Experimental and numerical study on vibration-based damage detection and localisation in laminated composite plates

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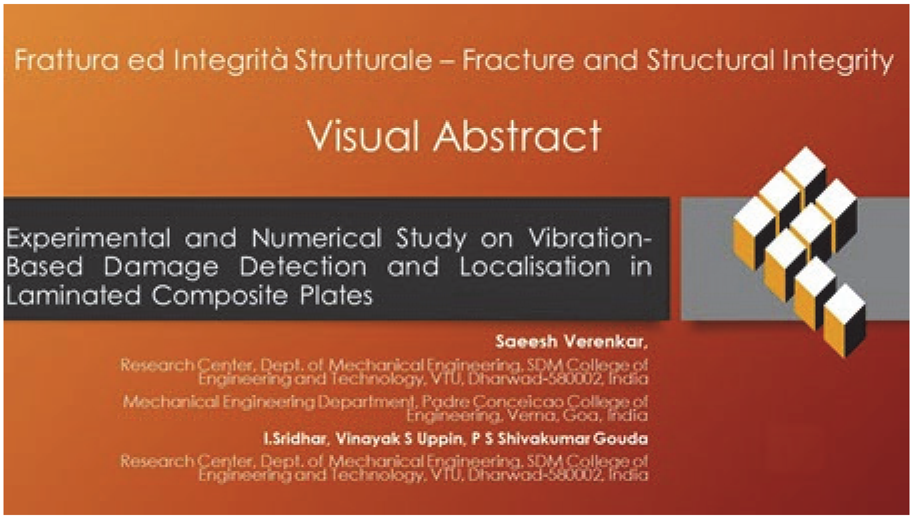

Frattura ed Integrità Strutturale – Fracture and Structural Integrity

Visual Abstract

Experimental and Numerical Study on Vibration-Based Damage Detection and Localisation in Laminated Composite Plates

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INTRODUCTION

Composite materials are widely utilized across aerospace, marine, civil, automotive, and railway sectors. Vulnerable to damages, especially delamination, such issues can compromise structural integrity and safety. To prevent catastrophic failures, early detection methods are essential to identify damage location and size. Vibration-based techniques for detecting and locating damage have been a prominent area of research, concentrating on alterations in modal frequencies and mode shapes as indicators of the presence, location, and extent of damage. Notably, comprehensive literature reviews on damage detection and SHM based on modal parameters have been conducted and summarized [1,2,3,4]. Their contributions have significantly advanced the field of vibration-based damage detection and enhanced the understanding of its practical applications in diverse engineering domains.

Natural frequencies and mode shapes are frequently used as modal parameters in damage detection due to their sensitivity to variations in the structural stiffness resulting from damage [1]. Lifshitz et al. [5] were pioneers in their endeavour to identify damage by studying alterations in the natural frequencies of structures. They specifically analysed the shifts in NFs caused by variations in dynamic moduli for the purpose of damage detection in elastomers. NFs provide valuable parameters to study for damage detection, and their measurement is a cost-effective and easily accessible experimental practice. Moreover, precise control of experimental conditions can significantly reduce uncertainties in measured frequencies, thereby enhancing the accuracy of the results.

Use of NFs for damage localization has certain drawbacks. They cannot be used alone for damage localization, as they reluctant provide precise information about the exact location of damage within the structure, only indicating its presence. Furthermore, NFs are highly sensitive to environmental and operational fluctuations, such as temperature changes and varying loads. These factors can introduce uncertainties in measurements, posing a challenge in distinguishing alterations attributed to damage from those resulting from external influences. Consequently, relying solely on natural frequencies for damage detection may compromise accuracy in scenarios affected by such variations.

In addition to NFs, mode shapes also offer valuable information for damage assessment [6]. However, compared to NFs, mode shapes are relatively less impacted by environmental factors. This characteristic makes mode shapes a more suitable option for damage detection. [7]. Modal Assurance Criterion (MAC)[8,9] is a method used, among others, for damage detection through mode shape comparisons. MAC quantifies the similarity between undamaged and damaged mode shapes. Although it's not highly sensitive to small differences and isn't suitable for precise damage localization, it can still indicate structural damage [9,11]. To improve detection and localization, researchers introduced the mode shape damage index (MSDI) algorithm. This method employs a modified MAC matrix (DMAC) to emphasize nearly identical mode shapes while excluding dissimilar ones [12]. Through finite element analysis (FEA) of different damage cases and boundary conditions on plate models, the MSDI method was found effective in accurately locating both single and multiple damages on plate-like structures. It's also capable of distinguishing damages of varying severity levels.

Some researchers explored displacement mode shapes [13] and their rotation [14] as damage indicators in beam and plate structures. Displacement mode shapes were less sensitive, while the first-order derivative of mode shapes showed better detection of damages at multiple locations. Enhancements in damage detection techniques utilizing experimental modal parameters have been explored [15]. A methodology for detecting damage in wind turbine blades has been introduced, leveraging dynamic analysis and information about the curvature differences in mode shapes [16]. An innovative method was proposed for online SHM in laminated composite plates using modal data and machine learning [17]. The authors [17] developed the "node-releasing technique" with the commercial FE code Ansys, enabling efficient damage detection for various crack types in Unidirectional Laminate (UDL) composite layered configurations. However, using mode shapes for damage detection has limitations, including the influence of vibration testing on large structures and the impact of sensor placement and noise effects.

The Modal Curvature Method (MCM) is a technique for damage detection and localization, monitoring changes in curvature from mode shapes due to damage. Initially developed to assess the relationship between curvature and flexural stiffness (EI), MCM demonstrates high sensitivity to damage [18]. MCM with the Modal Curvature Squared Method (MCSM)[19] facilitates easier identification of abnormal changes. Rucevskis et al [20] demonstrated damage detection in composite plate without the need of data from a healthy plate.

The main objective of this study is to deepen the understanding of detecting and pinpointing damage in composite plates using a thorough investigation that combines both experimental and numerical modal analysis techniques. The focus is on developing an innovative approach that leverages modal shapes and their spatial derivatives for accurately detecting and localizing delamination in laminated composite plates under various damage scenarios. The study also aims to contribute to the advancement of damage detection techniques for composite laminates, particularly in plate-like structures, thereby enhancing the capabilities of SHM applications.

DAMAGE DETECTION BASED ON NATURAL FREQUENCIES

Construction of Glass-Epoxy composite plate

The construction process involved creating a composite plate using eight layers of bi-directional woven E-Glass fabric of 200 GSM. Epoxy resin Lapox L12 was used as a matrix material, with K6 hardener used as curing catalyst. The composite laminate was meticulously prepared using the hand layup method, following a specific stacking sequence depicted in Fig. 1.

Upon stacking the initial four layers of fabric, a deliberate step was taken to introduce delamination. This was achieved by placing a 40 x 40mm Teflon film at the centre of the laminate, as illustrated in Fig. 1. Special care was taken during the casting process to ensure the proper formation of the delamination. Subsequently, additional layers of fabric were added according to the prescribed stacking sequence in order to complete the laminate structure.

Post-stacking, the composite laminate underwent a curing process under atmospheric conditions for a duration of 24 hours. Material properties for the composite laminate were evaluated using the rule of mixtures [17] and are summarized in Tab. 1.

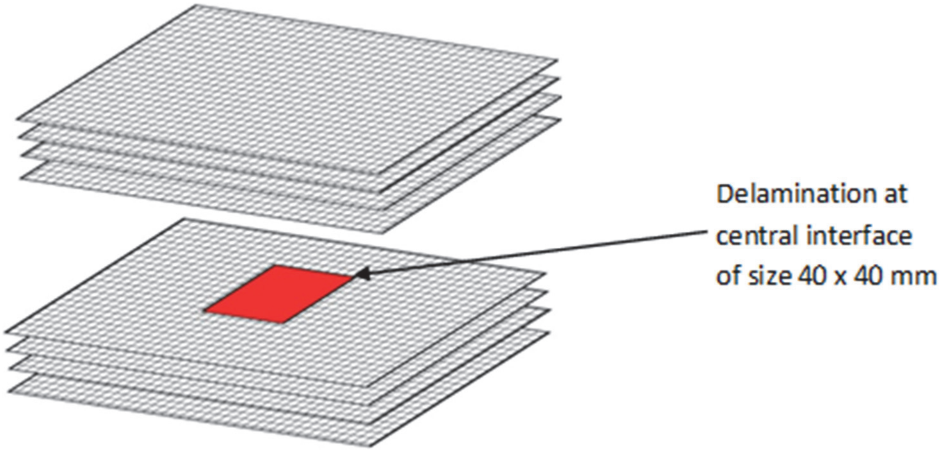


Figure 1: Stacking sequence and interface region in Glass-epoxy composite plate.

Material Properties	Values
E_{11}	39.51 GPa
$E_{12}= E_{13}$	6.38 Gpa
$G_{12}=G_{13}$	2.47 GPa
G_{23}	2.73 GPa
$\nu_{12=13}$	0.28
ρ	1900 kg/m³

Table 1: Properties of Glass-Epoxy composite plate.

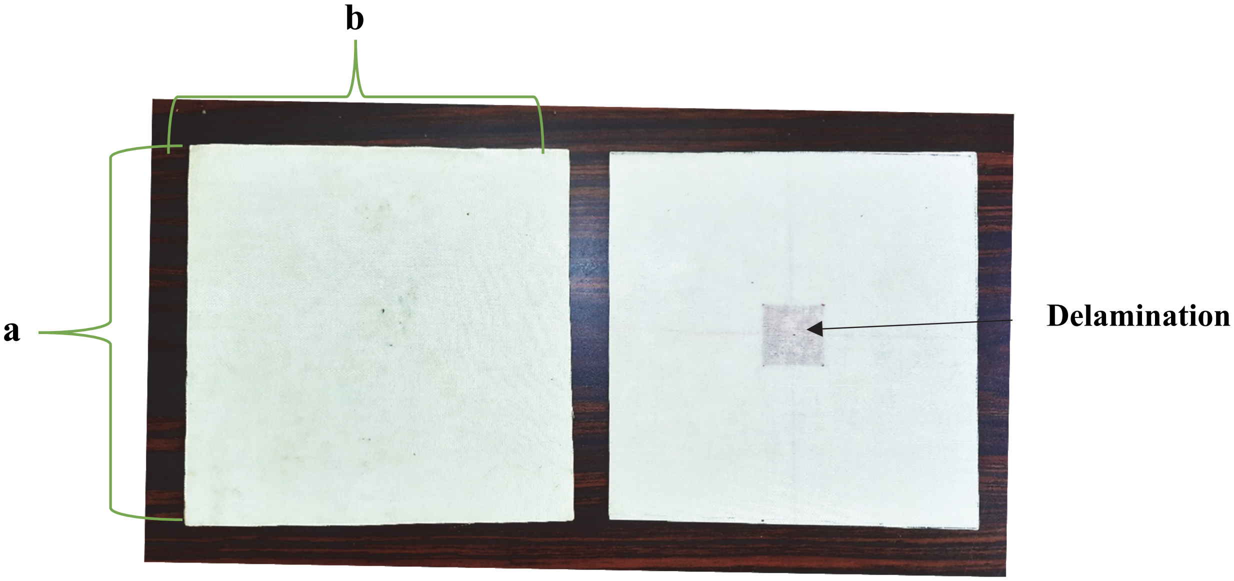


Figure 2: Composite plate of dimensions 250mm x 250 mm x 1.2mm.

Experimental Modal Analysis

The experimental investigation involved studying free vibration responses of a delaminated Glass-Epoxy composite plate. The CDAQ-9174 system from National Instruments was used for data acquisition. The plate dimensions were set to a = b = 250 mm (Fig. 2) and h = 1.2mm, with a delamination size of 40x40 mm inserted at the center of the laminate.

(a) (b)

Figure 3: (a) Elements of data acquisition system (b) Measuring points

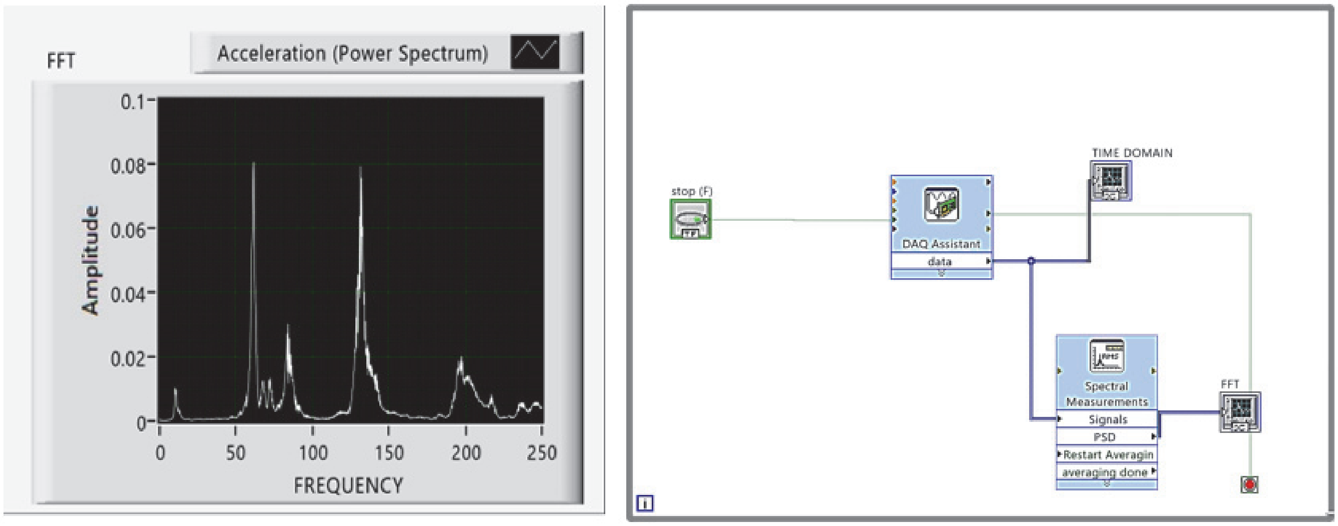
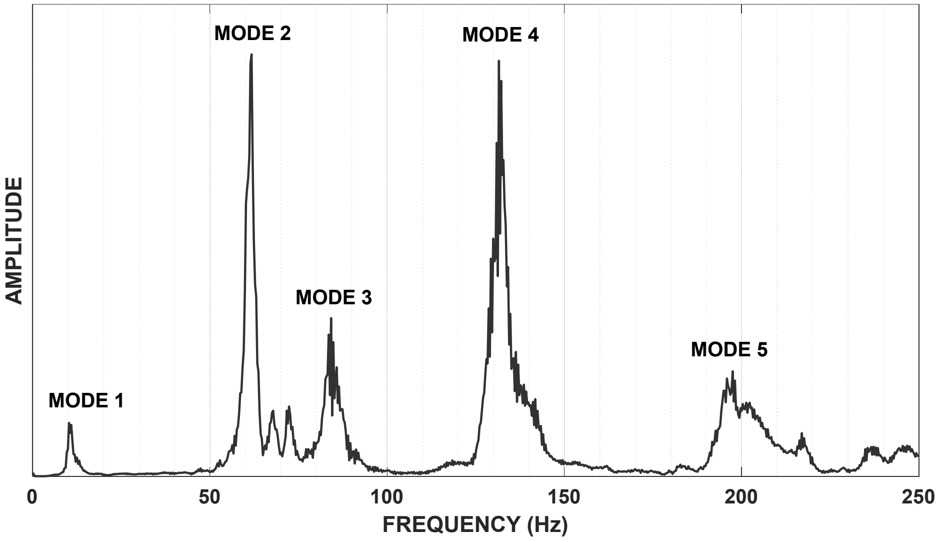
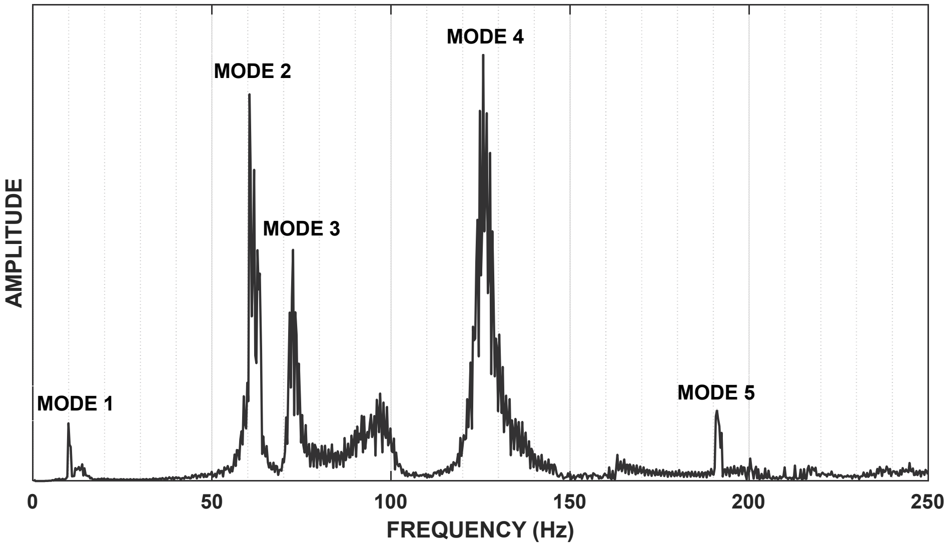


Figure 4: Front panel and block diagram of LabVIEW software

The experimental setup, as depicted in Fig. 3(a), included a fixture for CFFF support condition and an accelerometer mounted on the plate. Impact hammer excitation was applied at the defined points on the plate as shown in Fig. 3 (b), and the data was captured using the NI CDAQ-9174, a compact USB data acquisition system with four channels. The acquired signals were processed using LabVIEW software, where a virtual instrument (VI) program circuit was developed (Fig. 4) to facilitate input supply and output display on the computer screen. The LabVIEW software utilized a power spectrum module for fast Fourier transformation of the acceleration signals, enabling the extraction of frequency domain responses. The NFs of vibration for various modes were identified by analysing peaks in frequency response curve, as shown in Fig. 5.



(a)



(b)

Figure 5: Frequency Response Function (FRF) data for (a) before damage (b) after damage.

Numerical Modal Analysis

In this study, Finite Element Analysis (FEA) of composite plate is conducted using Ansys Composite PrepPost(ACP) module, a specialized software application for creating, preparing, and post-processing composite structures in FEA. ACP offers various features ideal for composite modeling, including the capability to model intricate shell and solid composite structures, conduct FE analysis, post-process results tailored to composites, perform parametric design analyses, and automate tasks using scripting.

The CAD geometry of the plate, adhering to the required dimensions shown in Fig. 2, is first constructed in Ansys Design Modeler. Material properties listed in Tab. 1 are then assigned to the model. Subsequently, the model is imported into ACP Pre-post, where layer sequences, fiber directions, and material properties are defined. To introduce delamination in the plate, an interface layer is created to facilitate node detachment at the desired location. Following this, a mesh is generated, and end conditions (CFFF) are applied to the model. Modal analysis is executed to visualize frequencies and mode shapes of plate pre- and post-damage, as illustrated in Fig. 6 and 7, respectively. Finally, the simulation results are verified with experimental analysis and presented in Tab. 2, indicating a reasonable agreement between experimental and FEA outcomes. This substantiates the validity of model for further numerical analysis.

	PRE-DAMAGE			POST-DAMAGE		
MODE NO	FEA	EXPERIMENTAL	% Difference	FEA	EXPERIMENTAL	% Difference
1	10.899	10.88	0.17	9.1272	9.24	1.22
2	62.434	61.76	1.08	59.257	60.8	2.60
3	78.811	84.16	6.79	77.904	72.64	6.76
4	125.32	131.52	4.95	123.49	125.76	1.84
5	191.86	197.44	2.91	181.85	191.04	5.05

Table 2: Comparison of Natural Frequencies (in Hz) for Plate Pre- and Post-Damage.

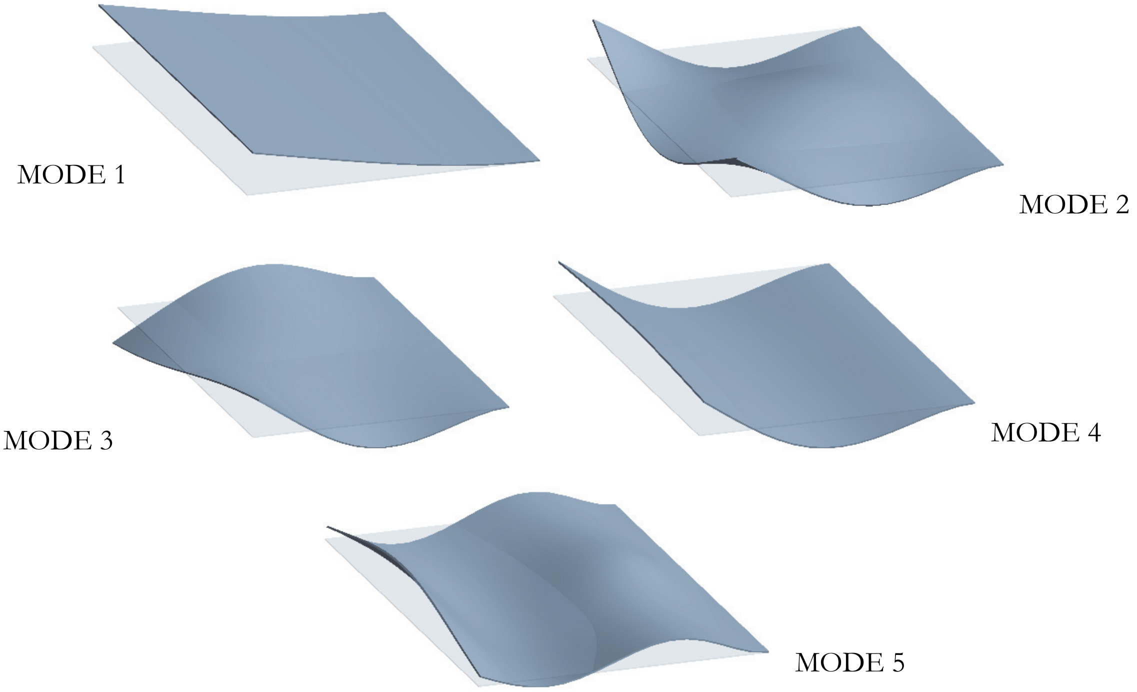


Figure 6: First five mode shapes of plate pre-damage.

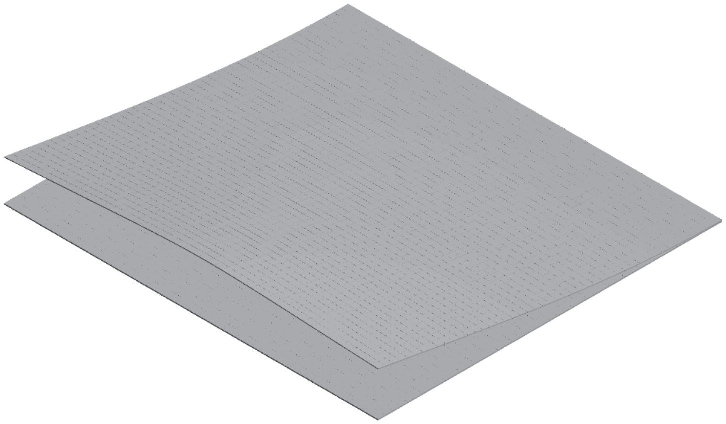


Figure 7: Separation of plies due to delamination.

Basic results and discussion

Detection and localization of small-scale damage like delamination in composite laminates pose significant challenges due to the inherent heterogeneity of composites. In this study, effect of delamination on natural frequencies of a square composite plate was investigated, focusing on first five modes. The plates were subjected to CFFF (Clamped-Free-Free-Free) boundary conditions.

Tab. 2 presents the findings on the percentage change in the fundamental natural frequency concerning the delamination size in the analysed composite plates. The results demonstrate that, overall, the impact of delamination on the fundamental natural frequency is minimal. However, the study reveals a more substantial change in frequency, especially for higher modes. These findings emphasize the importance of understanding the effect of delamination on different vibration modes and suggest that the fundamental mode is less sensitive to delamination compared to higher modes in composite plates. These insights contribute to a better understanding of structural behaviour of composite materials with delamination and have implications for engineering applications and damage assessment.

Therefore, based solely on the change in frequency, it is challenging to accurately locate delamination in composite laminates, especially when the delamination size is very small and when data from only the fundamental mode are available. Additional methods or approaches may be necessary to effectively detect and localize such small-scale damage in composite laminates.

DAMAGE DETECTION THROUGH MODE SHAPES

Presence of damage within a structure typically results in reduced stiffness and alterations in system modal characteristics, including NFs and mode shapes. Consequently, substantial changes in these modal parameters serve as indicators of structural damage. In recent times, several techniques have proven effective in detecting the presence and pinpointing location of damage in structures based on modal parameters. As emphasized in the preceding section, although changes in modal frequencies can often detect the existence of damage, localizing the damage necessitates knowledge about the more sensitive vibrational modes. In this study, the distinction in field derivatives is utilized as a damage indicator to precisely identify location of delamination in composite laminates.

DI based on mode shape

Mode shape, which illustrates transverse movement of a vibrating structure at a particular resonant frequency, is highly responsive to damage. Disparity between mode shapes of an intact structure and a damaged one serves as a noteworthy indicator for detecting damage. Most straightforward method for quantifying damage involves computing a damage index using mode shapes, as described by Eqn. (1)

$$\Delta\varphi^{i} = \left|\Delta\varphi_{i}^{d} - \Delta\varphi_{i}^{h}\right| \tag{1}$$

where $\Delta\varphi_{i}^{d}$ and $\Delta\varphi_{i}^{h}$ represent mode shapes of structure in its damaged and healthy states, respectively and 'i' is node number or measure point.

These mode shapes may contain some measurement noise, which can introduce local perturbations and create peaks in mode shape slope, curvature, and curvature square profiles. These noise-induced peaks could potentially be mistaken for damage or mask the peaks caused by real structural damage, leading to inaccurate detection.

To overcome this challenge average value of mode shape DI is calculated for each mode and each node or measured point using the following formula (2).

$$MS_{i} = \frac{1}{N}\sum_{n=1}^{N}\left(\Delta\varphi_{i}\right)_{n} \tag{2}$$

where N is the number of modes.

Damage index through Mode Shape Slope (MSS)

The slope of the mode shape indicates the rate of change of displacement. The Modal Slope Sensitivity, based on alterations in the slope of the mode shape, can be evaluated using Eqn. (3). This involves comparing the slopes of mode shapes between pre and post-damaged structures.

$$\Delta\varphi^{i} = \left|\Delta\varphi_{i}^{'d} - \Delta\varphi_{i}^{'h}\right| \tag{3}$$

Mode shape slope is derived by using the central difference approximation formula as shown in Eqn. (4)

$$\Delta\varphi^{'} = \frac{\left(\Delta\varphi_{i+1} - \Delta\varphi_{i-1}\right)}{2h} \tag{4}$$

where 'h' is the element length or distance between consecutive nodes.
Considering the average values for 'N' number of modes the Eqn. (4) can be re written as

$$MSS_{i} = \frac{1}{N}\sum_{n=1}^{N}\left(\Delta\varphi_{i}^{'}\right)_{n} \tag{5}$$

DI based on higher order derivatives of mode shape (DMS)

Derivatives of mode shapes, such as curvature, exhibit heightened sensitivity to damage in comparison to mode shapes alone. The absolute discrepancy between an intact and a damaged structure in terms of DMS, can be a significant indicator of damage. The DMS is attainable through the central finite difference method, and the DI is computed by contrasting the derivative mode shapes between sound and damaged structures. Moreno-García et al. [21] introduced the DFD (Difference in Field Derivatives) DI, which calculates derivatives up to fourth order in the x-direction for each mode.

Proposed DI

Various authors have employed the previously mentioned techniques to predict the occurrence of damage and its specific location. Typically, the mode shape and its derivatives, such as curvature-based techniques, were originally developed to detect damage in one-dimensional beam-like structures.

However, considering earlier points discussed, the proposed structural damage detection technique is specifically designed to precisely predict both the presence and location of damage in 2D plate-like structures. In contrast to the conventional methods intended for one-dimensional structures, this approach caters to the unique characteristics and complexities presented by plate-like configurations.

In this study, a DI as in Eqn. (10) is proposed based on differentiating the transverse displacement of each node on the plate. To numerically compute these derivatives, the Finite Difference method is employed, which approximates the derivatives using the displacement values and their uniform spacing in each direction. Some of the commonly used second and fourth order central finite difference approximations for first and second order derivative are mentioned in Eqn. (6)-(9)

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} + O\left(h^{2}\right) \tag{6}$$

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^{2}} + O\left(h^{2}\right) \tag{7}$$

$$f'(x) = \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h} + O\left(h^{4}\right) \tag{8}$$

$$f''(x) = \frac{-f(x+2h) + 16f(x+h) - 30f(x) + 16f(x-h) - f(x-2h)}{12h^{2}} + O\left(h^{4}\right) \tag{9}$$

For the current study second order central finite difference approximation of the first four derivatives is considered as for higher orders more points are needed which can affect the detection of damage at the edges. To quantify the extent of damage present in the structure, the absolute difference between the square of these derivatives is computed. This calculation is performed for each mode, and the results are then averaged over the number of modes considered in analysis. By averaging the absolute differences between the squares of the derivatives for all the modes, the study utilizes modified form of damage indices given in Eqn. (10) employed by Moreno-García et al. [21] to effectively assess and detect presence of damage in the plate-like structure. The proposed damage indice of Eqn. (10) provides valuable insights into the location and severity of damage, offering a robust and accurate method for SHM and damage detection.

$$DI = \frac{1}{N}\sum_{n=1}^{N}\left|\left(\frac{\partial^{p}\psi_{n}\left(x,y\right)}{\partial x^{p}}\right)^{2}-\left(\frac{\partial^{p}\Delta\tilde{\psi}_{n}\left(x,y\right)}{\partial x^{p}}\right)^{2}\right| \tag{10}$$

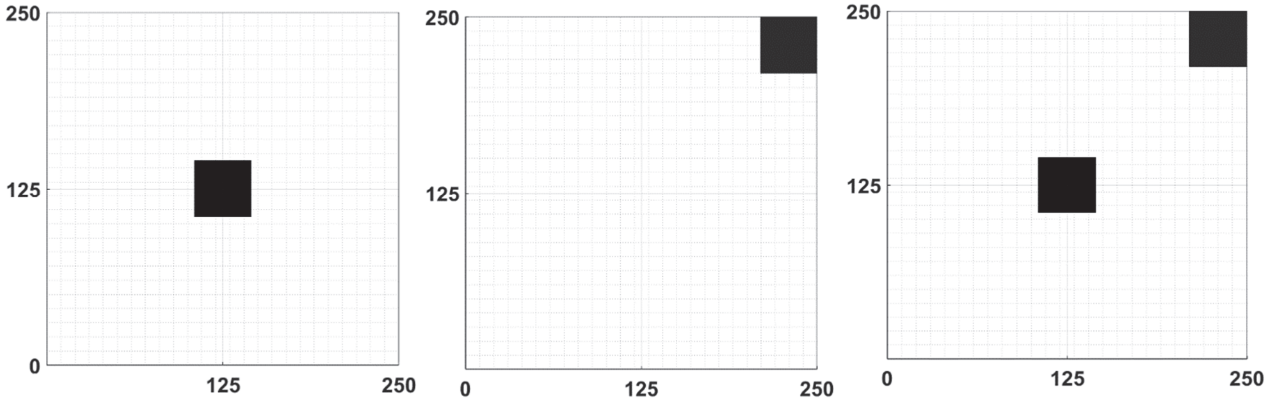
where p=1,2,3,4, $\Delta\psi_i$ and $\Delta\tilde{\psi}_i$ are the mode shape of the pre- and post-damaged state of a structure respectively and 'i' is the node number or measured point.

Finite Element (FE) analysis

To validate and assess the reliability of introduced DI, numerical modal analysis was conducted using the Finite Element method. The analysis was performed using FE software ANSYS and modal results obtained were further analysed for location detection using the MATLAB software. An average of 10 modes was considered for the current analysis. Three distinct case studies as shown in Fig. 8 are considered to detect damage in the plate-like structure:

a) Damage occurring at the centre of the plate.
b) Damage located at the corner of the plate.
c) Damage present both at the centre and corner of the plate.

Figs. 1 and 2 showcase the material properties and the layer sequence, respectively, of the plate used in the analysis.



(a) (b) (c)

Figure 8: Damage location on plate (a) centre (b) corner (c) corner and Centre

RESULTS AND DISCUSSIONS

A broad investigation into damage detection in composite plates using numerical modal analysis were done, the results and their discussions are presented for the case using fourth order derivative under the following section. The results demonstrate the efficiency and accuracy of proposed DI for detecting damage in the plate-like structure under different scenarios as shown in Fig. 9-11. For first case where the damage is located at centre of the plate, DI successfully and precisely identifies location of damage as shown in Fig. 9. Similarly for second case where the damage is at corner of the plate, from Fig. 10 it was observed that the DI accurately pinpoints the damaged region. These findings indicate that the proposed method performs well in single-damage detection scenarios.

However, in the third scenario involving multiple damage detection, a noticeable difference in the resolution of the DI is observed as indicated in Fig. 11. Damage located at centre of plate exhibits a relatively lower resolution compared to the damage situated at the corner of the plate. Though the accuracy increases as the number of modes considered are increased to 15 as shown in Fig. 12. The results overall affirm ability of proposed method to effectively detect and localize damage in two-dimensional plate-like structures, with its performance being particularly notable in single-damage scenarios. These outcomes highlight the potential practical applications of the DI in SHM and damage assessment.

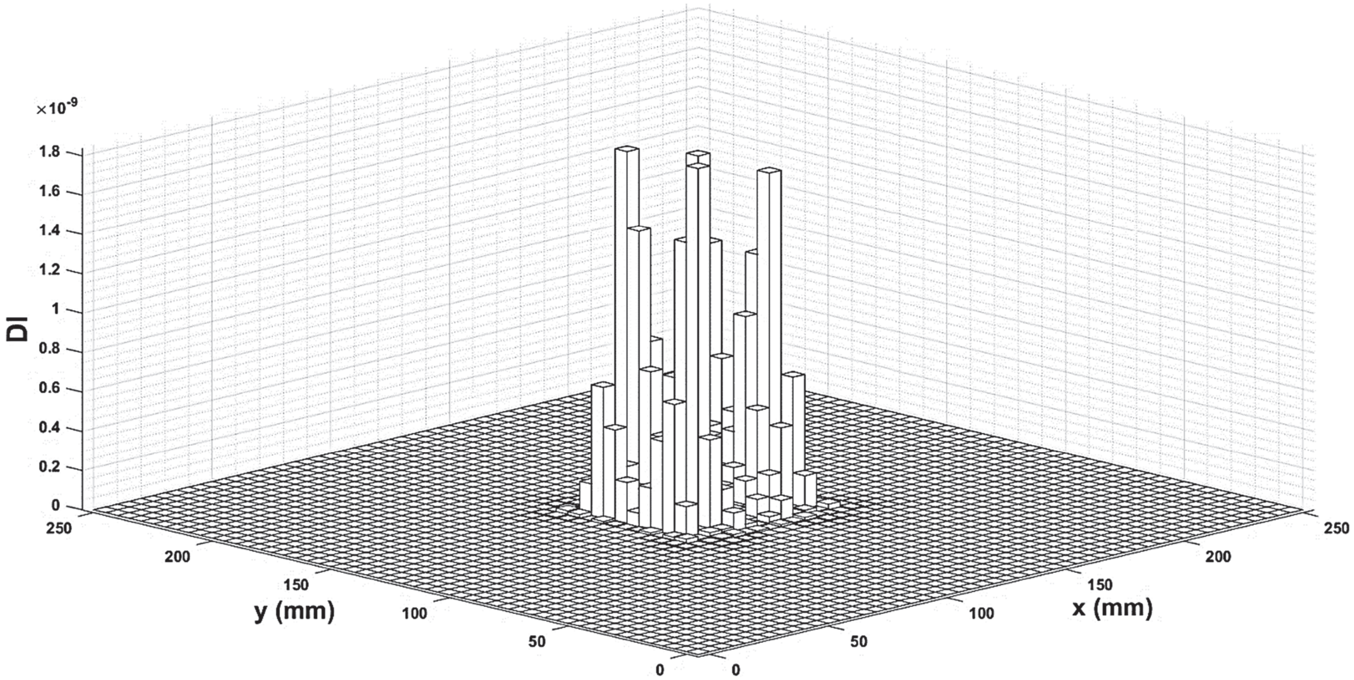


Figure 9: Damage detection for damage at centre of plate.

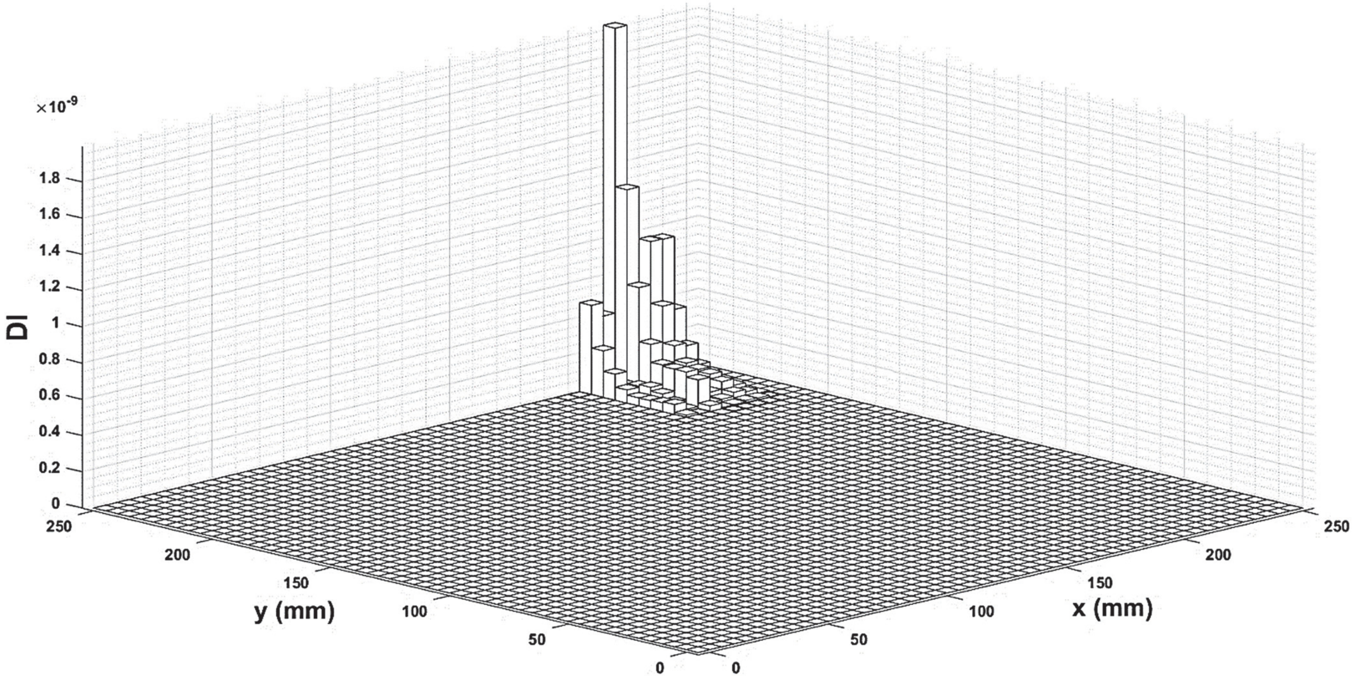


Figure 10: Damage location for damage at corner of plate

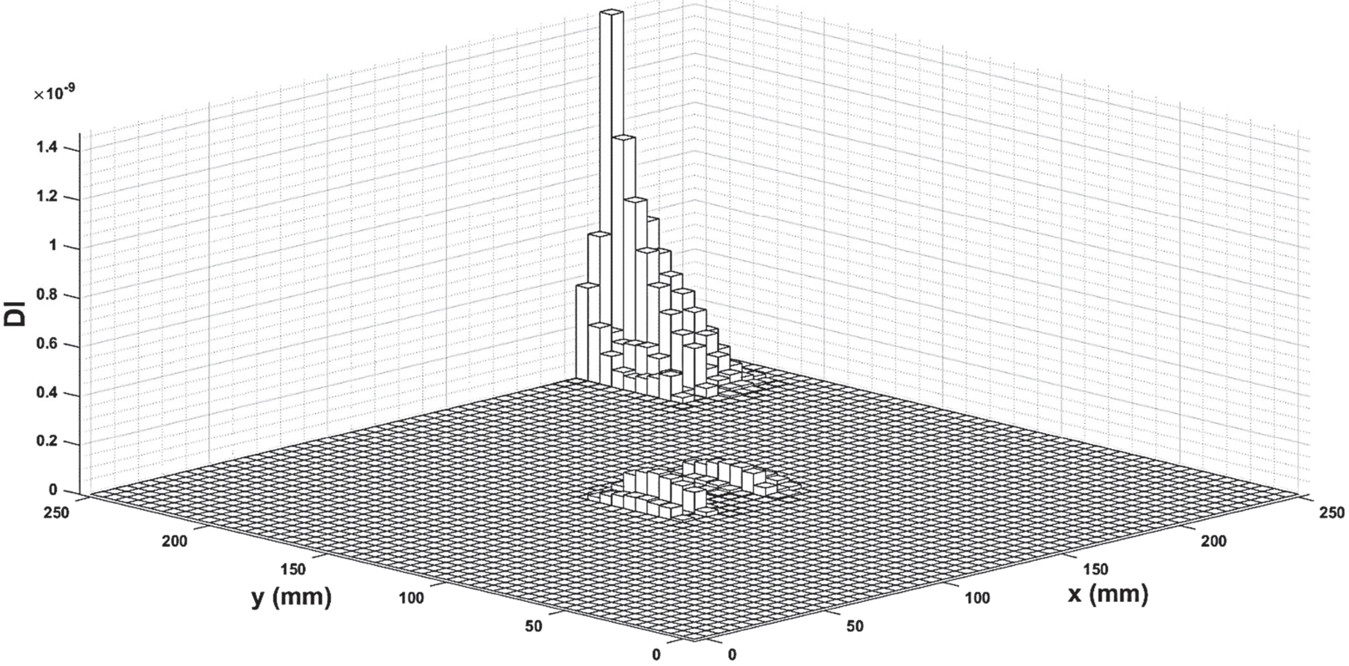


Figure 11: Damage location for damage at corner and centre of plate.

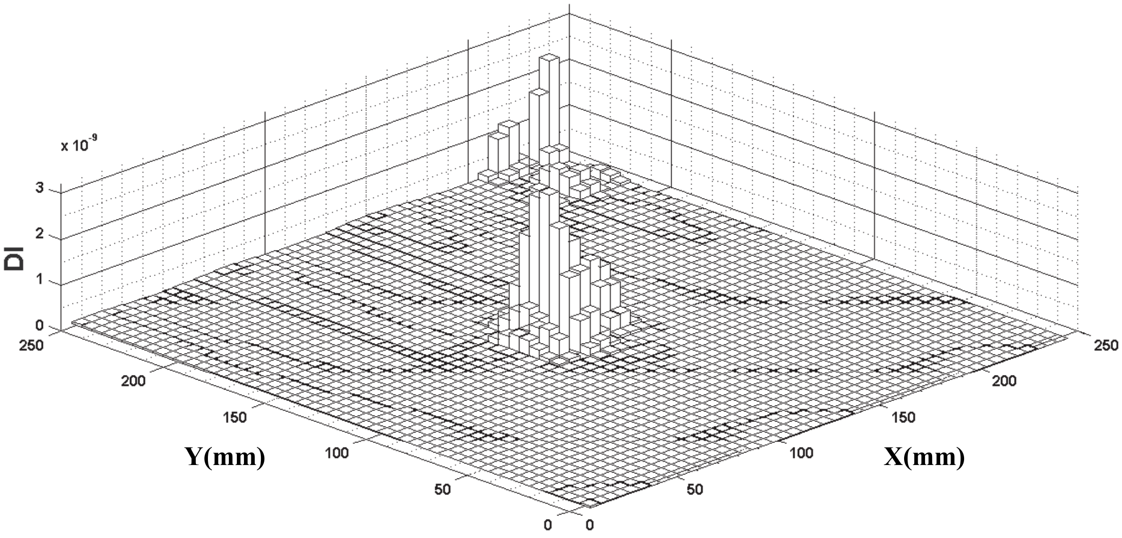


Figure 12: Damage location for damage at corner and centre of plate(N=15)

CONCLUSIONS

A comprehensive investigation of damage detection in composite plates were done successfully using experimental and numerical modal analysis. The FE method, implemented through ANSYS allows for a detailed analysis of the plate's modal characteristics before and after damage. The simulation results exhibit good agreement with experimental analysis, validating the model's accuracy for further numerical assessments.

The study highlights the impact of delamination on the fundamental natural frequency and higher modes in composite plates. While the fundamental mode shows minimal changes in frequency due to delamination, higher modes exhibit more significant alterations. sole reliance on frequency changes may pose challenges in precisely locating small-scale delamination in composite laminates. Therefore, additional complementary methods may be necessary for detecting such subtle damages effectively.

The research introduces the DI as an effective tool for damage detection in plate-like bodies using mode shape and its derivatives. It successfully identifies and accurately locates single-damage scenarios, as evidenced by the simulations for damage at centre and corner of plate. However, in cases of multiple damage detection, the DI exhibits varying resolutions, with higher sensitivity towards damage at the corner of the plate compared to the centre for N=10, but the resolution increases at centre of plate as the value of N is increased to 15. Nonetheless, overall performance suggests potential practical applications of the DI in SHM.

In conclusion, this investigation provides valuable insights into damage detection in composite plates and lays the foundation for future advancements in SHM and damage assessment. The proposed DI showcases promising capabilities, and with further refinements and integration with complementary techniques, it holds great potential to enhance the safety and reliability of composite structures in various engineering applications.

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