# Optical Skyrmions

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## ABSTRACT

We show that Skyrmions provide a natural language and tool with which to describe and model structured light fields. These fields are characterised by an engineered spatial variation of the optical field amplitude, phase and polarisation. In this short presentation there is scope only for dealing with the simplest (and perhaps most significant) of these namely those that can be designed and propagate within the regime of paraxial optics. Paraxial Skyrmions are most readily defined in terms of the normalised Stokes parameters and as such are properties of the local polarisation at any given point in the structured light beam. They are also topological entities and as such are robust against perturbations. We outline briefly how Skyrmionic beams have been generated to order in the laboratory. Optics gives us access, also, to the Skyrmion field and we present the key properties of this field and show how it provides the natural way to describe the polarisation of structured light beams.

**Keywords:** Structured light, topology, Skyrmions, Paraxial optics

## 1. INTRODUCTION

Structured light beams are prepared by controlling the amplitude, phase and also the polarisation of the propagating field [1-5]. They have been shown to exhibit intrinsically topological features such as phase and polarisation singularities and, for suitable structures, to produce very tightly focussed light with a large field in the propagation direction [6,7]. The simplest of these beams display a polarisation structure that is reminiscent of the Skyrmions that have been produced in the surfaces of magnetic media [8]. These magnetic Skyrmions have a magnetisation that is orthogonal to that at the edges of the sample but is protected from flipping by the surrounding arrangement of the spins. Remarkably, similar structures have also been prepared in the very different field of paraxial optics. These Skyrmionic beams retain the topological character of the magnetic Skyrmions without the interactions between adjacent areas of different polarisation. Moreover, they can also vary along the propagation direction and so allow us to explore and also to engineer Skyrmions in three spatial dimensions. Before proceeding we should note that Skyrmions were first introduced by Skyrme as a model for pions in nuclear physics [9,10]. Since then they have appeared, also, in wide range of physical phenomena including the theory of quantum liquids [11], photonic materials [12], fractional statistics [13], non-linear field theories [14] and cosmology [15]. Undoubtedly the greatest impact of Skyrmions to date, however, has been in the study of magnetic materials [8,16].

Our aim here is to provide an introduction to the physics of Skyrmions by reference to their appearance in paraxial optics. We shall describe how simple Skyrmionic beams can be constructed, both theoretically and also in the laboratory [17,18]. These beams are similar to the Poincaré beams, in which all possible polarisations appear at some point in the transverse plane [4], but they are subtly different. Skyrmions are topological features [19] that are characterised by an integer Skyrmion number which is a property of an underlying Skyrmion field. This field is constructed from the local (normalised) Stokes parameters and Skyrmion field lines have the physically appealing interpretation that they correspond to lines of constant polarisation [20]. Several features follow from this and the fact that the divergence of the Skyrmion field is zero [17]. The Skyrmion field, being in this sense transverse, can be written as the curl of a Skyrmion potential, in much the same way that the magnetic induction can be written as the curl of the vector potential, $\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}$. This feature provides a more accurate and topological way of measuring the Skyrmion number. We conclude with a brief discussion of how the ideas presented here can be extended beyond paraxial optics.

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## 2. SKYRMIONIC BEAMS

We start by considering a general, but paraxial, light beam with a spatially varying polarisation. This polarisation can conveniently be parameterised by three local Stokes parameters, which we can combine into a three-dimensional vector, $\mathbf{S} = (S_1, S_2, S_3)$, where $S_1$ is the difference between the horizontal and vertical linear polarisations, $S_2$ is the difference between the two diagonal linear polarisations and $S_3$ is the difference between the two circular polarisations. These three suffice to determine, uniquely, the polarisation at any given position in the beam [21]. When expressed in terms of these, the Skyrmion number, for a beam propagating in the $z$-direction and at $z$, takes the form

$$n(z) = \frac{1}{4\pi}\int \mathbf{S}\cdot\left(\frac{\partial \mathbf{S}}{\partial x}\times\frac{\partial \mathbf{S}}{\partial y}\right) dxdy\,, \tag{1}$$

where the integration covers the entire $x - y$ plane transverse to the direction of propagation. In all but a few (interesting) cases, this quantity will be an integer. The fact that it is an integer is an indication of its topological character [19] in that a quantity that is required to be an integer cannot vary continuously; it can only remain constant or jump discontinuously from one integer to another. The propagation of the Skyrmions we have studied and created display both is these characteristics [17,18].

It is convenient to employ bra-ket notation for the polarisation, effectively exploiting the mathematical similarity between the Poincaré sphere for polarisation and the Bloch sphere for a quantum spin-half particle. (We do not suggest that Skyrmions are necessarily quantum in nature; this description is nothing more than a representation of the Jones calculus for polarisation). With this, we can represent the local state of spin by the complex vector

$$|\psi(\mathbf{r})\rangle = \frac{|0\rangle + v(\mathbf{r})|1\rangle}{\sqrt{1+|v(\mathbf{r})|^2}}\,, \tag{2}$$

where $|0\rangle$ and $|1\rangle$ correspond to any two orthogonal polarisations, such as left and right circular polarisations.

It is simplest and most convenient, for our purposes, to prepare the two orthogonal and superposed polarisations in orthogonal Gauss-Laguerre modes. These modes have been the focus of much attention over the last thirty years because of they carry well defined quantities of optical orbital angular momentum [22]. This orbital angular momentum is simply $\ell\hbar$ for each photon, where the azimuthal dependence of the mode is $e^{i\ell\phi}$. The complex quantity $v(\mathbf{r})$ is simply the ratio of the two complex Laguerre-Gaussian modes and so carries the azimuthal phase dependence $e^{i\Delta\ell\phi}$, where $\Delta\ell$ is the difference between the values of $\ell$ for the two modes [17]. In most cases the value of the Skyrmion number is $\pm\Delta\ell$ where the sign of this quantity depends on the behaviour of the polarisation at the centre of the beam and at its edges.

To construct our Skyrmionic beams we first separate a pair of beams with horizontal and vertical polarisations. This is achieved by preparing a diagonally polarised Gaussian laser beam and separating two beams using a Wollaston prism. Each of these is allowed to fall on a different region of a digital micromirror device, which imposes on the first-order diffracted beam the required azimuthal phase structure. When the two beams are combined, we have the desired Skyrmionic beam. The spatially-varying polarisation pattern can then be analysed using a combination of wave-plates and a polariser [18]. In this fashion we have constructed and analysed a wide variety of Skyrmionic structures.

At their simplest, the Syrmionic beams include all possible polarisations at some point in the transverse plane. When this is the case, the beams are examples of the Poincaré beams [4]. There exist examples, however, of Poincaré beams with Skyrmion number zero and also bizarre non-integer Skyrmion beams that are not Poincaré beams [17] and lack the topological robustness of their integer Skyrmion counterparts. So it is clear that the two classes of beams, while having features in common, are in fact distinct.

## 3. SKYRMION FIELD

It is helpful and natural to introduce a Skyrmion field in the form

$$\Sigma_i = \frac{1}{2}\varepsilon_{ijk}\varepsilon_{pqr}S_p\frac{\partial S_q}{\partial x_j}\frac{\partial S_r}{\partial x_k}\,. \tag{3}$$

Here $\varepsilon_{ijk}$ andf $\varepsilon_{pqr}$ are the alternating symbols familiar from the cross-product. It follows that the Skyrmion number is simply

$$n(z) = \frac{1}{4\pi}\int \Sigma_z dxdy \,. \tag{4}$$

It is a key property of the Skyrmion field that it is divergenceless (or transverse) so that

$$\boldsymbol{\nabla}\cdot\boldsymbol{\Sigma} = 0 \,. \tag{5}$$

This means that, as with other divergenceless fields such as the magnetic induction $\mathbf{B}$, Skyrmion field lines can only be either closed loops or extend to infinity.

The Skyrmion number for a beam is stable, because it is an integer and therefore is a topological property. This does not mean, however, that it is imune to change. It can hop disconinuously from one integer value to another at a prescribed plane. This is most readily achieved by preparing the two contributing Laguerre-Gaussian modes to focus in different planes [17].

The Skyrmion field lines have a significance beyond their connection with Skyrmions. Skyrmion field lines are, in fact, lines of constant polarisation and so these form the skeleton of all paraxial structured beams. It is worth taking a few lines to prove this important result [20]. Consider a line of constant polarisation and let us construct a local Cartesian coordinate system $(u, v, w)$ such that $\mathbf{u}$ lies along the line of constant polarisation. This means that at this point the Stokes parameters do not vary in the $u$-direction, so that $\partial\mathbf{S}/\partial u = 0$. The three components of the Skyrmion field at this point are

$$\begin{aligned}
\Sigma_u &= \frac{1}{2}\varepsilon_{pqr}S_p\left(\frac{\partial S_q}{\partial v}\frac{\partial S_r}{\partial w} - \frac{\partial S_r}{\partial v}\frac{\partial S_q}{\partial w}\right)\\
\Sigma_v &= \frac{1}{2}\varepsilon_{pqr}S_p\left(\frac{\partial S_q}{\partial w}\frac{\partial S_r}{\partial u} - \frac{\partial S_r}{\partial w}\frac{\partial S_q}{\partial u}\right)\\
\Sigma_w &= \frac{1}{2}\varepsilon_{pqr}S_p\left(\frac{\partial S_q}{\partial u}\frac{\partial S_r}{\partial v} - \frac{\partial S_r}{\partial u}\frac{\partial S_q}{\partial v}\right),
\end{aligned} \tag{6}$$

from which it is clear that $\Sigma_v$ and $\Sigma_w$ are both zero and so the one remaining non-zero component of the Skyrmion field, $\Sigma_u$, points along the line of constant polarisation. It then follows, also, that lines of constant polarisation are either closed loops or extend to infinity.

## 4. SKYRMION POTENTIAL

The transverse form of the Skyrmion field means that we can write it as the curl of a further, albeit non-unique, field. This is the analogue of the relationship between the magnetic induction and the vector potential, $\mathbf{B} = \boldsymbol{\nabla}\times\mathbf{A}$. For the Skyrmion field we can write

$$\boldsymbol{\Sigma} = \boldsymbol{\nabla}\times\mathbf{V} \,. \tag{7}$$

A simple and suitable form for $\mathbf{V}$ is

$$V_i = \frac{S_3}{1-S_3^2}\left(S_2\nabla_i S_1 - S_1\nabla_i S_2\right). \tag{8}$$

The benefit of this is that we can replace the surface integral (4) for the Skyrmion number by a line integral (exploiting Stokes's theorem). The catch is that the Skyrmion potential can diverge and such divergent points must be excluded from the line integral, much as poles must be omitted from contour integration in the complex plane. Nevertheless, we have shown that this technique proves a method for extracting the Skyrmion number from experimental data with exceptional precision [18].

## 5. CONCLUSION

We have seen that there is a natural link between Skyrmions and the polarisation of paraxial structured light. The Skyrmion number is a topological property of such beams and as such is robust against many perturbations. It can change, but only discontinuously. The Skyrmion field itself has an important role to play in structured light whether or not there is a Skyrmion present; Skyrmion field lines are lines of constant polarisation. This is true for every possible polarisation and suggests a way to extend the analysis of polarisation patterns beyond the familiar C and L lines associated with circular and linear polarisation [1].

We may expect that the connection between Skyrmion field lines and polarisation patterns might extend to similar features in other branches of physics. Natural examples include the spatial spin patterns of electron and neutron beams and also gravitational waves, which like light have only two orthogonal polarisations. It is interesting to consider how optical Skyrmions and Skyrmion fields might be extended beyond the paraxial regime. We have made progress in this direction but further presentation of this idea will have to wait for another occasion.

Finally, we should point out that the paraxial Skyrmions presented hare are but one of a veritable zoo of topological features that are manifest in structured light beams. As a starting point for accessing this large (and growing) literature, we end with a few references from which to start exploring this literature [23-25].

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