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Minimization of the stress concentration in Formed Parts through Non-Parametric Optimization

Salam O. Dahi[1*], Badr Kamoon[2], Hamzah Kadhim Hasan[3]

[1,2]Al Furat Al Awsat Technical University/ Technical College Al-Mussaib
[3]Al Furat Al Awsat Technical University/ Karbala Technical Institute
**Email: Salam obaid@atu.edu.iq*

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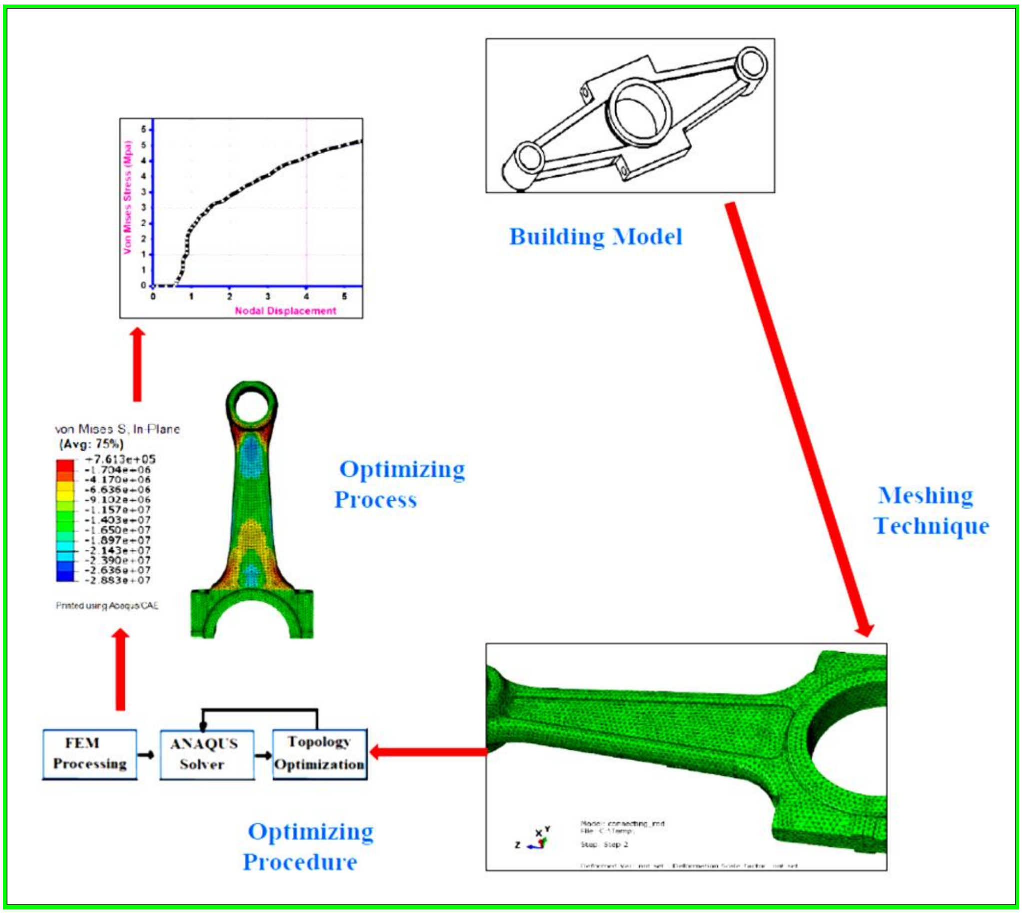
Abstract: Parametric and non-parametric are the main optimization methods that are used in various industrial fields. In non-parametric optimization, the process of manipulating the node locations (shape optimization) or removing mass without changing the node locations (topology optimization) is adopted to achieve a desired objective. This structural optimization is formulated as a non-parametric problem, and for analysis purposes, ABAQUS/CAE software is adopted for this approach. Manufacturing process like forming is always linked with stress concentration, especially in the sharp ends and variable cross sections like holes and fillets. The problems of representation and finding the optimal and better structural design of some known quantities such as reactions, loads and masses is not easy. A large deflection may be induced in a structure when experiencing severe mechanical loads. In this work, the numerical method has been presented to investigate a method for optimization of formed parts geometry. Numerical examination confirmed that high-stress concentrations are generated in many places. Material distribution is highly influenced by nonlinearity and the new layout will result in intermediate densities. In such cases, the nonlinear elasticity like nonlinear strain must be considered. As a result, the non-parametric optimization can offer good design flexibility to use the existing model with ease of setup and without the need for parameterization. It can provide a conceptual design that can reduce the structure's weight to the maximum extent in the early design stage. This work is going to optimize the design of the formed plates by reducing the volume while maximizing its stiffness. As a recommendation, in order to provide an attractive approach with suitable levels of structural performance, the combination of both optimization methods is the short way to achieve this aim.

Keywords: Numerical Analysis; Stiffness; Structure topology; Simulation; ABAQUS.

Graphical abstract



1. Introduction

Structural optimization is classified as parametric optimization and non-parametric optimization. Topology optimization has become a widely used tool in industrial design. It's a powerful tool used in various design areas such as optics, electronics, and structural mechanics. This type of optimization involves design element modification to create hard and soft structures, while shell element thicknesses will be adjusted in the sizing optimization. The resulting designs are seen as proposals and can differ greatly from those obtained through trial and error or incremental improvements. The use of topology optimization has become a popular method for reducing weight in casting parts during early product development. The sizing optimization differs from topology sizing in terms of the type of design variables used. Parametric optimization like sizing optimization usually uses metaheuristic methodologies in which the variables such as dimensions are used in defining a geometric model to achieve a desired objective.

In structural design, Non-parametric optimization is used to optimize the design of a structure without the availability of data about some parameters such as the shape or size of the structure. This includes maximizing stiffness and minimizing weight. Also, this approach is useful when the design is complex and difficult to model mathematically.

The sizing such as the volume of specific material or cross-section can immediately be related to stress minimization problems. Additionally, sizing optimization allows for the efficient design of sheet metal parts by modifying their thickness. It involves finding the optimal distribution of the material in a domain to achieve a specific objective function while adhering to certain constraints. A non-parametric approach can provide multiple options to optimize the weight, stiffness, and dynamic behavior of shell structures, making it particularly useful for designing parts.

Structural optimization has been adopted in the current forming process. This method is apt for this process and can be grouped into topology, size, and shape optimization. It can be used to minimize weight, stresses, and easily determination of the boundary conditions. Moreover, this method is seamlessly integrated with the processing capabilities, allowing for fast and dependable optimization setup and results.

2. Literature review and problem statement

Nowadays simplifying the modelling is possible by using the numerical solution. The Finite difference method and finite elements method are the particular discretizations of these substantial cases. The most important features in FEM modelling are approximating the domain of the specific design into subdomains as elements that represent the variables. This approximation consists of undetermined parameters and algebraic polynomials. The method of weighted residues is normally for converting the physical problem into a finite element numerical set in order to form matrix formation [1].

However, when nonlinearity is introduced, optimization must also account for buckling instability, which can significantly decrease the structure's load-carrying capacity. The demonstration of the ability to optimize a topology while accounting for the effects of buckling caused by thermal expansion is a clear indication of its potential for practical applications where the suppression of buckling under high temperatures is necessary [2].

The optimization process involves manipulating the density and Young modulus of each element, which can result in millions of design variables. Despite this, the optimization process generally takes only 15-80 iterations. Objective functions and constraints can be based on static analysis and frequency response analysis. The objective function can be minimized, or maximized, or a Min-Max formulation can be used [3].

Shape modification methods can be used to enhance the design process. It’s the most desirable way, especially from the standpoint of design. For the objectives of minimizing the values of von Mises stresses by using a non-parametric shape optimization problem, we can formulate the following equation [4].

In a non-parametric sizing method, the sensitivity-based algorithm is used to allow the handling of a large variable number. Sizing can be approached in two ways: free, where each shell element can obtain a separate thickness, or clustered, where a group of elements is clustered together to be a uniform shell thickness [5]. In modern times, numerical methods utilizing FEA software are commonly used for design analysis, and some nonlinear methods deal with deforming the structures by applying thermal loads only and without applying any mechanical load [6, 7].

To estimate the high-resolution parameters, the connections between low-resolution samples are used, and a limited number of simulations is required to implement this process. It has been demonstrated that the resolution approach produces nearly identical designs with much lower computational costs [8, 9].

Compliance is the primary objective function used in topology optimization, where the weight of saved materials serves as the constraint. On the other hand, structural optimization can have several multidisciplinary constraints such as stresses, displacements, eigenvalues, and the effectiveness of control surfaces, flutter speed, etc. [10, 11]. Industries can significantly decrease the time required for product development in design and production and consequently, reduce their expenses while enhancing their profits through the application of optimization methods [12].

In structural design, topology optimization can be considered an extension of size and shape optimization methods, which seek to find the optimal values of parameters defining members or configurations of a structure. The optimality criteria method through ANSYS software is commonly used for topology optimization. Structural analysis, which includes analytical, experimental, and numerical methods, is used to assess the behavior of engineering structures. Among the numerical methods, finite element analysis (FEM) is the most comprehensive technique used by engineers today. While the solution provided by FEM is approximate, it can be improved by using more elements to represent the model [13, 14].

Topology optimization and shape optimization are powerful tools that can be utilized in the initial design phase. The entire geometry is considered a variable, enabling large structural modifications such as the creation of new boundaries and/or holes. The optimal shape is not dependent on the initial guess due to this. The multi-objective optimization approach is a good way to minimize mass and maximize stiffness while ensuring that stress constraints are met everywhere except at stress concentration. This approach is useful for the conceptual design of structures. However, the obtained structure's geometry must be further modified by the designer to satisfy non-structural design requirements and eliminate stress concentration [15, 16].

The global stress levels are measured using the normal stress aggregation scheme. An efficient sensitivity number formulation is derived from the adjoin sensitivity of this measure. To stabilize the optimization process, both sensitivity numbers and topology variables are filtered due to the highly nonlinear stress behavior. Additionally, the filtered sensitivity numbers are further stabilized with their historical information. Through a series of 2D and 3D benchmark designs, the method has been demonstrated to be effective, feasible, and simple to implement [17, 18].

Topology optimization involves using mathematical methods to determine how to distribute materials within a fixed geometric domain while adhering to certain constraints, in order to optimize the mechanical performance of a structure [19, 20].

A phenomenon of stress concentration occurs when the variable cross-section material in a small area or a sharp corner is loaded. This can lead to failure under variable load at any time. The challenges related to stress concentration in these parts include parts materials designed to distribute stress more evenly, the manufacturing process, and recommended high-strength materials.

However, many current techniques for managing stress constraints in topology optimization do not effectively address the non-self-adroitness of this type of problem. The challenges related to stress concentration forming parts include parts materials designed to distribute stress more evenly, the manufacturing process, and recommended high-strength materials.

3. The aim and objectives of the study

The main purpose and objective of finite element model construction this modeling process and optimization are:

1- To explore and identify the weak points in the part during particle dislocations caused by extreme mechanical loading. This analysis process ultimately leads to a diagnosis of many unexpected errors before the tools are invested.

2- During this simulation process, some important parameters will be investigated and determined, and even up to many fabrication errors can be avoided.

3- The findings of the finite element model may lead to the determination and reduction of the equivalent stress concentration values and maximize the stiffness of the design area.

4- It also aims to identify the best combination of non-parametric variables to evaluate the impact of these process parameters on the optimization procedure.

4. Materials and Methods

4.1 Process Design Layout

To initiate the procedure, the primary step is to establish the geometry of the workpiece, which involves specifying all the essential dimensions for the modeling process. Additionally, it is crucial to define the material specifications and other geometrical properties.

Figure (1) and Figure (2) illustrate the part geometry includes all important dimensions and the part geometry in three dimensions respectively for the product understudy in this process.

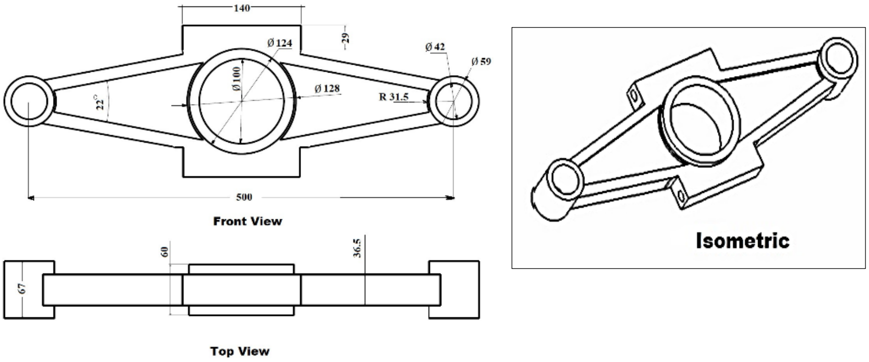


Figure 1. *The part geometry includes all important dimensions.*

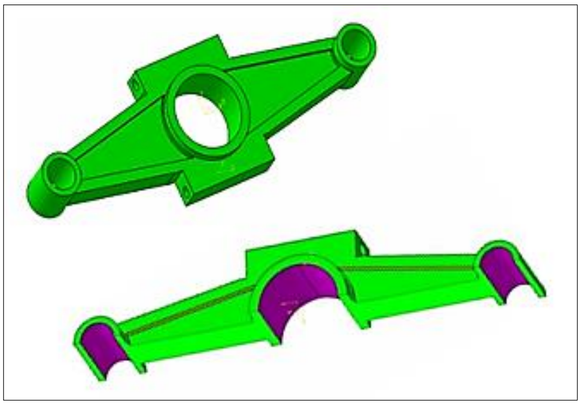


Figure 2. *Representation of the part geometry in a three-dimensional.*

The material of this sample is made of elastic steel with a density of (7800) kg/m3, (0.3) Poisson’s ratio, and a Young modulus value of (210 GPa).

4.2 Simulation and Analysis Process

The complex geometry is difficult to parameterize because it has an irregular mesh with sophisticated boundary conditions such as contacts, springs, and kinematic couplings. Consequently, non-parametric optimization is normally preferred in these cases. The optimization problems related to structural and topology optimization can be expressed as non-linear programming problems, but the variations in the statements depend on the type of objective function used, design constraints imposed, and the kind of design variables included in the optimization problem. The optimization process for determining the optimal structure involves interpolating a shape density function over quadrilateral elements, allowing both shape and topology to be variable. The simulation procedure of this model by ABAQUS /CAE involves many steps illustrated in Figure 3.

The low-resolution model is utilized to investigate the parameter space and estimate the high-resolution model and its sensitivity, which includes parameters related to structural load and stiffness. The low-resolution model helps identify parametric variations and allows for many simulations to be conducted.

Figure 4 shows the workflow based on ABAQUS finite element.

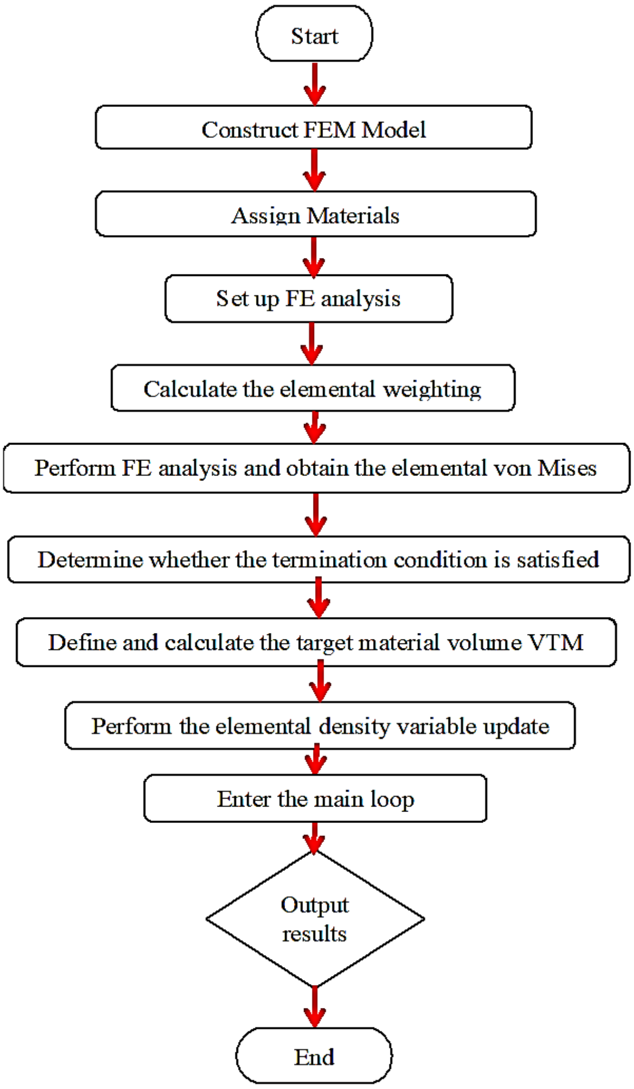


Figure 3. *Flowchart of the topology optimization procedure.*

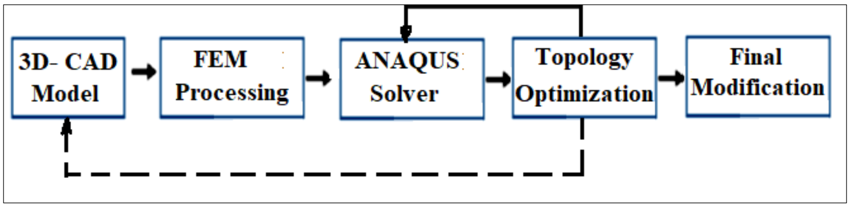


Figure 4. *Topology-optimized design process workflow based upon ABAQUS finite element.*

The loading and boundary including constrained the displacement of the node in the canter in the x- and y-directions. Also, the rotation is constrained for all nodes in all directions. Load a 17300 N is applied at the center. In order to regulate the thickness of elements, the optimization process establishes minimum and maximum thickness limits as boundary conditions. These limits can be determined as absolute values or relative to the original element thickness. The smoothing of the mesh is performed using an orphan mesh with linear tetrahedral

(C3D4) elements and is symmetrical with respect to the X-Z plane. It adjusts the position of the inner nodes in relation to the movement of the surface nodes to ensure a uniform thickness throughout. Figure 5 illustrates this mesh type.

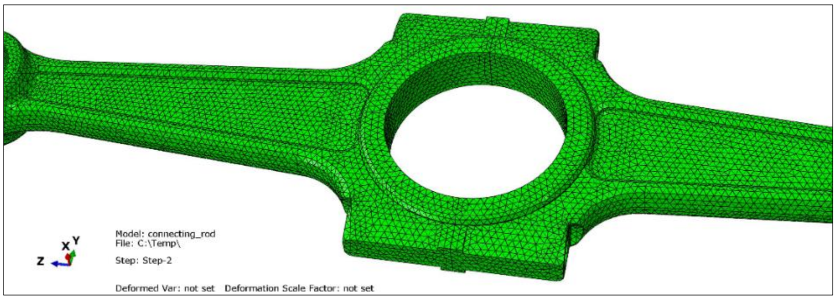


Figure 5. *An orphan mesh with linear tetrahedral (C3D4) elements.*

The tetrahedral (C3D4) element is the 3D normalization of a 2D triangular mesh and is commonly constructed as equilateral, such as in systems with curvatures. This element type is commonly used due to its ability to satisfy the requirements of numerical simulations.

The mesh generation greatly simplifies the structure definition which permits the sequential solution in a few steps. Sometimes the von Mises stress referred to a uniaxial tensile that would create distorted energy due to the non-linear behavior of the material [21].

$\partial\rho/ \partial t + \nabla *(\rho v) = 0;$

$\rho\, (\partial v / \partial t) + \rho v * (\nabla * v) = \nabla *\sigma + \rho\, g\, ;$

$\rho\, [(\partial\, e / \partial t + v * (\nabla e)] = \sigma : \varepsilon\, ,$

Where: -

$\varepsilon = D - \nabla *h + \rho\, r$ (1)

Where (D) = ½[$\nabla\, v + (\nabla v)c$] is the rate of deformation, (ε) is the strain rate, (σ) is the stress, (v) is the velocity, (ρ) is the density, (g) is the body force, (e) is the specific internal energy, (h) is the heat flux, and (r) is the internal heat source.

In the first step, the load in the z-direction is applied with a magnitude of 14000 N to the center node of the rod. In addition, 1750 N will be applied in the z-direction. In this model, some regions are required for applying loads and fixtures, so they will be excluded from any additional applied loads.

4.3 Finite Element Proceeding

For analysis purposes, ABAQUS topology optimization is adopted for this approach. In non-parametric optimization, to achieve a desired objective, the optimization process either manipulates the node locations (shape optimization) or removes mass without changing.

A standard CAE model normally contains different element types such as beams, solids, and shells.

Topology optimization is the distribution of given material resources within a specified spatial domain in a systematic design framework to achieve maximum stiffness.

The following function is used to calculate the von Mises stresses (σvms) [22]:

$\sigma_{vms} = [\sigma^2_1 + \sigma^2_2 + \sigma^2_3 - \sigma_1\, \sigma_2 - \sigma_2\, \sigma_3 - \sigma_3\, \sigma_1\,]^{0.5}$

$\sigma_{vms} = [\sigma^2_1 + \sigma^2_3 - \sigma_3\, \sigma_1\,]^{0.5}$

In terms of applied stresses in coordinate directions. Von-Mises criterion sometimes referred to as equivalent stress is used for account the material hardening.

$\sigma_{vms} = [\sigma^2_x - \sigma_x * \sigma_y + \sigma^2_y + 3\tau_{xy}\,]^{0.5}$ (2)

Where σ_y and σ_x are the components of the stress tensor σ in the directions of y and x respectively, and τ_{xy} represents the shear stress.

The evolutionary structural optimization method is proposed in this work as an evolutionary topology optimization method for stress minimization design. The design variables for an optimization problem represent the parameters to be changed during the optimization.

The optimal structure is determined using multiple objectives, which involve maximizing stiffness and minimizing mass. Compliance and mass are combined into an objective function that is minimized. The ratio between weight factors in the optimal design results in a more or less uniform stress distribution, except at stress concentrations which typically occur at the boundaries due to space constraints. To reduce stress concentrations, larger safety factors can be used and the feasible domain can be increased where possible. To estimate the values of these stresses and other corresponding parameters, element, and nodal labelling are very important to determine the effect at each node and element exactly. Figure 6, and Figure 7 show the distribution of elements and nodal labels overall on the geometry surface.

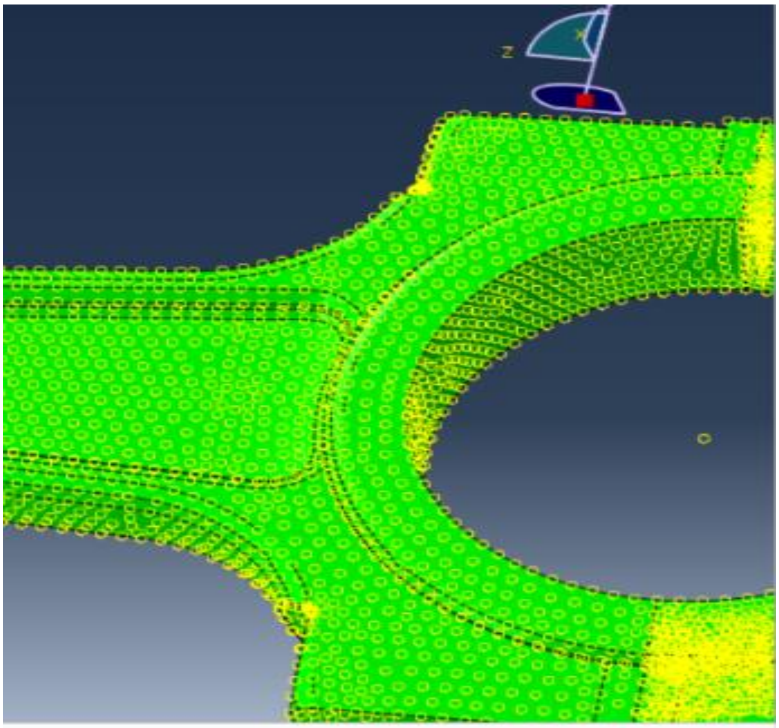


Figure 6. *Elements distribution over the central geometry surface.*

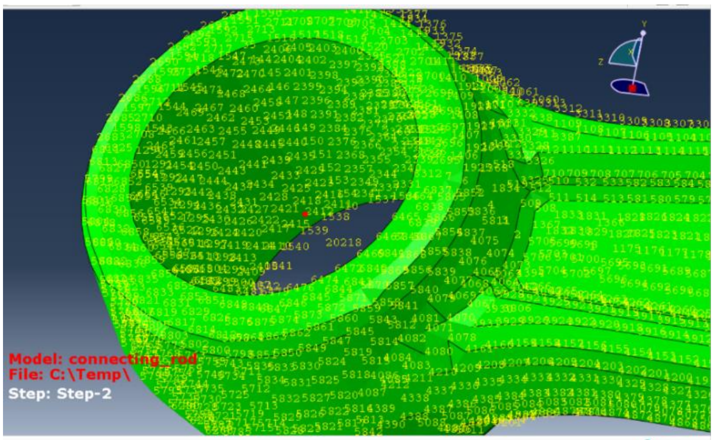


Figure 7. *The density of nodal distribution in the part end holes.*

The optimization will remove elements from the model by providing stiffness and mass to ensure that they don’t participate in the response of the structure.

The displacements of the surface nodes for shape optimization, in the design area, are considered the design variables. During topology optimization, the module either moves a node inward (shrinkage) or outward (growth) or leaves the position unchanged. The influence of restriction on a surface can move the node in the direction to be moved. Topology Optimization will interpolate the displacement of the corner nodes from the movement of mid-side nodes.

Topology optimization as non-parametric optimization is the most important material optimization approach adopted here as a method that can lead to better results based on time optimization and material reduction. In this paper, a topology optimization process is conducted to optimize the design of the formed rod by reducing the volume while maximizing its stiffness. However, it’s helpful for the identification of the ideal material optimization procedure and for the pursuit of lightweight structure solutions without compromising their initial strength

5. Results and Discussion

The current work is conducted by using ABAQUS/ CAE software to analyze the linearly elastic isotropic structures and determine compliance, optimized shape, deformed and unreformed shape, displacements, and stresses based on some criterion. A design response will calculate the volume and the sum of the important stresses overall of the elements in the design area. Structural topology optimization can be achieved and manipulate these multiple analytical and numerical tasks. The main challenge in this optimization is related to the type of design variables, which are binary and continuous design variables. The input file of the model contains the element in the form of an orphan mesh to define the model that is used by the optimization. The Analysis type performed here is static stress analysis.

Optimization constraints include constraining the intended topology from making any changes and allowing the optimization to implement a feasible and acceptable solution. It’s notable to say that during the optimization task, some design areas that are required as fixtures for applying loads will be modified and may be involving some excluded regions due to this reason.

Minimizing the maximum von Mises stress and determining the maximum von Mises stress is the main approach for creating this optimization. The orphan mesh with mesh smoothing is applied for this model in all design areas. The load value of 14000 N that is applied will cause to generate high von Mises stress values in the part and these stresses will raise with continuous loading. The contour plot in Figure 8 shows the distribution of initial von Mises stresses in the case of initial loading.

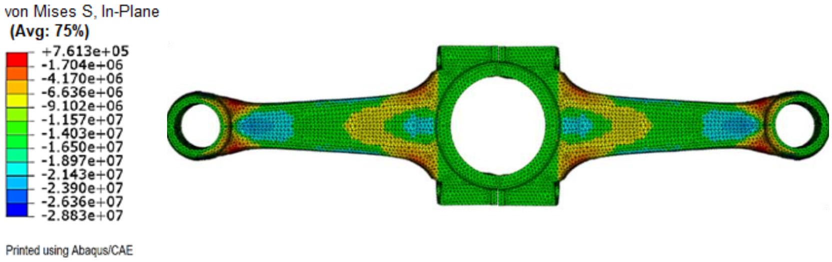


Figure 8. *The contour plot of Von Mises stress distribution during the loading case.*

Consequently, during the first step, and to reduce the maximum von Mises stress, the surface nodes will be adjusted and resulting in an optimized shape that leads to a decrease in the von Mises stress gradually. In the second optimization step, these stresses will be minimized slightly under the loading conditions. Figure 9 shows the contour plot of the change in the stress distribution after optimization during the second load case.

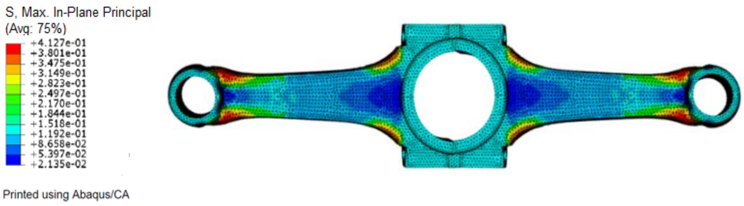


Figure 9. *The contour plot of stress distribution after optimization.*

As a result of the optimization process, the specified target volume and maximum stiffness are achieved. Also, as the volume of material is reduced, the strain energy increases, and the optimized design is about 60 % of the topology of the initial volume. The new design geometry formed by the topology optimization should not contain defects and undercuts.

The output plot and contour plot shows that, throughout the optimization process the volume design is kept constant. In the design area, the surface node’s position has been optimized so that the geometric constraints and specific size constraints of von Mises stress during pregnancy are minimized. In the first step, the applied load of (25000 N) will result in the maximum von Mises stress value in the rod. Consequently, during the first step, the maximum von Mises stress at the surface nodes is reduced. Figure 10 shows the concentration of stresses on the modal surfaces.

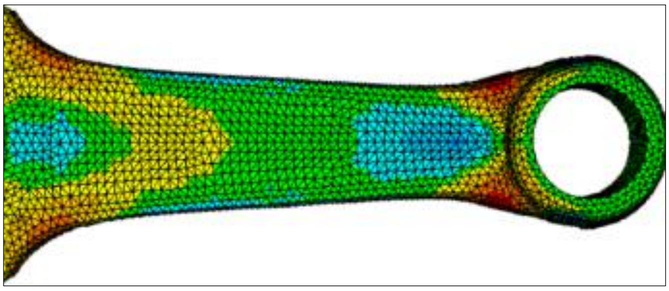


Figure 10. *Stress concentration on the modal surfaces.*

In the resulting optimized shape at the second step, the von Mises stress will slightly increase due to the effects of variable loading conditions. Results revealed, that life expectancy by using the stress-based topology optimization is about 5% higher than the compliance-based topology optimization. Also, it achieved a better prediction of fatigue life. It found the density of stress concentration will be very high in the corner and fillet zones as shown in Figure 11.

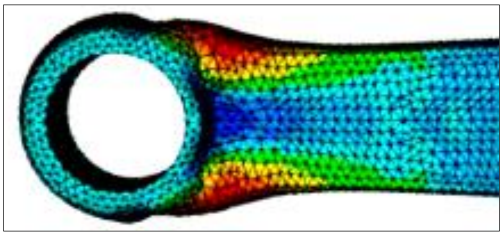


Figure 11. *The density of stress concentration in the fillet zones.*

Von Mises stress is mostly used to determine if a given material will yield at a specific load or not. Material yielding always depends on crystal dislocations and nodal displacement. The relationship between nodal displacement and Von Mises stress is illustrated in Figure 12.

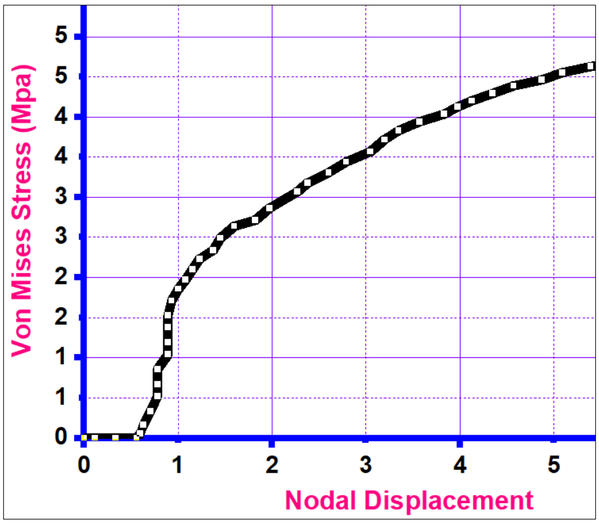


Figure 12. *The plot of the nodal displacement and Von Mises stress relationship.*

5.1 Discussion

The ease of setup and avoidance of restrictive parameterization with respect to design freedom is the main advantage of non-parametric shape optimization. The material properties in the areas of the loaded elements during optimization will be modified (adding elements or removing elements) till achieved the optimal solution. But the unloaded elements will be excluded and their properties still unchanged as explained in Figures (8, 9, 10, and 11).

According to the applied boundary conditions, and the model constraints, the optimized part should contain 75% of the initial volume of the original model.

A comparison of the analysis results of stress concentration has been made with the work [6]. The free-sizing results share similarities with topology optimization. The equivalent distribution stresses will be observed at many zones and, junctions, especially in strong bending and rectangular cross-section.

The distribution of the convergent thickness shows the regions with minimum and maximum allowable thickness.

Implementation of such results can be difficult, but it provides insight into areas that are important for stiffness [10].

Car parts designers can utilize these results to suggest ideas such as local reinforcement plates and tailor-welded blanks. Two elements can be characterized by exploring and validating potential benefits from the results of free sizing. A new clustering approach can be used to define the new cluster groups, and this new clustering of components can be used for repeating the sizing run. Due to the clustering difference, the distribution of the thickness will be different from that result from the first run.

The legend and scale for the thickness remain unchanged to enhance the comparison between the first and second runs based on the thickness distribution. Thus, the weight reduction achieved is twice more than the weight reduction that was achieved with the first sizing run with combined components. The method of nonlinear optimization presented will help in exploring a large and lighter design environment compared with other methods. The measure of nonlinear strain is associated with harmonious nonlinear finite element analysis sensitivities. Topology optimization reveals that the nonlinear elasticity is a remarkable property in this simulation.

6. Conclusions and Future Scope

6.1 Conclusions

The non-parametric module structure is an effective strategy for optimizing the thickness of shell structures of many automobile components. Shape and topology optimization is usually performed to reduce the part weight while maintaining the required stiffness and frequency level. It offers benefits such as no need for parameterization of individual shell thicknesses and efficient handling of millions of design variables, making it ideal for optimizing large-scale components. Structure topology can greatly influence any change in the stress state caused by deformations due to external mechanical loads. The nonlinear model can be constructed by decomposing deformed and un-deformed bodies. The optimization procedure involves finite element modeling of the part and simulation results evaluation.

During the analysis of this model, some conclusions are recorded below:

1- This nonlinear optimization approach is proposed to minimize the von Mises stress under some constraints like volume and assume maintaining the original shape.

2- For the specific shape design, it can be achieved without parameterization of design factors. It’s concluded that this optimization method is effective and especially suitable for improving the design process at the final stage.

3- Significant design can be achieved by integrating nonlinearity in topology optimization when applying mechanical load to the structure. The optimized design is about 60 % of the topology of the initial volume.

4- It is concluded from the results that the density of stress concentration will be very high in the corner and fillet zones. It also concluded that the stress-based topology optimization is about 5% higher than the compliance-based topology optimization

5- Compared to the starting design, approximately more than 10% of the same sizing thickness the weight is reduced. It is notable to say that the powerful concept of reducing the structural weight of the products is topological optimization.

As a recommendation, the non-parametric optimization offers a robust and efficient approach for lightweight, eliminates segregated materials, and enhances additive manufacturability. It also improves integrity, minimizing materials wastage, and providing significant weight reductions.

6.2 Future Scope

The future scope should involve experimental validation for these solutions where the data taken from the simulation would be compared to the corresponding data from the experiments. The optimization possibilities and options could expand through the creation of new processes that may combine or integrate with existing results. Furthermore, the layout of the taken optimization (parametric or nonparametric) could be compared to the original design space.

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Conflict of Interests
We have clearly acknowledged that this work received no specific grant from any funding agency.

The authors declare that they are self-sponsored and they didn't receive any financial support.

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Data Availability
All datasets were generated during the current study. These data are available from the corresponding author upon reasonable request.

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