Cosmological perturbations in extended electromagnetism: General gauge invariant approach

Roberto Dale[*]

Departamento de Física, Ingeniería de Sistemas y Teoría de la Señal, Universidad de Alicante, 03690, San Vicente del Raspeig, Alicante, Spain

Diego Sáez[†]

Departamento de Astronomía y Astrofísica, Universidad de Valencia, 46100 Burjassot, Valencia, Spain

A certain vector-tensor theory is revisited. It was proposed and analyzed as a theory of electromagnetism without the standard gauge invariance. Our attention is first focused on a detailed variational formulation of the theory, which leads to both a modified Lorentz force and the true energy-momentum tensor of the vector field. The theory is then applied to cosmology. A complete gauge invariant treatment of the scalar perturbations is presented. For appropriate gauge invariant variables describing the scalar modes of the vector field (A modes), it is proved that the evolution equations of these modes do not involve the scalar modes appearing in general relativity (GR modes), which are associated to the metric and the energy-momentum tensor of the cosmological fluids. However, the A modes modify the standard gauge invariant equations describing the GR modes. By using the new formalism, the evolution equations of the A perturbations are derived and separately solved and, then, the correction terms—due to the A perturbations—appearing in the evolution equations of the GR modes are estimated. The evolution of these correction terms is studied for an appropriate scale. The relevance of these terms depends on both the spectra and the values of the normalization constants involved in extended electromagnetism. Further applications of the new formalism will be presented elsewhere.

I. INTRODUCTION

In previous papers, an extended theory of electromagnetism was proposed [1] and developed [2–4]. The evolution of cosmological scalar perturbations was studied in [2]; nevertheless, the authors assumed that the *scalar* perturbations of the electromagnetic field do not affect metric perturbations evolution. They stated that this assumption holds both in the radiation dominated period and in the matter dominated era, that is to say, at any moment before dark energy domination. Hence, it affects the choice of the initial conditions for numerical integrations, which are always fixed in the radiation dominated era ($z \sim 10^8$), when the cosmologically significant scales had superhorizon sizes. Moreover, in paper [2], the conservation law of standard electromagnetism, $\nabla_\mu J^\mu = 0$, is assumed, e.g., to get Eq. (2.3) from Eq. (2.2). Nevertheless, this law is not an equation of extended electromagnetism, whose true conservation law (see below) admits solutions with $\nabla_\mu J^\mu \neq 0$. This fact is important in cosmology, where the law $\nabla_\mu J^\mu = 0$ implies that only vector modes are involved in the expansion of the current J^μ, whereas the condition $\nabla_\mu J^\mu \neq 0$ requires the existence of J^μ scalar modes. We have introduced one of these modes (see below) in our general calculations. We see that, in extended electromag-

netism, there are scalar modes in the expansions of both the electromagnetic field A^μ (A modes) and the current J^μ (J modes). Since we have no convincing arguments proving that, initially, at $z \sim 10^8$, all these modes are negligible against the small scalar modes of the radiation fluid, we cannot ensure that they do not affect metric perturbations. Hence, the A and J scalar modes should not be neglected *a priori* in order to get the metric and fluid initial conditions for numerical integrations. After these considerations it seems that the assumption used in [2] is a simplifying condition which would require further justification (if it exists). In this situation, it is obvious that a more general study of cosmological perturbations is worthwhile. It is performed in this paper, where a general complete treatment of the cosmological perturbations is developed in the framework of extended electromagnetism. Our approach has various relevant properties: (i) it is gauge invariant, (ii) it does not involve approximating conditions, (iii) it involves a $J^{(0)}$ scalar mode as it is required by the general conservation law of extended electromagnetism, and (iv) it uses appropriate scalar modes for the field A^μ which evolve independently of the scalar general relativity (GR) modes (metric and fluid modes).

This paper is structured as follows: The basic equations of the vector-tensor (VT) theory are derived—by using variational techniques—in Sec. II, where the energy-momentum tensor and the Lorentz force are calculated. The theory is applied to cosmology in Sec. III, where the

---

[*]rdale@dfists.ua.es
[†]diego.saez@uv.es

linear approximation is studied by using standard techniques based on decoupled scalar, vector, and tensor modes [5,6]. An analysis of the equations derived in Sec. III is performed in Sec. IV, where the equations satisfied by the scalar perturbations of the vector field are numerically solved, and the differences between the equations describing the evolution of the scalar modes in GR and in VT are studied. The Appendix contains some estimates in the framework of GR, which are useful in Sec. IV. Finally, Sec. V contains a general discussion and our main conclusions.

Let us finish this section fixing some notation criteria. Latin (Greek) indices run from 1 to 3 (0 to 3). The gravitational constant, the scale factor, the conformal time, and the Hubble constant are denoted G, a, τ, and H_0, respectively. Whatever function D may be, $D_{/\alpha}$ stands for its partial derivative with respect to the coordinate x^α, and D_B represents its background value. Units are chosen in such a way that the speed of light is $c = 1$. Spatial distances are given in megaparsecs.

II. THE THEORY: VARIATIONAL FORMULATION AND BASIC EQUATIONS

A charged isentropic perfect fluid is considered in the framework of the VT generalization of Einstein-Maxwell theory proposed in [1]. The basic equations are derived from the following action:

$$I = \int \left[\frac{R}{16\pi G} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \gamma(\nabla_\mu A^\mu)^2 \right.$$
$$\left. + J^\mu A_\mu - \rho(1 + \epsilon) \right] \sqrt{-g} \, d^4 x, \qquad (1)$$

where γ is an arbitrary parameter, R, $g_{\mu\nu}$, and g are the scalar curvature, the covariant metric components, and the determinant of the $g_{\mu\nu}$ matrix, respectively. The vector field of the theory is A^μ. The symbol ∇ stands for the covariant derivative and we define $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$. The electrical current is $J^\mu = \rho_q U^\mu$, where ρ_q is the density of electrical charge and U^μ is the four-velocity of the fluid world lines. Finally, for an isentropic perfect fluid, one can introduce a conserved energy density ρ [$\nabla_\mu(\rho U^\mu) = 0$] and an internal energy ϵ; so the fluid energy density is $\mu = \rho(1 + \epsilon)$ and the pressure is $P = \rho^2(d\epsilon/d\rho)$ (see [7]).

Some VT theories were proposed in the early 1970s (see [8,9]). All these theories were based on the action:

$$I = (16\pi G)^{-1} \int (R + \omega A_\mu A^\mu R + \eta R_{\mu\nu} A^\mu A^\nu$$
$$- \varepsilon F_{\mu\nu} F^{\mu\nu} + \tau \nabla_\nu A_\mu \nabla^\nu A^\mu + L_m) \sqrt{-g} \, d^4 x, \quad (2)$$

where ω, η, ε, and τ are arbitrary parameters and L_m is the matter Lagrangian, which couples matter with the fields of the VT theory. Actions (1) and (2) are equivalent for $\omega = 0$, $2\varepsilon - \eta = 8\pi G$, $\tau = \eta = 16\pi G\gamma$, and $L_m = J^\mu A_\mu - \rho(1 + \epsilon)$.

According to the variational techniques described in [7], three fields may be independently varied in the action (1). These fields are the vector field of the theory (A_μ), the flow lines of the fluid (U^μ), and the metric field ($g_{\mu\nu}$).

We first vary the field A_μ for fixed flow lines and metric (δ_A variations). Thus, we easily obtain the field equations for the A^μ field, whose form is

$$\nabla^\nu F_{\mu\nu} = J_\mu + J_\mu^A, \qquad (3)$$

where $J_\mu^A = -2\gamma \nabla_\mu(\nabla \cdot A)$ with $\nabla \cdot A = \nabla_\mu A^\mu$. Then, from these field equations one easily gets the relation:

$$\nabla^\mu J_\mu = -\nabla^\mu J_\mu^A, \qquad (4)$$

which indicates that the total current $J_\mu + J_\mu^A$ is conserved in the theory. This is the conserved current associated to the invariance of action (1) under the residual gauge transformation $A_\mu' = A_\mu + \partial_\mu \phi$ with $\partial^\mu \partial_\mu \phi = 0$.

In the second step, the flow lines are varied for fixed A_μ and $g_{\mu\nu}$ (δ_U variations) and, moreover, the densities ρ and ρ_q are adjusted to satisfy the equation $\nabla_\mu(\rho U^\mu) = 0$ and Eq. (4), respectively (see [7]). Since the right-hand side of Eq. (4) does not depend on U^μ, the following relation is satisfied $\delta_U(\nabla_\mu J^\mu) = \nabla_\mu(\delta_U J^\mu) = 0$. On account of these considerations, the following equations are easily obtained [7]:

$$(\mu + P)U^\mu \nabla_\mu U^\nu = -\nabla_\mu P(g^{\mu\nu} + U^\mu U^\nu) + F^{\mu\nu} J_\mu$$
$$+ (\nabla^\mu J_\mu^A) A^\nu. \qquad (5)$$

These equations describe the fluid evolution in the VT theory. The last term is the generalized Lorentz force, f^L, of the theory; hence, we can write

$$f_\nu^L = F_{\nu\mu} J^\mu + (\nabla^\mu J_\mu^A) A_\nu. \qquad (6)$$

Finally, the metric is varied whereas fields A_μ and U^μ are fixed (δ_g variations). For this kind of variations, Eq. (4) leads to the relation $\delta_g(\sqrt{-g} J^\alpha) = -\delta_g(\sqrt{-g} J^{A\alpha})$ and, then, from this relation and the identity $\nabla^\mu[A_\mu(\nabla \cdot A)] = (\nabla \cdot A)^2 + A_\mu \nabla^\mu(\nabla \cdot A)$, it follows that the Lagrangian densities $\gamma(\nabla_\mu A^\mu)^2 + J^\mu A_\mu$ [involved in action (1)] and $-\gamma(\nabla_\mu A^\mu)^2$ are fully equivalent. On account of this fact, δ_g variations lead to

$$G^{\mu\nu} = 8\pi G T^{\mu\nu}, \qquad (7)$$

where $G^{\mu\nu}$ is the Einstein tensor and $T^{\mu\nu}$ is the energy-momentum tensor of the charged fluid plus the electromagnetic field, whose form is

$$T^{\mu\nu} = (\mu + P)U^\mu U^\nu + P g^{\mu\nu} + F^\mu{}_\alpha F^{\nu\alpha} - \tfrac{1}{4} g^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}$$
$$+ 2\gamma[\{A^\alpha \nabla_\alpha(\nabla \cdot A) + \tfrac{1}{2}(\nabla \cdot A)^2\} g^{\mu\nu}$$
$$- A^\mu \nabla^\nu(\nabla \cdot A) - A^\nu \nabla^\mu(\nabla \cdot A)]. \qquad (8)$$

This energy-momentum tensor is to be compared with that given in [2] taking into account differences in the assumed signature.

Equations (3), (5), and (7) are the field equations of the theory. Any solution of these equations satisfies Eq. (4), and also the relations $\nabla_\mu T^{\mu\nu} = 0$. These last relations combined with all the field equations and Eq. (8) lead to

$$[U^\beta \nabla_\beta(\mu) + (\mu + P)\nabla_\beta U^\beta]U^\alpha = 2F^{\alpha\beta}J^A_\beta - 2A^\alpha \nabla^\beta J^A_\beta. \tag{9}$$

In the Einstein-Maxwell theory, which is formally obtained in the limit $J^A_\beta \to 0$, Eq. (9) reduces to $U^\beta \nabla_\beta(\mu) + (\mu + P)\nabla_\beta U^\beta = 0$ [which is identical to Eq. (3.9) in [7]].

After this detailed variational study, which has not been previously developed, we are concerned with cosmological applications.

III. BACKGROUND UNIVERSE AND COSMOLOGICAL PERTURBATIONS

From the field equations of Sec. II, one easily finds the equations describing a homogeneous and isotropic neutral flat universe, in which the line element is

$$ds^2 = a^2[-d\tau^2 + dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2]. \tag{10}$$

The vector field, A, and the four-velocity, U, of the cosmic fluid have the covariant components $[A_{0B}(\tau), 0, 0, 0]$ and $[-a(\tau), 0, 0, 0]$, respectively. The energy density is the critical one, whose present value is $\rho_{B0} = 3H_0^2/8\pi G$, and the density of charge is $\rho_{qB}(\tau) = 0$ for any time. In this background, Eq. (3) reduces to

$$\Xi_B \equiv (\nabla \cdot A)_B = -\frac{1}{a^2}\left[\dot{A}_{0B} + 2\frac{\dot{a}}{a}A_{0B}\right] = \text{const}, \tag{11}$$

and Eq. (8) leads to the relations

$$\rho^A_B = -P^A_B = -\gamma \Xi^2_B, \tag{12}$$

where quantities ρ^A_B and P^A_B are the background energy density and pressure of the vector field A^μ, respectively. The equation of state (12) proves that the energy density of the background field A^μ play the role of a cosmological constant. In order to have positive values of ρ^A_B the parameter γ must be negative. Hereafter units are chosen in such a way that $c = 8\pi G = 1$; thus, from Eqs. (7) and (8) one easily gets the following basic cosmological equation for the background evolution:

$$3\frac{\dot{a}^2}{a^2} = a^2(\rho_B + \rho^A_B), \tag{13}$$

$$-2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = a^2(P_B + P^A_B), \tag{14}$$

where ρ_B and P_B are the background energy density and pressure of the cosmological fluid (baryons plus dark matter and radiation). Hereafter, w and c_s^2 stand for the ratios P_B/ρ_B and $dP_B/d\rho_B$, respectively.

In the next sections, perturbations are described—in the usual way—with the formalism summarized in [5] (see

also [6]). There are three types of perturbations whose evolution is independent during the linear regime. They are the so-called scalar, vector, and tensor fluctuations, which may be expanded in terms of the scalar, $Q^{(0)}$, vector, $Q_i^{(1)}$, and tensor, $Q_{ij}^{(2)}$, harmonics, respectively.

A. Tensor perturbations

There are no tensor modes involved in the expansion of vectors A^μ and J^μ; hence, in the VT theory, tensor modes only appear in the expansions of the same quantities as in GR (metric and anisotropic part of the stress tensor) and, moreover, they satisfy the same equations as in GR. Therefore, we are mainly interested in scalar and vector modes. Metric tensor modes (gravitational waves) and anisotropic stress tensor components evolve as in GR, namely, they satisfy the equation:

$$\ddot{H}_T^{(2)} + 2\frac{\dot{a}}{a}\dot{H}_T^{(2)} + k^2 H_T^{(2)} = P_B a^2 \Pi_T^{(2)}, \tag{15}$$

where $\Pi_T^{(2)}Q_{ij}^{(2)}$ is the tensor part of the anisotropic stress tensor and $H_T^{(2)}Q_{ij}^{(2)}$ the tensor part of the metric (see [5]).

For negligible anisotropic stress, cosmological fluctuations evolving well outside the effective horizon ($\dot{a}/a \gg k$, see [5,10]) obey the equation $\ddot{H}_T^{(2)} + 2(\dot{a}/a)\dot{H}_T^{(2)} \simeq 0$, whose general solution is $\dot{H}_T^{(2)} \propto a^{-2}$; hence, $\dot{H}_T^{(2)}$ is a fast decaying mode. This means that well after reheating, e.g. at $z = 10^8$, superhorizon scales evolve in such a way that quantity $H_T^{(2)}$ is almost independent of time. This fact will be taken into account below.

B. Vector perturbations

In the case of a flat background, the vector harmonics can be written as follows [6]: $\vec{Q}^\pm = \vec{\epsilon}^\pm \exp(i\vec{k}\cdot\vec{r})$, where $\vec{k}$ is the wave number vector. A representation of vectors $\vec{\epsilon}^+$ and $\vec{\epsilon}^-$ is [11,12]

$$\epsilon_1^\pm = (\pm k_1 k_3/k - ik_2)/\sigma\sqrt{2}, \quad \epsilon_2^\pm = (\pm k_2 k_3/k + ik_1)/\sigma\sqrt{2},$$
$$\epsilon_3^\pm = \mp\sigma/k\sqrt{2}, \tag{16}$$

where $\sigma = (k_1^2 + k_2^2)^{1/2}$.

The first order vector part of the fields A_μ and J_μ may be written as follows:

$$A_\mu = [0, A^{(1)\pm}Q_i^{(1)\pm}], \tag{17}$$

$$J_\mu = [0, a^2 J^{(1)\pm}Q_i^{(1)\pm}]. \tag{18}$$

As it is done in standard electromagnetism (Einstein-Maxwell theory), we define the covariant components of the electric and magnetic fields (E_μ and B_μ) as follows:

$$E_\mu = F_{\mu\nu}U^\nu, \tag{19}$$

and

$$B_\mu = \tfrac{1}{2}(-g)^{-1/2}\epsilon_{\mu\nu\rho\lambda}F^{\rho\lambda}U^\nu, \qquad (20)$$

where quantities $\epsilon_{\mu\nu\rho\lambda}$ are the Levy-Civita symbols. Then, at first order in perturbation theory, one easily finds

$$F_{\mu\nu}=A_{\nu/\mu}-A_{\mu/\nu}=\begin{bmatrix} 0 & -aE_1 & -aE_2 & -aE_3 \\ aE_1 & 0 & aB_3 & -aB_2 \\ aE_2 & -aB_3 & 0 & aB_1 \\ aE_3 & aB_2 & -aB_1 & 0 \end{bmatrix}. \quad (21)$$

Equations. (16)–(18) and (21) lead to the following equations in momentum space:

$$E^{(1)\pm}=-\frac{1}{a}\dot{A}^{(1)\pm}, \qquad B^{(1)\pm}=\pm\frac{kA^{(1)\pm}}{a}. \quad (22)$$

Hereafter we use the following equivalences: $\vec{E}=(E_1,E_2,E_3)$, $\vec{B}=(B_1,B_2,B_3)$, and $\vec{J}=(J_1,J_2,J_3)$. Moreover, $\vec{\nabla}\cdot\vec{X}$ and $\vec{\nabla}\wedge\vec{X}$ stand for the ordinary divergence and curl of $\vec{X}$, respectively.

By using this notation and Eqs. (22) it is easily verified that, up to first order, the equation

$$a(\vec{\nabla}\wedge\vec{E})+\frac{\partial}{\partial\tau}(a\vec{B})=0 \qquad (23)$$

is satisfied in position space. Finally, since the ordinary divergence of any vector mode vanishes, we can write

$$\vec{\nabla}\cdot\vec{B}\equiv B_{i/i}=0. \qquad (24)$$

Let us now consider the field equation (3), which may be rewritten as follows:

$$[\ln(\sqrt{-g})]_{/\alpha}F^{\beta\alpha}+F^{\beta\alpha}_{/\alpha}=J^\beta-2\gamma g^{\beta\alpha}(\nabla\cdot A)_{/\alpha}. \quad (25)$$

Since $\nabla\cdot A$ and $\nabla\cdot J$ are scalars, they may be expanded in scalar harmonics with no contributions from vector modes; hence, the term $-2\gamma g^{\beta\alpha}(\nabla\cdot A)_{/\alpha}$ vanishes in the case of vector modes and, consequently, Eq. (25) reduces to those of Einstein-Maxwell theory. By using Eqs. (21) and (25), it may be easily verified that, for vector modes and up to first order, Einstein-Maxwell field equations and Eq. (25) may be written as follows:

$$a(\vec{\nabla}\wedge\vec{B})-\frac{\partial}{\partial\tau}(a\vec{E})=a^2\vec{J}, \qquad (26)$$

$$\vec{\nabla}\cdot\vec{E}\equiv E_{i/i}=0. \qquad (27)$$

Finally, from Eqs. (18), (22), and (26), the following equation describing the evolution of $A^{(1)\pm}$ is found:

$$\ddot{A}^{(1)\pm}+k^2A^{(1)\pm}=a^4J^{(1)\pm}. \qquad (28)$$

For a given function $J^{(1)\pm}(\vec{k},\tau)$, the solution of Eq. (28) gives $A^{(1)\pm}(\vec{k},\tau)$ and then, quantities $E^{(1)\pm}$ and $B^{(1)\pm}$ are fixed by Eqs. (22). From these last quantities we may calculate $\vec{E}(\vec{x},\tau)$ and $\vec{B}(\vec{x},\tau)$ by using the explicit

form (16) of the vector harmonics. The resulting $\vec{E}$ and $\vec{B}$ quantities satisfy the four Eqs. (23), (24), (26), and (27) in position space.

The equations of this section are valid in the VT theory under consideration as well as in the standard Einstein-Maxwell theory. It is due to the fact that vector modes do not contribute to the terms involving γ either in Eq. (25) or in Eq. (8), and these terms are responsible for all the differences between both theories. The predictions of these theories only may be different due to the scalar modes involved in the vector fields A^μ and J^μ, which are studied in the next section.

C. Scalar perturbations

For a flat background, the scalar harmonics are plane waves; namely, $Q^{(0)}=\exp(i\vec{k}\cdot\vec{r})$. The first order scalar contributions to vectors A_μ and J_μ are

$$A_\mu=[\alpha^{(0)}Q^{(0)},\beta^{(0)}Q_i^{(0)}], \qquad (29)$$

$$J_\mu=[0,a^2J^{(0)}Q_i^{(0)}], \qquad (30)$$

where $Q_i^{(0)}=(-1/k)Q_{/i}^{(0)}$.

Since we assume that the Universe is neutral up to first order, the component J_0 vanishes and, moreover, taking into account that the equation $\nabla\cdot J=0$ is not an equation of the VT theory, a scalar part $a^2J^{(0)}Q_i^{(0)}$ must be included in the expansion of J_i. Equations (29) and (30) are absolutely general.

Equations (21) and (29) may be combined to get

$$E^{(0)}=-\frac{1}{a}(k\alpha^{(0)}+\dot{\beta}^{(0)}), \qquad B^{(0)}=0. \quad (31)$$

Similarly, the scalar $\nabla\cdot A$ may be expanded in terms of scalar harmonics; namely, we can write

$$\nabla\cdot A=\Xi_B(1+\Xi^{(0)}Q^{(0)}). \qquad (32)$$

In order to calculate $\Xi^{(0)}\equiv(\nabla\cdot A)^{(0)}/\Xi_B$, we must use the relation $\nabla\cdot A=\partial A^\mu/\partial x^\mu+\Gamma^\mu_{\mu\nu}A^\nu$, which involves the Christoffeld symbols. It is then evident that $\Xi^{(0)}$ depends on the coefficients $\alpha^{(0)}$ and $\beta^{(0)}$ appearing in the expansion of A_μ [see Eq. (29)], and also on the coefficients involved in the expansion of the metric components $g_{\mu\nu}$, which appear in the Christoffeld symbols. If the resulting $\Xi^{(0)}$ is used to write Eqs. (7), (8), and (25) up to first order in scalar modes, all the equations are coupled among them and their solutions only have been found under special assumptions (see [2] and Sec. I). However, we have developed a method, in which we may first solve Eq. (25) and, then, the solution can be used to solve Eqs. (7) and (8). Let us now describe this method and derive the evolution equations for the scalar modes.

Coefficients $\alpha^{(0)}$ and $\beta^{(0)}$ are not the most suitable ones in order to expand the field equations. It is preferable the

use of the coefficients $E^{(0)}$ and $\Xi^{(0)}$, which are gauge invariant quantities. In terms of these variables, Eq. (25) reduces to

$$2\gamma a \Xi_B \dot{\Xi}^{(0)} = kE^{(0)}, \tag{33}$$

$$\dot{E}^{(0)} = -a^3 J^{(0)} - 2\gamma ka\Xi_B \Xi^{(0)} - \frac{\dot{a}}{a} E^{(0)}, \tag{34}$$

and Eq. (4) may be written as follows:

$$\ddot{\Xi}^{(0)} + 2\frac{\dot{a}}{a}\dot{\Xi}^{(0)} + k^2\Xi^{(0)} = -\frac{k}{2\gamma\Xi_B} a^2 J^{(0)}. \tag{35}$$

Since Eq. (4) is a consequence of Eq. (25), which is equivalent to Eq. (3), Eq. (35) may be easily obtained by combining Eqs. (33) and (34). Hence, functions $\Xi^{(0)}$ and $E^{(0)}$ may be found by solving Eqs. (33) and (34) for a given $J^{(0)}$ plus initial values of $\Xi^{(0)}$ and $E^{(0)}$ and, then, the resulting $\Xi^{(0)}$ and $E^{(0)}$ functions and the chosen $J^{(0)}$ will satisfy Eq. (35).

Equation (34) may be easily derived from Eqs. (33) and (35), consequently, we may also proceed as follows: Eq. (35) is solved for a given $J^{(0)}$ and initial values of $\Xi^{(0)}$ and $\dot{\Xi}^{(0)}$ and, then, the resulting solution $\Xi^{(0)}$ is used to get function $E^{(0)}$ by using Eq. (33). Obviously, Eq. (34) is satisfied by the $\Xi^{(0)}$ and $E^{(0)}$ functions we have found by solving Eqs. (33) and (35).

It is worthwhile to point out that Eqs. (15) and (35) have the same form. In fact, if we replace $H_T^{(2)}$ by $\Xi^{(0)}$ and $\Pi_T^{(2)}$ by $-kJ^{(0)}/2P_B\gamma$ in Eq. (15), the resulting equation is identical to Eq. (35). Condition $J^{(0)} = 0$ in Eq. (35) is equivalent to condition $\Pi_T^{(2)} = 0$ in (15). Hence, some previous conclusions about the evolution of gravitational wave modes would be also valid for the $\Xi^{(0)}$ evolution; in particular, for $J^{(0)} = 0$ and superhorizon scales, the relation $\dot{\Xi}^{(0)} \simeq 0$ holds.

In extended electromagnetism, function $J^{(0)}$ does not vanish *a priori*. Condition $J^{(0)} = 0$ implies the relation $\nabla^\mu J_\mu = 0$ in position space, but this relation is not a basic equation of the theory. For this reason, function $J^{(0)}$ is included in the equations derived in this section; nevertheless, there is no—at the moment—physically motivated rule to build up this function. Anyway, for a given $J^{(0)}$ (including the cosmological possibility $J^{(0)} = 0$), Eqs. (33) and (34) involve the constant $\gamma < 0$, the wave number k, the scale factor, functions $\Xi^{(0)}$ and $E^{(0)}$, and their first order derivatives. These equations do not involve scalar perturbations associated to the metric and the energy-momentum tensor. They may be easily solved for given values of γ and k, and initial values of $\Xi^{(0)}$ and $E^{(0)}$ [alternatively we may solve Eq. (35) for initial values of $\Xi^{(0)}$ and $\dot{\Xi}^{(0)}$]. The resulting function $\Xi^{(0)}$ appears in the expansion of Eqs. (7) and (8) in scalar harmonics (see below).

The gauge invariant formalism described in [5] is used in this paper; namely, the metric, the four-velocity, and the part of the energy-momentum tensor (8) being independent of the parameter γ are all expanded as follows (in the flat case):

$$g_{00} = -a^2(1 + 2\tilde{A}Q^{(0)}), \quad g_{0i} = -a^2\tilde{B}^{(0)}Q_i^{(0)},$$

$$g_{ij} = a^2[(1 + 2H_LQ^{(0)})\delta_{ij} + 2H_T^{(0)}Q_{ij}^{(0)}], \quad U_i = av^{(0)}Q_i^{(0)},$$

$$\rho = \rho_B(1 + \delta Q^{(0)}), \quad T_{ij} = P_B(1 + \pi_LQ^{(0)})\delta_{ij} + P_B\pi_T^{(0)}Q_{ij}^{(0)}. \tag{36}$$

Any other quantity as, e.g., U_0, T_{0i}, and so on, may be easily written in terms of the coefficients involved in these equations (see [5]), which may be combined to build up the following gauge invariant variables:

$$\eta = (w\pi_L - c_s^2\delta)/w, \qquad v_s^{(0)} = v^{(0)} - \frac{1}{k}\dot{H}_T^{(0)},$$

$$\epsilon_m = \delta + 3(1 + w)\frac{1}{k}\frac{\dot{a}}{a}(v^{(0)} - \tilde{B}^{(0)}),$$

$$\Phi_A = \tilde{A} + \frac{1}{k}\dot{\tilde{B}}^{(0)} + \frac{1}{k}\frac{\dot{a}}{a}\tilde{B}^{(0)} - \frac{1}{k^2}\left(\ddot{H}_T^{(0)} + \frac{\dot{a}}{a}\dot{H}_T^{(0)}\right), \tag{37}$$

$$\Phi_H = H_L + \frac{1}{3}H_T^{(0)} + \frac{1}{k}\frac{\dot{a}}{a}\tilde{B}^{(0)} - \frac{1}{k^2}\frac{\dot{a}}{a}\dot{H}_T^{(0)}.$$

The complementary part of the energy-momentum tensor (8), namely, the part depending on γ may be easily expanded in terms of scalar harmonics. The resulting expansion involves the variable $\Xi^{(0)}$ and its first order time derivative (or equivalently $\Xi^{(0)}$ and $E^{(0)}$).

In order to expand Eqs. (7) and (8) and the relation $\nabla_\nu T^{\mu\nu} = 0$, we use the same gauge invariant potentials and variables as in [5]) (see above), plus the gauge invariant variables $\Xi^{(0)}$, $E^{(0)}$, and $J^{(0)}$. The resulting equations reads as follows:

$$\frac{2k^2}{a^2}\Phi_H = \rho_B\epsilon_m - 2\gamma\Xi_B\left[\left(3\frac{\dot{a}}{a^3}A_{0B} + \Xi_B\right)\Xi^{(0)} + \frac{A_{0B}}{a^2}\dot{\Xi}^{(0)}\right], \tag{38}$$

$$-\frac{k^2}{a^2}(\Phi_A + \Phi_H) = P_B\Pi_T^{(0)}, \tag{39}$$

$$\dot{v}_s^{(0)} + \frac{\dot{a}}{a}v_s^{(0)} = k\Phi_A + \frac{k}{1+w}(c_s^2\epsilon_m + w\eta) - \frac{2wk}{3(1+w)}\Pi_T^{(0)}, \tag{40}$$

$$(\rho_Ba^3\epsilon_m)\dot{} = -ka^3(\rho_B + P_B)v_s^{(0)} - 2a^2\dot{a}P_B\Pi_T^{(0)}$$
$$- ka^3A_{0B}J^{(0)} - 3\gamma a^3\Xi_BA_{0B}(\rho_B + P_B)\Xi^{(0)}. \tag{41}$$

Equations (38)–(41) plus (13) and (14) may be combined to get the following equation for the evolution of ϵ_m:

$$(\rho_B a^3 \epsilon_m)^{\cdot\cdot} + (1 + 3c_s^2)\frac{\dot{a}}{a}(\rho_B a^3 \epsilon_m)^{\cdot} + \left[k^2 c_s^2 - \frac{1}{2}(\rho_B + P_B)a^2\right](\rho_B a^3 \epsilon_m)$$

$$= -k^2(P_B a^3 \eta) - 2\dot{a}(P_B a^2 \Pi_T^{(0)})^{\cdot} + \frac{2}{3}k^2(P_B a^3 \Pi_T^{(0)}) + 2\rho_B a^2\left[w - c_s^2 - (1 + c_s^2)\frac{\rho_B^A}{\rho_B}\right](P_B a^3 \Pi_T^{(0)})$$

$$- 2\gamma a^3 \Xi_B(\rho_B + P_B)(2A_{0B}\dot{\Xi}^{(0)} - \Xi_B a^2 \Xi^{(0)}) - ka^3\left(A_{0B}\dot{j}^{(0)} + \left[(2 + 3c_s^2)\frac{\dot{a}}{a}A_{0B} - a^2 \Xi_B\right]j^{(0)}\right). \qquad (42)$$

For $\Xi^{(0)} = 0$, $J^{(0)} = 0$, and $\rho_B^A = 0$, Eqs. (38)–(42) reduce to the equations derived by Bardeen in the flat case [5]. For $\Xi^{(0)} = 0$, $J^{(0)} = 0$, and $\rho_B^A = \rho_\Lambda \neq 0$, Eqs. (38)–(42) describe fluctuation evolution in a standard flat universe with a cosmological constant whose energy density is ρ_Λ. Finally, if $\rho_B^A = \rho_\Lambda$, and the two functions $J^{(0)}$ and $\Xi^{(0)}$ do not vanish at the same time, Eqs. (38), (41), and (42) contain new terms, which modify the equations describing perturbation evolution in flat universes with cosmological constant.

IV. ANALYZING THE BASIC DIFFERENTIAL EQUATIONS OF EXTENDED ELECTROMAGNETISM

In this section, it is assumed that the condition $J^{(0)} = 0$ holds in cosmology, which is equivalent to assume the well-known conservation law of Einstein-Maxwell theory ($\nabla_\mu J^\mu = 0$). Under this arbitrary assumption, we study the background equations (11)–(14), and the Eqs. (33)–(35) and (38)–(42) describing the evolution of the first order scalar perturbations in momentum space.

We begin with the background equations. Functions $\rho_B(\tau)$ and $P_B(\tau)$ are given by the formulas

$$\rho_B = \rho_{Br0}(1 + z)^4 + \rho_{Bm0}(1 + z)^3, \quad P_B = \rho_{Br0}(1 + z)^4/3, \qquad (43)$$

where $\rho_{Br0} = 8 \times 10^{-34}$ gr/cm^3 and $\rho_{Bm0} = 0.2726\rho_c$ are the present energy density of radiation and matter.

Moreover, the baryon density is assumed to be $\rho_{Bb0} = 0.0461\rho_c$, and the value of the Hubble constant is $H_0 = 100h$ Km s^{-1} Mpc^{-1} with $h = 0.704$. All these values are compatible with a certain version of the concordance model (see [13]). In this model, the dark energy density, ρ_B^A, is easily obtained from the relation $\rho_{Bm0} + \rho_{Br0} + \rho_B^A = 3H_0^2$, which is valid in flat backgrounds.

Equation (13) governing the evolution of the scale factor may be numerically solved for the above parameters; thus, the evolution of the scale factor is obtained. The resulting function $a(\tau)$ is necessary to study the remaining equations of the theory, namely, Eqs. (33)–(35) and also Eqs. (38)–(42).

From Eq. (12) one easily gets the relation $\Xi_B = \pm|\gamma|^{-1/2}(\rho_B^A)^{1/2}$ (negative γ). Once the value of $\rho_B^A = \rho_\Lambda$ is fixed (see above), this last relation leads to $\Xi_B \propto S_{gn}|\gamma|^{-1/2}$, where S_{gn} only may take on the values $+1$ and -1. For a given value of $|\gamma|$, only the absolute value of Ξ_B

may be obtained (its sign is arbitrary). On account of Eq. (11), we may also write the relation $A_{0B} \propto S_{gn}|\gamma|^{-1/2}$. In the background, quantities $|\gamma|$ and S_{gn} remain arbitrary.

Hereafter, D^{in} stands for the initial value of quantity D whatever it may be. Let us now consider Eqs. (33)–(35). We first solve Eq. (35) by using initial values $\Xi^{(0)\text{in}}$ and $\dot{\Xi}^{(0)\text{in}}$ at redshift $z = 10^8$. At this high redshift, the cosmological scales of interest (see below) are superhorizon ones; hence, taking into account the similarity between Eqs. (15) and (35) and the comments in the last paragraph of Sec. III A, the condition $J^{(0)} = 0$ assumed in this section allows us to take $\dot{\Xi}^{(0)\text{in}} = 0$. Only the initial value of $\Xi^{(0)}$ may be appropriately chosen to integrate Eq. (35). The values of $|\gamma|$ and S_{gn} are fully irrelevant to perform this integration. Hereafter, numerical calculations are performed for the spatial scale $\tilde{L} = 3 \times 10^3 h^{-1}$ Mpc, which reenters the effective horizon at present time (τ_0). Its wave number is $\tilde{k} \simeq 1.47 \times 10^{-3}$. This scale is useful for normalization in GR (see the Appendix). Equation (35) has been solved for the wave number $\tilde{k}$ with the initial condition $\Xi^{(0)\text{in}}(\tilde{L}) = 10^{-4}$ in position space. In order to write the corresponding initial condition in momentum space, we use the well-known relation [14]

$$\langle|X(x)|^2\rangle_L \simeq k^3\langle|X(k)|^2\rangle/2\pi^2, \qquad (44)$$

where X is an arbitrary quantity. Evidently, this relation must be particularized for $X = \Xi^{(0)\text{in}}$, $L = \tilde{L}$, and $k = \tilde{k}$ to calculate $\Xi^{(0)\text{in}}(\tilde{k})$. Either this last initial quantity (in momentum space) or $\Xi^{(0)\text{in}}(\tilde{L})$ (in position space) may be seen as a normalization constant. The $\Xi^{(0)}$ spectrum would be necessary to derive the initial value of $\Xi^{(0)}(k)$ for $k \neq \tilde{k}$.

The solution of Eq. (35) plus Eq. (33) allows us to calculate function $E^{(0)}$ which is proportional to $S_{gn}|\gamma|^{1/2}$.

Numerical integrations have given the functions $A_{0B}(z)$, $\Xi^{(0)}(\tilde{k}, z)$, and $E^{(0)}(\tilde{k}, z)$ represented in Fig. 1. These functions correspond to $|\gamma| = 1$, $S_{gn} = +1$, $k = \tilde{k}$, and $\Xi^{(0)\text{in}}(\tilde{L}) = 10^{-4}$. Since the dependence of these functions in terms of the parameters $|\gamma|$, S_{gn}, and $\Xi^{(0)\text{in}}(\tilde{k})$ is known (see above), Fig. 1 contains complete information about the scalar modes associated to the field A^μ for the scale $\tilde{k}$. The same may be done for any linear spatial scale with the help of an appropriate spectrum for $\Xi^{(0)\text{in}}$. Let us now study Eqs. (38)–(42) describing the evolution—in the framework of extended electromagnetism—of the scalar modes
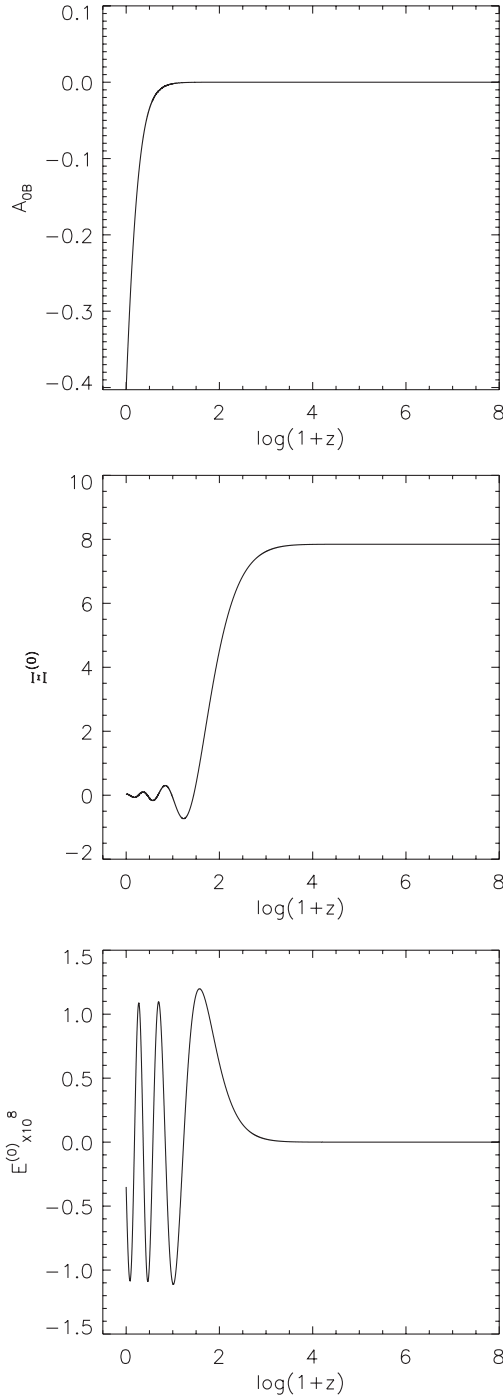
FIG. 1. Functions $A_{0B}(z)$ (top), $\Xi^{(0)}(\tilde{k}, z)$ (middle) and $E^{(0)}(\tilde{k}, z) \times 10^8$ (bottom) in terms of $\log(1 + z)$, for $\tilde{L} = 3000h^{-1}$ Mpc, $|\gamma| = 1$, $S_{gn} = +1$, and $\Xi^{(0)\text{in}}(\tilde{L}) = 10^{-4}$.

appearing in standard GR cosmology. Equations (38), (41), and (42) contain the terms:

$$\xi_1(\tau, k) = -2\gamma \Xi_B \left[\left(3\frac{\dot{a}}{a^3} A_{0B} + \Xi_B \right) \Xi^{(0)} + \frac{A_{0B}}{a^2} \dot{\Xi}^{(0)} \right], \quad (45)$$

$$\xi_2(\tau, k) = -3\gamma a^3 \Xi_B A_{0B}(\rho_B + P_B)\Xi^{(0)}, \quad (46)$$

$$\xi_3(\tau, k) = -2\gamma a^3 \Xi_B(\rho_B + P_B)(2A_{0B}\dot{\Xi}^{(0)} - \Xi_B a^2 \Xi^{(0)}), \quad (47)$$

respectively. These terms—which appear in extended electromagnetism but not in Einstein theory with cosmological constant—may be calculated, for the scale $\tilde{k}$, by using the integration data used to build up Fig. 1 and, then, these terms may be compared with appropriate terms involved in GR equations (for the same wave number).

Taking into account that Ξ_B and A_{0B} are proportional to $S_{gn}|\gamma|^{-1/2}$ and also that $\Xi^{(0)}$ does not depend on S_{gn} and $|\gamma|$, it is trivially proved that quantities ξ_1, ξ_2, and ξ_3 are also independent of S_{gn} and $|\gamma|$. For the wave number $\tilde{k}$, these quantities are proportional to the number $\Xi^{(0)\text{in}}(\tilde{L})$.

As it follows from Eqs. (38), (41), and (42), quantities ξ_1, ξ_2, and ξ_3 are to be compared with the GR values of the terms

$$\Upsilon_1(\tau, k) = \rho_B \epsilon_m, \quad (48)$$

$$\Upsilon_2(\tau, k) = -ka^3(\rho_B + P_B)v_s^{(0)}, \quad (49)$$

$$\Upsilon_3(\tau, k) = [\tfrac{1}{2}(\rho_B + P_B)a^2 - k^2 c_s^2](\rho_B a^3 \epsilon_m), \quad (50)$$

respectively. After the estimation of Υ_1, Υ_2, and Υ_3 in standard cosmology (based on GR), the three functions $r_i(\tau, k) = |\xi_i(\tau, k)/\Upsilon_i(\tau, k)|$ may be calculated. Evidently, for very small r_i values, GR and extended electromagnetism would lead to the same differential equations for the evolution of the GR scalar perturbations, whereas r_i values of the order of 10^{-3} or greater would suggest relevant differences with respect to GR. If differences are expected, the matter power spectrum $P(k)$ and the angular power spectra of the CMB should be accurately estimated by using numerical codes as CMBFAST [15] and CAMB [16]. These accurate calculations—under general enough initial conditions—are beyond the scope of this paper.

The r_i ratios will be estimated for the wave number $\tilde{k}$. The corresponding spatial scale is useful for normalization in GR, which is necessary to estimate the functions $\Upsilon_i(\tau, \tilde{k})$. The method used for the estimation of these functions and for normalization in standard GR cosmology are described in the Appendix.

The three functions $r_i(z, \tilde{k})$ are represented in Fig. 2 for $\Xi^{(0)\text{in}}(\tilde{L}) = 10^{-4}$. Two panels (left and right) show the evolution of each ratio r_i. The evolutions of the three ratios are similar. From $z = 10^8$ to $z \sim 10^2$, the chosen spatial scale is well outside the effective horizon and all the ratios increase without oscillations (see left panels); however, for $z < 10^2$, there are oscillations whose amplitudes grow as z decreases (see right panels). It is due to the fact that our spatial scale and that of the effective horizon come near as the redshift decreases (they have been chosen to be identical at $z = 0$). The maximum values of the ratios are
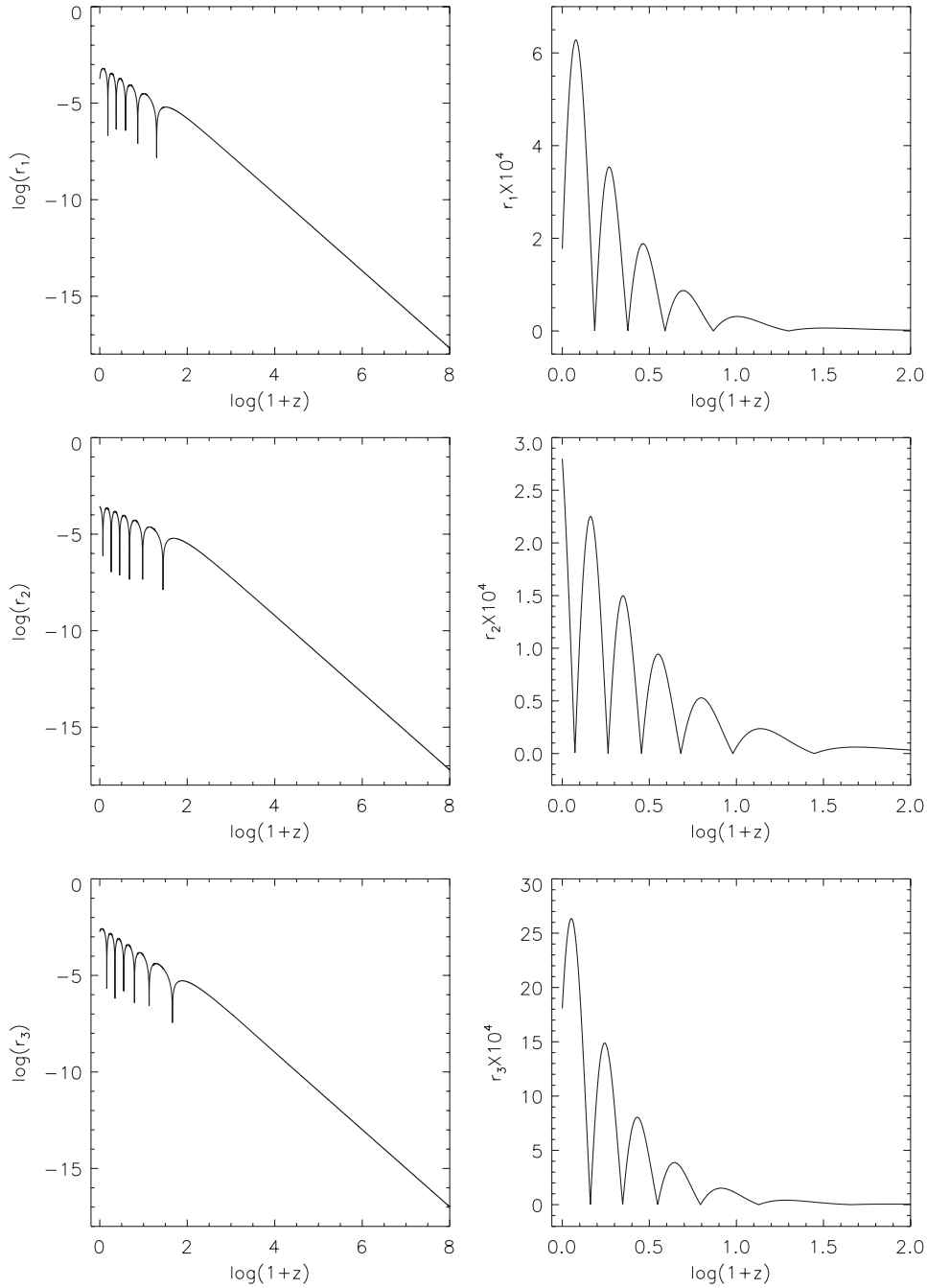
FIG. 2.   Left: functions $\log[r_1(\tilde{k}, z)]$ (top), $\log[r_2(\tilde{k}, z)]$ (middle) and $\log[r_3(\tilde{k}, z)]$ (bottom) in terms of $\log(1 + z)$, from $z = 10^8$ to $z = 0$. The wave number $\tilde{k}$ is the same as in Fig. 1. Right: functions $r_1(\tilde{k}, z) \times 10^4$ (top), $r_2(\tilde{k}, z) \times 10^4$ (middle) and $r_3(\tilde{k}, z) \times 10^4$ (bottom) in terms of $\log(1 + z)$, from $z = 10^2$ to $z = 0$, for the same wave number as in the left panels.

$r_1 \simeq 6.3 \times 10^{-4}$, $r_2 \simeq 2.8 \times 10^{-4}$, and $r_3 \simeq 2.6 \times 10^{-3}$. These maximum values are all reached close to $z = 0$. They are small enough to ensure that, for $\Xi^{(0)\text{in}}(\tilde{L}) = 10^{-4}$, the chosen scale evolves as in GR.

V. DISCUSSION AND CONCLUSIONS

An exhaustive variational formulation of extended electromagnetism has been presented in Sec. II. In particular,

the energy-momentum tensor of the vector field A^μ has been found. This tensor has been analyzed to conclude that the last term of Eq. (8) has not the same sign as in previous calculations [2]. Our sign appears as a result of the relation $\delta_g(\sqrt{-g}J^\alpha) = -\delta_g(\sqrt{-g}J^{A\alpha})$, which is consistent with the conservation law of $J_\mu + J_\mu^A$. This sign is only compatible with a negative constant γ (see Sec. III). Our formulation also leads to Eq. (6) giving the components of the Lorentz force and to Eq. (9).

We have emphasized that, for vector and tensor linear perturbation of the Friedman-Robertson-Walker universe, extended electromagnetism is fully equivalent to Einstein-Maxwell theory. Although this fact was already known (see [2]), our detailed but brief study of vector and tensor perturbations is necessary to describe our approach to the evolution of scalar modes and, moreover, this study exhibits novel aspects. Note that (i) our brief considerations about tensor perturbations have been used to study the evolution of some scalar modes [analogy between Eqs. (15) and (35)], (ii) our analysis of vector perturbations leads to new relations in momentum space derived from Eqs. (16) and, (iii) this analysis suggests the definition of the gauge invariant quantity $E^{(0)}$, which plays a crucial role in our description of the scalar modes.

A new approach to deal with the evolution of the scalar modes in extended electromagnetism has been described. It is gauge invariant and fully general. This formalism leads to Eqs. (33)–(35) and (38)–(42), which are the most important findings of this paper.

The scalar modes associated to the vector field A^{μ} are assumed to be $\Xi^{(0)}$ and $E^{(0)}$. Then, in Sec. III C, it is proved that the evolution equations of these gauge invariant modes [Eqs. (33)–(35)] do not involve any other scalar mode (excepting $J^{(0)}$). These simple equations may be easily solved by using standard numerical methods for any given function $J^{(0)}$.

In Sec. IV, where the simplifying condition $J^{(0)} = 0$ is assumed, the solution of Eqs. (33)–(35) may be found by using the initial value of $\Xi^{(0)}$ at $z \sim 10^8$, whereas the initial values of both $\dot{\Xi}^{(0)}$ and E^0 [related by Eq. (33)] may be neglected. The solution corresponding to a particular wave number $\tilde{k}$ (reentering the effective horizon at the present time) is presented in Fig. 1. There is no problem to integrate Eqs. (33)–(35) for any other scale. Since we have the numerical solutions of these equations, the correction terms ξ_i appearing in Eqs. (38)–(42) may be numerically treated as known functions of k and τ and, consequently, these last equations only involve—as unknown functions—the scalar modes appearing in GR. Their numerical solution has not been found in this paper, where the correction terms ξ_i have been compared with appropriate terms of the GR equations (Y_i) for suitable values of k and $\Xi^{(0)\text{in}}$. For the chosen scale $\tilde{k}$ and $\Xi^{(0)\text{in}}(\tilde{L}) < 10^{-4}$, the scalar modes would evolve as in GR. The spectrum of $\Xi^{(0)}$ and the standard power spectrum $P(k)$, at initial time, would be necessary to perform similar comparisons for all the cosmological scales. If all the scales are found to evolve as in GR, both theories are equivalent from the cosmological point of view; on the contrary, some appropriate code, as e.g., CMBFAST or CAMB, may be modified to estimate—in the VT—the angular power spectrum of the CMB, the matter power spectrum, and so on.

In the background, functions $A_{0B}(\tau)$ and $\Xi_B(\tau)$ cannot be fully fixed. Both functions are proportional to $S_{gn}|\gamma|^{-1/2}$,

but these parameters are arbitrary. Moreover, function $\Xi^{(0)}$ and the correction terms ξ_i defined in Eqs. (45)–(47) are independent of parameters S_{gn} and $|\gamma|$. Hence, cosmological considerations cannot fix the values of these parameters. It is not surprising, since it is well known (see [9]) that the theories based on the Lagrangian (2) with $J^{\mu} = 0$ (no currents) cannot completely fix the vector field.

Condition $J^{(0)} = 0$ has been assumed to be valid in cosmology; nevertheless, this condition is not strictly required by extended electromagnetism. The question is: What would be a scalar $J^{(0)}$ current in cosmology? More research about this scalar mode and its meaning is being carried out.

Finally, let us discuss in detail the fact that our energy-momentum tensor and that found in [2] have opposite signs. The possible consequences of this difference deserve special attention.

If the relation $\gamma = 2\xi$ is satisfied, our Lagrangian (with γ) is identical to that used in [2] (including ξ). In spite of this fact, opposite signs appear in the energy-momentum tensors. As it is explained in Sec. II and summarized in the first paragraph of this section, our sign is obtained—from the common Lagrangian—with right variational calculations based on the true conservation law of the theory. Equation (4) is actually the conservation law satisfied in extended electromagnetism; however, if we take $\nabla^{\mu}J_{\mu} = 0$, which is not an equation of the theory, but the conservation law of standard electromagnetism, the opposite sign is easily found in the resulting energy-momentum tensor. This sign is not right.

For an arbitrary positive ξ value and for the corresponding negative value $\gamma = -\xi/2$, the energy-momentum tensor in [2] is identical to our energy-momentum tensor. Hence, the Einstein equations are also indistinguishable for these values of ξ (positive) and γ (negative). However, the field equations of the vector field A^{μ} are different for the same values, namely, for $\gamma = -\xi/2$.

Equation (3) may be written in the form $\nabla^{\nu}F_{\mu\nu} = J_{\mu} - 2\gamma\nabla_{\mu}(\nabla \cdot A)$, where $\nabla \cdot A = \nabla_{\mu}A^{\mu}$, and these field equations are to be compared with the equations $\nabla^{\nu}F_{\mu\nu} = J_{\mu} - \xi\nabla_{\mu}(\nabla \cdot A)$ appearing in [2]. This comparison shows that the terms $-2\gamma\nabla_{\mu}(\nabla \cdot A)$ and $-\xi\nabla_{\mu}(\nabla \cdot A)$—which modify the field equations of standard electromagnetism—have opposite signs for $\gamma = -\xi/2$. Since the resulting A^{μ} field equations are different, distinct predictions seem to be unavoidable in general nonlinear applications of extended electromagnetism. The discussion of these nonlinear cases is beyond the scope of this paper, where we are concerned with cosmological linear applications of the theory.

Actually, both signs lead to the same conclusions for first order perturbations of Minkowski and Robertson-Walker space-times, namely, for the cases considered, e.g., in [17,18]. It is due to the fact that, according to Eqs. (11), (12), and (35), the signs of Ξ_B and $\Xi^{(0)} \equiv (\nabla \cdot A)^{(0)}/\Xi_B$ are arbitrary and, consequently, for $\gamma = -\xi/2$, these signs

may be chosen to make identical the scalar parts of the terms $-2\gamma\nabla_\mu(\nabla \cdot A)$ and $-\xi\nabla_\mu(\nabla \cdot A)$. Thus, the linearized A^μ field equations derived in our paper become equivalent to those of previous papers [2,17,18]. Since the energy-momentum tensors are also identical for $\gamma = -\xi/2$, the modes of positive energy coincide, and the conclusions of papers [17,18] (with $\xi > 0$) may be also obtained here for $\gamma < 0$. However, only our signs are right and, in general, only our A^μ field equations should be applied in nonlinear cases.

ACKNOWLEDGMENTS

APPENDIX: ESTIMATING THE Y_i FUNCTIONS IN GR

Since WMAP observations strongly suggest that cosmological perturbations are adiabatic, only the case $\eta = 0$ is considered in this section.

For cosmological perturbations evolving outside the effective horizon ($k < aH/2\pi$), the evolution is essentially independent of the microphysics. This means that the anisotropic stress due to neutrinos may be neglected ($\Pi_T^{(0)} = 0$), and also that, in spite of the tight coupling between photons and baryons (see [10]) at $z > 1100$, the transfer of energy and momentum between these two species may be forgotten and, consequently, the corresponding fluids may be treated as independent. This means that, for superhorizon scales, a good enough estimate of functions Y_i may be done by solving Eqs. (38)–(42) for $\Pi_T^{(0)} = \eta = 0$. In this case, Eqs. (38)–(42) lead to

$$\ddot{\Psi}_m + (1 + 3c_s^2)\frac{\dot{a}}{a}\dot{\Psi}_m + \left[k^2 c_s^2 - \frac{1}{2}(\rho_B + P_B)a^2\right]\Psi_m = 0,$$
$$(A1)$$

$$\dot{\Psi}_m = -ka^3(\rho_B + P_B)v_s^{(0)}, \quad (A2)$$

$$\dot{v}_s^{(0)} + \frac{\dot{a}}{a}v_s^{(0)} = \frac{1}{a}\left[\frac{kc_s^2}{(\rho_B + P_B)a^2} - \frac{1}{2k}\right]\Psi_m, \quad (A3)$$

$$\Phi_H = \frac{\Psi_m}{2ak^2}, \quad (A4)$$

where $\Psi_m = \rho_B a^3 \epsilon_m$. This system of equations must be solved together with the background equations for appropriate initial conditions at $z = 10^8$.

The background is a flat universe with cosmological constant. The energy densities of matter, radiation and vacuum correspond to the concordance model (see above). The background differential equations may be easily integrated to get $a(\tau)$, $\rho_B(\tau)$, and $P_B(\tau)$.

The integration of Eqs. (A1)–(A3) only requires ϵ_m^{in} and $v_s^{(0)in}$ at $z = 10^8$. In fact, from the first of these values one easily obtains Ψ_m^{in}, and the initial value of $\dot{\Psi}_m$ may be then obtained by substituting $v_s^{(0)in}$ into Eq. (A3). The second order differential equation (A1) may be integrated by using Ψ_m^{in} and $\dot{\Psi}_m^{in}$. Function $\Psi_m(\tau)$ is then known and $v_s^{(0)in}$ may be used to solve Eq. (A3) and get function $v_s^{(0)}(\tau)$.

Since ϵ_m and $v_s^{(0)}$ are gauge invariant quantities, their initial values may be calculated in any gauge. We have used the synchronous gauge to perform this calculation. For superhorizon scales, Eq. (96) of Ref. [10] may be used to easily get the following initial conditions:

$$\delta_\gamma^{in} = -\frac{2}{3}C(k\tau^{in})^2, \quad \delta_c^{in} = \delta_b^{in} = \frac{3}{4}\delta_\gamma^{in}, \quad v_c^{(0)in} = 0,$$

$$v_\gamma^{(0)in} = v_b^{(0)in} = -\frac{1}{18}Ck^3(\tau^{in})^3, \quad H_L^{in} = \frac{1}{6}C(k\tau^{in})^2, \quad (A5)$$

$$H_T^{(0)in} = -6C\left(1 + \frac{1}{18}\right)(k\tau^{in})^2,$$

where the conformal time τ^{in} is that corresponding to the chosen initial redshift $z = 10^8$, C is a normalization constant, and the subscripts γ, b, and c stand for photons, baryons, and cold dark matter, respectively. The fluid formed by these three components has the following density contrast and peculiar velocity [10]:

$$\delta = (\rho_{Bb}\delta_b + \rho_{Bc}\delta_c + \rho_{B\gamma}\delta_\gamma)/\rho_B,$$

$$v^{(0)} = [(\rho_{Bb} + P_{Bb})v_b^{(0)} + (\rho_{Bc} + P_{Bc})v_c^{(0)}$$

$$+ (\rho_{B\gamma} + P_{B\gamma})v_\gamma^{(0)}]/(\rho_B + P_B). \quad (A6)$$

The initial values of ϵ_m and $v_s^{(0)}$ may be easily calculated taking into account Eqs. (37), (A5), and (A6). Equations. (A1)–(A3) may be then solved.

The estimation of quantities Y_i requires normalization. The question is: How can we find a good enough value of
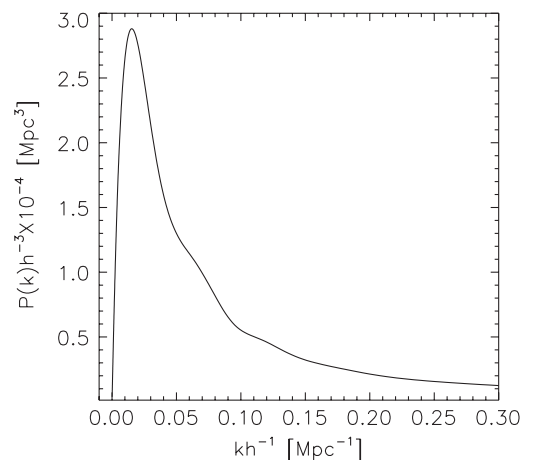

FIG. 3. Matter power spectrum—estimated with CMBFAST— for the chosen version of the concordance model.

the normalization constant C? It is well known that, for superhorizon scales, the gauge invariant quantity

$$\zeta = \frac{2}{3} \frac{\Phi_H + (aH)^{-1}\dot{\Phi}_H}{1+w} + \Phi_H \left[1 + \frac{2}{9}\left(\frac{k}{aH}\right)^2 \frac{1}{1+w} \right] \quad (A7)$$

is conserved [19] and, moreover, at horizon crossing, the relation

$$\delta(k, \tau) = O(1)\frac{\zeta}{1+w} \quad (A8)$$

is satisfied, where $O(1)$ is a number of order unity (see [19] and references cited therein). Then, normalization may be achieved as follows: in a first step, the matter power spectrum at present time $P(k, \tau_0)$ is obtained, by using CMBFAST, for a certain version of the concordance model

(see [20] for details). The resulting spectrum is represented in Fig. 3. From it, we can estimate $P(\tilde{k}, \tau_0)$. In a second step, Eqs. (A1)–(A3) are numerically solved for the scale $\tilde{k}$ and for an arbitrary C value and, then, by combining Eqs. (A2), (A4), and (A7), the function $\zeta(\tilde{k}, \tau)$ may be easily calculated. Finally, in a last step, the value of the normalization constant C is fixed. It is done by using Eq. (A8) to calculate $\delta(\tilde{k}, \tau_0)$, and taken into account that the resulting $\delta(\tilde{k}, \tau_0)$ quantity must be identical to $P^{1/2}(\tilde{k}, \tau_0)$ for the right C value.

The $\delta(\tilde{k}, \tau_0)$ value obtained from the spectrum of Fig. 3 and Eq. (44) may be easily used to estimate the contrast $\delta(\tilde{L}, \tau_0)$ in position space, and the resulting value is close to 10^{-3} as it is expected for this scale.

---

[1] J. Beltrán Jiménez and A. L. Maroto, J. Cosmol. Astropart. Phys. 03 (2009) 016.

[2] J. Beltrán Jiménez, T. S. Koivisto, A. L. Maroto, and D. F. Mota, J. Cosmol. Astropart. Phys. 10 (2009) 029.

[3] J. Beltrán Jiménez and A. L. Maroto, Phys. Rev. D **83**, 023514 (2011).

[4] J. Beltrán Jiménez and A. L. Maroto, Prog. Theor. Phys. Suppl., **190**, 33 (2011).

[5] J. M. Bardeen, Phys. Rev. D **22**, 1882 (1980).

[6] W. Hu and M. White, Phys. Rev. D **56**, 596 (1997).

[7] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time*, Cambridge Monographs on Mathematical Physics (Cambridge University Press, New York, 1999).

[8] C. M. Will, *Theory and Experiment in Gravitational Physics* (Cambridge University Press, New York, 1993).

[9] C. M. Will, Living Rev. Relativity **9** (2006) [http://www.livingreviews.org/lrr-2006-3].

[10] C. P. Ma and E. Bertschinger, Astrophys. J. **455**, 7 (1995).

[11] J. A. Morales and D. Sáez, Phys. Rev. D **75**, 043011 (2007).

[12] J. A. Morales and D. Sáez, Astrophys. J. **678**, 583 (2008).

[13] N. Jarosik *et al.*, arXiv:1001.4744.

[14] E. D. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, Reading, MA, 1994).

[15] U. Seljak and M. Zaldarriaga, Astrophys. J. **469**, 437 (1996).

[16] A. Lewis, A. Challinor, and A. Lasenby, Astrophys. J. **538**, 473 (2000).

[17] J. Beltrán Jiménez and A. L. Maroto, J. Cosmol. Astropart. Phys. 02 (2009) 025.

[18] J. Beltrán Jiménez and A. L. Maroto, Phys. Lett. B **686**, 175 (2010).

[19] R. H. Brandenberger, Rev. Mod. Phys. **57**, 1 (1985).

[20] R. Dale and D. Sáez, Astrophys. Space Sci. **337**, 439 (2012).