

A Bayesian updating of crack distributions in steam generator tubes

Atualização Bayesiana da distribuição de trincas em tubos de gerador de vapor

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Abstract

The structural failure of steam generator tubes is a common problem that can affect the availability and safety of nuclear power plants. To minimize the probability of occurrence of failure, it is needed to implement maintenance strategies such as periodic nondestructive inspections of tubes. Thus, a tube is repaired or plugged whenever it has detected a crack which a threshold size is overtaken. In general, uncertainties and errors in crack sizes are associated with the nondestructive inspections. These uncertainties and errors should be appropriately characterized to estimate the actual crack distribution. This work proposes an Bayesian approach for updating crack distributions, which in turn allows computing the failure probability of steam generator tubes at current and future times. The failure criterion is based on plastic collapse phenomenon, and the failure probability is computed by using the Monte-Carlo simulation. The failure probability at current and future times is in good agreement with the ones presented in the literature.

Keywords

Steam generator tubes • Crack distribution • Bayesian updating

Resumo

A ruptura de um dos tubos do gerador de vapor é um problema que pode afetar a disponibilidade e segurança das usinas nucleares. Para reduzir ao mínimo a probabilidade da ocorrência desse problema, deve-se implementar uma estratégia de manutenção com inspeções periódicas dos tubos do gerador de vapor, por meio de técnicas não-destrutivas. Com isso, um tubo é reparado ou tamponado sempre quando o tamanho da trinca detectada ultrapassa um valor crítico. Em geral, incertezas na detecção e erros de medição estão associados às técnicas não-destrutivas. Essas incertezas e erros devem ser caracterizados propriamente para se estimar acuradamente a distribuição dos tamanhos de trincas. Neste trabalho, propõe-se aplicar uma abordagem probabilística Bayesiana para atualizar a distribuição dos tamanhos de trinca, e a partir da distribuição obter a probabilidade de falha dos tubos do gerador de vapor em momentos presente e futuro. O critério de falha é baseado no fenômeno do colapso plástico, e a probabilidade de falha é computada através da simulação de Monte-Carlo. Os resultados de probabilidade de falha em momentos presente e futuro estão em boa concordância com valores encontrados na literatura.

Palavras-chave

Tubos do gerador de vapor • Distribuição de trincas • Atualização Bayesiana

1 Introduction

Steam generator (*SG*) is an important equipment for the operational performance and safe condition of nuclear power plants of pressurized water reactor (*PWR*) type. The *PWR* plants are considerably safe, but a failure in some

SG components has the potential for releasing nuclear materials into the environment.

The *SG* tubes are components subjected to deterioration mechanisms, such as stress corrosion cracking which gives rise to initiation and propagation of crack-like flaws. Such flaws are spots where nuclear materials from the primary circuit pass through the *SG* tubes to the secondary circuit. One way to remediate this radioactive release is to plug or repair the cracked tubes. This technical procedure assures a safe condition for the *PWR* plants, however it may affect the operational performance of the plants. An optimal balance between safe conditions and operational performance is achieved based on a reliable evaluation of crack distributions in the *SG* tubes [1].

The updating of crack distributions begins by performing periodic nondestructive inspections (*NDI*) on the *SG* tubes. The result of such inspections reveals the presence of cracks, as well the size of the cracks. However, detection and measurement of cracks are subjected to uncertainties and errors. Hence, attention should be given to the assessment of the effect of *NDI* on the crack distribution. The effectiveness of *NDI* is a function of the probability of detection of the cracks [2].

At current times, crack distributions can be determined by using an appropriate detection curve of the *NDI* and the Bayes theorem for specializing crack distributions. The evaluation of actual crack distributions is known as condition monitoring assessment. As long time goes by, stress corrosion cracking acts to propagate the crack sizes, and a new crack distribution will be found at future times. Based on the crack size growth it is possible to estimate future crack distributions [3]. The evaluation of future crack distributions is known as operational assessment. Finally, the structural integrity of *SG* tubes is assessed by calculating the failure probability based on the plastic collapse criterion. In this criterion, crack sizes are significant parameters considered as time-dependent random variables. The method for obtaining the failure probability is the Monte-Carlo simulation [4]. Numerical results from the Monte-Carlo simulation have shown the efficiency of the Bayesian update procedure as a way to adapt the decision rule of the aperiodic maintenance policy [5, 6]. In summary, structural reliability is assessed as condition monitoring and operational assessments based on actual and future crack distributions, respectively. The results of failure probability of *SG* tubes are in good agreement with ones in the literature.

In Section 2, the procedure used for updating crack distributions is presented. In Section 3, the fundamentals of the structural reliability assessment are described. Next, the results of failure probability are calculated and discussed in Section 4. Finally, some conclusions are presented in Section 5.

2 Updating of crack distributions for *SG* tubes

To assess the structural reliability of *SG* tubes at current and future times, crack distributions must be updated based on the crack information obtained from the *NDI* data.

A flowchart illustrating the step-by-step of this work is shown in Fig. 1, in which each step corresponds to a distinct crack distribution. First, the detected crack distribution is estimated by employing statistical methods. Second, the actual-detected crack distribution is obtained by using a detection curve of the *NDI*. Third, the actual crack distribution is inferred using the Bayesian approach. And finally, the future crack distribution is obtained by considering the crack growth rate over the time.

2.1 The detected crack distribution

As several distribution models may be suggested to describe some data, goodness-of-fit tests should be used to select which model is the best one [7]. The Anderson-Darling goodness-of-fit test is sensitive to deviations in tails of the distribution models. There is a proper statistic that is used to compare the fit of the distribution models [8]. To conclude that one distribution model is the best, the related Anderson-Darling statistic must be substantially lower than others.

The goodness-of-fit measures how well a distribution model accounts for some data through hypothesis tests. The hypotheses to be tested are whether some data follows a distribution model (null hypotheses), or not. The probability of getting a result more extreme if the null hypothesis is true is the so-called *p*-value. When the *p*-value in the Anderson-Darling test is less than a given value (usually 0.05), the null hypothesis is rejected [9].

After the *NDI* is performed, it is necessary to find which distribution model is better fitted to the detected crack data. In this work, the Anderson-Darling test is used to determine the probability density function of the detected crack size $f_0(x)$.

2.2 The actual-detected crack distribution

There are random errors in comparisons between detected and actual crack sizes from *NDI* on the *SG* tubes. Thus, detected crack sizes must be calibrated with actual crack sizes. Plots of actual crack size versus detected crack size reveal that a linear trend fits the crack data obtained from *NDI*. The linear regression gives a good first-order approximation for the detection curve [10]. Hence, the actual-detected crack size is

METHODOLOGY

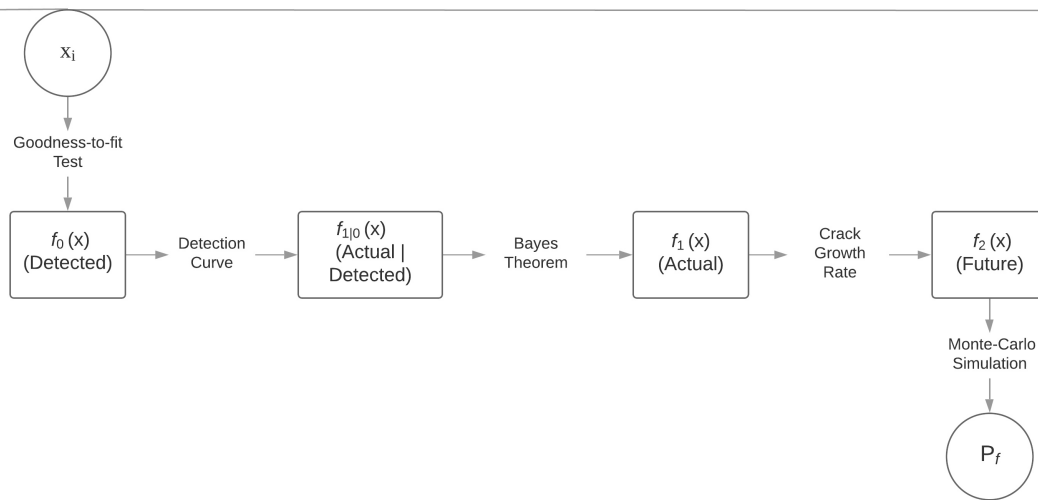


Figure 1: Methodology for updating of crack distributions.

$$x_{1|0} = \alpha + \beta x_0 + \epsilon, \tag{1}$$

where x_0 is the detected crack size, α and β are the coefficients of the linear regression, and ϵ is the calibration error with zero mean and standard deviation σ_ϵ .

The mean value and variance of the actual-detected crack size are respectively obtained from Eq. (1), and given by

$$\mu_{1|0} = \alpha + \beta \mu_0, \tag{2}$$

and

$$\sigma_{1|0}^2 = \beta^2 \sigma_0^2 + \sigma_\epsilon^2, \tag{3}$$

where μ_0 and σ_0 are the mean value and standard deviation of the detected crack size, respectively. The probability density function of the actual-detected crack size $f_{1|0}(x)$ corresponds to the same distribution model for the detected crack size, but is adjusted by the use of the mean value and variance obtained from Eqs. (2) and (3).

2.3 The actual crack distribution

Because of the presence of some small-sized cracks, the *NDI* is not able to detect all cracks that exist in the *SG* tubes. The probability density function of the actual-detected crack size $f_{1|0}(x)$ based only on the detected crack data may not represent necessarily the complete actual crack distribution. The Bayes Theorem in probability theory can be used to back figure the actual crack distribution [6].

According to the Bayes Theorem, the relationship between the probability density functions of the actual-detected crack size and the actual crack size are written as

$$f_{1|0}(x) = \frac{p_0(x)f_1(x)}{\int_0^\infty p_0(x)f_1(x)dx}, \tag{4}$$

where $p_0(x)$ is the probability of detecting a crack of a given size x .

After the integration at the denominator of Eq.(4), the result is a constant. Hence, the probability density function of the actual crack size can be estimated by

$$f_1(x) = K \frac{f_{1|0}(x)}{p_0(x)}, \tag{5}$$

where

$$K = \int_0^{\infty} p_0(x)f_1(x)dx. \quad (6)$$

Since $f_1(x)$ is an unknown function, the constant K must be determined by integrating the Eq. (5),

$$K \int_0^{\infty} \frac{f_{1|0}(x)}{p_0(x)} dx = \int_0^{\infty} f_1(x)dx = 1. \quad (7)$$

That means

$$K = 1 / \int_0^{\infty} \frac{f_{1|0}(x)}{p_0(x)} dx. \quad (8)$$

Thus, the probability density function of the actual crack size, after one operational cycle, is determined through Eq. (5), and the mean value and variance of the actual crack size can be estimated respectively as

$$\mu_1 = \int_0^{\infty} xf_1(x)dx, \quad (9)$$

and

$$\sigma_1^2 = \int_0^{\infty} (x - \mu_1)^2 f_1(x)dx. \quad (10)$$

2.4 The future crack distribution

Once the stress corrosion cracking acts to propagate the crack, a crack size growth occurs in *SG* tubes. Thus, future crack distributions are to be estimated after two operational cycles of the nuclear power plant.

Assuming that the crack growth rate is linear, a future crack size can be determined by the following equation:

$$x_2 = x_1 + (\Delta x)t, \quad (11)$$

where Δx is the crack size growth per operational cycle, and t is the number of operational cycles.

The mean value and variance of the future crack size are respectively obtained from Eq. (11), and given by

$$\mu_2 = \mu_1 + (\mu_{\Delta x})t, \quad (12)$$

and

$$\sigma_2^2 = \sigma_1^2 + (\sigma_{\Delta x}^2)t^2. \quad (13)$$

where $\mu_{\Delta x}$ and $\sigma_{\Delta x}$ are the mean value and standard deviation of the crack size growth per operational cycle. The probability density function of the future crack size $f_2(x)$ corresponds to the same distribution model for the actual crack size, but is adjusted by the use of the mean value and variance obtained from Eqs. (12) and (13).

3 Failure probability of *SG* tubes

Crack acceptance criteria are applied to assess the structural integrity of *SG* tubes containing cracks. In this work, the crack acceptance criterion is based on the limit load analysis, which assumes that the plastic collapse is the prevailing failure mode due to the very high ductility of the materials.

The main loading that leads an *SG* tube to failure is the pressure difference across the tube wall. Considering a partial through-wall axial crack, located outside an *SG* tube, between two support plates or between the first support plate and the tube sheet, EPRI [11] establishes a limit load criterion given by the following equation:

$$p_b = 0.58(s_y + s_u) \frac{t}{R_i} \left(1.104 - \frac{L}{L + 2t} h \right), \quad (14)$$

where p_b is the burst pressure difference across the tube wall, s_y is the yield stress, s_u is the ultimate strength, t is the tube wall thickness, R_i is the inlet tube radius, L is the crack length, and h is the crack depth ratio.

Governing parameters of the problem are defined as basic variables which are get together in the basic vector \mathbf{x} . The space of basic variables \mathbf{D} may be divided into failure and safety regions. The failure region is defined by $\mathbf{D}_f = \{\mathbf{x} | g(\mathbf{x}) \leq 0\}$, where $g(\mathbf{x})$ represents the failure limit-state function, and \mathbf{x} is the basic vector. Notice that

$g(\mathbf{x}) = 0$ is the interface between failure and safety regions, and so it is called failure limit-state surface. For the limit load criterion, the failure limit-state function is given by

$$g(\mathbf{x}) = 0.58(s_y + s_u) \frac{t}{R_i} \left(1.104 - \frac{L}{L + 2t} h \right) - p_b. \quad (15)$$

In the reliability assessment, the basic variables are modeled as random variables, and the failure probability is determined by [12]

$$P_f = \int_{\mathbf{D}_f} f(\mathbf{x}) d\mathbf{x}, \quad (16)$$

where $f(\mathbf{x})$ is the joint probability density function of the basic variables. One of the most accurate methods for calculating the failure probability is the Monte Carlo simulation, which is based on the relative frequency in which the failure limit-state function is violated [13]. This is the method employed in this work to do so.

4 Results and discussions

A typical SG in a PWR nuclear power plant subjected to hypothetical operating conditions is taken a study case. The NDI data consists of the location, morphology, and sizes of each detected crack of the P14 and P15 outages of successive operational cycles. The crack sizes of interest in this work are length and depth ratio. Thus, the basic vector in the failure limit-state function is $\mathbf{x} = (L, h)$. Table 1 shows the NDI data of the P14 outage.

The normal, Weibull, exponential, and lognormal distributions are tested by applying the Anderson-Darling goodness-of-fit test to the NDI data of the P14 outage. Since the Weibull distribution provides the largest p -value, it is considered the best distribution for the detected crack depth ratio. For the detected crack length, the lognormal distribution has the largest p -value, followed by the exponential. Since the exponential distribution is the most common to represent crack length, it is considered the best distribution for this size.

The probability density function of the detected crack length of the P14 outage is

$$f_0(L) = \frac{1}{8.7} \exp\left(-\frac{L}{8.7}\right), \quad (17)$$

and of the detected crack depth ratio is

$$f_0(h) = \frac{0.45}{11.4} \left(\frac{h}{11.4}\right)^{-0.55} \exp\left[-\left(\frac{h}{11.4}\right)^{0.45}\right]. \quad (18)$$

The detected crack distribution can be related to the actual-detected distribution through the called detection curve, which is approximated by a linear regression. The coefficients of this linear regression used in the present work are shown in Table 2.

Adjusting the detected crack distribution by the use of the mean value and variance obtained according to Eqs. (2) and (3), we obtain the actual-detected crack distribution. Thus, the probability density function of the actual-detected crack length of the P14 outage is

$$f_{1|0}(L) = \frac{1}{8.7} \exp\left(-\frac{L}{8.7}\right), \quad (19)$$

and of the actual-detected crack depth ratio is

$$f_{1|0}(h) = \frac{0.45}{11.4} \left(\frac{h}{11.4}\right)^{-0.55} \exp\left[-\left(\frac{h}{11.4}\right)^{0.45}\right]. \quad (20)$$

The probability density function of the actual crack size can be estimated by using the Bayes Theorem, for which is necessary to know the POD curve. A log-logistic function with two parameters, usually used for the POD curve is given by [15]:

$$p_0(x) = \frac{1}{1 + \exp[a + b \log(x)]}, \quad (21)$$

where $a = 16.4$ and $b = -9.3$ are the parameters of the POD curve [14].

Table 1: *NDI* data of the *P14* outage.

<i>SG</i>	<i>Row</i>	<i>Column</i>	<i>Location</i>	<i>Origin</i>	<i>Type</i>	<i>L</i> (mm)	<i>h</i> (% TW)
1	2	48	01H	OD	Axial	11.46	61.0
1	4	71	01H	OD	Axial	7.75	65.0
1	4	73	01H	OD	Axial	9.51	61.0
1	5	76	01H	OD	Axial	9.61	42.0
1	6	38	01H	OD	Axial	8.52	51.0
1	6	44	01H	OD	Axial	7.68	36.0
1	6	74	01H	OD	Axial	13.98	52.0
1	7	26	01H	OD	Axial	7.35	47.0
1	7	40	01H	OD	Axial	12.89	52.0
1	7	43	01H	OD	Axial	7.26	44.0
1	7	47	01H	OD	Axial	13.02	47.0
1	7	72	01H	OD	Axial	11.26	45.0
1	9	39	01H	OD	Axial	10.22	53.0
1	10	37	01H	OD	Axial	14.14	54.0
1	10	93	01H	OD	Axial	10.63	42.0
1	12	34	01H	OD	Axial	8.02	38.0
1	12	60	01H	OD	Axial	5.39	54.0
1	14	21	01H	OD	Axial	8.0	44.0
1	14	41	01H	OD	Axial	6.21	47.0
1	14	46	01H	OD	Axial	12.87	49.0
1	15	77	01H	OD	Axial	12.5	47.0
1	16	67	01H	OD	Axial	9.37	45.0
1	16	77	01H	OD	Axial	8.55	67.0
1	18	79	01H	OD	Axial	9.0	40.0
1	19	89	01H	OD	Axial	11.71	39.0
1	19	91	01H	OD	Axial	9.51	41.0
1	22	98	01H	OD	Axial	7.8	35.0
1	23	90	01H	OD	Axial	9.95	45.0
1	25	87	01H	OD	Axial	6.89	53.0
1	26	73	01H	OD	Axial	11.83	67.0
1	26	85	01H	OD	Axial	11.43	57.0
1	30	65	01H	OD	Axial	11.46	41.0
1	30	65	01H	OD	Axial	12.9	40.0
1	30	86	01H	OD	Axial	7.94	43.0
1	34	53	01H	OD	Axial	18.15	65.0
1	35	56	01H	OD	Axial	11.01	43.0
1	35	60	01H	OD	Axial	15.37	77.0
1	35	60	01H	OD	Axial	15.89	67.0
1	35	75	01H	OD	Axial	10.87	43.0
1	41	64	01H	OD	Axial	18.39	50.0
1	41	65	01H	OD	Axial	17.25	53.0
1	44	50	01H	OD	Axial	11.23	46.0
1	44	51	01H	OD	Axial	9.05	47.0
1	45	77	01H	OD	Axial	18.32	36.0
1	49	64	01H	OD	Axial	7.02	46.0
1	11	35	02H	OD	Axial	15.72	58.0
1	13	7	TEH	ID	Axial	7.2	100.0
1	16	45	TSH	ID	Axial	4.75	93.0
1	20	69	TSH	OD	Axial	4.26	40.0
1	22	62	TSH	OD	Axial	15.94	40.0
1	34	50	TSH	OD	Axial	5.56	43.0
1	35	14	TSH	ID	Axial	4.38	100.0

Table 2: The coefficients of the linear regression [14].

x	α	β	σ_ϵ
$L (mm)$	0.64	2.794	4.5
$h (\% TW)$	1.116	-2.27	9.1

Using the Bayes Theorem as expressed by Eq. (5), the probability density function of the actual crack length of the P14 outage after one operational cycle is

$$f_1(L) = \frac{1}{4.2} \exp\left(-\frac{L}{4.2}\right), \tag{22}$$

and of the actual crack depth ratio is

$$f_1(h) = \frac{0.74}{14.2} \left(\frac{h}{14.2}\right)^{-0.26} \exp\left[-\left(\frac{h}{14.2}\right)^{0.74}\right]. \tag{23}$$

Figure 2 shows the curves for the probability density function of the crack size, comparing the detected and actual distributions. It can be seen that the detected crack distributions are well fitted to the large-sized cracks. On the other hand, the actual crack distributions emphasize small-sized cracks which were not detected by the NDI.

The future crack distribution is estimated considering the crack growth rate by the use of the mean value and variance obtained according to Eqs. (12) and (13). The statistics of the growth rate for the crack length and crack depth ratio are shown in Table 3.

Table 3: The crack size growth per operational cycle [16].

x	$\mu_{\Delta x}$	$\sigma_{\Delta x}$
$L (mm)$	2.54	2.34
$h (\% TW)$	5.0	25.0

Thus, the probability density function of the future crack length of the P14 outage after two operational cycles is

$$f_2(L) = \frac{1}{6.8} \exp\left(-\frac{L}{6.8}\right), \tag{24}$$

and of the future crack depth ratio is

$$f_2(h) = \frac{0.92}{21.2} \left(\frac{h}{21.2}\right)^{-0.08} \exp\left[-\left(\frac{h}{21.2}\right)^{0.92}\right]. \tag{25}$$

Figure 3 shows the curves for the probability density function of the crack size, comparing the actual and future distributions. The future distributions provide a decrease in the probability of occurrence of small-sized cracks, and an increase in the one of large-sized cracks. This behavior is in good agreement with the the crack growth process over time.

From the knowledge of the crack distributions, it is possible to assess the structural reliability of SG tubes. A partial through-wall axial crack is assumed to be present outside the SG tube. A pressure difference across the tube wall is applied as the unique loading condition. The data of the tube geometry and loading condition are found in Table 4.

The structural reliability is assessed as condition monitoring and operational assessments. The attribute of structural reliability is the failure probability of SG tubes, calculated by using the Monte-Carlo simulation. The results are presented in Table 5. There is a decrease in the failure probability from the actual crack distribution to the future crack distribution. The order of magnitude of the failure probability for the future crack distribution is the same as the one presented by [17], for instance.

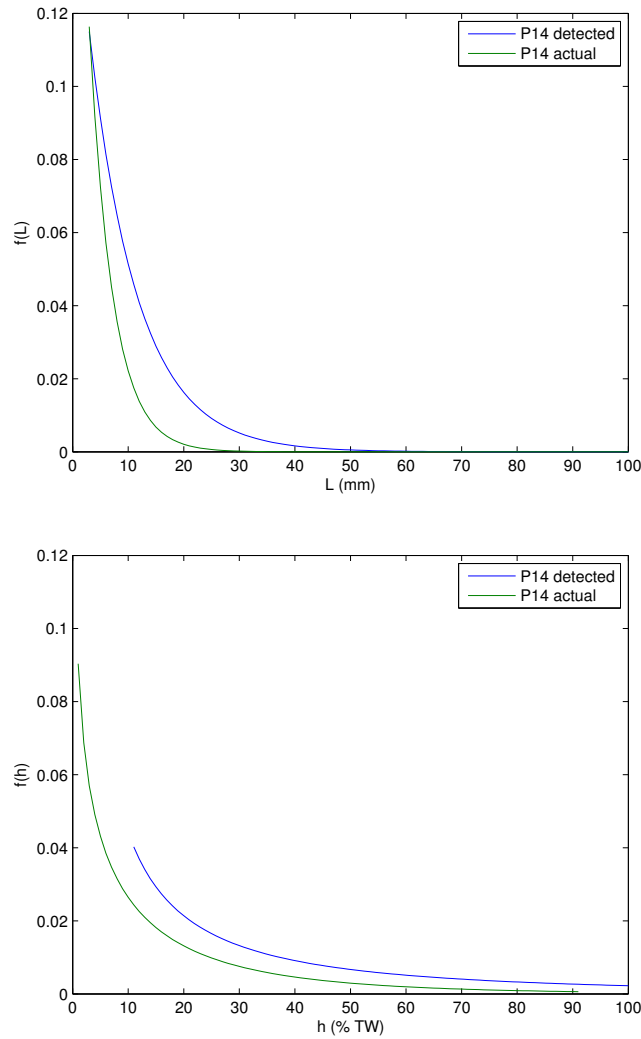


Figure 2: The detected and actual probability density functions of *P14* outage: crack length (top) and crack depth ratio (bottom).

Table 4: The geometry and loading conditions [16].

R_i	t	p_b	$s_y + s_u$
8.4327 mm	1.0923 mm	9.42 MPa	1010.8 MPa

Table 5: The actual and future failure probabilities of the *P14* outage.

Crack distribution	P_f
<i>P14</i> actual	8.2×10^{-3}
<i>P14</i> future	1.4×10^{-2}

The *NDI* data of the *P15* outage is obtained after one operational cycle has passed from the *P14* outage. Table 6 shows the *NDI* data of the *P15* outage. Thus, comparisons between the future crack distribution of the *P14* outage

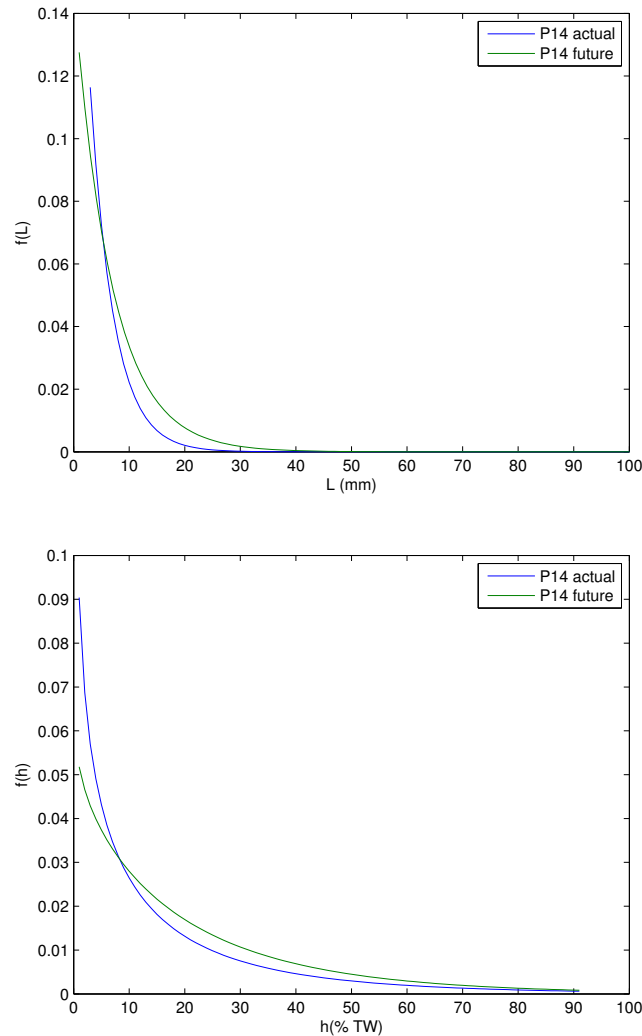


Figure 3: The actual and future probability density functions of the *P14* outage: crack length (top) and crack depth ratio (bottom).

and the actual crack distribution of the *P15* outage can be done.

The failure probability for the actual crack distribution of the *P15* outage is presented in Table 7. The failure probabilities for the *P14* future and *P15* actual crack distributions present the same order of magnitude.

Based on the results for the *P14* future and *P15* actual crack distributions, the Bayesian approach for updating of crack distributions can be appropriate to predict the failure probability at the next operational cycles. The failure probability for the crack distributions at the next operational cycles after the *P15* outage is shown in Table 8. We can note an increase in the failure probability over time. With this increase, plugging or repairing the cracked *SG* tubes is required to assure a safe condition for the *PWR* plants.

Figure 4 shows the curves for the probability density function of crack sizes considering several operational cycles of the *P15* outages. There is a decrease in the probability of occurrence of small-sized cracks, while there is an increase for large-sized cracks.

Conclusions

The goal of this work is to present a methodology for updating the crack distributions of the *SG* tubes, obtained from the *NDI* performed in *PWR* nuclear power plants. Considering the updated crack distributions, the structural reliability of the *SG* tubes is assessed in order to predict the level of safety of the *PWR* plants.

The *NDI* data are obtained at the *P14* and *P15* outages of a *PWR* plant. Distribution models must be adjusted to the *NDI* data for the statistical representation of the detected crack sizes in the *SG* tubes. The Anderson-Darling

Table 6: *NDI* data of the *P15* outage.

<i>SG</i>	<i>Row</i>	<i>Column</i>	<i>Location</i>	<i>Side</i>	<i>Type</i>	<i>L</i> (mm)	<i>h</i> (% TW)
1	4	71	01H	OD	Axial	7.11	39.0
1	5	76	01H	OD	Axial	4.7	36.0
1	7	26	01H	OD	Axial	5.49	39.0
1	7	47	01H	OD	Axial	11.37	38.0
1	7	72	01H	OD	Axial	7.13	42.0
1	12	60	01H	OD	Axial	5.07	36.0
1	16	45	TSH	ID	Axial	3.41	66
1	20	69	TSH	OD	Axial	3.43	39.0
1	22	62	TSH	OD	Axial	16.27	42.0
1	34	50	TSH	OD	Axial	5.33	33.0
1	35	14	TSH	ID	Axial	3.52	89.0

Table 7: The actual failure probabilities of the *P15* outage.

<i>Crack distribution</i>	P_f
<i>P15</i> actual	8.9×10^{-2}

Table 8: The future failure probability of the *P15* outage.

<i>Operational cycle</i>	P_f
<i>1st</i>	8.9×10^{-2}
<i>2nd</i>	1.4×10^{-1}
<i>3rd</i>	1.9×10^{-1}
<i>4th</i>	2.4×10^{-1}
<i>5th</i>	2.9×10^{-1}

goodness-of-fit test is used to choose the probability density function of the detected crack sizes. For the crack depth ratio of the *P14* outage, the goodness-of-fit test appoints to the Weibull distribution, and for the crack length to the exponential distribution.

The Bayesian updating allows to link the probability density functions between detected and actual crack sizes. This important mathematical tool can update correctly the crack distribution. Since small-sized cracks are not detected by the *NDI*, the detected crack distribution does not represent statistically these cracks. After the Bayesian updating, the actual crack distribution can cover the complete crack distribution.

The methodology for updating the crack distributions is effective, and permits the calculation of the failure probability of *SG* tubes for the *P14* and *P15* outages, in good agreement with results found in the literature. And the failure probability increases as long the time is considered. Therefore, the proposed methodology can be used as a tool for assessing the structural reliability of *SG* tubes after several operational cycles.

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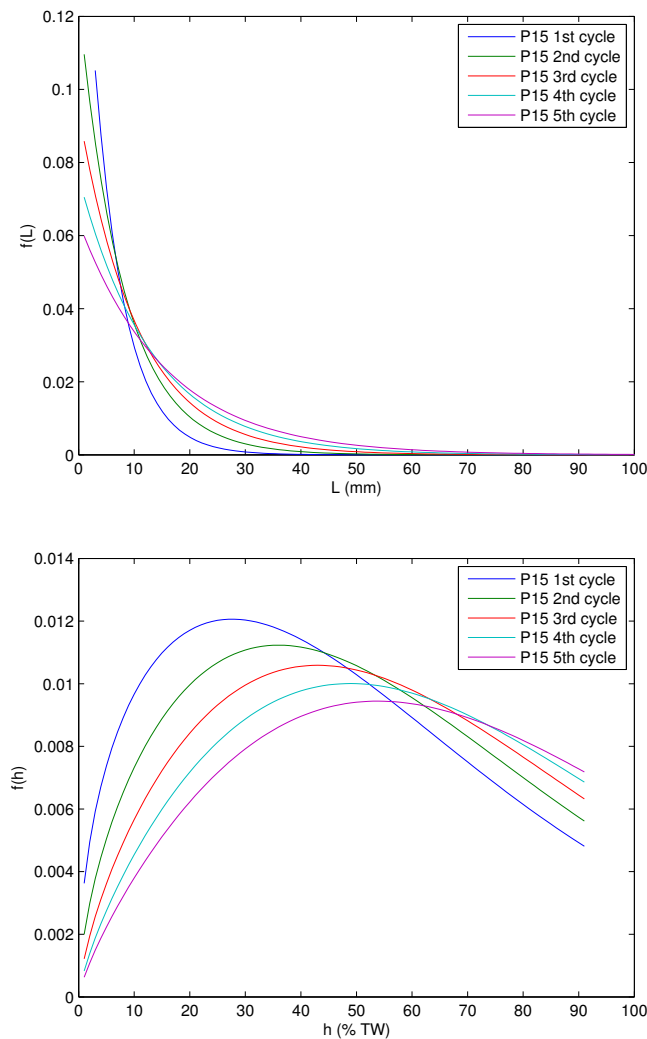


Figure 4: The probability density function of the P15 outage for operational cycles: crack length (top) and crack depth ratio (bottom).

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