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Development of Digital Twins to Support the Functioning of Cyber-physical Systems

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Abstract

The peculiarities of developing a digital twin for cyber-physical systems using an analytical model of the investigated process are considered. The proposed approach takes into account the conceptual uncertainty of the analytical model parameters with the subsequent passive identification by adapting the analytical model to the dynamic characteristics of the real physical process. In the article, the digital twin development is carried out on the analytical model air heating process example with the help of an electric calorifier. The analytical model uncertain parameters of the electric heater are analyzed and an integral quality index is proposed to evaluate the dynamic model adequacy in the process of air heating. For electric heater model adaptation, the passive identification algorithm of uncertain parameters is developed, in which deviations of the mathematical model uncertain coefficients are minimized. A numerical study of the considered approach has been carried out. It is shown that uncertain parameters' passive identification of the model belongs to the problem of single-extremal optimization. The considered modeling examples confirmed the effectiveness of the proposed approach to digital twins' development for cyber-physical systems.

Keywords: mathematical model, uncertain parameters, state space, digital twin, identification, quality criterion, cyber-physical system, electric heater.

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1 Introduction

Cyber-physical systems (CPS) are distributed systems with a deep interconnection between their physical and computational elements. CPS can be described as intelligent systems that include computational (hardware and software) and physical components, integrated and closely interacting with each other to reflect the changing state of the real world. CPS are integrations of computation, networking, and physical processes [1]. The "brain" of the system includes billions of nodes in the form of AI and other technologies. It receives data from sensors in the real world, analyzes this data, and uses it to further control physical elements. The guaranteed functioning of CPS is based on the general problem minimization of multi-factor risks, the margin of permissible risk, forecast of the destabilizing dynamics of risk factors, principles, hypotheses, and axioms that are directly related to the analysis of abnormal situations, accidents, and disasters. The key idea of the strategy is based on the main principle: to provide timely and reliable detection and estimation of risk factors, prediction of their development during a certain period of operation, and timely identification and elimination of the causes of abnormal situations before failures and other undesirable consequences occur and prevention of the transition from normal to an abnormal mode [2]. The fundamentally important peculiarities of CPS functioning are the following: sets of risk factors and sets of situations are largely unlimited; a set of risk situations is in principle not a complete group of random events; a threshold restriction of time for decision forming is a top priority; the problem is not completely formalized; indicators of a multifactor risk estimation are not determined; criteria of a multipurpose risk minimization are not determined. The communication with computational systems and different types of sensors is implemented online in real-time. Joint actions of CPS components determine the properties and special features of the mode of functioning of a complex system at any moment of time.

To ensure the reliable operation of the CPS, a digital twin (DT) is created, which accompanies the operation of the CPS throughout its life cycle [3, 4]. When adopting a control strategy, DT allows for adequate displaying of the dynamics of the physical process, predicting the behavior, detecting system malfunctions, finding modifications in the structure of the physical process by observable effects, and ensuring efficient and uninterrupted operation of CPS.

2 Relative papers

DT refers to a new innovative toolkit that helps exploit advanced scenarios of the Internet of Things (IoT) [5]. This toolkit is used to create digital copies of physical objects. These physical objects can be factories, power grids, transportation systems, buildings, cities, and more.

The number of publications on DT has increased dramatically over the past six years [6]. In total, approximately 8693 articles had been published on this topic by September 2022, but only 29 articles were published in 2016, the number had increased to 2997 by 2021 [7]. Thus, the DT development concept belongs to modern scientific trends.

One of the fundamental works on the standardization of DT development is the Industrial Internet Reference Architecture (IIRA) reference model proposed by IIC [8]. The document describes guidelines for the systems development, applications, and solutions using IoT in industry and infrastructure solutions. This architecture is abstract and provides general definitions for various stakeholders, system decomposition order, design patterns, and terms glossary. The IIRA model relates at least four stakeholder viewpoints (levels): business; usage; operation; and implementation. Each level focuses on DT functional model implementation, the structure, interfaces, and interactions between the DT components, and the DT model's system interaction with external elements of the environment to support CPS functioning. The DT technology includes (but is not limited to) combinations of the program object: physical model and data; analytical model and data; temporal variable archives; transactional data; master data; and visual models and calculations. DT creation concept has a multifaceted architecture and correspondingly complex mathematical support for implementation. Many DT-related technologies have been patented. Google Patents returned 6181 results for this query dated 2003-01-01. The largest patent holders are large corporations: Siemens AG (13.4% of all patents); General Electric (9.8%); Beijing University of Aeronautics and Astronautics (3.4%). A study that conducted a cluster analysis of DT patents from the Webpat and Derwent databases found 140 records by 2018 [9].

In 2023, the concept of a digital twin is evolving into something more subtle and incredibly practical: an executable digital twin (xDT). Simply put, xDT is a digital twin on a chip. xDT uses data from a (relatively) small number of sensors embedded in a physical product to perform real-time simulations using reduced-order models. Based on the data from a small number of sensors, it can predict the physical state anywhere within the object (even in places where sensors cannot be placed) [10]. The world's first digital twin city was created for Singapore in 2014 at a cost of US$73 million. In 2022, the system was replaced with an advanced version that includes data from sensors, drones, government agencies, etc. [11]. The city of Zurich has its own DT with a detailed 3D map of roads and underground and above-ground facilities [12]. According to the World Economic Forum 2022, by 2030, the technology of DT will have saved $289 billion on city planning and construction; in 2020, investments in DT of Chinese cities exceeded $380 billion [13].

The mathematical description of DT can be obtained using statistical modeling, machine learning, or analytical modeling techniques. Methods of statistical modeling can be divided into three groups [14]: regression analysis models; classification models; and anomaly detection models. The method choice depends on the size, quality, and data nature, as well as on the problem type and the process knowledge being modeled. For technological processes in CPS, analytical models are often used, which have valuable properties in engineering [15]. In [16], the CPS development using deterministic models, which have proven to be extremely useful, is discussed. Deterministic mathematical models of CPS are based on differential equations and include synchronous digital logic and single-threaded imperative programs. However, CPS combines these models in such a way that determinism is not preserved.

The practice of using analytical models to describe the functioning of technological processes indicates that ready-to-use models are extremely rare because such models are developed under conditions of conceptual uncertainty that need to be disclosed for a particular physical process. Conceptual uncertainty arises from the knowledge incompleteness of the physical environment, process, or system. Conceptual uncertainty is complex [17], conceptual uncertainty examples are an uncertainties combination: objectives; operation of process; structure modeled system; system elements interaction, or interaction with the external environment, and others. For analytical models, the above uncertainties are complicated by information uncertainty, caused by methodological uncertainty (complex processes are linearized when modeling), measurement distortion (due to inaccuracy and inertia of sensors and the presence of disturbances), and other factors.

3 Research problem statement

Considering the above, the publication's aim is to develop a DT model for manufacturing plants by applying the analytical model of air heating on an electric heater in conditions of conceptual uncertainty. The problem of conceptual uncertainty disclosure in contentive statement is reduced to a problem of system-coordinated disclosure of a set of diverse uncertainties on the basis of unified principles, techniques, and criteria. This set includes the uncertainty of parameters for each type of electric heater, uncertainty of its physical and mechanical characteristics, and situational uncertainty of risks in the process of operation. Such uncertainty refers to the conceptual one [17]; being distinct from information uncertainty, it represents a unified complex of the lack of information, ambiguity, and contradictoriness of interconnected and interdependent elements of a specified set of polytypic uncertainties.

4 Models and Methods

4.1 Analytical model of air heating on electric heater. Analysis of model parameters

Let's consider the analytical model of air heating on an electric heater, which is proposed in [18]:

$$\begin{cases} T_E \frac{d\,\Delta\theta_E}{dt} + \Delta\theta_E = k_0\,\Delta N_E + k_1\,\Delta\theta_A, \\ T_A \frac{d\,\Delta\theta_A}{dt} + \Delta\theta_A = k_2\,\Delta\theta_E + k_3\,\Delta\theta_{A0} + k_4\,\Delta G_A, \\ T_d \frac{d\,\Delta d_A}{dt} + \Delta d_A = k_5\,\Delta d_{A0} + k_6\,\Delta G_A; \end{cases} \tag{1}$$

here $T_E = \frac{c_E M_E}{K_E}$, $K_E = \alpha_0 F_0$, $k_0 = \frac{1}{K_E}$, $k_1 = 1$; $T_A = \frac{c_A M_A}{K_A}$, $K_A = c_A G_A + \alpha_0 F_0$, $k_2 = \frac{\alpha_0 F_0}{K_A}$, $k_3 = 1 - k_2$, $k_4 = \frac{c_A(\theta_{A0} - \theta_A)}{K_A}$; $T_d = \frac{\omega V_A}{G_A}$, $k_5 = 1$, $k_6 = \frac{d_{A0} - d_A}{G_A}$.

To solve the system of differential equations (1), the state space can be used:

$$X' = AX + BU, \tag{2}$$

$$X = \begin{bmatrix} \Delta\theta_A \\ \Delta d_A \\ \Delta\theta_E \end{bmatrix}; A = \begin{bmatrix} -1/T_A & 0 & k_2/T_A \\ 0 & -1/T_d & 0 \\ k_1/T_E & 0 & -1/T_E \end{bmatrix};$$

$$B = \begin{bmatrix} k_3/T_A & 0 & k_4/T_A & 0 \\ 0 & k_5/T_d & k_6/T_d & 0 \\ 0 & 0 & 0 & k_0/T_E \end{bmatrix}; U = \begin{bmatrix} \Delta\theta_{A0} \\ \Delta d_{A0} \\ \Delta G_A \\ \Delta N_E \end{bmatrix}.$$

For models (1) and (2), the parameters classification is proposed in the block diagram form, which is shown in Figure 1. The analysis of numerical values of the model parameters allows us to conclude that thermophysical values of material flows and constructional materials of the electric heater are determined with high accuracy from handbooks on thermophysical properties of substances and materials; these parameters refer to blocks 1 – 3 (see Fig. 1).

The modifiable in general case uncertain parameters of the model are in Block 4. Formally, for model (2), there are six changing parameters α_0, G_A, θ_{A0}, θ_A, d_{A0}, and d_A, on which all coefficients of mathematical model (2) depend. The task of numerical values finding of the parameters α_0, G_A, θ_{A0}, θ_A, d_{A0}, and d_A should be solved in conceptual uncertainty conditions because these parameters are linked into a single complex of the mathematical model interrelated parameters.

For example, an increase in air flow G_A leads to an increase in the heat transfer coefficient α_0; exactly the same effect can be achieved by increasing the air humidity d_{A0}. The heat transfer coefficient α_0 depends on many factors and can significantly change its value depending on air moisture d_A; air flow rate G_A; temperature difference $\theta_E - \theta_A$; design features of the heat exchange surface, and other factors.

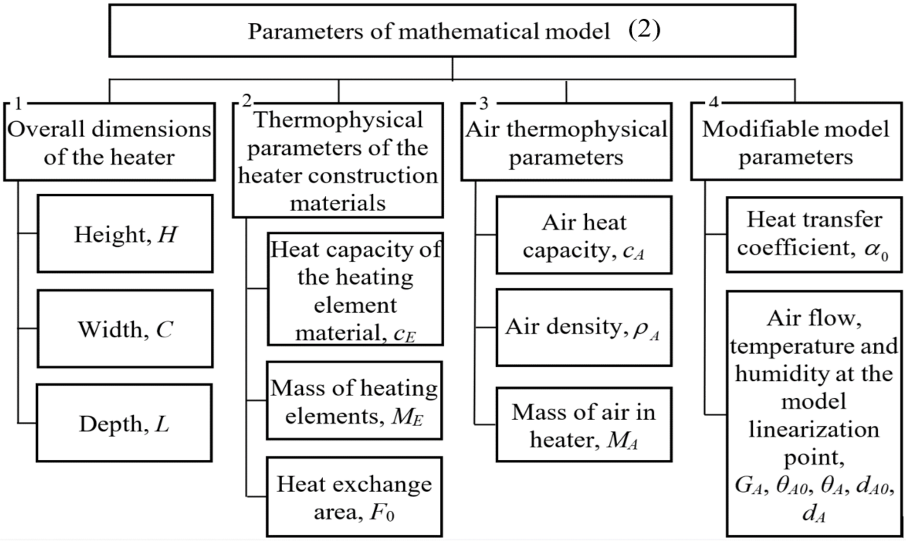

Figure 1. Block diagram of the analytical model parameters classification of electric heater

Thus, the same numerical value α_0 has an infinite number of combinations of the considered parameters. Note that heat transfer coefficient α_0 depends on many factors and there is no sensor to measure this parameter. This parameter is a research subject in thermal engineering, which is based on experimental studies and similarity theory [19]. To solve such problems, it is necessary to formulate a strategy for developing an analytical model, which will later be used to design the DT model. In these conditions, a choice of an analytical model from the available set, the uncertain parameters analysis of the model and their justification, and the method of identifying uncertain parameters is a nonformalizable procedure, and only the researcher can carry it out. The result depends on the competence, skill, experience, intuition, and other individual qualities of an actual researcher who is carrying out the given procedure.

For example, for model (2), the list of uncertain parameters contains six items, which complicates the search. However, the parameters G_A,

θ_{A0}, θ_A, d_{A0}, and d_A can be measured using CPS sensors and thus solve the uncertainty problem of these quantities. In this case, for model (2), it is necessary to reveal the uncertainty of only one parameter α_0, which greatly simplifies the search task.

4.2 Passive identification of mathematical model parameters

Analytical model (1) is obtained using the studied laws of heat and mass transfer. For adapting the model to the concrete process of air heating, specification of its parameters is required. Identification of model parameters can be carried out using active or passive experiment on the operating equipment. Passive identification methods are most often used, since there is no need to expend additional production resources in the course of the experiment and this approach is justified by its cost-effectiveness. To identify the model in the state space (2), the Kalman filter is most often used, or the least squares method (LSM) and its modifications are used [20]. In our case we will apply LSM, taking into account its universality.

Formally, model (2) has six changing parameters α_0, G_A, θ_{A0}, θ_A, d_{A0}, and d_A (see Fig. 2). In the passive identification process, it is sufficient to specify α_0 and G_A. Other uncertain parameters of the electric heater model θ_{A0}, θ_A, d_{A0}, and d_A can be estimated using the CPS sensors. As an identification criterion, using dependence that minimizes the error square of the state variables, measured values of the physical process X, and the output vector $\overline{X}$ estimates of the model being identified (2) for the same input action U:

$$I = M\left\{\int_{t_0}^{t_0+t_f} \left(X-\overline{X}\right)^{\mathrm{T}} Q \left(X-\overline{X}\right)\, dt\right\} \to \min, \tag{3}$$

here t_0 is the initial time of the trend, t_f is the duration of the trend sampling, Q is the unit square matrix, T is the matrix transpose operator, M is the mathematical expectation operator, which takes into account industrial perturbations.

The general block diagram of passive identification of the analytical model parameters (2) is shown in Figure 2. The search criterion (3) is calculated numerically when implementing the identification algorithm. Therefore, it is recommended to use zero-order numerical optimization methods to identify the model parameters [21]. Numerical methods require significant computational resources, so the identification algorithm can be implemented within a decision support system (DSS) [22].

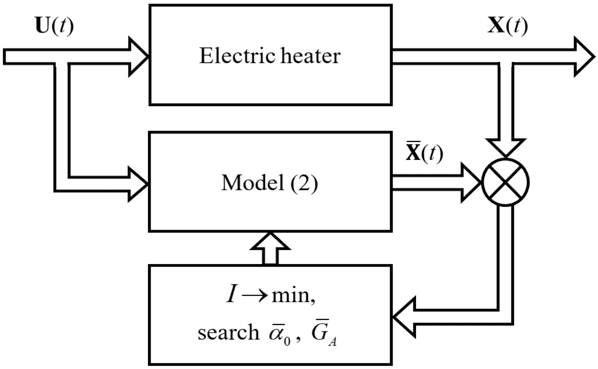

Figure 2. The block diagram of passive identification of the model parameters (2)

The passive parametric identification algorithm of the dynamic model (2) consists of the following steps:

1) the CPS sensors monitor the vector of input influence $U(t)$ and output state $X(t)$ of the physical process in real-time, the initial data are processed in the DSS; the database of input and output states for the physical process is formed;

2) the output state $\overline{X}(t)$ of the mathematical model with the initial values of the parameters $\bar{\alpha}_0$ and $\bar{G}_A$ is estimated by the time trends of the input action $U(t)$;

3) the identification criterion (3) is defined, using vectors of output $X(t)$ and the identifiable physical process states $\overline{X}(t)$;

4) the parameters $\bar{\alpha}_0$ and $\bar{G}_A$ of the identified model (2) are numerically optimized using criterion (3);

5) if the minimum of criterion (3) is found, then proceed to the design of DT, else continue to minimize the identification criterion and go to Step 2.

In the identification process, it is necessary to take into account the numerical method peculiarities for minimization of criterion (3). Also, for qualitative identification, the time trends t_f of input and output states should be several times longer than the physical process transient's duration. To eliminate overflow effects of the numerical minimization algorithm of criterion (3), it is necessary to set search limits for identifiable parameters $\bar{\alpha}_0$ and $\bar{G}_A$, based on the mathematical model physical feasibility.

4.3 Results estimation of mathematical model parameters identification

The proposed numerical identification algorithm was investigated using MatLAB. To model the time trends of the operating electric heater X(t), the reference model (2) was used with numerical values of matrices:

$$A = \begin{bmatrix} -2.631 & 0 & 0.268 \\ 0 & -2.357 & 0 \\ 0.179 & 0 & -0.178 \end{bmatrix}; B = \begin{bmatrix} 2.362 & 0 & -21.98 & 0 \\ 0 & 2.358 & 0 & 0 \\ 0 & 0 & 0 & 0.0036 \end{bmatrix}. \tag{4}$$

To simulate production disturbances, a random signal with an amplitude of ± 0.2 was mixed into the reference output variables of the vector $X(t)$. In the identified model, the initial values of the parameters $\bar{\alpha}_0 = 110$ and $\overline{G}_A = 0.15$ differed significantly from the reference values $\alpha_0 = 161$ and $G_A = 0.43$. Therefore, the numerical values of matrices A and B of the reference model (2) were significantly different from the numerical values of the matrices of the identified model:

$$\bar{A} = \begin{bmatrix} -1.265 & 0 & 0.167 \\ 0 & -1.097 & 0 \\ 0.111 & 0 & -0.111 \end{bmatrix}; \bar{B} = \begin{bmatrix} 1.099 & 0 & -21.98 & 0 \\ 0 & 1.097 & 0 & 0 \\ 0 & 0 & 0 & 0.0036 \end{bmatrix}. \tag{5}$$

For the numerical identification of $\bar{\alpha}_0$ and $\bar{G}_A$, the MatLAB function fminsearch(...) was used, where the simplex Nelder–Mead optimization method is applied. The main results of the numerical study are presented in Figures 3 and 4.

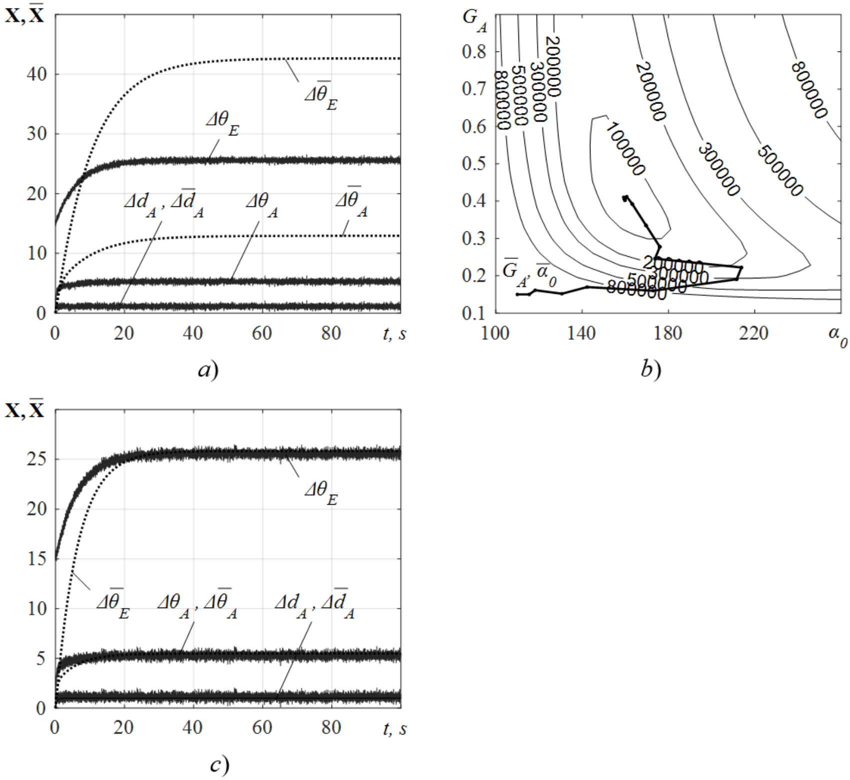

Figure 3. The parametric identification of $\bar{\alpha}_0$ and $\overline{G}_A$ with step influence $U\left(t\right) = \left[1, 1, -0.2, 1\right]^T$:
a) the simulation of transients before identification;
b) the identification trajectory of parameters $\bar{\alpha}_0$ and $\overline{G}_A$ by criterion (3);
c) the simulation of transients after identification parameters $\bar{\alpha}_0$ and $\overline{G}_A$

Figure 3 shows the case when the initial conditions of the reference model $X\left(0\right) = \left[\ 2,\quad 1,\quad 15,\ \right]^{\mathrm{T}}$ and the identified model $\bar{X}\left(0\right) =$

$\begin{bmatrix} 0, & 0, & 0, \end{bmatrix}^{\mathrm{T}}$ are significantly different. Also, the identification parameters are significantly different: $\alpha_0 = 161$, $G_A = 0.43$; $\bar{\alpha}_0 = 110$, $\overline{G}_A = 0.15$. With the step influence of the input vector $U(t) = \begin{bmatrix} 1, & 1, & -0.2, & 1 \end{bmatrix}^{\mathrm{T}}$, there were obtained the transients shown in Figure 3(a). The difference between the output values of the reference $X(t)$ and the identifiable model $\bar{X}(t)$ is quite significant. Figure 3(b) shows the isolines surface of criterion (3) and its minimization trajectory. It is seen that the functional has one extremum in the area of the model physical practicability, so any numerical optimization method can be used as an optimization method. According to the proposed identification algorithm, $\bar{\alpha}_0 = 160.54$ and $\overline{G}_A = 0.4$ were determined. The found parameter values are quite close to the reference ones $\alpha_0 = 161$ and $G_A = 0.43$. Figure 3(c) shows the time characteristics of the state variables of the reference $X(t)$ and the identified $\bar{X}(t)$ model after identification under the input influence $U(t) = \begin{bmatrix} 1, & 1, & -0.2, & 1 \end{bmatrix}^{\mathrm{T}}$ on both models. Based on the simulation results, it can be concluded that the proposed algorithm for passive identification of the electric heater has good convergence in the case of different initial conditions and the presence of random perturbations.

Fig. 4 (a) shows the case when the reference model is in the stationary state $X(0) = \begin{bmatrix} 2.3, & 0, & 22.5 \end{bmatrix}^{T}$ during the presence of random perturbations. This state is provided by the input influence vector $U(t) = \begin{bmatrix} 0, & 0, & 0, & 1 \end{bmatrix}^{T}$. The initial conditions of the identifiable model are zero $\overline{X}(0) = \begin{bmatrix} 0, & 0, & 0 \end{bmatrix}^{T}$. According to the simulation condition, the parameters of the identified model $\bar{\alpha}_0 = 250$, $\overline{G}_A = 0.8$ are significantly different from the reference model $\alpha_0 = 161$, $G_A = 0.43$. Fig. 4 (b) shows the surface isolines of criterion (3) and its minimization trajectory. During the identification process, the values of the parameters $\bar{\alpha}_0 = 157.28$, $\overline{G}_A = 0.42$ are optimized. As in the first study, the found values of the parameters are quite close to the reference ones. Fig. 4 (c) depicts the temporal characteristics of the state variables of the reference $X(t)$ and the identifiable $\overline{X}(t)$ model after identification under the input influence $U(t) = \begin{bmatrix} 0, & 0, & 0, & 1 \end{bmatrix}^{T}$. According to the modeling results can conclude that the proposed passive identification algorithm has good convergence in the case of the

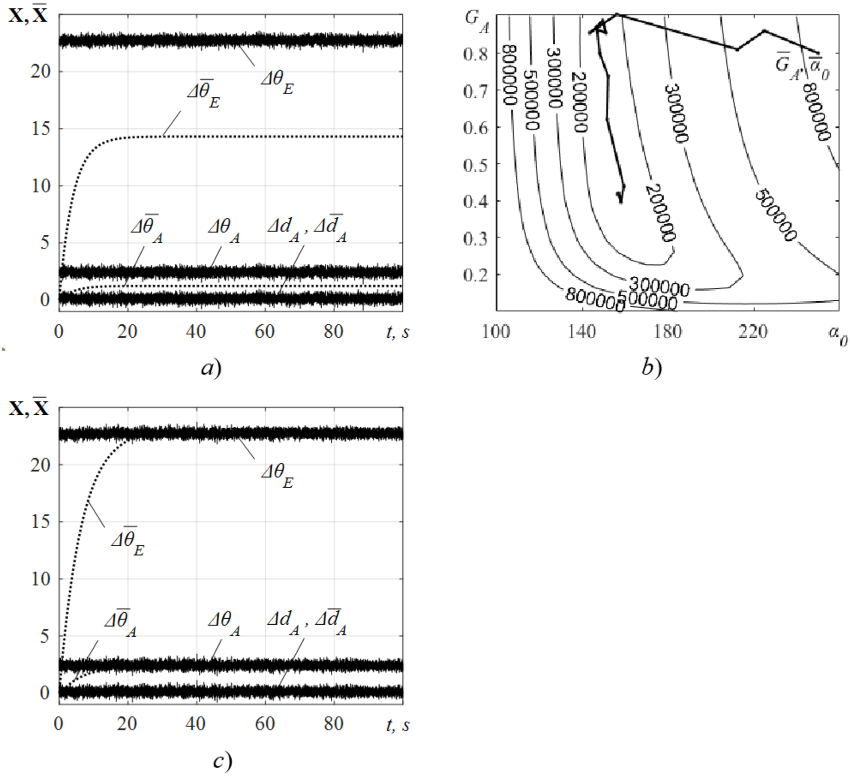

Figure 4. The parametric identification of $\bar{\alpha}_0$ and $\overline{G}_A$ with step influence $U(t) = [0, 0, 0, 1]^T$:

a) the simulation of transients before identification;

b) the identification trajectory of parameters $\bar{\alpha}_0$ and $\overline{G}_A$ by criterion (3);

c) the simulation of transients after identification parameters $\bar{\alpha}_0$ and $\overline{G}_A$

object stationary and the random perturbations presence.

5 Approach to developing a digital twin of CPS under conceptual uncertainty

The Generalized structural scheme of the DT development procedure based on the analytical model of physical process is presented in Figure 5. This takes into account the parametric uncertainty of the physical process mathematical description.

In the first stage of development, it is necessary to conduct a literature analysis in the applied field of research for the physical process. This will help to determine the model structure, existing advantages, and disadvantages. As a rule, the analytical model of the studied process has the system form of differential, difference, or algebraic equations.

The practice of using analytical models shows that ready-to-use models are very rare. Even tested models require adjustment of parameters in order to adapt them to specific conditions of use. Thus, when DT is developed for a particular physical process, the researcher needs to determine the uncertainty ”physical limits of that process” in the numerical values form of mathematical model parameters. To do this, the researcher needs to perform passive identification of the mathematical model parameters. A very important role at this stage is played by the data quality for the model identification, so the formation of the database should be guided by the known requirements of informativeness, synchroneity, and correctness.

The last step in DT model development is the identified model discretization. Here it is necessary to set correctly the sampling time for the mathematical model. On the one hand, the sampling time should not be small in order to ensure the information distribution over the CPS network. On the other hand, a large sampling time will lead to the loss of intermediate information for short-term forecasts. The obtained numerical model, even of a sufficiently adequate high degree, does not yet guarantee a prediction estimate of high quality if the basic uncertainties for the mathematical model of the physical process are not taken into account. Therefore, after designing DT, it is necessary to check the possibility of using it for solving the assigned forecasting tasks.

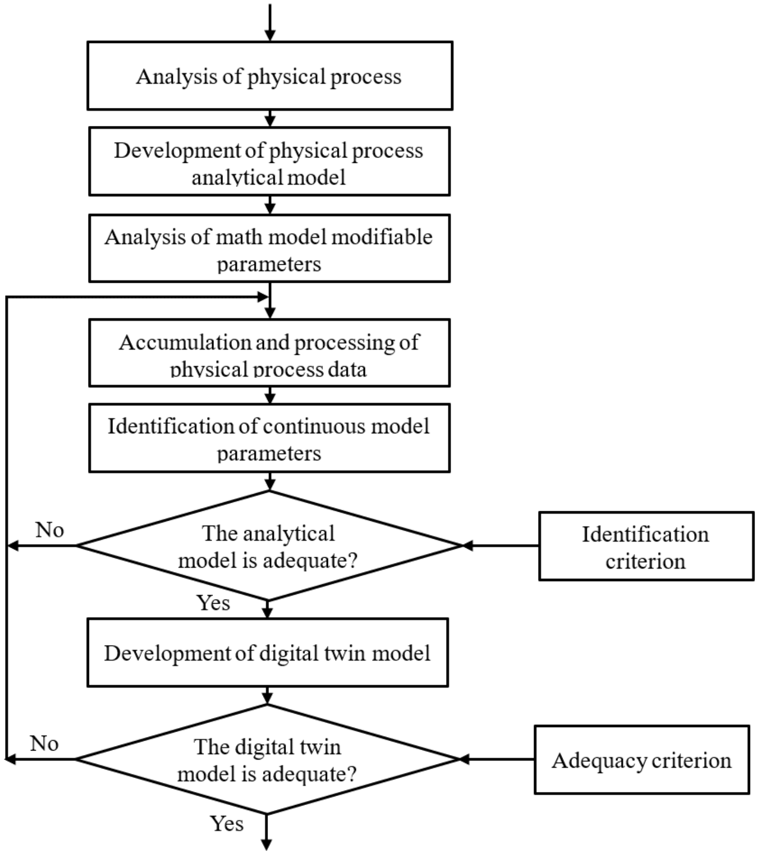

Figure 5. The structural scheme of the model DT development procedure

An important characteristic for DT is to determine the received prediction quality. Often, the quality of forecast estimates is determined with the help of LSM. However, LSM is one of many possible statistics that depends on the data scale. Therefore, only this characteristic is not enough for the analysis of a qualitative prediction. The quality of linear and pseudo-linear models is assessed using several statistical quality criteria [20] since each criterion has its own specific purpose and characterizes one property of a prediction evaluation. Therefore, DT developer must comprehensively study a physical process, existing mathematical models, possible perturbing effects on a physical process and justify the use of adequacy criteria for DT in conditions of conceptual uncertainty.

5.1 Digital twin development for the air heating process by an electric heater

The DT development will be carried out on the basis of analytical model (2). In order to obtain adequate computational data at the first stage of DT synthesis, it is necessary to identify the continuous model of the electric heater, using the passive identification algorithm discussed above. Also, it is highly desirable to reduce the computational resources of DT. It is known from modeling theory that computational resources for simulation differential equations are more than for their discrete analogs based on difference equations. Therefore, let us consider a discrete representation of a continuous model (2).

The mathematical model (2) can be represented in a discrete form [23]

$$\bar{X}_{k+1} = \bar{A}_d \bar{X}_k + \bar{B}_d U_k, \tag{6}$$

here $\bar{A}_d = e^{\bar{\mathrm{A}} T_{KV}}$, $\bar{B}_d = \int_0^{T_{KV}} e^{\bar{\mathrm{A}}(T_{KV}-t)}\, \bar{B}\ dt$, T_{KV} is the sampling period.

Thus, the DT synthesis methodology for the electric heater consists of steps:

1) the uncertain parameters identification ($\bar{\alpha}_0$ and $\overline{G}_A$) of a mathematical model (2) by the considered algorithm;
2) transition from the continuous model (2) to the discrete model (6), which is DT;
3) if, during operation, the DT accuracy has deteriorated (due to non-stationarity of the physical process), then go to Step 1 to identify the parameters of the model.

5.2 Results of the digital twin simulation

The proposed methodology was used to develop and simulate the DT of an electric heater using the MatLAB software package. Let's consider the example of DT simulation using the model in state space (2) [24]. MatLAB was used to calculate the matrices $\overline{A}_d$ and $\overline{B}_d$ of DT (6). Simulation results are shown in Figure 6.

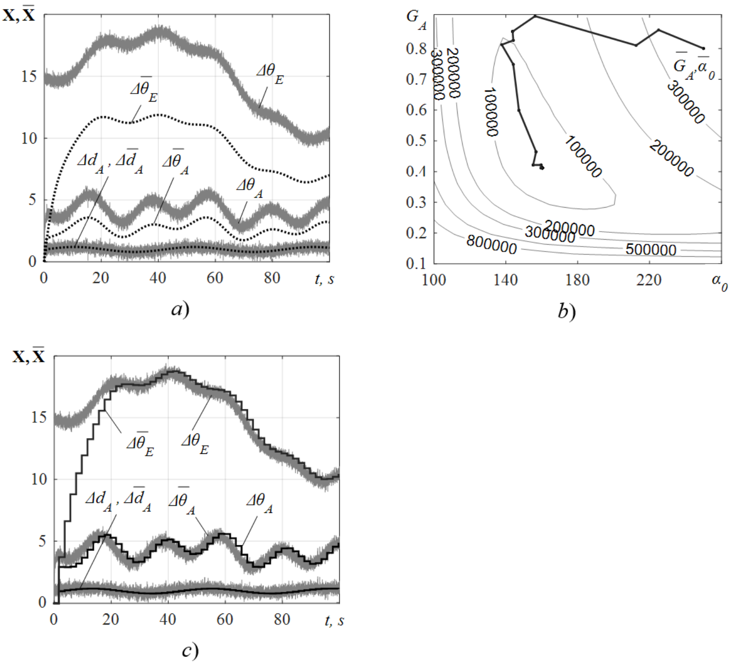

Figure 6. The DT development for the electric heater HE 36/2:
a) the simulation of transients before identification;
b) the identification trajectory of parameters $\bar{\alpha}_0$ and $\overline{G}_A$ according to the proposed algorithm;
c) the simulation of transients for the physical model (2) and DT (6)

Figure 6 (a) simulates the case with input influence

$$U(t) = \begin{bmatrix} 1+0.5\sin(0.15t) & 1+0.2\sin(0.15t) & -0.2+0.1\sin(0.3t) & 0.5+0.2\sin(0.05t) \end{bmatrix}^T,$$

and initial conditions for the physical model $X(0) = \left[\ 2,\ \ 1,\ \ 15,\ \right]^T$, $\alpha_0 = 161$, $G_A = 0.43$ and the identifiable model $\bar{X}_k(0) = \left[0,\ \ 0,\ \ 0,\right]^T$, $\bar{\alpha}_0 = 250$, $\overline{G}_A = 0.8$. Figure 6 (b) shows the surface isolines of criterion (6) and the minimization trajectory of its parameters $\bar{\alpha}_0$, $\overline{G}_A$, which resulted in the finding $\bar{\alpha}_0 = 160.3$, $\overline{G}_A = 0.41$. After identification of model (2), using MatLAB function c2d(...) numerical values of DT model matrices (6) are calculated for the sampling period $T_{KV} = 2$:

$$\bar{A}_d = \begin{bmatrix} 0.0133 & 0 & 0.0829 \\ 0 & 0.0125 & 0 \\ 0.0552 & 0 & 0.7238 \end{bmatrix};$$

$$\bar{B}_d = \begin{bmatrix} 0.9037 & 0 & -9.0374 & 0.547 \\ 0 & 0.9875 & 0 & 0 \\ 0.221 & 0 & -2.2097 & 6.1755 \end{bmatrix}.$$

Figure 6 (c) shows the time characteristics of the state variables for the reference model X and the DT $\bar{X}_k$ of the electric heater.

6 Conclusions

Today there is no single approach to the processes of production plants management. Development and implementation of CPS are individual tasks for each enterprise. Systems implementation for operational management of production processes inevitably leads to the creation of multidimensional dynamic systems that are able to interact effectively in a single information management space. The solution of such problems is considered in the concept of CPS, as this technology initially assumes the symbiosis of computational and physical processes.

Digitalization of physical environments and processes implies the use of complex mathematical models with uncertain parameters, which need to be corrected in order to self-adapt to specific application conditions. As an example, the passive identification technique of the electric heater analytical model with subsequent synthesis of a digital twin for the CPS is considered. Based on system analysis methodology, the results obtained are summarized and an approach to the digital twin development using the analytical model under conditions of conceptual uncertainties is proposed.

The peculiarity of the proposed approach is the several key parameters identification of the analytical model, which are refined in the passive identification process. The use of a physical process analytical model makes it possible to abandon the search for all its parameters. It is known that in modern methods of system analysis, the choice of structure and type of model plays an important role in further research and may require a lot of time and additional information for building an adequate model. In the proposed approach, the structure of the analytical model is known, for which only the key uncertain parameters are identified from the measured variables of the real physical process.

Examples of parameter identification for the model (2) in the state space are given. It is shown that the uncertain parameters identification of the model in the state space belongs to the problem of single-extremal optimization. The procedure for synthesizing the model of the electric heater digital twin is proposed and numerically investigated. The simulation results confirmed the effectiveness of the proposed procedure for creating a digital twin using an analytical model.

The use of DT in CPS makes it possible to identify bottlenecks in technological processes, improve product quality, and reduce the risks of abnormal operation throughout the life cycle of equipment. DT is used for the prediction of equipment operation modes and self-diagnostics, as well as optimization of the physical system structure. This approach provides a high-precision assessment of the plant's production capacity when drawing up the production program.

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