

RESEARCH ARTICLE

Single item stochastic lot sizing problem considering capital flow and business overdraft

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This paper introduces capital flow to the single item stochastic lot sizing problem. A retailer can leverage business overdraft to deal with unexpected capital shortage, but needs to pay interest if its available balance goes below zero. A stochastic dynamic programming model maximizing expected final capital increment is formulated to solve the problem to optimality. We then investigate the performance of four controlling policies: (R, Q) , (R, S) , (s, S) and (s, \bar{Q}, S) ; for these policies, we adopt simulation-genetic algorithm to obtain approximate values of the controlling parameters. Finally, a simulation-optimization heuristic is also employed to solve this problem. Computational comparisons among these approaches show that policy (s, S) and policy (s, \bar{Q}, S) provide performance close to that of optimal solutions obtained by stochastic dynamic programming, while simulation-optimization heuristic offers advantages in terms of computational efficiency. Our numerical tests also show that capital availability as well as business overdraft interest rate can substantially affect the retailer's optimal lot sizing decisions.

Keywords: stochastic lot sizing; capital flow; business overdraft; stochastic dynamic programming; genetic algorithm

1. Introduction

Capital shortage is a key factor affecting the growth of many small and medium enterprises (SMEs). In contrast to large companies, due to sales volume and operation scales, it is not easy for SMEs to obtain external capital in the form of loans or venture capital investment. In addition, SMEs usually lack the capital to absorb large losses. A survey of 531 businesses that went bankrupt during the calendar year 1998 in Bradley (2000) pointed out that inadequate financial planning was one of main reasons for their business failing. A report by Coughtrie, Morley, and Ward (2009) showed shortage of capital accounted for 17% of company bankruptcies in Australia in 2008. Elston and Audretsch (2011) found that 84% of high-tech entrepreneurs in the US had experienced a shortage of capital at some time.

Numerical experiments in Zeballos, Seifert, and Protopappa-Sieke (2013) demonstrate that the lack of access to short-term debt drastically inflates working capital requirements and lowers cash flows. Business overdraft is an easy and effective short-term option for many companies to solve cash shortages. It can ensure a company has funds in place and available immediately when something unexpected happens. After an agreement between a company and a bank is made, overdraft occurs when money is withdrawn from a bank account when the available balance goes below zero. The company pays interest for the negative balance at an agreed rate. A survey (Sophie Doove 2014) about SMEs in the 28 countries of the European Union showed that SMEs preferred to use bank overdraft, bank loan and trade credit. A report by Ipsos (2017) based on the surveys of over 1000 SMEs from 2014 to 2016 in UK revealed that business overdraft accounted for about 20% — and ranked second — of all external finances in the three years.

Given the above account, it is important for a manager to take capital flow and external financing into

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account when making operational decisions. In this respect, we contribute to the literature on stochastic lot sizing as follows.

- We incorporate capital flow and one kind of external financing, i.e. overdraft, in the stochastic lot sizing problem and formulate a stochastic dynamic programming model to obtain optimal solutions.
- We discuss four inventory controlling policies for this problem and use simulation-genetic algorithm to obtain approximate values of the controlling parameters.
- We introduce a simulation-optimization heuristic inspired by the approach originally introduced by Askin (1981).
- We conduct a comprehensive numerical study to compare stochastic dynamic programming, simulation-genetic algorithm and simulation-optimization heuristic.

The rest of this work is structured as follows. Section 2 reviews the related literature. Section 3 describes the problem setting. Section 4 provides the stochastic dynamic programming model. Section 5 gives the simulation-genetic algorithm approach and Section 6 presents the simulation-optimization heuristic. Section 7 adopts some numerical examples to show the influence of capital flow and overdraft to optimal policy structure. A computational study and its results are detailed in Section 8. Finally, Section 9 draws conclusions and outlines future research directions.

2. Literature review

Due to its practical relevance, a large body of literature has emerged on the lot sizing problem over the last few decades. After the pioneering work by Wagner and Whitin (1958) for the single item lot sizing problem with deterministic demands, many extensions were investigated for lot sizing problems, such as multi item settings, capacity constraints, and stochastic demand. Here we limit our discussion to key studies that are relevant in the context of our discussion.

The stochastic lot sizing problem has been thoroughly investigated in the inventory control literature Axsäter (2007). Under mild assumptions, Scarf (1959) proved the optimal policy takes the well-known (s, S) form, in which s denotes the reorder point and S is the order-up-to-level. Silver (1978) and Askin (1981) proposed simple heuristics based on the least period cost method for the problem under penalty cost, which are stochastic extensions of the well-known Silver-Meal's heuristic (Silver and Meal 1973). Bookbinder and Tan (1988) discussed three control strategies: the "static uncertainty" strategy, in which lot sizing decisions including review intervals and order quantities for each period must be made at the beginning of the first period; the "static-dynamic uncertainty" strategy, in which review intervals are fixed at the beginning but order quantities for each period are not determined until this period comes; and the "dynamic uncertainty" strategy, which allows the retailer to decide dynamically at each period whether or not to place an order and how much to order.

Many subsequent studies focused on modeling and solving the stochastic lot sizing problem under the three strategies introduced in Bookbinder and Tan (1988). In the context of the "static uncertainty" strategy, Guan et al. (2006) developed a branch-and-cut algorithm; Tempelmeier and Herpers (2011) presented modifications of several well-known dynamic lot sizing heuristics for solving the single item stochastic problem; Koca, Yaman, and Aktürk (2015) used second order cone programming to obtain solutions for a capacitated single item lot sizing problem with controllable processing time. In the context of the "static-dynamic uncertainty" strategy, Tarim and Kingsman (2004) proposed a mathematical programming approach to compute near-optimal "static-dynamic uncertainty" policy parameters under service level constraints; Tarim and Kingsman (2006) extended the previous study to a penalty cost setting, by using a piecewise linear approximation of the cost function; Tempelmeier (2007) proposed a β service level constraint and a new computation of holding costs for the problem; Özen, Dođru, and Tarim (2012) considered the problem with dynamic fixed-ordering, holding costs and dynamic penalty costs and proved that the optimal policy is base-stock policy; Rossi, Kilic, and Tarim (2015) applied a new piecewise approximation method in Rossi et al. (2014) to handle a range of service level measures as well as lost sales. The "dy-

dynamic uncertainty” strategy was initially investigated in Askin (1981), who developed an early heuristic; Federgruen and Zipkin (1984) addressed the stationary demand case, and Bollapragada and Morton (1999) extended the previous study to handle the non-stationary demand case.

Relevant works taking capital flow or financing into account in inventory management problems are the following. Buzacott and Zhang (2004) analyzed the importance of joint consideration of production and financial decisions via a news vendor model. Chao, Chen, and Wang (2008) investigated a multi period news vendor problem constrained by cash flow and proved the optimal policy is a base stock policy. Gong, Chao, and Simchi-Levi (2014) extended the model by incorporating short term financing. Zeballos, Seifert, and Protopappa-Sieke (2013) built a periodic review inventory problem with working capital constraints, payment delay and multiple sources of financing. Wuttke et al. (2016) considered the supplier’s supply chain finance adoption decisions within a diffusion model and obtained some insights regarding a buyer’s optimal decisions regarding timing and payment terms. The above mentioned works are not for lot sizing problem and there are no fixed ordering costs in these models. Considering capital flow constraints, Chen and Zhang (2015) formulated a single item model for deterministic demands with trade credit and devised a dynamic algorithm with heuristic adjustments to solve it.

Our literature survey reveals that no work has so far investigated the stochastic lot sizing problem under non-stationary stochastic demand, capital flow, and external financing. This, together with our initial discussion on the relevance of this topic, motivates our study.

3. Problem description

For convenience, the notations adopted in this paper are listed in Table 1. Relevant notations will be introduced when needed.

In our problem, demand is stochastic and non-stationary. For each period t , its demand is represented by D_t , which is a non-negative random variable with probability density function f_t , cumulative distribution function F_t , mean μ_t , variance σ_t^2 . Random demand is assumed to be independent over the periods. Unmet demand in any given period is back ordered and satisfied as soon as the replenishment arrives. Excess stock is transferred to next period as inventory and the sell back of excess stock is not allowed.

We assume initial capital quantity of the retailer is B_0 ; order delivery lead time is zero; selling price of the product is p and the retailer receives payments only when the products are delivered to its customers. A fixed cost a is charged when placing orders, regardless of the order quantity, and R_t is a 0-1 variable to determine whether the retailer makes order at period t ; a variable cost v is charged on every order unit. End-of-period inventory level for period t is I_t , and we set I_t^+ to represent $\max\{I_t, 0\}$ and I_t^- to represent $\max\{-I_t, 0\}$. A variable inventory holding cost h is charged on every product unit carried from one period to the next; per unit stock-out penalty cost is π ; at the beginning of each period t , its present capital is B_{t-1} , if its initial capital is below zero, the retailer has to pay interests with a rate of b .

The actual sales quantity in period t is $\min\{D_t + I_{t-1}^-, Q_t + I_{t-1}^+\}$, where $D_t + I_{t-1}^-$ is demand plus backorder in period t and $Q_t + I_{t-1}^+$ is the total available stock in period t . End-of-period capital B_t for period t is defined as its initial capital B_{t-1} , plus payments by customers for the realized demand in this period $p \min\{D_t + I_{t-1}^-, Q_t + I_{t-1}^+\}$, minus the payments to suppliers and this period’s fixed ordering cost, holding and backorder costs $vQ_t + aR_t + hI_t^+ + \pi I_t^-$, and minus the interest paid if its initial capital is negative $b \max\{-B_{t-1}, 0\}$. Full expression of B_t is given by Eq. (1), and the inflows and outflows of capital from period $t - 1$ to period $t + 1$ is detailed shown by Figure 1.

$$B_t = B_{t-1} + p \min\{D_t + I_{t-1}^-, Q_t + I_{t-1}^+\} - (vQ_t + aR_t + hI_t^+ + \pi I_t^-) - b \max\{-B_{t-1}, 0\}, \quad (1)$$

As for the final capital in the whole planning horizon, we defined it as the end-of period capital B_T , minus the interest paid if B_T is negative, which is:

$$B_{T+1} = B_T - b \max\{-B_T, 0\}. \quad (2)$$

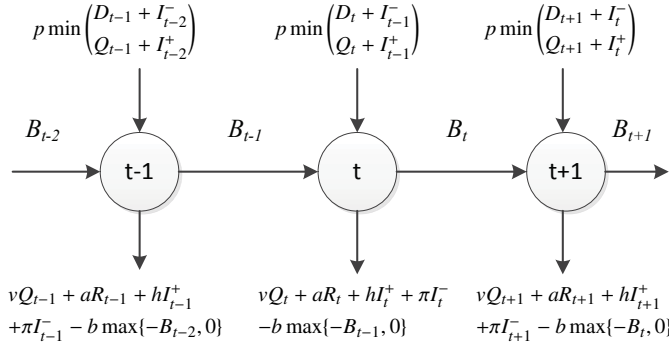


Figure 1. Inflows and outflows of capital at periods $t - 1, t, t + 1$

Our aim is to find a replenishment plan that maximizes the expected final capital increment, i.e. $E(B_{T+1}) - B_0$.

Table 1. Notations adopted in our paper.

Notations	Description
Indices	
t	Period index, $t = 1, 2, \dots, T$
Problem parameters	
B_0	Initial capital
I_0	Initial inventory, we assume $I_0 = 0$
I_t^+	$\max\{I_t, 0\}$
I_t^-	$\max\{-I_t, 0\}$
p	Product selling price
a	Fixed ordering cost
v	Unit variable ordering cost
h	Unit inventory holding cost
π	Unit penalty cost for back orders
b	Interest rate for negative initial capital
Random variables	
D_t	Random demand at period t with probability density function $f_t(D_t)$, cumulative distribution function $F_t(D_t)$, mean μ_t , variance σ_t^2
State variables	
I_t	End-of-period inventory for period t
B_t	End-of-period capital for period t
Decision variables	
Q_t	Order quantity at the beginning of period t
R_t	whether the retailer orders at period t
S_t	Order up to level at the beginning of period t , and $S_t = I_{t-1} + Q_t$
\bar{s}_t	Threshold of the inventory level for (s, S) policy
\bar{Q}_t	Maximum order quantity for (s, \bar{Q}, S) policy

4. A stochastic dynamic programming approach

In this section, we apply stochastic dynamic programming (SDP) to solve the problem.

States. The system state at the beginning of period t is represented by initial inventory I_{t-1} and initial capital quantity B_{t-1} . Note that I_{t-1} and B_{t-1} can be negative.

Actions. The action at period t is the production quantity Q_t , given initial inventory I_{t-1} and initial capital quantity B_{t-1} .

State transition function. The inventory and capital at the end of period t is determined by initial inventory I_{t-1} , initial capital B_{t-1} , demand D_t and action Q_t . The state transition of the system for B_t and I_t is described by the following equations.

$$B_t = B_{t-1} + p \min \{D_t + I_{t-1}^-, Q_t + I_{t-1}^+\} - (a\delta_t + hI_t^+ + \pi I_t^- + vQ_t) - b \max\{-B_{t-1}, 0\}, \quad (3)$$

$$I_t = I_{t-1} + Q_t - D_t, \quad (4)$$

Capital transition equation is same as the expression of end-of-period capital B_t .

Immediate profit. The immediate profit for period t is the expected capital increment during this period. Given B_{t-1} , I_{t-1} and Q_t , immediate profit ΔB_t can be expressed as

$$\Delta B_t^{Q_t}(I_{t-1}, B_{t-1}) = E(B_t) - B_{t-1}, \quad (5)$$

Functional Equation. Define $F_t(B_{t-1}, I_{t-1})$ as the maximum expected capital quantity increment during periods $t, t+1, \dots, T$, when the initial inventory and capital of period t are I_{t-1} and B_{t-1} , respectively. The functional equation is expressed as:

$$F_t(B_{t-1}, I_{t-1}) = \max_{Q_t \geq 0} \{\Delta B_t + F_{t+1}(B_t, I_t)\}, \quad t = 1, 2, \dots, T, \quad (6)$$

By the definition of final capital in Eq. (2), the boundary condition for the functional equation is:

$$F_{T+1}(B_T, I_T) = -b \max\{-B_T, 0\}. \quad (7)$$

Our aim is to maximise the expected capital increment $F_1(B_0, I_0)$ over the planning horizon given initial capital B_0 and inventory I_0 .

4.1 A numerical example

Here we give a numerical example to show the results of SDP. Values of parameters are listed in Table 2. For convenience, there are 3 periods in total, and two levels of demand in each period with equal probability 0.5. Details of demand levels are presented in Table 3.

Table 2. Parameter values.

B_0	I_0	p	a	v	h	π	b
5	0	5	10	1	1	2	20%

Table 3. Demand values.

	D_1	D_2	D_3
level 1	1	1	1
level 2	2	2	2

From SDP, optimal expected final capital increment is 1.75. For 5 random demand realization samples, results of SDP are shown by Table 4.

5. A simulation-based genetic algorithm approach

Because of states explosion, it is time consuming for stochastic dynamic programming to obtain a final result. Therefore, we propose four inventory controlling policies based on three strategies of stochastic lot sizing problem (Bookbinder and Tan (1988)) to solve our problem.

Table 4. Solutions of SDP for 5 demand realization samples.

	D_1	D_2	D_3	Q_1	Q_2	Q_3	$E(B_{T+1}) - B_0$
sample 1	2	1	2	0	5	0	3.8
sample 2	2	1	1	0	5	0	-2.2
sample 3	2	2	2	0	5	0	3.0
sample 4	1	1	2	0	4	0	1.4
sample 5	1	2	1	0	4	0	3.0

- (1) **(R, Q) policy.** This is a “static uncertainty” strategy, in which the review intervals and ordering quantities for replenishment periods are fixed at the beginning of planning horizon. For this policy, the decision variables are review intervals R_t and ordering quantities Q_t .
- (2) **(R, S) policy.** This is a “static-dynamic” strategy, in which the review intervals are fixed at the beginning of the planning horizon, but the order quantities are not fixed. Order quantity Q_t is related with the inventory level at the review intervals I_{t-1} and the fixed order-up-to levels S_t , which is $\max\{0, S_t - I_{t-1}\}$.
- (3) **(s, S) policy.** This is a “dynamic uncertainty” strategy policy: (s, S) policy, in which the review intervals and ordering quantity are neither fixed. If initial inventory level at period t is above s_t , order nothing in this period; else order to the order-up-to level S_t .
- (4) **(s, \bar{Q}, S) policy.** This is a variant of the previous “dynamic uncertainty” strategy, in which we impose an upper bound \bar{Q} for the maximum ordering quantity.

Details of order quantity Q_t in (s, S) policy is presented by the formula below.

$$Q_t = \begin{cases} 0 & I_{t-1} \geq s_t \\ \max\{0, S_t - I_{t-1}\} & I_{t-1} < s_t. \end{cases} \quad (8)$$

For the (s, \bar{Q}, S) policy, order quantity for each period t is determined by Eq. (9).

$$Q_t = \begin{cases} 0 & I_{t-1} \geq s_t \\ \max\{\bar{Q}_t, \max\{0, S_t - I_{t-1}\}\} & I_{t-1} < s_t. \end{cases} \quad (9)$$

Since our problem explicitly models capital flows, one may be inclined to adopt an inventory policy in which decisions are dependent on available stock and capital at the beginning of each period. However, because end-of-period capital of each period is, in fact, solely determined by its initial capital, order quantity, and previous demand realizations, as we will show, a control policy that only takes into account initial stock level in each period — essentially an (s, S) policy — and that sets an upper bound to the maximum possible replenishment quantity is able to achieve near-optimal performances. Furthermore, even in absence of an upper bound to the maximum possible replenishment quantity, an (s, S) policy appears to perform well.

5.1 Genetic algorithm settings

It is difficult to obtain values of controlling parameters for these policies from mathematical optimization directly. Therefore, we solve this problem by Monte-Carlo simulation and genetic algorithm to obtain near-optimal parameter values. Informally speaking, our approach proceeds as follows. We generate a sufficiently large random demand samples and obtain values of controlling parameters by maximizing the average final capital for these samples via genetic algorithm; then we simulate the controlling parameters over 100,000 benchmark random demands to estimate final expected capital increment.

Genetic algorithm (GA) is a global search procedure that searches from one population of solutions to another, in which good solutions are selected to evolve to the next generation while some mutations take place to avoid falling into local optimum traps (Dorsey and Mayer 1995). For our problem, besides the

lower and upper bounds of the controlling parameters, there are no extra constraints. It is not difficult for genetic algorithm to generate feasible populations and find a good solution. Settings of genetic algorithm we adopted are listed below; note that to assess the validity of these settings we will carry out stability analysis (Kaut et al. 2007) in our computational study.

- (1) **Fitness function, bounds of variables.** The fitness function is the maximum average final capital increment of the samples. R_t is a 0-1 variable; lower bound for Q_t and \overline{Q}_t are 0; lower bounds for s_t and S_t are negative enough numbers; upper bounds for Q_t , \overline{Q}_t , s_t , and S_t are large enough numbers.
- (2) **Population.** Population type is double vector real coding and population size is 200. Initial populations are randomly generated between the lower and upper bounds of each controlling parameter.
- (3) **Fitness scaling.** Fitness scaling is based on the rank of fitness scores of each individual population, which can remove the effect of the spread of the raw scores. The rank of an individual is its position in the sorted scores. An individual with rank r has scaled score $1/\sqrt{r}$. This scaling method makes poorly ranked individuals more nearly equal in scores.
- (4) **Selection.** We use roulette selection method to chooses parents for the next generation based on their scaled fitness values.
- (5) **Reproduction and crossover.** The minimum number of elite individuals that are guaranteed to survive to the next generation is 10. Other children are produced by crossover of the selected parents. The Crossover rate is 0.8. Crossover method is scattered, which first creates a random binary vector and then selects the genes where the vector is a 1 from the first parent, and the genes where the vector is a 0 from the second parent, and combines the genes to form a child.
- (6) **Mutation.** The mutation method is Gaussian, which add a random number to the selected parents. The random number is generated from a Gaussian distribution with mean 0 and shrinking standard deviation. Initial standard deviation δ_0 is 10 and the shrinking of standard deviation for the k_{th} generation is shown by the following equation, in which *generations* is the maximum generations initially set for the genetic algorithm to perform.

$$\delta_k = \delta_{k-1} \left(1 - \frac{k}{\text{generations}}\right). \quad (10)$$

- (7) **Migration.** Migration takes place every 20 generations forward, and the best 20% individuals from one subpopulation replace the worst individuals in another subpopulation.
- (8) **Stopping criteria.** Maximum generations is 10000 and function tolerance is 0.000001. If the average change in the fitness function value over 50 generations is less than function tolerance, the algorithm stops.

5.2 A numerical example

For the same numerical example solved by SDP, solutions of genetic algorithm for the four controlling policies are given by Table 5.

Table 5. Solutions of genetic algorithm.

							$E(B_{T+1}) - B_0$
policy (s, \overline{Q}, S)	$s_1 = -1$ $\overline{Q}_1 = 9$	$s_2 = 0$ $\overline{Q}_2 = 7$	$s_3 = 4$ $\overline{Q}_3 = 8$	$S_1 = 7$	$S_2 = 3$	$S_3 = 0$	1.29
policy (s, S)	$s_1 = 0$	$s_2 = 7$	$s_3 = 0$	$S_1 = 5$	$S_2 = 3$	$S_3 = 3$	1.29
policy (R, S)	$R_1 = 0$	$R_2 = 1$	$R_3 = 0$	$S_1 = 0$	$S_2 = 3$	$S_3 = 0$	1.29
policy (R, Q)	$R_1 = 0$	$R_2 = 1$	$R_3 = 0$	$Q_1 = 0$	$Q_2 = 5$	$Q_3 = 0$	0.65

For the same 5 random demand realization samples, final capital increments of four controlling policies are provided by Table 6. It shows, policy (s, \overline{Q}, S) , policy (s, S) and (R, S) obtain the same final expected capital increment as SDP, while policy (R, Q) performs a little worse.

Table 6. Expected final capital increment for 5 demand realization samples.

	D_1	D_2	D_3	(s, \bar{Q}, S)	(s, S)	(R, S)	(R, Q)
sample 1	2	1	2	3.8	3.8	3.8	3.8
sample 2	2	1	1	-2.2	-2.2	-2.2	-2.2
sample 3	2	2	2	3.0	3.0	3.0	3.0
sample 4	1	1	2	1.4	1.4	1.4	-2.0
sample 5	1	2	1	3.0	3.0	3.0	0.0

6. A simulation-optimization heuristic approach

In this section, we present a simulation-optimization heuristic for our problem that is based on Silver's heuristic (Silver 1978). To describe the simulation-optimization heuristic, we first define the concept of *replenishment cycle*.

Definition 1. A *replenishment cycle* (t, r) is the time interval between two consecutive replenishments executed at period t and at period $r + 1$; the last replenishment cycle is (t, T) .

The idea behind Silver's lot sizing heuristic is to view the average cost per period as a function of the length of a replenishment cycle. Assuming this function is convex with respect to the length of the replenishment cycle, it is possible to determine the optimal length of the next replenishment cycle. With regard to our profit-maximization problem, we view the average capital increment per period is concave to the replenishment cycle length. For the computation of capital increment in a replenishment cycle, we have the following property.

Lemma 1. The expected total capital increment during replenishment cycle (t, r) given initial inventory I_{t-1} and initial capital B_{t-1} , is concave in ordering quantity Q_t for $Q_t > 0$.

Proof. Total expected capital increment in replenishment cycle (t, r) is

$$\Delta B_{t,r}^{Q_t}(I_{t-1}, B_{t-1}) = \Delta B_t^{Q_t}(I_{t-1}, B_{t-1}) + \Delta B_{t+1}^0(I_t, B_t) + \cdots + \Delta B_r^0(I_{r-1}, B_{r-1}), \quad (11)$$

For period t ,

$$\Delta B_t^{Q_t}(I_{t-1}, B_{t-1}) = E\left(p \min\{D_t + I_{t-1}^-, Q_t + I_{t-1}^+\} - (a + hI_t^+ + \pi I_t^- + vQ_t) - b \max\{-B_{t-1}, 0\}\right), \quad (12)$$

Given I_{t-1} and B_{t-1} , it can be easily proved that the above formula is concave in Q_t . For period $n \in \{t + 1, t + 2, \dots, r\}$,

$$\Delta B_n^0(I_{n-1}, B_{n-1}) = E\left(p \min\{D_n + I_{n-1}^-, I_{n-1}^+\} - (hI_n^+ + \pi I_n^-) - b \max\{-B_{n-1}, 0\}\right). \quad (13)$$

The above function is separable, and its separable items are all concave piecewise linear functions of Q_t : the first term $p \min\{D_n + I_{n-1}^-, I_{n-1}^+\}$ is concave in Q_t , the second term $-(hI_n^+ + \pi I_n^-)$ is concave in Q_t , and the third term $-b \max\{-B_{n-1}, 0\}$ is also concave because B_{n-1} is concave. This function is therefore concave in Q_t . \square

For the simulation-optimization heuristic, we use Monte-Carlo simulation to approximate $E(I_t)$ in a replenishment cycle by randomly generating an appropriate number of demand samples. Using this sample-based approximation we can estimate expected total capital increment of the replenishment cycle and therefore develop a simulation-based extension of Silvers heuristic. We apply line search to compute the optimal order quantity Q_t for replenishment cycle (t, r) as well as the associated expected total capital increment based on Lemma 6. We increase the value of r , starting from t , until the expected average capital increment per period first decreases. A pseudo-code of the algorithm is given in Algorithm 1. Computational

complexity of our simulation-optimization heuristic is comparable to that discussed for the approach in (Gutierrez-Alcoba et al. 2017), which is $O(NT \log(\bar{Q}/\lambda))$, where N is the number of samples used, T is the length of planning horizon, \bar{Q} is the upper bound of ordering quantity and λ is the step length used in the line search.

Algorithm 1: Simulation-optimization heuristic

Data: current period t , initial inventory I_{t-1} , initial capital B_{t-1}

Result: optimal order quantity Q_t^*

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1  $r \leftarrow t$ ;
2  $\Delta B^* \leftarrow -\infty$ ;
3 do
4    $Q \leftarrow \arg \max_{Q \geq 0} \Delta B_{t,r}^Q(I_{t-1}, B_{t-1})$ ;
5    $\Delta B \leftarrow \Delta B_{t,r}^Q(I_{t-1}, B_{t-1}) / (r - t + 1)$ ;
6   if  $\Delta B \geq \Delta B^*$  then
7      $Q^* \leftarrow Q$ ;
8      $\Delta B^* \leftarrow \Delta B$ ;
9   end
10   $r \leftarrow r + 1$ ;
11 while  $\Delta B \geq \Delta B^*$  and  $r \leq T$ ;

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6.1 A numerical example

In the same numerical example as SDP and genetic algorithm, expected final capital increment by simulation-optimization heuristic is -1.25. Results of simulation-optimization heuristic for the same 5 random demand realization samples are provided by Table 7.

Table 7. Solutions of simulation-optimization heuristic for 5 demand realization samples.

	D_1	D_2	D_3	Q_1	Q_2	Q_3	$E(B_{T+1}) - B_0$
sample 1	2	1	2	0	4	0	-1.0
sample 2	2	1	1	0	4	0	1.0
sample 3	2	2	2	0	4	0	-2.0
sample 4	1	1	2	0	3	0	-3.2
sample 5	1	2	1	0	3	0	-2.0

Compared with SDP and four controlling policies by genetic algorithm, simulation-optimization heuristic performs a little worse for this numerical example.

7. Numerical examples

In this section, we illustrate the importance of modeling capital flows and business overdraft. By using a selection of instances, we demonstrate the impact of changes in problem parameters on the structure of the control action.

We consider 6 periods, demands in each period are independent and follow Poisson distribution. Mean values of the demands of each period are listed in Table 8, other relevant parameter values are given by Table 9.

If $I_0 = 0$, $B_0 = 0$, optimal final capital increment is -3.98. If $I_0 = 0$, $B_0 = 20$, optimal final capital increment is 5.52. If demand realizations are same as expected demands, optimal policy for the two case with different initial capital is presented by Table 10.

Table 8. Mean values of demands.

period	1	2	3	4	5	6
\bar{D}_t	3	4	3	5	4	3

Table 9. Parameter values.

I_0	p	a	v	h	π	b
0	4	12	2	1	3	20%

Table 10. Optimal solutions for two different initial capital.

	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$	$E(B_{T+1}) - B_0$
$Q_t (B_0 = 0)$	0	9	0	11	0	0	-3.98
$Q_t (B_0 = 20)$	9	0	0	12	0	0	5.52

Table 10 shows available capital may have substantial impact on the structure of the optimal policy and maximum final capital increment. If $I_0 = 0$, $B_0 = 0$, but interest rate $b = 5\%$, optimal final capital increment is 4.90. With same demands realizations, solutions of the two cases with different interest rates are given in Table 11.

Table 11. Optimal solutions for two different interest rates.

	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$	$E(B_{T+1}) - B_0$
$Q_t (b = 5\%)$	0	9	0	11	0	0	4.90
$Q_t (b = 20\%)$	8	0	0	13	0	0	-3.98

Results in Table 11 show interest rate can also substantially affect the structure of optimal policy. Other combinations of parameters, e.g. “contribution margin” that is selling price minus unit variable production cost, can also influence optimal policy. With $I_0 = 0$, $B_0 = 0$, $b = 20\%$, optimal solutions for two different margins are presented by Table 12 (for margin = 3, $v = 2$, $p = 5$; for margin = 5, $v = 1$, $p = 6$).

Table 12. Optimal solutions for two different margins.

	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$	$E(B_{T+1}) - B_0$
$Q_t (\text{margin}=3)$	0	10	0	9	0	0	-7.94
$Q_t (\text{margin}=5)$	8	0	0	13	0	0	44.78

The above results reveal that capital availability and interest rates impact retailer’s optimal policy structure and expected final capital increment in the stochastic lot sizing problem.

8. Computational study

In this section we present an extensive computational study to investigate the effectiveness of our approaches.

8.1 Test bed

The test beds are adopted from Rossi, Kilic, and Tarim (2015). There are 10 demand patterns for numerical analysis: 2 life cycle patterns (LCY1 and LCY2), 2 sinusoidal patterns (SIN1 and SIN2), 1 stationary pattern (STA), 1 random pattern (RAND), 4 empirical patterns (EMP1, EMP2, EMP3, EMP4). To ensure SDP can solve these instances in reasonable time, we rescale original demands in Rossi, Kilic, and Tarim (2015) and select 6 successive periods for testing. The values of expected demands for different patterns are given by Table 13 and Figure 2, respectively.

Table 13. Expected demands of different patterns.

demand pattern	1	2	3	4	5	6	Total
STA	7	7	7	7	7	7	42
LCY1	8	7	6	5	4	3	33
LCY2	2	3	4	5	6	7	27
SIN1	8	5	2	1	2	5	23
SIN2	5	6	7	8	7	6	39
RAND	8	4	1	3	1	3	20
EMP1	1	3	8	4	8	7	31
EMP2	1	4	7	3	5	8	28
EMP3	3	8	4	4	6	2	27
EMP4	3	1	5	8	4	4	25

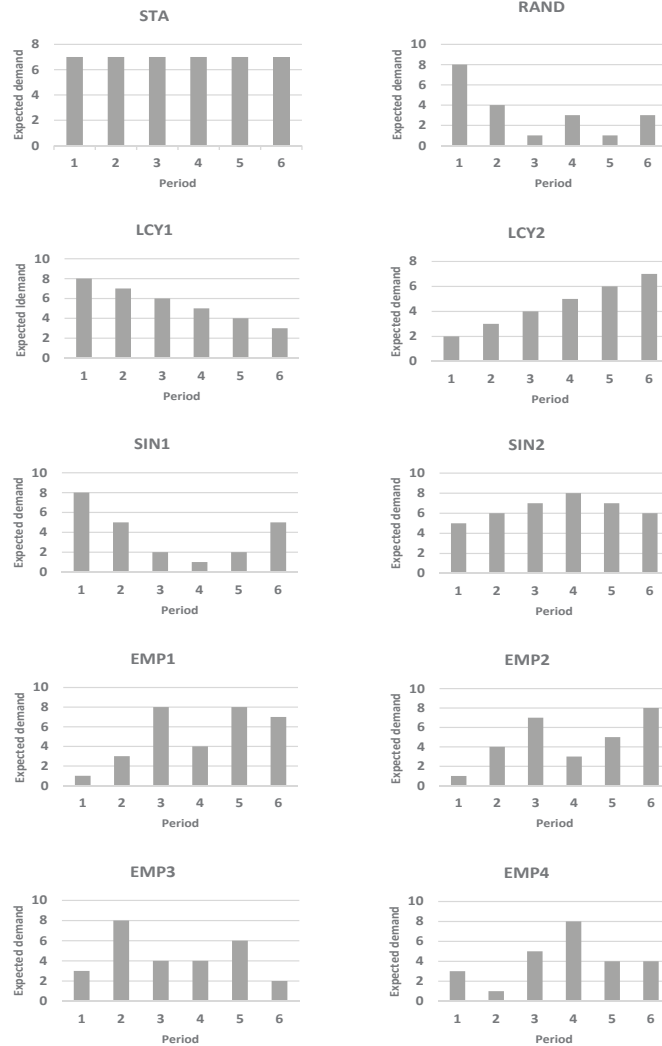


Figure 2. Demand patterns in our computational analysis.

In our tests, demands follow Poisson distribution. Two possible levels are set for each parameter as listed by Table 14, except unit holding cost h , which is fixed to be 1. The values of parameters are set to make most numerical instances encounter capital shortage in the first three periods. For each set of those parameters combinations, 10 demand patterns are tested. Therefore, There are 640 numerical cases in total.

Table 14. Parameter values.

	B_0	p	a	v	π	b
level 1	0	5	10	1	2	0.05
level 2	20	10	15	2	4	0.2

8.2 GA Stability test

We assess stability of solutions obtained by genetic algorithm on random samples by analysing in-sample and out-of-sample stability. In-sample stability means the final capital increment of genetic algorithm should not vary across demands scenarios of the same size. Out-of-sample stability refers to that when the solutions are simulated on the benchmark demands, the expected final capital increment should be same among all demands scenarios (Kaut et al. 2007).

We randomly select 32 cases from the test beds to do stability tests. For a given size of scenarios for each case, we run genetic algorithm for 10 different samples. Like Sexton, Dorsey, and Johnson (1998), mean of standard deviation (STD) and mean of root mean square error (RMSE) of the 32 cases are adopted as the stability criteria. Results of stability tests are presented by Table 15 and Figure 3.¹

Table 15. In-sample and out-of-sample tests results

scenarios	In-sample test								Out-of-sample test							
	Mean STD				Mean RMSE				Mean STD				Mean RMSE			
	$(s\bar{Q}S)$	(sS)	(RS)	(RQ)	$(s\bar{Q}S)$	(sS)	(RS)	(RQ)	$(s\bar{Q}S)$	(sS)	(RS)	(RQ)	$(s\bar{Q}S)$	(sS)	(RS)	(RQ)
100	0.93	1.19	1.66	2.40	3.24	2.60	6.22	11.65	1.64	1.29	1.45	1.34	3.24	2.99	5.48	11.21
500	0.70	0.87	1.21	1.49	3.94	2.77	6.26	11.74	1.40	1.06	1.17	1.15	3.39	2.80	5.41	11.14
1000	0.56	0.64	1.21	1.44	3.92	2.70	6.20	11.59	0.65	0.75	1.19	1.35	2.53	2.65	5.36	10.97
1500	0.55	0.65	1.32	1.52	4.09	2.84	6.24	11.78	0.52	0.72	1.31	1.35	2.35	2.63	5.37	11.21

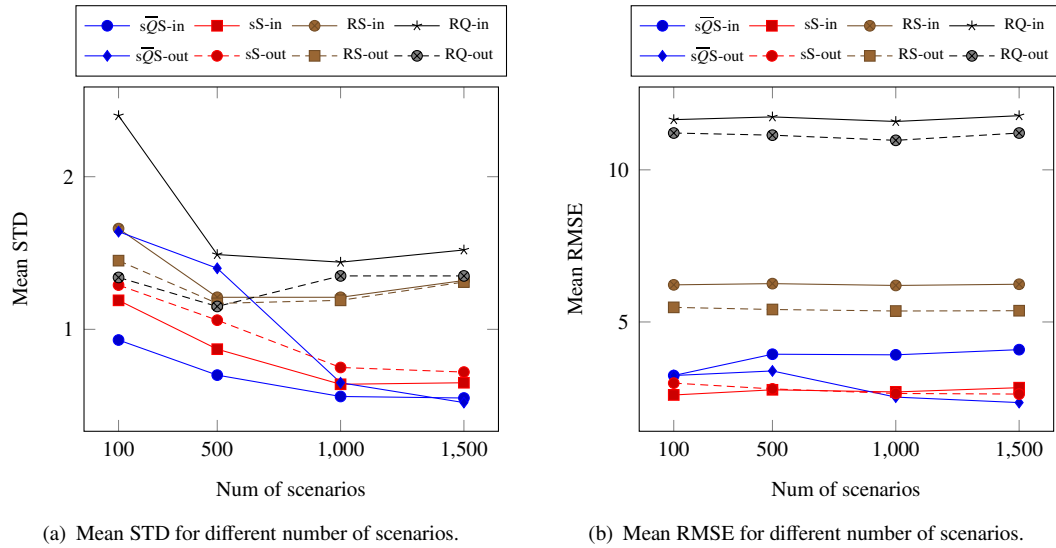


Figure 3. Stability test results for different number of scenarios.

The results of stability test show when the number of scenarios is 1000 or more, both mean STD and mean RMSE of genetic algorithm for the four controlling policies tend to be stable. Therefore, we select 1000 scenarios as the number of samples for genetic algorithm to solve our problem.

¹In Figure 3, "in" means in-sample, "out" means out-of-sample

8.3 Results and discussion

In this section we present an extensive comparison of SDP, simulation-genetic algorithm and simulation-optimization heuristic pivoting on the test bed. Results are given by Table 16 and Table 17. As performance criterion, we adopt root mean square error (RMSE) and mean absolute percentage error (MAPE) between the optimal capital increment and the simulated capital increment under each given policy. Both RMSE and MAPE are employed because RMSE does not show as clearly as MAPE on how close simulated results are to optimal, while MAPE may result in some high percentage errors when optimal capital increments are near 0 even though optimal value and simulated values are close. The simulation confidence level is 95%; details about the confidence intervals in the computational study for RMSE and MAPE are provided in Appendix 1.

Table 16. Pivot table for the computational study—RMSE.

	Root Mean Square Error (RMSE)					Cases
	Sim-opt	GA- $s\bar{Q}S$	GA- sS	GA- RS	GA- RQ	
Ini capital						
0	14.30	2.31	2.23	6.64	16.20	320
20	12.80	3.96	3.88	5.89	15.60	320
Interest rate						
0.05	13.60	3.24	3.22	6.24	15.40	320
0.2	13.61	3.25	3.11	6.31	16.40	320
Selling price						
5	11.70	2.36	2.22	5.40	14.30	320
10	15.30	3.94	3.89	7.04	17.40	320
Fixed order cost						
10	11.00	2.87	2.83	6.26	16.40	320
15	15.80	3.59	3.47	6.29	15.40	320
Vari order cost						
1	13.70	3.31	3.29	6.24	15.10	320
2	13.50	3.18	3.04	6.32	15.70	320
Penalty cost						
2	13.90	3.79	3.68	5.46	15.40	320
4	13.30	2.58	2.56	6.99	16.40	320
Demand pattern						
EMP1	16.10	3.01	4.68	5.13	15.10	64
EMP2	15.40	3.19	4.32	5.63	15.20	64
EMP3	8.18	2.83	3.69	5.43	15.50	64
EMP4	16.10	3.33	2.19	7.28	15.50	64
LCY1	16.40	1.81	2.75	8.64	16.00	64
LCY2	18.60	1.74	3.26	4.44	15.20	64
RAND	11.40	4.37	1.64	8.57	15.50	64
SIN1	10.60	4.06	1.82	8.78	15.50	64
SIN2	12.20	4.55	3.19	7.38	15.70	64
STA	4.06	2.04	2.84	7.11	15.60	64
General	13.60	3.25	3.17	6.28	15.90	640
Aver. Time(s)	0.04	194.39	184.88	42.31	46.91	

Table 16 and Table 17 both show that policies (s, S) and (s, \bar{Q}, S) solved by genetic algorithm, in general perform better than other approaches (RMSE: 3.17 and 3.25, respectively; MAPE: 5.68% and 5.59%, respectively), followed by policy (R, S) (RMSE: 6.28, MAPE: 26.72%), simulation-optimization heuristic (RMSE: 13.60, MAPE: 53.67%) and policy (R, Q) (RMSE: 15.90, MAPE: 66.63%). Considering the confidence levels, performance of policy (s, S) and policy (s, \bar{Q}, S) are essentially identical. For the four controlling policies, it can be concluded that their performance is related with their flexibility. Since policy (s, S) and policy (s, \bar{Q}, S) are based on "dynamic uncertainty" strategy, which is most flexible, they perform best for the problem, while the least flexible policy (R, Q) has worst performance. It is however surprising that enforcing a maximum order quantity \bar{Q} does not seem to be beneficial, and that an (s, S) policy

Table 17. Pivot table for the computational study—MAPE.

	Mean Absolute Percentage Error (MAPE)					Cases
	Sim-opt	GA- $s\bar{Q}S$	GA- sS	GA- RS	GA- RQ	
Ini capital						
0	67.31%	4.42%	4.78%	31.43%	74.77%	320
20	40.04%	6.95%	6.39%	22.02%	58.49%	320
Interest rate						
0.05	44.57%	4.84%	5.02%	22.42%	50.05%	320
0.2	62.78%	6.53%	6.16%	31.02%	83.22%	320
Selling price						
5	98.88%	9.37%	9.16%	49.39%	123.30%	320
10	8.47%	2.00%	2.02%	4.06%	9.97%	320
Fixed order cost						
10	33.64%	3.37%	3.93%	15.51%	42.06%	320
15	73.71%	8.00%	7.25%	37.94%	91.20%	320
Vari order cost						
1	16.14%	2.77%	2.8%	7.77%	19.06%	320
2	91.21%	8.60%	8.37%	45.67%	114.20%	320
Penalty cost						
2	34.43%	5.00%	4.91%	15.15%	40.17%	320
4	72.92%	6.37%	6.27%	38.30%	93.09%	320
Demand pattern						
EMP1	2.52%	1.76%	1.68%	6.06%	19.88%	64
EMP2	104.25%	10.29%	9.67%	18.82%	63.27%	64
EMP3	26.38%	2.44%	2.25%	11.18%	31.43%	64
EMP4	6.82%	2.39%	2.20%	8.19%	21.95%	64
LCY1	92.92%	5.15%	4.51%	61.07%	123.76%	64
LCY2	31.74%	7.00%	5.00%	89.91%	193.40%	64
RAND	29.04%	5.14%	6.01%	5.48%	23.29%	64
SIN1	39.55%	7.72%	8.08%	10.54%	35.18%	64
SIN2	23.44%	8.02%	6.73%	11.13%	46.73%	64
STA	180.06%	6.99%	9.56%	46.15%	107.43%	64
General	53.67%	5.68%	5.59%	26.72%	66.63%	640
Aver. Time(s)	0.04	194.39	184.88	42.31	46.91	

with parameters carefully selected seems to provide competitive performances. Conversely, a simulation-optimization heuristic similar to that originally proposed by Silver and Meal (1973) — which proved to be effective when the item is perishable (Gutierrez-Alcoba et al. 2017) — performs poorly under capital flow.

The performance of different approaches does not seem to be affected by different parameter levels under the criterion RMSE; however, it is affected by the margin of product — selling price and unit variable ordering cost under the MAPE criterion. Finally, the performance of the simulation-optimization heuristic varies substantially across different demand patterns.

In terms of computation times, the simulation-optimization heuristic runs faster than genetic algorithm, with average computation time less than one second (0.04s). Among the policies solved via genetic algorithm, policy (R, S) runs fastest (42.31s), followed by policy (R, Q) (46.91s), policy (s, S) (184.88s), policy (s, \bar{Q}, S) (194.39s).

9. Conclusion

In this paper, we discussed a stochastic lot sizing model considering capital flow and business overdraft. In addition to obtaining optimal solutions via stochastic dynamic programming, we presented four controlling policies whose parameters have been computed via genetic algorithm, and a simulation-optimization heuristic. Our computational study demonstrate that policies (s, S) and (s, \bar{Q}, S) feature the smallest RMSE

and MAPE with respect to the optimal solutions obtained by stochastic dynamic programming, followed by policy (R, S) , simulation-optimization heuristic and policy (R, Q) . Simulation-optimization heuristic is, however, the fastest approach. Numerical examples illustrate that capital availability, sales contribution margin and interest rates may substantially impact the optimal policy structure. These factors should be therefore taken into account by managers when making lot sizing decisions.

Future research could proceed in several directions. When banks provide business overdraft, there is usually an upper limit for the borrowing capital in each period or floating interest rates for different amounts of borrowing capital; this is an extension that is worth investigating. Other extensions may consider different financing behaviors, such as trade credit, long term loans, and leasing. Finally, the optimal policy structure under capital flow constraints may also be explored.

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Appendix A. Confidence levels for the computational study.

Table A1 provides details of confidence levels in our computational study under criterion RMSE and Table A2 gives the confidence intervals under criterion MAPE.

Table A1. Details of confidence levels for the computational study—RMSE.

	0.95 Confidence intervals—RMSE					Cases
	Sim-opt	GA-sQS	GA-sS	GA-RS	GA-RQ	
Ini capital						
0	[13.30,15.60]	[2.15,2.50]	[2.07,2.42]	[6.16,7.20]	[15.08,17.60]	320
20	[11.89,13.90]	[3.68,4.29]	[3.60,4.21]	[5.47,6.38]	[15.46,16.90]	320
Interest rate						
0.05	[12.61,14.70]	[3.01,3.51]	[2.99,3.49]	[5.79,6.76]	[14.34,16.70]	320
0.2	[12.63,14.80]	[3.02,3.52]	[2.89,3.37]	[5.86,6.84]	[15.19,17.70]	320
Selling price						
5	[10.85,12.70]	[2.19,2.56]	[2.06,2.41]	[5.01,5.85]	[13.29,15.50]	320
10	[14.17,16.60]	[3.65,4.27]	[3.61,4.21]	[6.53,7.63]	[16.11,18.80]	320
Fixed order cost						
10	[10.23,11.90]	[2.66,3.11]	[2.63,3.07]	[5.81,6.78]	[15.20,17.70]	320
15	[14.63,17.10]	[3.33,3.89]	[3.22,3.76]	[5.84,6.82]	[14.33,16.70]	320
Vari order cost						
1	[12.68,14.80]	[3.08,3.59]	[3.05,3.57]	[5.79,6.76]	[14.04,16.40]	320
2	[12.65,14.70]	[2.95,3.44]	[2.82,3.29]	[5.86,6.85]	[16.46,18.10]	320
Penalty cost						
2	[12.86,15.00]	[3.52,4.11]	[3.41,3.98]	[5.07,5.92]	[14.34,16.70]	320
4	[12.38,14.50]	[2.40,2.80]	[2.37,2.77]	[6.49,7.58]	[15.19,17.70]	320
Demand pattern						
EMP1	[13.76,19.50]	[2.57,3.64]	[3.99,5.66]	[4.37,6.20]	[12.90,18.30]	64
EMP2	[13.12,18.60]	[2.73,3.86]	[3.68,5.22]	[4.80,6.81]	[12.92,18.30]	64
EMP3	[6.98,9.90]	[2.41,3.42]	[3.15,4.46]	[4.63,6.57]	[13.22,18.70]	64
EMP4	[13.75,19.50]	[2.84,4.03]	[1.87,2.65]	[6.21,8.80]	[12.95,18.40]	64
LCY1	[14.01,19.90]	[1.54,2.18]	[2.35,3.33]	[7.37,10.40]	[13.67,19.40]	64
LCY2	[15.88,22.50]	[1.49,2.11]	[2.78,3.94]	[3.78,5.37]	[12.94,18.30]	64
RAND	[9.73,13.80]	[3.73,5.29]	[1.39,1.98]	[7.31,10.40]	[13.25,18.80]	64
SIN1	[9.05,12.80]	[3.46,4.91]	[1.55,2.20]	[7.49,10.60]	[13.20,18.87]	64
SIN2	[10.42,14.80]	[3.88,5.50]	[2.72,3.86]	[6.29,8.92]	[13.42,19.00]	64
STA	[3.47,4.91]	[1.74,2.47]	[2.42,3.43]	[6.06,8.60]	[13.58,19.30]	64
General	[12.89,14.40]	[3.07,3.44]	[3.00,3.35]	[5.95,6.64]	[15.09,16.80]	640

Table A2. Details of confidence levels for the computational study—MAPE.

	0.95 Confidence intervals—MAPE					Cases
	Sim-opt	GA-sQS	GA-sS	GA-RS	GA-RQ	
Ini capital						
0	±36.63%	±1.26%	±2.00%	±15.36%	±32.11%	320
20	±14.58%	±1.87%	±1.39%	±12.49%	±32.08%	320
Interest rate						
0.05	±16.44%	±1.52%	±1.13%	±10.62%	±17.70%	320
0.2	±35.86%	±1.94%	±2.17%	±16.69%	±41.74%	320
Selling price						
5	±38.83%	±2.19%	±2.38%	±19.49%	±44.55%	320
10	±0.49%	±0.11%	±0.10%	±0.18%	±0.33%	320
Vari order cost						
1	±2.25%	±0.30%	±0.27%	±0.94%	±1.65%	320
2	±38.98%	±2.20%	±2.39%	±19.57%	±44.77%	320
Penalty cost						
2	±15.23%	±1.05%	±0.94%	±8.92%	±19.49%	320
4	±36.30%	±2.01%	±2.26%	±17.59%	±40.81%	320
Demand pattern						
EMP1	±0.58%	±0.13%	±0.15%	±1.86%	±4.23%	64
EMP2	±63.98%	±5.53%	±5.31%	±10.09%	±33.15%	64
EMP3	±8.11%	±0.30%	±0.21%	±3.33%	±8.90%	64
EMP4	±0.97%	±0.37%	±0.34%	±3.52%	±5.96%	64
LCY1	±72.12%	±3.04%	±1.94%	±48.27%	±100.05%	64
LCY2	±14.39%	±3.26%	±2.00%	±75.57%	±170.44%	64
RAND	±10.12%	±1.78%	±2.32%	±1.31%	±8.03%	64
SIN1	±17.08%	±5.98%	±3.89%	±4.35%	±13.87%	64
SIN2	±9.49%	±4.44%	±3.16%	±3.79%	±17.49%	64
STA	±169.98%	±4.38%	±9.23%	±40.71%	±105.10%	64
General	±19.69%	±1.13%	±1.22%	±9.88%	±22.64%	640