as Proposition 5.1.

Proposition 5.1. *The main contractor sets the optimal incentive payment a_s^{II} and the optimal effort level e_m^{II}, which induces effort level e_s^{II} for different parameter values, as follows:*

1. $c \leq br$

   (a) For $a_0 \geq c$: $a_s^{II} = c$ and $e_m^{II} = e_s^{II} = 1$.

   (b) For $2c - r \leq a_0 < c$: $a_s^{II} = br - \frac{4a_0(br-c)}{4a_0 - r + \sqrt{r^2 - 8a_0(c-a_0)}}$ and $e_m^{II} = e_s^{II} = 1 - \frac{r - \sqrt{r^2 - 8a_0(c-a_0)}}{4a_0} \geq 0.5$.

   (c) For $a_0 < 2c - r$: $a_s^{II} = br - \frac{4a_0(br-c)}{r - \sqrt{r^2 - 8a_0(r-c)}}$ and $e_m^{II} = e_s^{II} = \frac{r - \sqrt{r^2 - 8a_0(r-c)}}{4a_0} < 0.5$.

2. $c > br$

   (a) For $a_0 \geq c$: $a_s^{II} = 2c - br$ and $e_m^{II} = e_s^{II} = 1$.

   (b) For $2c - r \leq a_0 \leq c$: $a_s^{II} = 2c - br > c$ and $e_m^{II} = 1 - \frac{r - \sqrt{r^2 - 2a_0(c-a_0)}}{a_0}, e_s^{II} = 1$.

   (c) For $a_0 \leq 2c - r$: $a_s^{II} = 0$ and $e_m^{II} = \frac{r - \sqrt{r^2 - 2a_0(r-c)}}{a_0}, e_s^{II} = 0$.

The fixed fee F_s^{II} is set such that the subcontractor's IR constraint of Equation (18) *is binding.*

Figure 7 shows the optimal project execution under the II contract combination as described in Proposition 5.1. We divide the graph into two parts to discuss the results for $c \leq br$ and $c > br$ separately.

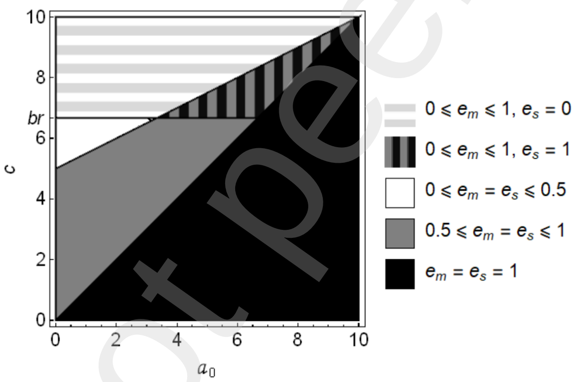


Figure 7: II contract combination: Optimal project execution (for $b = 2/3, r = 10$)

For $c \leq br$, that is, when the subcontractor's unit effort cost is less than his perceived share of the unit rework cost, the main contractor and subcontractor always execute identical non-zero efforts. When the end user's unit incentive payment is large, $a_0 \geq c$, the main contractor gives the subcontractor a strong incentive $a_0 \geq a_s = c$ covering the unit effort cost, and thus, both of them exert maximum effort levels. When the end user's incentive payment is smaller, $a_0 < c$, the main contractor pays the subcontractor less than the unit effort cost as an incentive payment, which results in lower yet identical effort levels. Given that the main contractor and subcontractor perceive a different rework cost, identical effort can be achieved only by distorting the incentive payment to the subcontractor upwards, that is, $a_0 \leq a_s \leq c$.

For $c > br$, that is, when the subcontractor's unit effort cost is larger than his perceived share of the unit rework cost, the main contractor and subcontractor do not always execute the same effort levels. In Subsection 4.1, we show that when $c > br$, the subcontractor chooses only maximum or zero effort, and the main contractor cannot incentivize the subcontractor's interior levels of effort. Thus, the main contractor either chooses a high incentive payment or no incentive payment; she selects the option that gives her a higher revenue. When the end user's incentive is large enough, or $a_0 \geq 2c - r$, the main contractor gives a high incentive payment $a_s = 2c - br$ to induce the subcontractor's maximum effort. The main contractor, however, may or may not choose to exert maximum effort, depending on whether the end user's incentive payment is larger than the unit effort cost. When $a_0 < 2c - r$, the main contractor gives zero incentive $a_s = 0$, and thus, the subcontractor makes no effort, while the main contractor maintains a strictly positive effort. In summary, the main contractor and subcontractor exert asymmetrical efforts, with either one of them exerting more effort than the other depending on the problem parameters.

The main contractor always sets the minimal a_s needed to induce the desired effort level from the subcontractor. The optimal incentive payment is weakly decreasing in b, the portion of the rework cost borne by the subcontractor, because the rework cost and incentive payment are substitute mechanisms to encourage the subcontractor to exert effort. If the perceived rework cost is higher, the subcontractor is more motivated to exert high effort even in the absence of an incentive payment. The incentive payment is independent of a_0 when a corner solution is optimal, and is increasing in a_0 when an interior solution is optimal. When an interior solution is optimal, the main contractor offers the subcontractor a higher incentive payment than what she received from the end user ($a_s \geq a_0$), to mitigate the distortion arising from the subcontractor's lower perceived rework cost. When a corner solution is optimal, however, the optimal incentive payment to the subcontractor could be either more or less than the incentive payment the main contractor received from the end user.

5.2. End User's Contracting Decision

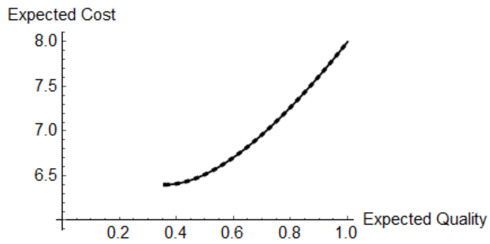
In the II contract combination, the end user offers a fixed fee F_0 and an incentive payment a_0 to minimize its expected cost while meeting the main contractor's IC and IR constraints. The end user's optimization problem for a given a_0 is

$$\begin{aligned}
\min_{F_0 \geq 0} C(F_0, a_0) \quad &= \quad F_0 + \mathrm{E}_{\epsilon_m, \epsilon_s}\left[a_0(e_m^* + e_s^* - \epsilon_s - \epsilon_m)^+\right], \\
\text{s.t.} \quad & \quad \text{IC: } \{F_s^*, a_s^*, e_m^*\} = \underset{F_s, a_s \geq 0; 0 \leq e_m \leq 1}{\arg\max} \Omega_m(F_s, a_s, e_m), \\
& \quad \text{IR: } \Omega_m(F_s^*, a_s^*, e_m^*) \geq 0.
\end{aligned}$$

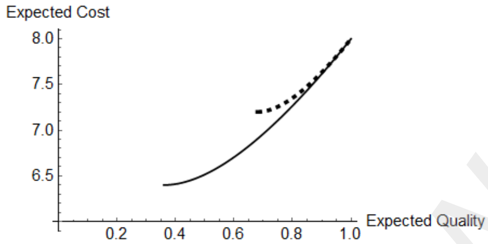
The end user's cost is affected by the effort e_m^* and e_s^*, which are determined by the main contractor and subcontractor, respectively, according to Proposition 5.1. Since both efforts are independent of F_0, we can make the IR constraint binding to obtain the optimal F_0 for a given value of a_0, as follows:

$$F_0 = -\mathrm{E}_{\epsilon_m, \epsilon_s}\left[a_0(e_m^* + e_s^* - \epsilon_s - \epsilon_m)^+\right] + c\,(e_m^* + e_s^*) + \frac{r}{2}(e_m^* - 1)^2 + \frac{r}{2}(e_s^* - 1)^2. \qquad (19)$$

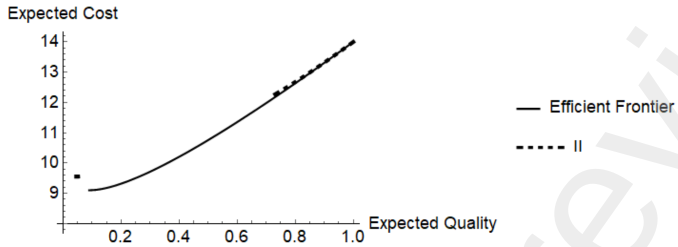
By varying the incentive payment a_0, the end user can induce different effort levels and achieve different expected project qualities. We explore the full range of incentive payment, that is, $a_0 \in [0, c]$, and plot the expected project cost as a function of the expected project quality in



(a) Case I: $c \leq br$ ($c = 4, b = 2/3$)



(b) Case II: $br < c \leq \max\{br, \frac{r}{2}\}$ ($c = 4, b = 1/3$)



(c) Case III: $c > \max\{br, \frac{r}{2}\}$ ($c = 7, b = 2/3$)

Figure 8: II contract combination: End user's expected cost and quality (for $r = 10$)

Figure 8. By setting the appropriate incentive payment a_0, the end user can opt for a quality–cost trade-off if it prefers. In panel (a), we observe that the end user can achieve the first-best efficient frontier if the subcontractor's perceived rework cost exceeds the effort cost ($br > c$), but not otherwise (panels (b) and (c)). Nevertheless, tailoring an incentive contract always enables the end user to induce a variety of different quality and cost outcomes, giving the end user more control over project execution. We compare all the contract combinations in the next section.

6. Contract Comparison

In this section, we first compare the project execution and each party's profit across the different contracts and quantify the value-added by using an incentive payment in EPC contracts. Then, we discuss the impact on quality and cost risk from the end user's perspective.

6.1. Project Execution: The (In)efficient Frontier

We compare the cost and quality outcomes under the FF, FI, and II contract combinations with the efficient frontier. The differences among these cases help us understand when the incentive contracts can effectively alleviate the moral hazard problem.

Proposition 6.1. 1. *The FF contract combination reaches the lowest efficient point if and only if (iff) $b = 1$.*

2. *The FI contract combination reaches the lowest efficient point iff $\frac{c}{r} \leq b \leq 1$.*
3. *The II contract combination reaches any point on the efficient frontier iff $\frac{c}{r} \leq b \leq 1$. Furthermore, it always reaches the highest efficient point iff $a_0 \geq c$.*

Proposition 6.1 explores each contract combination in turn. All the three contract combinations have the following in common: when $b = 1$, the efficient frontier can be reached. Indeed, when the subcontractor is fully accountable for the rework cost, there is neither a distortion of his incentives nor a negative externality for the main contractor. Thus, the principal can shift the risk to a risk-neutral agent without suffering from moral hazard. This extends the classical result found in economic theory (Baiman et al., 2000; Balachandran and Radhakrishnan, 2005; Laffont and Martimort, 2009) to the nested structure of EPC contracts.

Proposition 6.1 also explains what happens in the presence of a negative externality (i.e., for $b < 1$). The externality means that the principal might no longer be able to achieve the efficient frontier with all contract combinations. If the main contractor does not sign an incentive contract with her subcontractor, that is, the FF contract combination, the efficient frontier can no longer be achieved. However, as long as at least one contract in the nested relationship is an incentive contract, it may be possible to reach the efficient frontier as long as the externality is not too large, that is, $1-b \leq \frac{r-c}{r}$. While the FI contract combination achieves the lowest efficient point only, the II contract combination can achieve the entire efficient frontier. Finally, for the II contract combination, a sufficiently large incentive from the end user, or $a_0 \geq c$, eliminates moral hazard completely by incentivizing maximum effort for both contractors, thereby reaching the highest point on the efficient frontier.

In all other cases (i.e., $b < \frac{c}{r}$ and $a_0 < c$), none of the contract combinations can achieve the efficient frontier, which means that the end user obtains a lower expected quality and/or pays a higher cost than under centralized planning. Thus, Proposition 6.1 establishes the following boundary condition for the ability of incentive contracts to eliminate moral hazard: The externality caused by the sharing of the rework cost between the subcontractor and main contractor must be smaller than a threshold.

6.2. Project Outcomes: Cost Risk

The discussion in the previous subsection on the ability to achieve the efficient frontier considers *expected* quality and *expected* cost to the end user. However, fixed-fee EPC contracts are advocated by the FIDIC with the objective of reducing the end user's risk exposure. Therefore, we compare the cost variability under the centralized execution on the first-best efficient frontier and the different contract combinations numerically. We use the following project parameters: effort cost $c \in \{4, 7\}$, rework cost $r = 10$, sharing parameter $b \in \{1/3, 2/3\}$, and the end user's unit incentive payment $a_0 \in [0, c]$. We use our analytical results to calculate the optimal effort levels of the two contractors, given the optimal contract terms from the main contractor to the subcontractor under all three contract combinations (FF, FI, and II). We simulate 10,000 independent and identically distributed quality shocks ϵ_m and ϵ_s from a uniform distribution defined over $[0, 1]$ to calculate the realized cost under each scenario.

We start by exploring the cost variability under centralized planning, that is, for the efficient frontier in Figure 9; we find that as the expected quality increases, the cost variability *decreases*. The end user's cost consists of two components: effort cost and rework cost. The former is deterministic and set when the contractors' effort levels are chosen by the centralized planner. The rework cost, however, is stochastic, as it depends on the outcome of the random quality shock. This feature explains the behavior of the average cost and its variability. To achieve a higher expected quality, the central planner must set a higher effort level, which comes at a higher deterministic cost of effort; this simultaneously decreases the likelihood and the amount of rework, thereby decreasing the variability of the end user's cost. Consequently, we observe that as the total expected cost (and quality) increases, the cost variability decreases. Furthermore, quality and cost realization are inversely related: Whenever the quality realization is less than the minimum quality requirement, the contractors must engage in rework to meet the quality threshold whereas high quality realizations incur zero rework cost. Accordingly, under low initial cost of effort and after accounting for the cost of rework, the end user may even pay more for the delivery of a project of minimum quality than for a project with maximum efforts (where rework does not occur). Thus, a risk-averse end user with strong aversion to cost uncertainty may prefer to set a higher effort level and pay a higher expected cost to reduce cost variability.

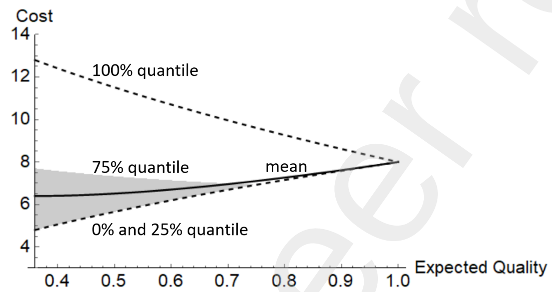
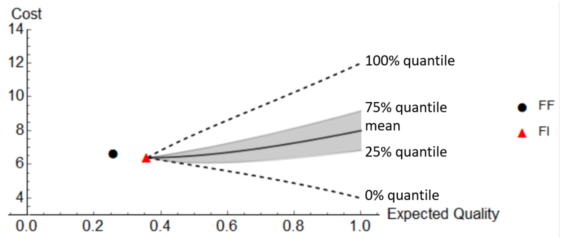
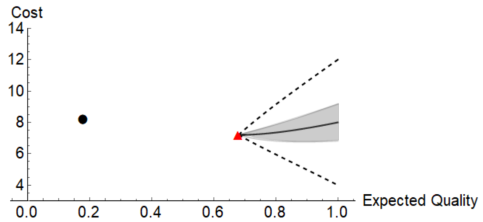


Figure 9: First-best execution: Cost variability on the efficient frontier (for $c = 4, r = 10$)

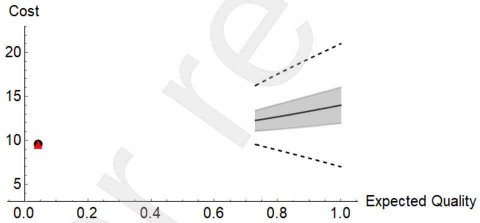
Next, we show the cost outcomes under the different contract combinations in Figure 10 and compare these with Figure 9. Under the FF and FI contract combinations, there is no cost variability for the end user. Whenever the lowest efficient point is achieved—that is, $b = 1$ for fixed contracts only or $c/r \leq b \leq 1$ for the FI contract combination—the end user's fixed fee is the lowest efficient expected cost. The II contract combination allows for a larger range of outcomes, as the end user can vary the incentive a_0 to encourage different effort levels from the contractors. We focus on panel (a) to highlight the most interesting feature of the cost variability under the II contract combination. While the II contract combination can achieve the efficient frontier whenever $c/r \leq b \leq 1$, that is, the same expected quality–cost trade-off as the central planner, the cost variability *increases* in the expected quality. This is because the end user's cost under the II contract combination consists of a fixed fee and an incentive payment, whereas the central planner's cost consists of effort cost and rework cost. Under the II contract combination, the cost variability is caused by the incentive payment, which varies with the project quality outcome. Higher levels of expected quality correspond to a higher level of quality variability, because quality shortfalls—and the compulsory rework to achieve constant, minimum quality—become less likely. This greater quality variability directly leads to higher cost variability. This contrasts with the efficient frontier at which the variability in cost outcomes is driven by the amount of rework cost. On the efficient frontier, the variability of rework cost is larger for lower quality efforts, as rework is incurred only when the quality falls below the threshold. The qualitative insights about cost variability discussed above hold for the other panels in Figure 10, except for the fact that the expected cost for a given quality under all contract combinations is larger than the expected cost on the efficient frontier.



(a) Case I: $c \leq br$ $(c = 4, b = 2/3)$



(b) Case II: $br < c \leq \max\{br, \frac{r}{2}\}$ $(c = 4, b = 1/3)$



(c) Case III: $c > \max\{br, \frac{r}{2}\}$ $(c = 7, b = 2/3)$

Figure 10: Cost variability under all contract combinations (for $r = 10$)

End users turn to EPC as a means of reducing their risk exposure; however, it is important to acknowledge that the II contract combination increases the end user's cost risk compared to the FF or FI contract combinations. Nevertheless, noteworthily, as the cost level varies commensurately with the quality outcome, the end user pays more for projects of better quality and less for projects of lower quality. This seems more desirable than the central planner's quality–cost variability, where the end user may have to pay more for lower quality outcomes than for higher quality outcomes.

7. Extension of Limited-Liability Subcontractor

Subcontractors are typically smaller firms that may have limited liability. A subcontractor with limited liability requires his *lowest* realized profit—not just his expected profit—to meet his reservation utility. We determine the optimal contract terms under the II contract combination and compare the results with the unlimited liability case. The subcontractor with limited liability is concerned about the downside of the potential rework cost; we therefore also consider the possibility that the main contractor optimizes the sharing parameter b to control the subcontractor's risk exposure. We limit our analysis to the II contract combination, as it is the best-performing contract under the unlimited liability case.

7.1. Optimal II Contract and Project Execution

The main contractor optimizes her profit by choosing her effort level e_m and the contract terms F_s and a_s for the subcontractor, taking into account the subcontractor's IC and IR constraints. Compared to the model in Equations (16)–(18), the only change can be found in the IR constraint (i.e., Equation (18)). We replace Equation (18) with the following equation, which ensures that the subcontractor's worst outcome meets his reservation utility:

$$\min_{\epsilon_s} \left\{ F_s + a_s(e_s^* - \epsilon_s)^+ - ce_s^* - br(\epsilon_s - e_s^*)^+ \right\} \geq 0. \tag{20}$$

An analysis of the subcontractor's IR constraint shows that while the incentive payment serves to increase effort—and reduces rework cost—the fixed fee must always be sufficient to compensate for the rework cost of the worst-case random shock. The resulting optimal effort levels are shown in Figure 11 as a function of the end user's incentive payment and the cost of effort.

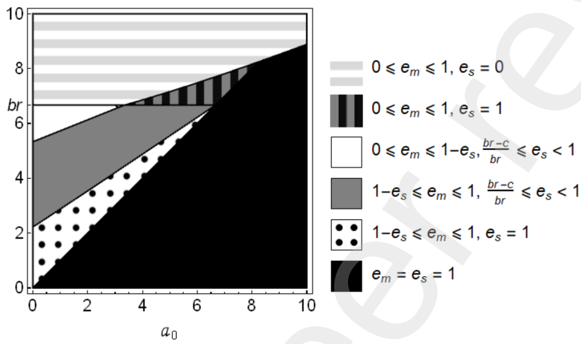


Figure 11: II contract combination: Optimal project execution with a limited-liability subcontractor (for $r = 10, b = 2/3$)

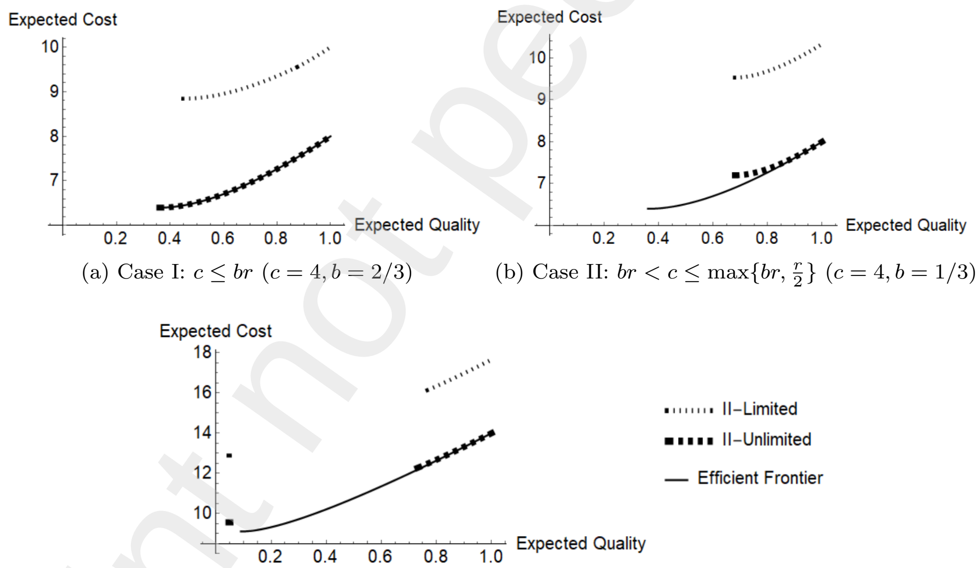
Similar to previous figures, we divide our discussion into two parts, depending on whether the cost of effort is larger than or smaller than the perceived rework cost. When the cost of effort is less than the perceived rework cost (i.e., $c \leq br$), the limited liability of the subcontractor causes the two contractors to exert asymmetric efforts, with the subcontractor making higher efforts than the main contractor (except when both contractors exert maximum efforts, i.e., when the end user's incentive is very large). The subcontractor also exerts a higher effort than under the case with unlimited liability. This over-investment occurs because it is relatively cheap to exert effort. The main contractor offers a higher incentive payment to meet the subcontractor's limited liability constraint, which increases the subcontractor's effort. This reduces the subcontractor's maximum rework cost under the worst case, which in turn allows the main contractor to set a low fixed fee. A high level of subcontractor effort increases the main contractor's expected incentive payment; this, therefore, causes the main contractor to similarly exert more effort whenever the subcontractor's effort level is higher than under the first-best effort.

When the cost of effort exceeds the perceived rework cost (i.e., $c > br$), the main contractor is unable to induce an interior effort level from the subcontractor, who either exerts full or no effort. However, limited liability now leads the subcontractor to invest weakly less than a subcontractor with unlimited liability. When the rework cost is cheap compared to the effort cost, the main contractor must pay a very high incentive to induce the subcontractor to exert any effort. However, the main contractor must offer a fixed fee that covers the maximum effort cost to meet the subcontractor's reservation utility under the worst possible quality shock. The high expected incentive payment coupled with a large fixed fee is too costly for the main contractor who, thus, prefers to induce zero effort and offer only a fixed fee covering the rework cost. As low subcontractor effort decreases the main contractor's expected incentive payment, the main contractor also exerts lower effort than the first-best effort.

The end user's optimization problem is similar to the case with unlimited liability in Subsection 5.2. We solve the end user's problem to obtain the optimal value of F_0 as a function of the incentive fee a_0.

$$F_0 = -\,\mathrm{E}_{\epsilon_m,\epsilon_s}\left[a_0(e_m + e_s - \epsilon_s - \epsilon_m)^+\right] + c\,(e_m + e_s) + \frac{r}{2}(e_m - 1)^2 + \frac{r}{2}(e_s - 1)^2 + \frac{a_s}{2}(e_s)^2 + \frac{br}{2}(1 - e_s^2). \tag{21}$$

We plot the end user's expected cost and quality trade-off in Figure 12. Under limited liability, the end user can never achieve the efficient frontier. Economic theory predicts that when an agent is risk averse, moral hazard cannot be fully alleviated. This continues to hold even if the end user does not directly contract with the risk-averse party, but its risk-neutral agent does.



(a) Case I: $c \le br$ $(c = 4, b = 2/3)$

(b) Case II: $br < c \le \max\{br, \frac{r}{2}\}$ $(c = 4, b = 1/3)$

(c) Case III: $c > \max\{br, \frac{r}{2}\}$ $(c = 7, b = 2/3)$

Figure 12: Subcontractor with limited liability: End user's expected cost and quality under the II contract combination (for $r = 10, a_0$ increasing from 0 to c)

7.2. Optimal Rework Cost Sharing

Limited liability increases the cost to the main contractor, as they must now cover the subcontractor's losses when rework is needed. However, the main contractor can also mitigate the impact of quality shocks on the subcontractor's revenue by manipulating the rework cost-sharing parameter b. We know that for a subcontractor with unlimited liability, the optimal value of the sharing parameter b is always 1, such that the subcontractor is solely responsible for his own quality shortfalls. This eliminates externalities and guarantees that the efficient frontier can be reached for all contract combinations.

If the subcontractor has limited liability, however, the optimal value of the sharing parameter b is no longer always 1 and the main contractor may prefer to share the subcontractor's rework cost. Under limited liability, the sharing parameter simultaneously acts as a disincentive to the subcontractor and a risk-allocation mechanism, and is subject to opposing forces. A higher b offers a stronger incentive to the subcontractor and encourages a higher effort by making rework more costly. This reduces the main contractor's cost burden from the subcontractor's rework cost both directly (through a lower share in the cost) and indirectly (through a lower quality shortfall). However, a higher b also decreases the net revenue the subcontractor obtains in the worst case, which increases the fixed fee the main contractor must pay to meet the subcontractor's participation constraint. The optimal sharing parameter b balances the impact on the fixed fee and the main contractor's share of the subcontractor's rework cost. We show numerically that project parameters exist, such that the optimal sharing parameter b is strictly less than 1.

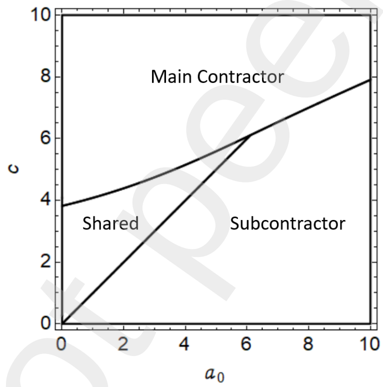
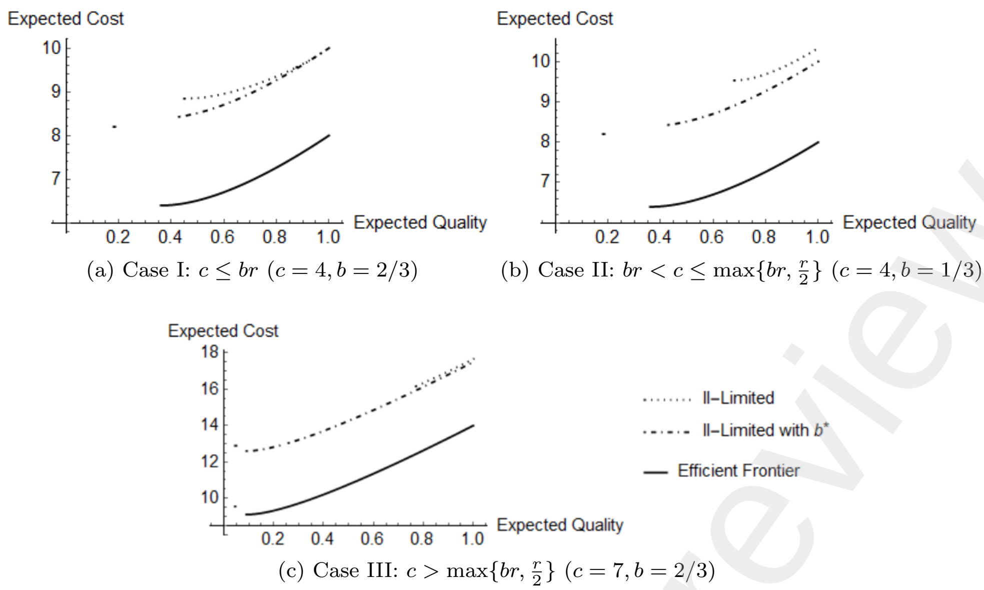


Figure 13: Optimal allocation of the subcontractor's rework cost under limited liability (for $r = 10$)

Figure 13 shows who should bear the subcontractor's rework cost as a function of the incentive a_0 and the effort cost c (for a given rework cost r) under the limited liability case. If the incentive a_0 is large enough relative to c, both contractors exert maximum effort, such that the rework cost is never incurred. Consequently, rework does not enter the subcontractor's participation constraint, and the main contractor shifts all rework costs to the subcontractor to avoid undermining the subcontractor's incentive to exert effort. Conversely, if the cost of effort is large enough—and thus, the optimal effort level is low and the rework cost is high—the main contractor prefers to bear the full rework cost ($b = 0$); this insulates the subcontractor from the rework cost and keeps the fixed fee low. For lower effort cost and lower incentive payment from the end user, the main contractor shares the rework cost to balance the incentive effect of the anticipated rework cost for the subcontractor with an increase in the fixed fee payable by the main contractor. In this case, the optimal sharing parameter of the rework cost is $b = \frac{c}{r}$, leading to symmetric contractor efforts.



(a) Case I: $c \leq br$ $(c = 4, b = 2/3)$

(b) Case II: $br < c \leq \max\{br, \frac{r}{2}\}$ $(c = 4, b = 1/3)$

(c) Case III: $c > \max\{br, \frac{r}{2}\}$ $(c = 7, b = 2/3)$

Figure 14: Subcontractor with limited liability: End user's expected cost and quality under the II contract combination with an optimal sharing parameter (for $r = 10$)

By decreasing the main contractor's cost, the optimal sharing parameter reduces the payment of the end user to the main contractor. We show the impact of the improvement in the end user's quality–cost trade-off in Figure 14. Optimizing the rework cost-sharing parameter allows the end user to obtain a larger range of expected quality levels at a lower expected cost, albeit still falling short of achieving the efficient frontier.

In conclusion, the limited liability of the subcontractor introduces inefficiency into the nested contractual relationship. Our results show that this source of inefficiency can be reduced by introducing rework cost sharing (an externality), which shifts some of the cost from the subcontractor to the main contractor. While such rework cost sharing is always detrimental when both parties are risk neutral, it can be beneficial in the case of limited liability; this is because it shifts some of the risk from the risk-averse subcontractor to the risk-neutral main contractor.

8. Managerial Implications and Conclusion

This study investigates the impact of the most popular contract form for EPC projects, the fixed-fee contract, on the end user's project cost and quality within a series of nested contractual relationships; these relationships involve an end user, a main contractor, and a representative subcontractor. We explicitly model the subcontractor's rework to remedy quality shortfalls and incorporate the main contractor's sharing of the rework cost. We analyze the impact on the end user's risk exposure and extend the model to consider a subcontractor with limited liability.

We find that the fixed-fee contract cannot achieve the first-best trade-off due to moral hazard and rework cost sharing (a negative externality). Allowing the main contractor, or even the end user, to offer an incentive contract would improve the end user's ability to achieve the cost–quality trade-off under the first-best effort, even in the presence of rework cost sharing. We find that limited liability of the subcontractor predictably increases the end user's cost, since the main contractor offers a higher fixed fee and/or incentive pay to ensure the subcontractor's participation. However, the main contractor can lessen the increase in cost by manipulating the externality and choosing to bear some—if not all—of the subcontractor's rework cost, thereby shouldering some of the subcontractor's risk. This does not need to come at the expense of quality, as limited liability by itself is an incentive for the subcontractor to increase effort level. The shared rework cost causes a negative externality to the nested structure and distorts the execution of the project involving an unlimited-liability subcontractor; however, both the end user and main contractor benefit from an optimized sharing of rework cost with a limited-liability subcontractor.

The above findings provide the following *managerial insights* on EPC contracting for both end users and main contractors. Irrespective of the type of contract offered by the end user, the main contractor should choose an incentive contract to induce the subcontractor's effort; this is because the main contractor can earn a higher profit under an incentive contract than under a fixed-fee contract. For the end user, an incentive contract is a better choice if the end user values quality highly and wants to achieve a higher level of quality than that a fixed-fee contract allows, but the fixed-fee contract is better if the end user wants to have zero cost risks.

Our study uses a stylized model to examine the cost and quality trade-offs. The model can be extended in several future research directions. One is to consider multiple subcontractors in the project outsourcing and study the interactions among more project stakeholders. The second is to model an imperfect inspection process and to evaluate its impact on the contract design and project outcomes. Last, more complex project structures, in which the tasks may be scheduled in multiple periods, are also worthy of study.

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Disclosure statement

The authors report there are no competing interests to declare.

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Appendix A. Proof

Proof of Lemma 3.1

In the efficient frontier, due to symmetric cost and uncertainty, the optimal values of effort levels e_m, e_s are symmetric. Thus, given the assumption that $\epsilon_i \sim U[0,1]$, for an effort level $e = e_m = e_s$, the expected cost and quality are:

$$\mathrm{E}[C] = 2ce + r(1-e)^2 = r\left(e - \frac{r-c}{r}\right)^2 + \frac{c(2r-c)}{r}, \mathrm{E}[q] = e^2,$$

thus, $e = \sqrt{E[q]}$. The expected cost is minimized at $\underline{e} = (r-c)/r$, achieving: $\mathrm{E}[C(\underline{e})] = \frac{c(2r-c)}{r}$ and $\mathrm{E}[q(\underline{e})] = \left(\frac{r-c}{r}\right)^2$.

1. For $0 < \tilde{q} \le \left(\frac{r-c}{r}\right)^2$, to minimize $E[c]$, we obtain $E[q] = \left(\frac{r-c}{r}\right)^2$ and $e_m = e_s = \underline{e} = (r-c)/r$. The constraint that $E[q] \ge \tilde{q}$ is satisfied and non-binding.
2. For $\tilde{q} \ge \left(\frac{r-c}{r}\right)^2$, we use the binding constraint to obtain $E[q] = \tilde{q}$, and $e_m = e_s = \sqrt{\tilde{q}}$.

To replace e_m, e_s with $\sqrt{E[q]}$, we obtain the function $\mathrm{E}[C] = 2c\sqrt{E[q]} + r(1 - \sqrt{E[q]})^2$.

Proof of Lemma 3.2

1. *IC constraint (Eq (3)): subcontractor's effort*

   The subcontractor exerts optimal effort level to maximize his profit function (Eq (1)): $\Omega_s^F(e_s) = \mathrm{E}_{\epsilon_s}[F_s - ce_s - br(\epsilon_s - e_s)^+] = F_s - ce_s - \frac{br}{2}(e_s-1)^2$. As the second order derivative on e_s is negative, the optimal effort level is: $\frac{\partial \Omega_s^F(e_s)}{\partial e_s} = -c - br(e_s - 1) = 0 \Longleftrightarrow e_s = \left(\frac{br-c}{br}\right)^+$.

2. *main contractor's decisions*

   The main contractor's profit is decreasing in F_s, so through binding IR constraint (Eq (4)) we obtain $F_s^{FF} = ce_s^{FF} + \frac{br}{2}(e_s^{FF} - 1)^2$.

   We substitute F_s^{FF} into the main contractor's expected profit (Eq (2)): $\Omega_m^{FF}(e_m) = F_0 - ce_m - \frac{r}{2}(e_m - 1)^2 - ce_s^{FF} - \frac{r}{2}(e_s^{FF} - 1)^2$.

   As the second order derivative on e_m is negative, the optimal effort level is as follows:

   $$\frac{\partial \Omega_m^{FF}(e_m)}{\partial e_m} = -c - r(e_m - 1) = 0 \Longleftrightarrow e_m^{FF} = \frac{r-c}{r}$$

Proof of Lemma 4.1

Under the incentive contract, the subcontractor sets his effort level to maximize his profit:

$$\max_{0 \le e_s \le 1} \Omega_s^I(e_s) = \mathrm{E}_{\epsilon_s}\left[F_s + a_s(e_s - \epsilon_s)^+ - ce_s - br(\epsilon_s - e_s)^+\right] = F_s + \frac{a_s}{2}(e_s)^2 - ce_s - \frac{br}{2}(e_s - 1)^2.$$

The first and second order derivatives on e_s are as follows.

$$\Omega_s^{I'}(e_s) = a_s e_s - c - br(e_s - 1), \Omega_s^{I''}(e_s) = a_s - br.$$

We discuss the solutions under concave and convex conditions as below.

1. *$a_s \leq br$: Ω_s^I is concave.* A local maximum is obtained by the FOC: $e_s = \frac{br-c}{br-a_s}$.

   (a) For $c \leq br$ and $a_s \geq c$, $\frac{br-c}{br-a_s} \geq 1$. Thus $e_s^* = 1$.

   (b) For $c \leq br$ and $a_s \leq c$, $\frac{br-c}{br-a_s} \in [0,1]$. Thus $e_s^* = \frac{br-c}{br-a_s}$.

   (c) For $c \geq br$, $\frac{br-c}{br-a_s} \leq 0$. Thus $e_s^* = 0$.

2. *$a_s \geq br$: Ω_s^I is convex.* The boundary values $e_s = \{0, 1\}$ are optimal depending on their profit:

$$\Omega_s^I(1) = F_s + \frac{a_s}{2} - c, \Omega_s^I(0) = F_s - \frac{br}{2}.$$

   (a) For $a_s \geq 2c - br$, $\Omega_s^I(1) \geq \Omega_s^I(0)$. Thus $e_s^* = 1$.

   (b) For $a_s \leq 2c - br$, $\Omega_s^I(1) \leq \Omega_s^I(0)$. Thus $e_s^* = 0$.

Together, we obtain the optimal solutions shown in Lemma 4.1 for the subcontractor's problem.

Proof of Lemma 4.2

Under the FI contract, the main contractor decides $\{F_s, a_s, e_m\}$, for the subcontractor's optimal $e_s^*(a_s)$ (see Eq (11) – (13)). We first determine F_s as a function of a_s, e_s^*. Then we divide the model into two subproblems, $\Omega_m^{FI1}(e_m)$ and $\Omega_m^{FI2}(a_s)$, as e_m is independent from a_s.

1. *Optimal F_s as a function of other decision variables*

   Through the binding IR constraint (Eq (13)), we obtain the optimal F_s: $F_s(a_s, e_s^*) = ce_s^* - \frac{a_s}{2}(e_s^*)^2 + \frac{br}{2}(e_s^* - 1)^2$.

2. *Reduced optimization problem $\Omega_m^{Reduced}(e_m, a_s)$*

   After substituting $F_s(a_s, e_s^*)$ in $\Omega_m^{FI}(F_s, a_s, e_m)$, we obtain:

$$\Omega_m^{Reduced}(e_m, a_s) = F_0 - ce_m - \frac{r}{2}(e_m - 1)^2 - ce_s^* - \frac{r}{2}(e_s^* - 1)^2.$$

   In this objective function, e_m is independent of a_s, with constraints on a_s, e_s^* only. Thus, we divide the optimization model into two subproblems, $\Omega_m^{Reduced}(e_m, a_s) = \Omega_m^{FI1}(e_m) + \Omega_m^{FI2}(a_s)$.

   (a) *main contractor's effort: $\Omega_m^{FI1}(e_m)$*

$$\max_{0 \leq e_m \leq 1} \quad \Omega_m^{FI1}(e_m) \quad = F_0 - ce_m - \frac{r}{2}(e_m - 1)^2.$$

   The optimal solution is $e_m^{FI} = \frac{r-c}{r}$.

(b) *main contractor's incentive payment: $\Omega_m^{FI2}(a_s)$ and IC constraint (Eq (12))*

Eq (12) is equivalent to Eq (10) in Lemma 4.1.

$$\max_{a_s \geq 0} \Omega_m^{FI2}(a_s) = -ce_s^* - \frac{r}{2}(e_s^* - 1)^2, \quad \text{s.t.} \quad Eq(10).$$

We solve the second subproblem for $c > br$ and $c \leq br$ separately.

i. *If $c > br$, then*

$$\begin{cases} \text{if } a_s \leq 2c - br : e_s^*(a_s) = 0 \text{ and } \Omega_m^{FI2}(a_s) = -\frac{r}{2}; \\ \text{if } a_s \geq 2c - br : e_s^*(a_s) = 1 \text{ and } \Omega_m^{FI2}(a_s) = -c. \end{cases}$$

For $c \leq \frac{r}{2}$, $e_s^* = 1$ is optimal. Set the lowest possible incentive, $a_s^{FI} = 2c - br$.

For $c > \frac{r}{2}$, $e_s^* = 0$ is optimal. Set the lowest possible incentive, $a_s^{FI} = 0$.

ii. *If $c \leq br$, then*

$$\begin{cases} \text{if } a_s \leq c : e_s^*(a_s) = \frac{br-c}{br-a_s} \text{ and } \Omega_m^{FI2}(a_s) = -ce_s^*(a_s) - \frac{r}{2}(e_s^*(a_s) - 1)^2; \\ \text{if } a_s \geq c : e_s^*(a_s) = 1 \text{ and } \Omega_m^{FI2}(a_s) = -c. \end{cases}$$

By FOC, we obtain the solution: $a_s = \frac{rc(1-b)}{r-c}$, $e_s = \frac{r-c}{r}$, $\Omega_m^{FI2}(\frac{rc(1-b)}{r-c}) = -c + \frac{c^2}{2r}$. As $\Omega_m^{FI2}(\frac{rc(1-b)}{r-c}) \geq \Omega_m^{FI2}(c)$ is always satisfied, the local maximum shown above is optimal for $c \leq br$ and $a_s^{FI} = \frac{rc(1-b)}{r-c}$.

Finally, we obtain e_m^{FI}, a_s^{FI} and the induced e_s^{FI} under all parameter conditions. F_s^{FI} can be expressed as $F_s^{FI}(a_s^{FI}, e_s^{FI}) = ce_s^{FI} - \frac{a_s^{FI}}{2}(e_s^{FI})^2 + \frac{br}{2}(e_s^{FI} - 1)^2$.

Proof of Proposition 5.1

Under the II contract, the main contractor decides $\{F_s, a_s, e_m\}$, and her profit is affected by e_s^* as shown by Eq (16). We follow the same steps as in the Proof of Lemma 4.2.

1. *Optimal F_s* : Set the IR constraint (Eq (13)) to be binding to find:

$$F_s(a_s, e_s^*) = ce_s^* - \frac{a_s}{2}(e_s^*)^2 + \frac{br}{2}(e_s^* - 1)^2. \tag{A.1}$$

2. *Reduced optimization problem $\Omega_m^{II}(e_m, a_s)$*

Substituting F_s and using $e_s^*(a_s)$ (Eq (10))shown in Lemma 4.1, we obtain:

$$\max_{0 \leq e_m \leq 1; a_s \geq 0} \Omega_m^{II}(e_m, a_s) = \mathrm{E}_{\epsilon_m, \epsilon_s}\left[F_0 + a_0(e_m + e_s^* - \epsilon_s - \epsilon_m)^+\right] - ce_m - \frac{r}{2}(e_m - 1)^2 - ce_s^* - \frac{r}{2}(e_s^* - 1)^2, \quad \text{s.t. } Eq(10).$$

Next, we solve under two parameter conditions $c > br$ and $c \leq br$.

(a) *If $c > br$, then*

$$e_s(a_s) = \begin{cases} 0, & \text{if } a_s \leq 2c - br; \\ 1, & \text{if } a_s \geq 2c - br. \end{cases}$$

As $e_s(a_s)$ can only take on two values, we directly compare the profit under those two values:

$$\Omega_m^{II}(e_m, a_s) = \begin{cases} \Omega_m^{II}(e_m, a_s|a_s \le 2c-br) = F_0 + \frac{a_0}{6}(e_m)^3 - ce_m - \frac{r}{2}(e_m-1)^2 - \frac{r}{2}, & \text{if } a_s \le 2c-br; \\ \Omega_m^{II}(e_m, a_s|a_s \ge 2c-br) = F_0 + a_0\left[\frac{e_m}{2} + \frac{1}{6} - \frac{1}{6}(e_m-3)(e_m)^2\right] - ce_m - c - \frac{r}{2}(e_m-1)^2, & \text{if } a_s \ge 2c-br. \end{cases}$$

Set the lowest possible a_s in each interval, and then optimize over the effort level e_m.

i. *For $a_s = 0 \le 2c - br$*, we optimize:

$$\max_{0\le e_m\le 1} \Omega_m^{II}(e_m, 0) = F_0 + \frac{a_0}{6}(e_m)^3 - ce_m - \frac{r}{2}(e_m-1)^2 - \frac{r}{2}. \tag{A.2}$$

The first and second order derivatives of Ω_m^{II} on e_m are as follows.

$$\frac{\partial\Omega_m^{II}(e_m,0)}{\partial e_m} = \frac{a_0}{2}(e_m)^2 - c - r(e_m-1), \frac{\partial^2\Omega_m^{II}(e_m,0)}{(\partial e_m)^2} = a_0e_m - r.$$

Through $\frac{\partial\Omega_m^{II}(e_m,0)}{\partial e_m} = 0$, we obtain an interior local maximum, $e_m = \frac{r-\sqrt{r^2-2a_0(r-c)}}{a_0} \ge 0$; the boundary point $e_m = 1$ could be a maximum point. Thus a direct comparison of the profits under the local maximum and boundary value for e_m yields:

$$\max_{0\le e_m\le 1} \Omega_m^{II}(e_m, 0) = \begin{cases} \Omega_m^{II}\left(\frac{r-\sqrt{r^2-2a_0(r-c)}}{a_0}, 0\right), & \text{if } a_0 \le 2c; \\ \Omega_m^{II}(1,0) = F_0 + \frac{a_0}{6} - c - \frac{r}{2}, & \text{if } a_0 \ge 2c. \end{cases} \tag{A.3}$$

ii. *For $a_s = 2c - br \ge 2c - br$*, we optimize:

$$\max_{0\le e_m\le 1} \Omega_m^{II}(e_m, 2c-br) = F_0 + a_0\left[\frac{e_m}{2} + \frac{1}{6} - \frac{1}{6}(e_m-3)(e_m)^2\right] - ce_m - c - \frac{r}{2}(e_m-1)^2 \tag{A.4}$$

The first and second order derivatives of Ω_m^{II} on e_m are as follows.

$$\frac{\partial\Omega_m^{II}(e_m,2c-br)}{\partial e_m} = a_0\left(\frac{1}{2} - \frac{(e_m)^2}{2} + e_m\right) - c - r(e_m-1), \frac{\partial^2\Omega_m^{II}(e_m,2c-br)}{(\partial e_m)^2} = a_0(-e_m+1) - r.$$

We obtain an interior local maximum by FOC and the boundary maximum ($e_m = 1$):

$$\max_{0\le e_m\le 1} \Omega_m^{II}(e_m, 2c-br) = \begin{cases} \Omega_m^{II}\left(1 - \frac{r-\sqrt{r^2-2a_0(c-a_0)}}{a_0}, 2c-br\right), & \text{if } a_0 \le c; \\ \Omega_m^{II}(1, 2c-br) = F_0 + a_0 - 2c, & \text{if } a_0 \ge c. \end{cases} \tag{A.5}$$

iii. *Comparison and optimum*

So far we found $e_m^*(a_s)$; next we compare Eqs (A.3) and (A.5) to find a_s^*.

$$\max_{a_s\ge 0}\left\{\max_{0\le e_m\le 1} \Omega_m^{II}(e_m, a_s)\right\} = \max\left\{\max_{0\le e_m\le 1} \Omega_m^{II}(e_m, 0), \max_{0\le e_m\le 1} \Omega_m^{II}(e_m, 2c-br)\right\}.$$

A. *If $a_0 \le c$, then*

$$\max_{a_s\ge 0}\left\{\max_{0\le e_m\le 1} \Omega_m^{II}(e_m, a_s)\right\} = \max\left\{\Omega_m^{II}\left(\frac{r-\sqrt{r^2-2a_0(r-c)}}{a_0}, 0\right), \Omega_m^{II}\left(1 - \frac{r-\sqrt{r^2-2a_0(c-a_0)}}{a_0}, 2c-br\right)\right\}.$$

By envelope theorem, the derivatives of the two objective functions on a_0 (as seen in Eqs (A.2) and (A.4)) are:

$$\frac{\partial\Omega_m^{II}\left(\frac{r-\sqrt{r^2-2a_0(r-c)}}{a_0},0\right)}{\partial a_0}=\frac{\partial\Omega_m^{II}\left(e_m,0\right)}{\partial a_0}=\frac{(e_m)^3}{6}\leq\frac{1}{6}$$

$$\text{and}\quad\frac{\partial\Omega_m^{II}\left(1-\frac{r-\sqrt{r^2-2a_0(c-a_0)}}{a_0},2c-br\right)}{\partial a_0}=\frac{e_m}{2}+\frac{1}{6}-\frac{1}{6}(e_m-3)(e_m)^2\geq\frac{1}{6}.$$

For $a_0=2c-r$, it is easy to verify that $\Omega_m^{II}\left(\frac{r-\sqrt{r^2-2a_0(r-c)}}{a_0},0\right)=\Omega_m^{II}\left(1-\frac{r-\sqrt{r^2-2a_0(c-a_0)}}{a_0},2c-br\right)$.

As the derivative of the objective function over a_0 is always larger for $\Omega_m^{II}\Big(1-\frac{r-\sqrt{r^2-2a_0(c-a_0)}}{a_0},\ 2c-br\Big)$, we have:

For $2c-r\leq a_0\leq c:\ e_m^{II}=1-\frac{r-\sqrt{r^2-2a_0(c-a_0)}}{a_0}, a_s^{II}=2c-br, e_s^{II}=1$;
For $a_0\leq 2c-r:\ e_m^{II}=\frac{r-\sqrt{r^2-2a_0(r-c)}}{a_0}, 1, a_s^{II}=0, e_s^{II}=0$.

B. *If $c\leq a_0\leq 2c$, then*

$$\max_{a_s\geq 0}\left\{\max_{0\leq e_m\leq 1}\Omega_m^{II}(e_m,a_s)\right\}=\max\left\{\ \Omega_m^{II}\left(\frac{r-\sqrt{r^2-2a_0(r-c)}}{a_0},0\right),\Omega_m^{II}(1,2c-br)\ \right\}.$$

By envelope theorem, the derivatives of the two objective functions on a_0 (as seen in Eqs (A.2) and (A.5)) are:

$$\frac{\partial\Omega_m^{II}(\frac{r-\sqrt{r^2-2a_0(r-c)}}{a_0},0)}{\partial a_0}=\frac{\partial\Omega_m^{II}(e_m^*(a_0),0)}{\partial a_0}=\frac{\partial\Omega_m^{II}(e_m^*,0)}{\partial a_0}=\frac{1}{6}(e_m^*)^3<1,\frac{\partial\Omega_m^{II}(1,2c-br)}{\partial a_0}=1.$$

As $\Omega_m^{II}(1,2c-br)\geq\Omega_m^{II}\left(\frac{r-\sqrt{r^2-2a_0(r-c)}}{a_0},0\right)$ when $a_0=c$, this inequality is also true for $a_0\geq c$. Therefore, the optimal solution is $e_m^{II}=1, a_s^{II}\geq 2c-br, e_s^{II}=1$.

C. *If $a_0\geq 2c$, then*

$$\max_{a_s\geq 0}\left\{\max_{0\leq e_m\leq 1}\Omega_m^{II}(e_m,a_s)\right\}=\max\left\{\ \Omega_m^{II}(1,0),\Omega_m^{II}(1,2c-br)\ \right\}.$$

As $a_0\geq 2c$ and $r\geq c$, $F_0+a_0-2c\geq F_0+\frac{a_0}{6}-c-\frac{r}{2}$ is true. Set $e_m^{II}=1, a_s^{II}=2c-br, e_s^{II}=1$.

In summary, the optimal solutions for $c>br$ are as follows.

For $a_0\geq c:\ e_m^{II}=1, a_s^{II}=2c-br, e_s^{II}=1$;
For $2c-r\leq a_0\leq c:\ e_m^{II}=1-\frac{r-\sqrt{r^2-2a_0(c-a_0)}}{a_0}, a_s^{II}=2c-br, e_s^{II}=1$;
For $a_0\leq 2c-r:\ e_m^{II}=\frac{r-\sqrt{r^2-2a_0(r-c)}}{a_0}, 1, a_s^{II}=0, e_s^{II}=0$.

(b) *If $c\leq br$, then*

$$e_s(a_s)=\begin{cases}\frac{br-c}{br-a_s}, & \text{if } a_s\leq c;\\ 1, & \text{if } a_s\geq c.\end{cases}$$

Assuming that the main contractor sets the lowest a_s inducing the desired effort level $e_s^*(a_s)$, we can limit ourselves to $a_s\in[0,c]$ with $e_s^*(a_s)=\frac{br-c}{br-a_s}$. For $e_s=\frac{br-c}{br-a_s}$, there is a one-to-one correspondence between e_s and a_s. Thus we can solve $\overline{\Omega}_m(e_m, e_s)$ instead of $\Omega_m^{II}(e_m, a_s)$. We obtain the optimal e_m, e_s by solving the following model.

$$\begin{aligned}\max_{0\le e_m, e_s\le 1} \overline{\Omega}_m(e_m, e_s) &= \mathrm{E}_{\epsilon_m,\epsilon_s}\left[F_0 + a_0(e_m + e_s - \epsilon_s - \epsilon_m)^+\right] - ce_m - \frac{r}{2}(e_m-1)^2 - ce_s - \frac{r}{2}(e_s-1)^2,\\ \text{s.t. } a_s(e_s) &= br - \frac{br-c}{e_s} \ge 0 \Longleftrightarrow e_s \ge \frac{br-c}{br}.\end{aligned}$$

We first maximize the objective function and then check the solution feasibility. Since the objective function is symmetric on e_m, e_s, we set $e_m = e_s = e$ and obtain:

$$\max_{0\le e\le 1} \widehat{\Omega}_m(e) = \mathrm{E}_{\epsilon_m,\epsilon_s}\left[F_0 + a_0(2e - \epsilon_s - \epsilon_m)^+\right] - 2ce - r(e-1)^2.$$

To simplify $\mathrm{E}_{\epsilon_m,\epsilon_s}\left[F_0 + a_0(2e - \epsilon_s - \epsilon_m)^+\right]$, we divide the solving process into two parts.

$$\max_{0\le e\le 1} \widehat{\Omega}_m(e) = \max\left\{\max_{0\le e\le \frac{1}{2}} \widehat{\Omega}_m(e), \max_{\frac{1}{2}\le e\le 1} \widehat{\Omega}_m(e)\right\}.$$

i. *For* $0 \le e \le \frac{1}{2}$

$$\max_{0\le e\le \frac{1}{2}} \widehat{\Omega}_m(e) = F_0 + \frac{4a_0}{3}(e)^3 - 2ce - r(e-1)^2.$$

The first and second order derivatives of $\widehat{\Omega}_m$ on e are as follows.

$$\widehat{\Omega}_m'(e) = 4a_0(e)^2 - 2c - 2r(e-1), \widehat{\Omega}_m''(e) = 8a_0e - 2r.$$

Local maximum: If $r^2 - 8a_0(r-c) \ge 0$, the optimal effort over the interval $[0, \frac{1}{2}]$ is either interior—for $a_0 \le 2c - r$—or the upper boundary of that interval—for $a_0 \ge 2c - r$: $e = \min\left\{\frac{r-\sqrt{r^2-8a_0(r-c)}}{4a_0}, \frac{1}{2}\right\}$.

If $r^2 - 8a_0(r-c) < 0$ or $a_0 \ge \frac{r^2}{8(r-c)}$, then $\widehat{\Omega}_m'(e) > 0$ and the optimal effort is $e = 1/2$. Given that $c \le r$ implies that $2c - r \le \frac{r^2}{8(r-c)}$, we obtain the optimal objective function values as follows.

$$\max_{0\le e\le \frac{1}{2}} \widehat{\Omega}_m(e) = \begin{cases} \widehat{\Omega}_m\left(\frac{r-\sqrt{r^2-8a_0(r-c)}}{4a_0}\right), & \text{if } a_0 \le 2c-r;\\ \widehat{\Omega}_m(\frac{1}{2}) = F_0 + \frac{a_0}{6} - c - \frac{r}{4}, & \text{if } a_0 \ge 2c - r.\end{cases} \tag{A.6}$$

ii. *For* $\frac{1}{2} \le e \le 1$

$$\max_{\frac{1}{2}\le e\le 1} \widehat{\Omega}_m(e) = F_0 + a_0\left[-\frac{1}{3} + e - \frac{1}{6}(-4+2e)(-1+2e)^2\right] - 2ce - r(e-1)^2.$$

The first and second order derivatives of $\widehat{\Omega}_m$ on e are as follows.

$$\widehat{\Omega}_m'(e) = -2a_0(1 - 4e + 2e^2) - 2c - 2r(e-1), \widehat{\Omega}_m''(e) = -8a_0(e-1) - 2r.$$

Local maximum: If $r^2 - 8a_0(c - a_0) \ge 0$, the optimal effort over the interval $e \in [\frac{1}{2}, 1]$ is either an interior local maximum or on the boundaries of that interval: $e = \max\left\{\frac{1}{2}, \min\left\{1 - \frac{r-\sqrt{r^2-8a_0(c-a_0)}}{4a_0}, 1\right\}\right\}$.

If $r^2 - 8a_0(c - a_0) < 0$, then the optimal effort $e = 1/2$ for $a_0 \leq 2c - r$ and $e = 1$ for $a_0 \geq c$. Combining all above cases, the optimal objective function values are as follows.

$$\max_{\frac{1}{2} \leq e \leq 1} \widehat{\Omega}_m(e) = \begin{cases} \widehat{\Omega}_m\left(\frac{1}{2}\right) = F_0 + \frac{a_0}{6} - c - \frac{r}{4}, & \text{if } a_0 \leq 2c - r; \\ \widehat{\Omega}_m\left(1 - \frac{r - \sqrt{r^2 - 8a_0(c - a_0)}}{4a_0}\right), & \text{if } 2c - r \leq a_0 \leq c; \\ \widehat{\Omega}_m(1) = F_0 + a_0 - 2c, & \text{if } a_0 \geq c. \end{cases} \tag{A.7}$$

iii. *Comparison and feasibility check*

It is easy to verify that the first-order derivative is continuous at $e = \frac{1}{2}$, and hence it cannot be an optimum unless it is an interior solution or if $a_0 = 2c - r$. Combining all the solutions and their conditions together, the optimal solutions for the relaxed problem are:

A. For $a_0 \geq c, e_m^{II} = e_s^{II} = e = 1, a_s^{II} = a_s(1) \geq c$;

B. For $c > a_0 \geq 2c - r, e_m^{II} = e_s^{II} = e = 1 - \frac{r - \sqrt{r^2 - 8a_0(c - a_0)}}{4a_0}, a_s^{II} = a_s(1 - \frac{r - \sqrt{r^2 - 8a_0(c - a_0)}}{4a_0})$;

C. For $a_0 < 2c - r, e_m^{II} = e_s^{II} = e = \frac{r - \sqrt{r^2 - 8a_0(r - c)}}{4a_0}, a_s^{II} = a_s(\frac{r - \sqrt{r^2 - 8a_0(r - c)}}{4a_0})$.

Next, we need to check whether the optimal solutions to the relaxed unconstrained problem satisfy the constraint $e_s = e \geq \frac{br - c}{br}$.

$$e = \frac{r - \sqrt{r^2 - 8a_0(r - c)}}{4a_0} \geq \frac{br - c}{br}, \text{ if } (1 - b)bcr^2 + 2a_0(c - br)^2 \geq 0 \text{ (True since } 0 \leq b \leq 1);$$

$$e = 1 - \frac{r - \sqrt{r^2 - 8a_0(c - a_0)}}{4a_0} \geq \frac{br - c}{br}, \text{ if } \frac{br - c}{br} \leq \frac{1}{2};$$

$$e = 1 - \frac{r - \sqrt{r^2 - 8a_0(c - a_0)}}{4a_0} \geq \frac{br - c}{br}, \text{ if } \begin{cases} \frac{br - c}{br} \geq \frac{1}{2} \\ a_0\left(1 - \frac{2c^2}{b^2 r^2}\right) + c\left(\frac{1}{b} - 1\right) \geq 0 \end{cases} \text{ (True);}$$

$$e = 1 \geq \frac{br - c}{br}, \text{ since } c \leq br.$$

Therefore, the optimal solutions to the relaxed model is also optimal to the original model.

Finally, we obtain e_m^{II}, a_s^{II} and the induced e_s^{II} under all parameter conditions. F_s^{II} can be expressed as $F_s^{II}(a_s^{II}, e_s^{II}) = ce_s^{II} - \frac{a_s^{II}}{2}(e_s^{II})^2 + \frac{br}{2}(e_s^{II} - 1)^2$ as shown by equation (A.1).

Proof of Proposition 6.1

We prove the three points successively.

1. *If $b = 1$, then*

FF contract: by Lemma 3.2, we have $e_m^{FF} = \frac{r - c}{r}$ and $e_s^{FF} = \frac{br - c}{br} = \frac{r - c}{r}$, which are the same with the efforts on the lowest efficient point as per Lemma 3.1.

FI contract: by Lemma 4.2, we have $e_m^{FI} = e_s^{FI} = \frac{r - c}{r}$, which are the same with the efforts on the lowest efficient point as per Lemma 3.1.

II contract: Proposition 5.1 shows the optimal effort levels under II contract. The effort levels of II contract are lowest when $a_0 = 0$, and are increasing on a_0. Those effort levels when $a_0 = 0$ and $b = 1$ shown as below are the same with the efforts on the lowest efficient point.

When $a_0 = 0$:

$$e_m^{II} = e_s^{II} = \lim_{a_0 \to 0} \frac{r - \sqrt{r^2 - 8a_0(r-c)}}{4a_0} = \lim_{a_0 \to 0} \frac{\frac{4(r-c)}{\sqrt{r^2-8a_0(r-c)}}}{4} = \frac{r-c}{r}, \text{ for } c \le br = r.$$

Therefore, whenever $b = 1$, FF, FI and II all reach the efficient frontier.

2. *If* $b < 1$ *and* $c \le \max\{a_0, br\}$*, then*

   FF contract: by Lemma 3.2, we have $e_m^{FF} = \frac{r-c}{r}$ and $e_s^{FF} = \frac{br-c}{br} < \frac{r-c}{r}$, since $b < 1$ and the subcontractor invests less than on the lowest efficient point.

   FI contract: same as in point 1.

   II contract: (1) For $c \le br$, the above proof for $b = 1$ under II contract has shown that the lowest effort levels of II contract are the same with the efforts on the lowest efficient point. With positive a_0, we have larger and still symmetric effort levels $e_m^{II} = e_s^{II}$. Therefore, for $c \le br$, II contract reaches the efficient frontier. (2) For $c \le a_0$, $e_m^{II} = e_s^{II} = 1$. Therefore, they reach the efficient frontier.

   Therefore, if $b < 1$ and $c \le \max\{a_0, br\}$, FI and II contracts reach the efficient frontier. FF contract leads to a lower total effort than the lowest efficient point.

3. *If* $b < 1$ *and* $c > \max\{a_0, br\}$*, then*

   FF contract: same as in point 2.

   FI contract: by Lemma 4.2, we have $e_m^{FI} \neq e_s^{FI}$, so the efforts no longer on the efficient frontier. Specifically the effort levels are:

$$e_m^{FI} = \frac{r-c}{r} \quad \text{and} \quad e_s^{FI} = \begin{cases} 1, & \text{if } br < c \le \max\{br, \frac{r}{2}\}; \\ 0, & \text{if } c > \max\{br, \frac{r}{2}\}. \end{cases}$$

   II contract: by Proposition 5.1, we have $e_m^{II} \neq e_s^{II}$, so the efforts no longer on the efficient frontier. Specifically, the effort levels are:

   For $a_0 \ge 2c - r : e_m^{II} = 1 - \frac{r - \sqrt{r^2 - 2a_0(c-a_0)}}{a_0} < 1, e_s^{II} = 1$;

   For $a_0 \le 2c - r : e_m^{II} = \frac{r - \sqrt{r^2 - 2a_0(r-c)}}{a_0} > 0, e_s^{II} = 0$.

   Therefore, if If $b < 1$ and $c > \max\{a_0, br\}$, FI and II contracts cannot reach the efficient frontier. FF leads to a lower total effort than the lowest efficient point.