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Regional Mapping of the Geoid Using GNSS (GPS) Measurements and an Artificial Neural Network

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Abstract: The determination of the orthometric height from geometric leveling has practical difficulties that, despite a number of scientific and technological advances, passed a century without substantial modifications or advances. Currently, the Global Navigation Satellite System (GNSS) has been used with reasonable success for orthometric height determination. With a sufficient number of benchmarks with known horizontal and vertical coordinates, it is often possible to adjust using the least squares method mathematical expressions that allow interpolation of geoid heights. The objective of this study is to present an alternative method to interpolate geoid heights based on the technique of Artificial Neural Networks (ANNs). The study area is the Brazilian state of São Paulo, and for training the ANN the authors have used geoid height information from the EGM08 gravity model with a grid spacing of 10 minutes of arc. The efficiency of the model was tested at 157 points with known geoid heights distributed across the study area. The results were also compared with the Brazilian Geoid Model (MAPGEO2004). Based on those 157 benchmarks it was possible to verify that the model generated by ANNs provided a mean absolute error of 0.24 m in obtaining a geoid height value. Statistical tests have shown that there was no difference between the means from known geoid heights and geoid heights provided by the

neural model for a significance level of 5%. It was also found that ANNs provided an improvement of 2.7 times in geoid height estimates when compared with the MAPGEO2004 geoid model.

1. Introduction

Artificial Neural Networks (ANNs) are clusters of processing units (known as neurons or nodes) interconnected and structured, whose operation is analogous to a neural structure from intelligent organisms [1]. ANNs derive their computing power from their distributed massively parallel structure and their ability to learn and generalize, making possible the resolution of complex problems in different knowledge areas [2].

The operation of ANNs is inspired by the human brain [2]; though ANNs have been used successfully in a variety of different application areas. According to [3], due to their non-linear structure, ANNs can represent more complex features from data, which is not always possible using statistical techniques or traditional deterministic methods. For [1], the major advantage of ANNs over conventional methods is that there is no need to know the intrinsic theory of the problem, nor the necessity to analyze the relationships that are not fully known among the variables involved in modeling.

In the geosciences area, ANNs have been used to model complex phenomena involving variables difficult to obtain. However some ANN applications involve easily obtainable variables for the solution of problems, but which are usually difficult to solve using conventional mathematical methods. In evapotranspiration and surface temperature modeling [4-10]; geophysics in lithological classification [11-17]; soil science—through the concept of pedotransfer function, which consist in modeling a problem with easily obtainable laboratory variables—there has been growing interest in ANN applications [18-20]. In the case of GPS, for example, a neural model to estimate the L_1 and L_2 carrier waves using as input information pseudorange data from the RBMC (Brazilian Network for Continuous Monitoring) was proposed [21]; and ANNs have also been used to estimate the value of TEC (Total Electron Content) using data from continuous monitoring GPS stations [22].

The geoid shape is directly related to the Earth gravity field. However, the ellipsoid is a mathematical surface with shape and dimensions close to that of the geoid and used as the reference surface for geodetic survey calculations. The geoid and ellipsoid surfaces are not coincident or parallel, and the separation between them is called 'geoid height', which can reach 100 m in some parts of the world. The relative inclination from these surfaces in extreme cases is of the order of 1' (one arc-minute) [23].

The importance of precise geoid modeling has increased in the last years with the advent of satellite positioning systems (such as GNSS). As is well known, the heights provided by GNSS are referred to the reference ellipsoid instead of the geoid. To convert these ellipsoidal heights to orthometric heights knowing the relation between the geoid and ellipsoid surfaces is necessary.

There are different methods to determine the geoid based on observed gravity data. Among these methods mention can be made of the application of Stokes' integral [24], the method of least squares collocation or Fourier transform [25]. These geoid determination methods are based on functional and/or stochastic relationships between gravity anomalies and geoid height. Some authors performed studies using ANNs for geoid modeling [26-29], with satisfactory results.

Seager *et al.* [28] using an ANN of back-propagation type, modeled the geoid heights in a certain region in Australia, obtaining an average error of 0.166 m and a maximum error of 0.711 m. Tierra *et al.* implemented an ANN method of free-air anomaly interpolation in a region in Ecuador and the results were found to be better than using kriging [29]. A similar study was conducted by [27] in an area of $3° \times 3°$ in the city of Imbituba, in the Brazilian state of Santa Catarina. The results obtained by free-air anomaly estimation were similar to using kriging interpolation. Maia [26] developed a neural model to obtain geoid heights with errors less than 0.5m in a region in the state of S ão Paulo, bounded by latitude $−19°$ to $−26°$ and longitude $−54°$ to $−44°$, whose variables were latitude, longitude and free-air anomaly from the EGM96 model [30].

Reinke *et al.* modeled the geometric geoid employing easily obtainable information [31]. This work was developed in the metropolitan region of the Brazilian city of S ão Paulo and data employed in ANNs training were input UTM coordinates, and in the output geoid height in the MAPGEO2004 system. The developed ANN was tested at 21 vertices with known geoid heights and the mean squared error that was obtained was 0.10 m.

The objective of this study was to evaluate the efficiency of an ANN of the back-propagation type for geoid height interpolation in the Brazilian state of S ão Paulo. The training was of the supervised type with geoid height information (on a 10 by 10 arc-minute grid) from the EGM08 model [32], validated on 157 benchmarks with known geoid heights distributed uniformly across the whole state.

2. Data and Methodology

For a better understanding of the method employed in this work it is necessary to introduce some important concepts.

2.1. Structure of Neural Networks

ANNs are used in areas as diverse as neurosciences, mathematics, statistics, physics, computer sciences and engineering; and their main objective is to solve complex problems based on the structure and functioning of the human brain. Also known as conexionism or distributed or parallel processing systems, ANN is an alternative technique to conventional algorithmic computing because it is not based on rules [2].

An ANN can be considered as a set of artificial neurons with local processing capacity, a connection topology that defines how these neurons are connected, and a learning rule [15].

The artificial neuron is the information processing unit—the fundamental element for the operation of the ANN—but still primitive if compared to those found in the brain [2]. The artificial neurons, as well as the biological neurons, have input connections (dendrites), output connections (axons) and an internal process that generates an output signal in response to the input signal.

The artificial neurons (Figure 1) are formed by:

- Input signals (x_1, x_2 and x_m) or input information, which might come from the environment or from the activation of other neurons.
- A set of weights (w_{k1}, w_{k2}, w_{km}), which describe the connection forces; that can be positive, representing excitatory junctions; or negative, inhibiting the activation of the neuron. When there is no connection between two neurons the synaptic weight is null.
- Sum function (Σ), which represents the summation of the input signals multiplied by their respective weights, constituting a linear combiner.
- Activation function [$\varphi(.)$], which restricts the output amplitude of the neuron, in an interval normalized between [0;1] or [−1,1].
- Output signal (y_k), which is the result generated by the neuron.

Figure 1. Structure of artificial neurons used in the Artificial Neural Network (ANN) [2].
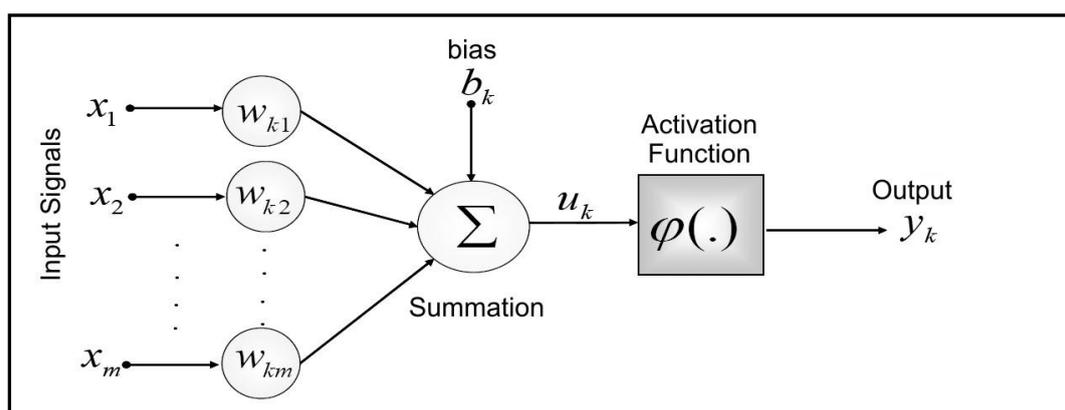
Every piece of input information has an associated weight, also known as the synaptic weight, which mathematically represents its degree of importance for that neuron [15]. The input signals of the neurons are multiplied by their synaptic weight, and the summation of this result added to the *bias* forms the input information of the neuron:

$$u_k = \sum_{j=1}^{n} w_{kj} \cdot x_j + b_k, \tag{1}$$

where x_j are the input signals; w_{kj} is the synaptic weight of the neuron k; b_k is the bias; and u_k is the linear combiner output of the model. The bias is a parameter external to the neuron; and its function is to avoid the emergence of errors when the input data is null. In addition, it increases or decreases the net input of the activation function by being either positive or negative [2].

The output value of the linear combiner (u_k or v_k) will be compared with the limit value for the neuron activation. If the value is higher, the neuron will activate, otherwise it will remain inactive. The activation function of internal order of the neuron, is increasing, continuous and working as a restraining element to the output amplitude of the neuron [15]. The output of a neuron in terms of the induced local field (or activation potential), defined by the activation function, has a normalized amplitude in the interval [0;1] or [−1.1] [2].

Each neuron has an activation function associated with it, responsible for the intensity of the signal to be transmitted by the connections to the neurons of the adjoining layers [2]. The linear and sigmoid

activation functions are most frequently used. The linear activation function is the one that does not limit the network output and is used only to store data input and output. The linear function is presented as:

$$\varphi(v) = k.v \tag{2}$$

where k is a constant.

The sigmoid activation function is the most popular, and assumes a continuous value interval between 0 and 1. In addition it has an adequate balance between linear and non-linear behavior [2].

The sigmoid activation functions are used in the Multilayer Perceptron [2]. The logistic function, described by Equation (3) and the hyperbolic tangent function represented by Equation (4), are also examples of sigmoid functions [2]. The difference between them is that the logistic function adopts values between 0 and 1, whereas the hyperbolic tangent function adopts values between −1 and +1.

$$\varphi(v) = \frac{1}{1 + \exp(-a \cdot v)} \tag{3}$$

$$\varphi(v) = \tanh(v) \tag{4}$$

in which v_k is the linear combiner output of the model and a is the inclination parameter of the function.

Among the different ANN models, the Multilayer Perceptron Model (MLP) is particularly popular. In the MLP there is an input layer (those neurons receive external stimulation), one or more intermediary layers, and the output layer (which provides the network result). The structure of such an ANN is indicated in Figure 2.

Figure 2. Multilayer Perceptron Model (MLP) network example.
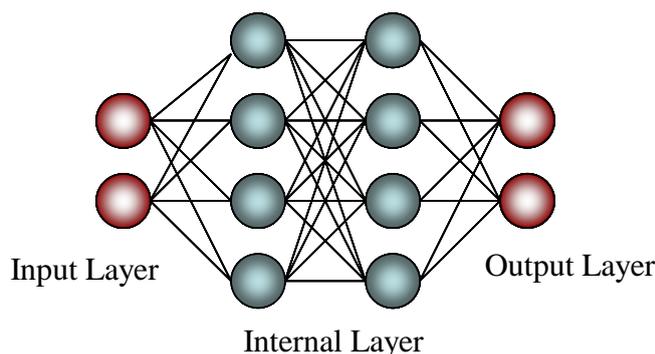
The lack of internal layers in a MLP network limits its application to the treatment of linearly separable problems. Only by adding at least one intermediary layer does it become possible to treat non-linear problems. The internal layers, inherent to the MLP model, transform the problem described by the set of network input data into a representation suitable for the output layer [2].

Error back-propagation is an algorithm for supervised ANN training (when the network output values are known) which has been often used for the solution of complex problems [2,15]. The learning of a network basically consists of an adjustment of the synaptic weights in order to match the knowledge acquired by the network. In simple terms, the algorithm works by comparing the acquired output for a determined input pattern with the desired output (value provided to the network during training) and, in a second phase, adjusts the network weights to match the obtained errors. This process

repeats until a predefined acceptable error is obtained (average quadratic error) according to Equation (5) or the maximum number of defined iterations is reached [15].

$$J = \frac{1}{2} \sum_{i=1}^{p} \left(y_d^i - \left(w \cdot x^i \right) \right)^2 , \tag{5}$$

where x^i is the input information, y_d^i is the output information, and w is the synaptic weight.

MLP with supervised training—using the back-propagation algorithm—has been successfully employed in solving several problems [16]. In some applications, such as complex problems involving large networks, the training process using the back-propagation algorithm is slow. Because of this limitation several modifications have been proposed to accelerate the training time and to increase its performance. The most employed variations of the back-propagation algorithm are: back-propagation with momentum, Quickprop, Levemberg-Marquardt, second order momentum, Newton, and Resilient Back-propagation. More details about those algorithms can be found in [2].

For geoid modeling the most employed algorithm is the Levemberg-Marquardt variety. Training by Levenberg–Marquardt results in the network being slower to perform more calculations, but with a reduction in the number of iterations [8,26,31].

The development of an ANN involves the training and validation stages. Network training consists of calculating the outputs from input data based on weight adjustments. The validation stage consists of providing the trained network with a set of input-output pairs, different from those used during the learning process [15].

In order to obtain good results in the training and validation stages, the input and output data must undergo some preprocessing, which consist mainly of data normalization. Data normalization is required so as to prevent variables with values of a higher magnitude order from dominating or distorting the network weights. The most common normalization intervals are [0;1] and [−1;+1], and the interval definition depends on the type of activation function to be employed (linear, sigmoid, *etc*).

After the validation and training stages, the network can be used to generate responses to other information sets, since all information of the training set are stored in the neurons after the adjustment of their synaptic weights.

The implementation of the ANN to estimate geoid heights was based on criteria established by [9], where the author specifies that the definition of the best ANN framework is empirical, via a trial and error process where tested input variables, number of intermediary layers, number of neurons per layer, the learning algorithm and the number of training cycles, are each trialed. Among the many possibilities the ANN with the smallest error in the training and validation stages is the one with the most probable structure to solve the problem which is being modeled.

The network topology employed in this study belongs to the class of multiple layer networks. In the input layer, the number of neurons was defined using geodesics (latitude and longitude). The number of intermediary layers and neurons in each layer was also defined through experiments. Successive refinements were performed to obtain the final structure of the most adequate network, that is, the one showing a good performance in the training stage and a good generalization capacity, evaluated by the network performance in the validation stage. The output layer was formed by only one neuron, having as output information the geoid height calculated from EGM08 [32].

2.2. Geoid Height for Training and Simulation of Neural Network

The study area is São Paulo, as presented in Figure 3, where all benchmarks used in ANN simulation have GPS coordinates in the SIRGAS (Geocentric Reference System for the America) reference system. Geoid heights are from Brazilian Geography and Statistic Institute (IBGE) database, which has an accuracy of ±0.10 m [33].

Figure 3. Study area and distribution of GPS leveling points.
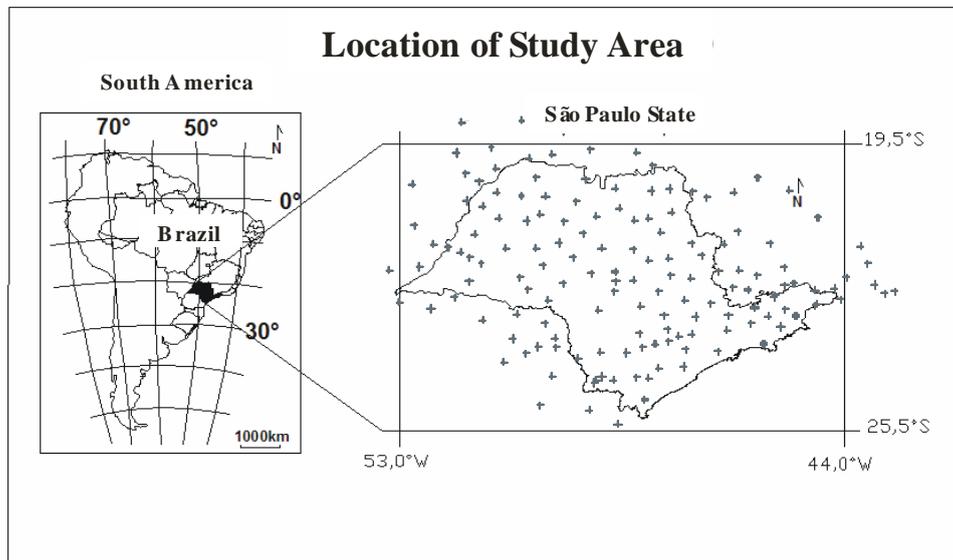
For ANN training the geoid height information from the EGM08 model was used. The data generated on the $10' \times 10'$ grid consisted of 2,035 points. The EGM08 model contains the spherical harmonic coefficients of the Earth's gravity field complete to degree and order 2,159, a total of 2,401,333 coefficients [32].

The training stage was performed using the Neural Network Toolbox module of the program MATLAB 7.0.1, and the number of intermediary layers, number of neurons per layer, the learning algorithm and the number of training cycles were empirically tested until the best network topology was found.

After the training stage, the tests were conducted with 157 GPS leveling data points. The geoid heights that were obtained were compared with the known values provided by the Brazilian Geoid Model (MAPGEO2004). According to [34], MAPGEO2004 has a resolution of 10' and was computed using the modified Stokes integral and the Fast Fourier Transform technique (FFT), using the following data:

- Average Helmert Anomalies on a $10' \times 10'$ grid in continental areas, obtained from Brazilian Geography and Statistic Institute (IBGE) and other organizations in Brazil or neighboring countries.
- Free-air anomalies derived from satellite altimetry data in oceanic areas.
- Digital terrain model with $1' \times 1'$ resolution obtained from topographic map digitisation.
- The EGM96 geopotential model to degree and order 180 [33].

3. Results and Discussion

The authors performed 350 training and validation tests to determine the best topology network for estimating the geoidal height for the state of S ão Paulo. During the training and simulation stages different network topologies were tested, altering the number of intermediary layers, the number of neurons, the activation function and the training algorithm. In general, the best results were obtained with the values normalized between [0;1] and consequently with the logistic sigmoid activation function (Equation (3)) that assumes values between 0 and 1. The sigmoid activation function is the most commonly employed in geoid modeling studies [8,26,29,31], and is usually associated with training algorithms of the back-propagation type [8].

During the performance of the tests, the best performance in the training stage did not always correspond to the best results in the validation stage. This might happen when the network memorizes the training data but loses its generalization capacity, a phenomenon called 'overfitting'. This might happen because the network topology is not adequate for representing the physical complexity of the problem, or because the training set is not representative of the field [2].

The best network was obtained by using the Levenberg-Maquart learning algorithm, which is a variation of the back-propagation algorithm. Regarding the number of cycles, experiments were performed using a maximum of 1,000 cycles in the training stage. However with 600 good results were obtained in the validation stage. The ANN that exhibited the best result in the validation stage was the one of [2-2-1] type with 5 neurons in the first intermediary layer and 4 in the second intermediary layer (see Figure 4).

Figure 4. Structure of the Artificial Neural Network (ANN) used for geoid height modeling.
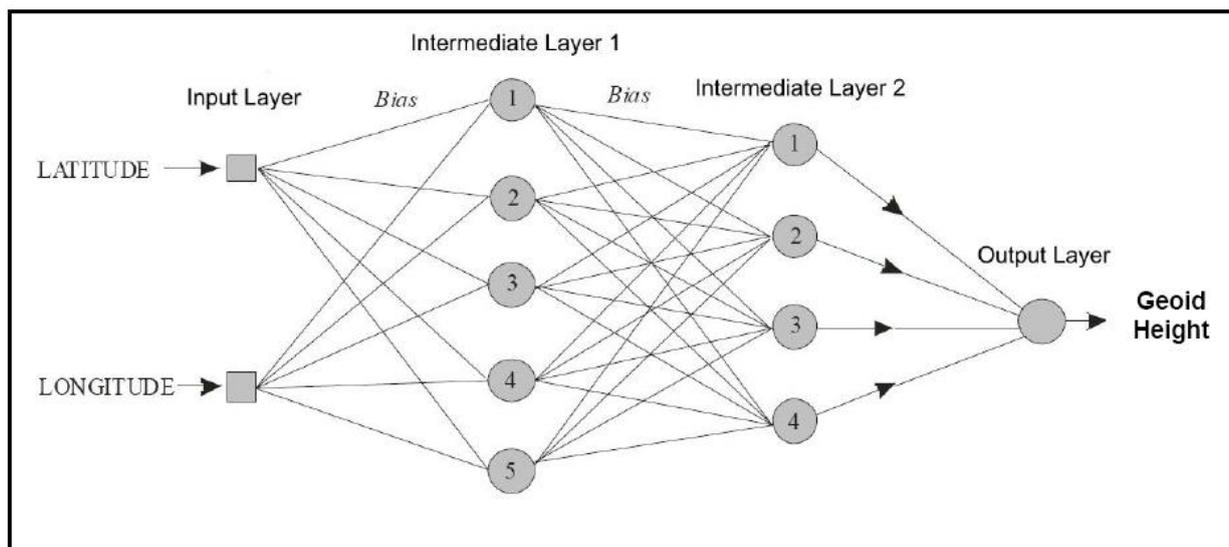
The values of known geoid heights and those obtained by the ANNs for the 157 points are plotted in Figure 5.

Figure 5. Known geoid heights and those estimated by the Artificial Neural Networks (ANNs) for the 157 GPS leveling points of the validation set.
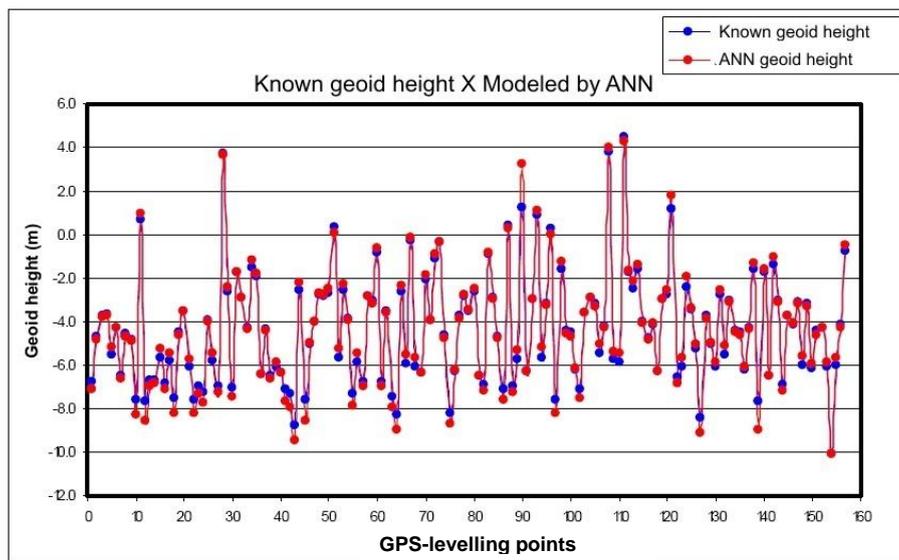
The geoid heights and those obtained by the ANNs (Figure 4) are similar. When working with neural networks it is usual to perform a linear regression to check if the ANN was able to learn with data provided in the validation stage. Almost always the determination coefficient (R^2) calculated value is close to 1, but, independent from the correlation, what is analyzed is how the dispersion of the dots happens along the adjusted straight line. For an efficiently trained neural network, the linear regression will show—in the validation stage—a very homogeneous dispersion of the dots almost coincident with the straight line.

The similarity between the values of geoid heights calculated by the ANNs (*y*) and the known values (*x*) can be seen from the results obtained with simple linear regression (Table 1). The equation obtained is of the type $y = a + bx$ and the nearer to 1 the regression determination coefficient (R^2) is, the better is the capacity of the ANN to estimate the geoid height.

Table 1. Results of the simple linear regression between known geoid heights and those estimated by the Artificial Neural Networks (ANNs).

Equation[1]	R^{2} [2]	SQR[3]
$y = 1.061x + 0.2545$	0.9865	18.86

(1) Linear regression performed for a confidence interval of 95% for the linear coefficient. (2) Determination coefficient. (3) Sum of squared differences between known geoidal heights values and geoidal heights obtained by ANNs.

The line angular coefficient (1.061) and the determination coefficient (R^2) are near the value of 1 (see Figure 6) indicating that the model generated by the ANN had a good performance in the training stage and, consequently, a good capability of estimating geoid heights during the validation process.

Figure 6. Linear regression between values from known geoid heights and those modeled by the Artificial Neural Networks (ANNs).
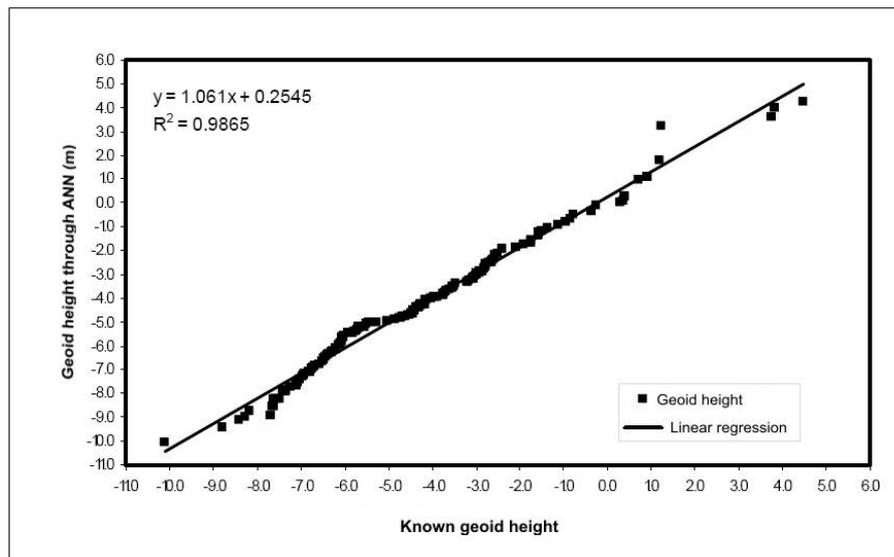

The differences (discrepancies) between known geoid heights and those obtained by the ANNs are shown in Figure 7. It can be observed that the distribution of discrepancies is more or less homogeneous, in positive and negative values. A tendency analysis using the "*t-Student*" test was performed, confirming that for a 10% significance level the neural model did not present a tendency in the discrepancies. The largest error generated by the network in the validation stage was for benchmark number 47 where the discrepancy between the known geoid height and the one determined by the ANN was 0.97 m. The RMS of the 157 GPS leveling point validation set was 18.86. A hypothesis test was conducted in order to determine if the average geoid height at the 157 GPS levelling points is the same as the average of known geoid heights. The *t-Student* test, for a 5% significance level, has shown that the averages between known geoid heights and those determined by the ANNs are statistically the same.

Figure 7. Discrepancies (differences) between known geoid heights and those obtained by the Artificial Neural Networks (ANNs) at the 157 GPS leveling points.
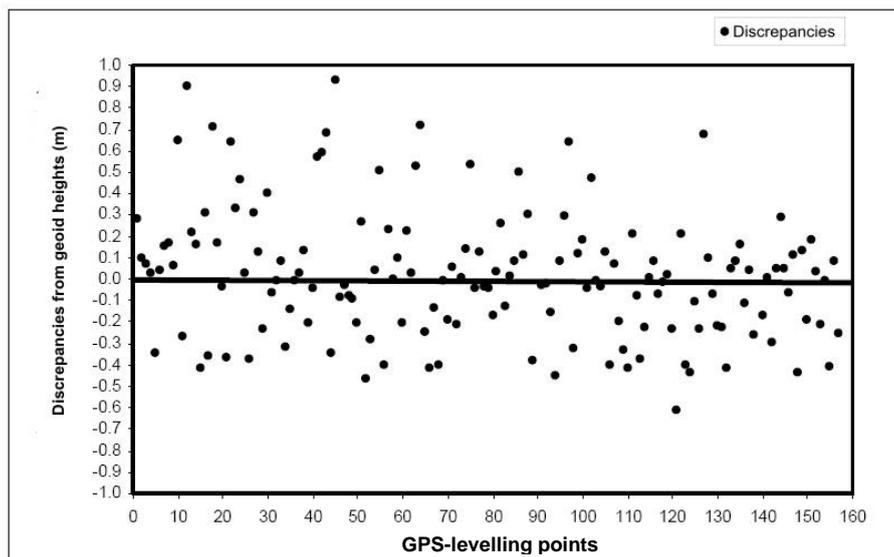
All results presented here indicate that there is a consistency of the ANN in obtaining geoid heights. The authors also analyzed how much the model represented by the ANNs is more efficient than the Brazilian Geoid Model (MAPGEO2004). The geoid height values obtained from the 157 GPS levelling points were compared with the known geoid heights (Figure 8) and it was observed that the results were less consistent than the results obtained by the ANNs.

Figure 8. Known geoid heights and those estimated by MAPGEO2004 for the 157 GPS leveling points of the validation set.
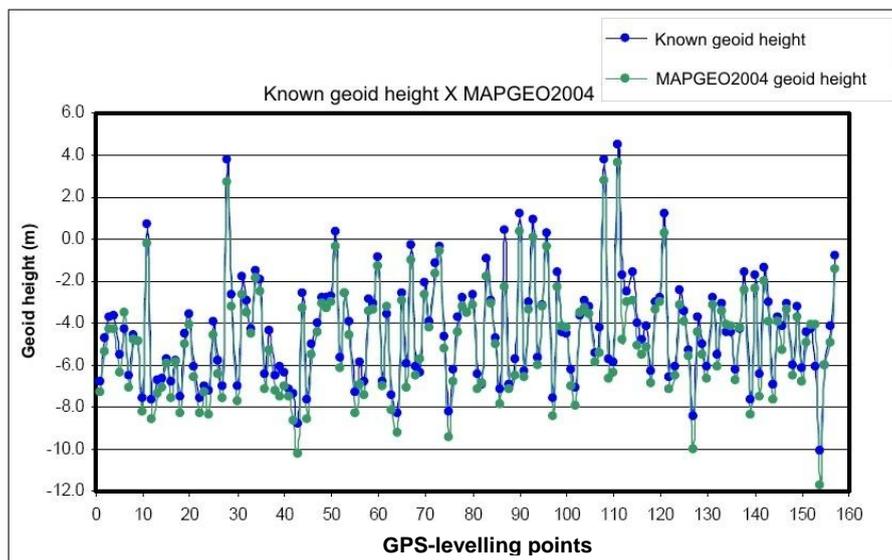
The linear regression analysis (Table 2) between known geoid heights and geoid heights obtained by MAPGEO2004 has demonstrated that MAPGEO2004 might have a systematic error for geoid heights for the state of S ão Paulo.

Table 2. Results of the simple linear regression between known geoid heights and those estimated by the Brazilian geoidal model (MAPGEO2004).

Equation[1]	R^{2} [2]	SQR [3]
y = 0.9918x − 0.6087	0.9607	92.39

(1) Linear regression performed for a confidence interval of 95% for the linear coefficient. (2) Determination coefficient. (3) Sum of squared differences between known geoidal heights values and geoidal heights obtained by MAPGEO2004.

By analyzing only the determination coefficient (R^2) in Table 2 it is not so evident that the model represented by the ANNs is more efficient than MAPGEO2004. However, when an analysis is made of the differences between the known geoid heights and the ones obtained by MAPGEO2004 it can be seen that the Sum of Squared Differences (SQR) has increased compared to the model of the ANNs. This fact can be explained because MAPGEO2004 has been found to have a systematic error in all Brazilian regions [8,26,31]. This fact can also be confirmed from the linear regression plot in Figure 9.

Figure 9. Linear regression between values from known geoid heights and those modeled by MAPGEO2004.
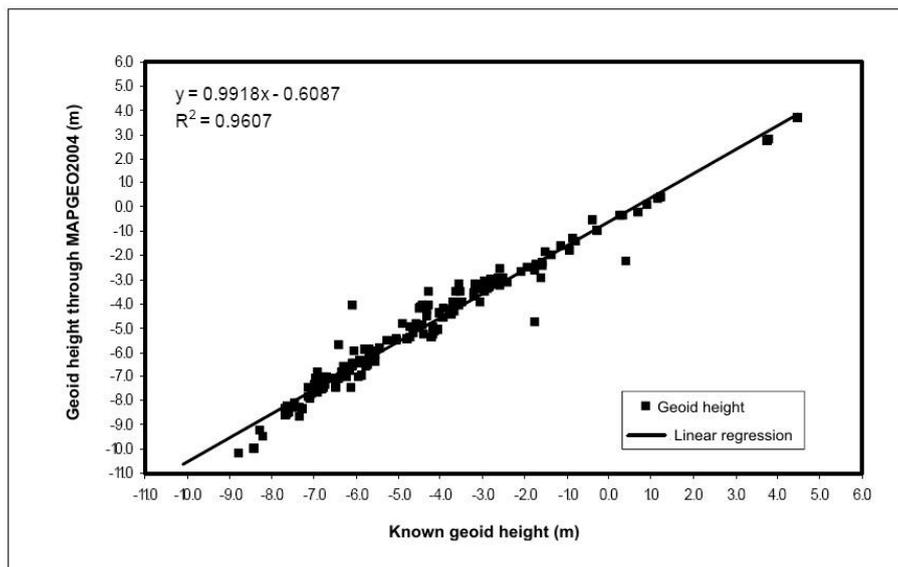
Unlike the linear regression obtained by the ANNs model, the geoid height dispersions are less uniform along the adjusted line. To better illustrate this fact, see the dispersion graph in Figure 10, displaying the differences between the known geoid heights and the ones obtained by MAPGEO2004. A systematic error in the Brazilian geoid model for the state of S ão Paulo can be clearly seen.

Figure 10. Discrepancies (differences) between known geoid heights and those obtained by MAPGEO2004 at the 157 GPS leveling points.
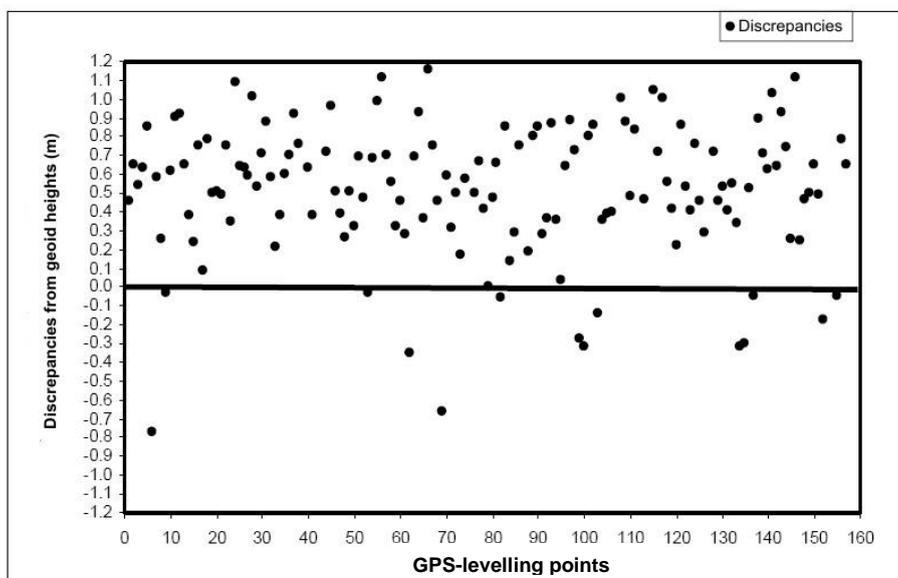
The discrepancies between known geoid heights and those obtained by MAPGEO2004 had a higher concentration of positive values than negative values. From a tendency analysis using the "*t-Student*" test it was observed that, for a 10% significance level, MAPGEO2004 exhibited a tendency in the

results, already confirmed in Figures 9 and 10, which confirms a systematic error in geoid height from this model for the state of São Paulo.

The same test was applied to the ANNs model and no tendency for a systematic error was observed. The systematic error detected in the MAPGEO2004 might be associated with data errors such as: systematic errors in Helmert average anomalies in continental areas obtained from gravimetric information from the Brazilian Geography and Statistic Institute, systematic errors in the digital model of the terrain obtained from the topographic charts, or errors in the anomalies derived from satellite radar altimetry in oceanic areas.

In absolute values, the average geoid height discrepancy obtained by the ANNs was 0.24 m, with a standard deviation of ±0.26 m; while MAPGEO2004 gave an average discrepancy of 0.64 m, with a standard deviation of ±0.42 m. From these results it can be seen that there was a significant improvement, with an error reduction of approximately 2.7 times.

The superiority of the ANNs, with respect to MAPGEO2004, in estimating geoid heights might be attributed to the fact that ANNs can approximate some relations between the input and output variables that many times are unknown or not perceived. Furthermore, the ANNs are less vulnerable to approximate data and/or the presence of distorted data.

4. Conclusions

This paper proposes a geoid height determination method for the Brazilian state of São Paulo based on an ANN trained in a supervised manner using information from the EGM08 model. Data from a $10' \times 10'$ grid were employed, a total of 2035 points. The variables involved in the ANN model were positional information such as ANN input (latitude and longitude), and geoid height was the ANN output.

The network with the best performance in the training and validation stages was obtained employing the Levemberg-Marquardt algorithm (which is a variation of the back-propagation algorithm), the logistic sigmoid activation function in all layers and data interval normalized between [0;1]. The training stage was concluded with 600 cycles. The network structure was formed by two intermediary layers, the first formed by 5 neurons and the second by 4 neurons.

To validate the model the authors employed 157 GPS leveling points with known geoid heights distributed uniformly across the state of São Paulo. The ANN model was also found to be more efficient for obtaining geoid heights in comparison to the Brazilian Geoid Model (MAPGEO2004). Comparing the absolute errors generated by the artificial network model and by MAPGEO2004, on average, the proposed method improved the estimated geoid height accuracy by a factor of approximately 2.7.

Linear regression analysis has shown that the model generated by the ANN had a good performance in the training stage and, consequently, a good capacity in estimating geoid heights during the validation process. The *t-Student* test, for a significance level of 5%, has shown that the averages of the known geoid heights and those determined by the ANNs for the 157 GPS leveling points were statistically the same.

Using the *t-Student* test, with a significance level of 10%, it was observed that the ANN model did not have a systematic error tendency. On the other hand, MAPGEO2004 has an error tendency that might be due to the information employed in its development.

This is a preliminary study, and new experiments are being performed in order to improve the ANN efficiency in the geoid height determination process. Thus, new neural models are being evaluated, including the thickening of the grid of points. In the network simulation and training stages, new algorithms are being tested.

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