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Upper bounds for the resultant and Diophantine applications

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for $l \geq 5$, where

$$(r) = \frac{D}{2r} \log(2D_F) + \frac{rd}{2} (2m \log l + \log(2D_G)) :$$

We choose

$$r = 1 + \frac{s \frac{D \log(2D_F)}{d(2m \log l + \log(2D_G))}}{1} :$$

Therefore

$$\frac{D}{r} \leq \frac{s \frac{D \log(2D_F)}{d(2m \log l + \log(2D_G))}}{1} \tag{3:9}$$

and

$$rd \leq d + \frac{s \frac{D \log(2D_F)}{d(2m \log l + \log(2D_G))}}{1} \tag{3:10}$$

Moreover (3.8) implies

$$r \leq \frac{D}{3d} + \frac{D}{3d} \frac{D}{d} \tag{3:11}$$

By (3.9), (3.10) and (3.11) we have

$$(r) \leq \frac{d}{2} \log(2D_G) + \frac{p}{d} \frac{D \log(2D_F)}{d(2m \log l + \log(2D_G))} :$$

tu

4. Lower bounds for $\sum_{j \in h} Q_{kj}$.

In this section we assume that F and G are integral co-prime polynomials. Then

$$j \text{Res}(F; G) \leq 1; \quad (F) \leq M(F)^2; \quad (G) \leq M(G)^2:$$

Hence theorem 1.1 and theorem 3.1 provides lower bounds for $\sum_{j \in h} Q_{kj}$. As a simple example, choose $\epsilon = m = 1$ in theorem 1.1. We find the following improved version of (1.6):

Corollary 4.2. Let $\alpha, \beta \neq 0$ be non-conjugate algebraic numbers of degrees d_1 and d_2 . Then

$$j \leq j^{-1} e^{(2D)^{1+d_2} M(\alpha)^{2d_1} M(\beta)^{2d_2}} \exp \left(\frac{q}{dD \log(2DM(\alpha)^2) \log 2DM(\beta)^2} \right)$$

provided that

$$3d \max \left\{ \frac{\log(2DM(\alpha)^2)}{\log(2DM(\beta)^2)}; 1 \right\} \leq D:$$

G_p is irreducible (see [R] p. 139). Hence, for any real x , the polynomials G and $F = G_p$ are square-free and coprime. We choose $\epsilon = (\log d)^{-2} = (\log \log d)^{-1}$. By the Prime Number Theorem,

$$D := \deg F = \frac{d}{p} \frac{d(\log d)^2}{2(\log \log d)^2} \frac{d^2}{2}; \quad (6.1)$$

$$\log M(F) = \frac{(\log d)^4 \log M(G)}{4(\log \log d)^3} \log d; \quad (6.2)$$

and, since $\log | \text{Res}(F; G) | \leq D \log p$ by lemma 2 of [D],

$$\log | \text{Res}(F; G) | \leq \frac{d \log p}{p} \frac{d(\log d)^2}{\log \log d}; \quad (6.3)$$

We also have $D \leq 9d$ and, by (6.2),

$$\max \left\{ \frac{\log(2DM(F)^2)}{\log(2DM(G)^2)}; 1 + \frac{2 \log M(F)}{\log d} \right\} \leq \frac{D}{3d};$$

Therefore we can apply theorem 1.1 (with $\ell = m = 0$), which gives

$$\log | \text{Res}(F; G) | \leq \frac{d}{2} \log(2D) + d \log M(F) + (D + d) \frac{\log(2DM(F)^2) \log(2DM(G)^2)}{dD \log(2DM(F)^2) \log(2DM(G)^2)}; \quad (6.4)$$

By (6.1) and (6.2) we have

$$\frac{d}{2} \log(2D) + d \log M(F) + (D + d) \log M(G) \leq 2d \log d \quad (6.5)$$

and

$$\frac{dD \log(2DM(F)^2) \log(2DM(G)^2)}{2 \frac{d^2(\log d)^4}{4(\log \log d)^2} \left(2 + \frac{(\log d)^3 \log M(G)}{(\log \log d)^3} \right)}; \quad (6.6)$$

Substituting (6.3), (6.5) and (6.6) into (6.4) we obtain

$$\frac{d(\log d)^2}{\log \log d} \leq 2d \log d + \frac{d(\log d)^2}{2 \log \log d} \left(2 + \frac{(\log d)^3 \log M(G)}{(\log \log d)^3} \right) + \frac{2d(\log d)^2}{2 \log \log d} \left(2 + \frac{(\log d)^3 \log M(G)}{(\log \log d)^3} \right);$$

whence

$$\log M(G) \leq \frac{4}{3} \frac{\log \log d}{\log d} \left(2 + \frac{\log \log d}{\log d} \right)^3;$$

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