Mathematical modeling of particle movement in laminar flow in a pipe

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Abstract. The article provides basic information about the principles of calculations for solving the system of Navier–Stokes equations using the control volume method. The calculation of the incompressible fluid velocity field and pressure is found using the SIMPLE algorithm and MacCormack method. Particle trajectories were determined using the Lagrangian approach.

1 Introduction

Introduction Currently, velocity profiles have been determined for laminar steady motion of a Newtonian fluid in channels with a wide variety of cross-sectional shapes [1]. The flow of a continuous medium in channels and elements of technical devices is of practical interest for a number of applications [2-6].

Calculation within the framework of the complete Navier-Stokes equations of laminar flows at low and moderate Re, as a rule, does not involve overcoming any fundamental difficulties. However, as Re increases (Re>1000), obtaining reliable results becomes a problem, which greatly limits the possibilities of numerical modeling of those laminar flows for which the boundary layer approximation is unsuitable, and, in particular, the study of the loss of stability of laminar flow.

In scientific research the basis of computational science serves as a mathematical model of the physical phenomenon that interests us. The equations of the mathematical model are translated into discrete algebraic form, amenable to numerical solution. Discrete algebraic equations describe a computational model which, when translated into a sequence of machine instructions, provides a simulation program for the computer. After this, the computer and the program make it possible to study the evolution of the model physical system in computational experiments.[15]

The term "particle models" is common to the class computational models in which a discrete description of physical phenomena involves the use of interacting particles. The name itself arose because in most applications these particles can be directly associated with physical objects. Each particle has a set of attributes, such as mass, charge, vorticity, position, momentum.

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In addition to direct methods, which by their nature are the most accurate, there are simpler and therefore more applicable models:

Euler-Euler models.

Euler-Lagrange models.

Euler-Euler models.

A widely used method is to create transfer laws on based on the volume fractions of two phases in each computational control volume, resulting in a continuous representation of both phases. This approach is called two-fluid model or also called Euler-Euler model. In progress [12] one of the first to mention such a concept as interpenetrating continuums.

The key idea behind building a two-phase model is that multiphase flow is represented as a set of interacting continuums. Behavior of each component phase of a multiphase model described by the laws of conservation of mass, momentum and energy. Phase interaction relative to each other is taken into account by adding special (algebraic or differential) terms.

Euler-Lagrange models.

The Lagrangian approach, also known as the "dilute phase approach", used when the amount of dispersed phase is small and does not interfere with movement continuous phase [13-14]. In the Lagrangian approach, three main ones can be distinguished modeling method:

•Direct Numerical Simulation (DNS),

• Large Eddy Simulation (LES),

• Reynolds averaging by the Navier-Stokes method (RANS) [2]

DNS modeling approach requires particles to be smaller on the scale of Kolmogorov. This requirement greatly limits the use of DNS. low Reynolds numbers or very small particles. To overcome this is a limitation, LES modeling can be used. Application as DNS, Likewise, LES is limited to dilute systems, where collisions and hydrodynamic interactions can be neglected and are assumed one-way connection between the dispersed and carrier phases [10]. In recent studies [3] one-way coupled Eulerian-Lagrangian models have been used to study dilute solid-liquid flows weigh it. Reviews of Lagrangian-Eulerian methods for multiphase flows have been given in works [11]

This can only be achieved with a very fine mesh, which therefore leads to unrealistically high computational time and memory costs. Therefore, a subgrid drift velocity model was developed to allow use coarser meshes [12]. All phenomena on a scale smaller than numerical grid, require first of all modeling of flow turbulence fluids using RANS or LES methods. However, such models need extensions to account for two-way communication.

In this article, a mathematical model of the movement of dispersed particles in a flow is created and numerical results are obtained. For this purpose, the speed of water movement was determined using the Navier-Stokes equation [7]. The trajectories of dispersed particles were determined using the Lagrangian approach.

2 Physical and mathematical formulation of the problem

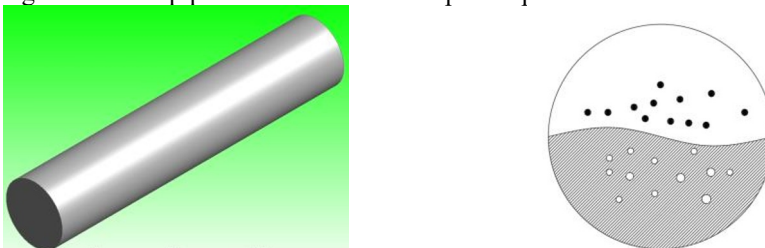In Fig. Figure 1 shows pipeline of the flow of dispersed particles

Fig. 1. Schematic view of the flow

$$\begin{cases} \dfrac{\partial U}{\partial x} + \dfrac{\partial rV}{r\partial r} = 0 \\[2mm] \dfrac{\partial U}{\partial t} + U\dfrac{\partial U}{\partial x} + V\dfrac{\partial U}{\partial r} + \dfrac{\partial p}{\rho\partial x} = \nu\left(\dfrac{\partial^2 U}{\partial x^2} + \dfrac{\partial^2 U}{\partial r^2} + \dfrac{\partial U}{r\partial r}\right) + \mathrm{K}\left(U - U_p\right) \\[2mm] \dfrac{\partial V}{\partial t} + U\dfrac{\partial V}{\partial x} + V\dfrac{\partial V}{\partial r} + \dfrac{\partial p}{\rho\partial r} = \nu\left(\dfrac{\partial^2 V}{\partial x^2} + \dfrac{\partial^2 V}{\partial r^2} + \dfrac{\partial V}{r\partial r} - \dfrac{V}{r^2}\right) - g + \mathrm{K}\left(V - V_p\right) \\[2mm] \dfrac{dU_p}{dt} = K(U - U_p), \quad \dfrac{dV_p}{dt} = K(V - V_p) - g \end{cases}$$

<div align="right">(1)</div>

Here U, V -radial and axial velocities of two-phase flow, U_p, V_p - radial and axial velocities of dispersed particles, p – hydrostatic pressure, K -phase interaction coefficient, g - acceleration of gravity:

The coefficient of phase interaction during laminar motion is determined as follows.

$$K = \frac{18\rho\nu}{\rho_\rho \delta^2}$$

Here ρ_ρ - dispersed particle density, ρ - fluid density, δ - diameter of dispersed particles.

$$\mathrm{Re}_d = \frac{\vartheta \cdot d}{\nu}$$ - Reynolds number for a dispersed particle, ν - coefficient of kinematic viscosity.

The vector representation of system of equations (1) has the following form.

$$\frac{\partial \Phi}{\partial t} + U\frac{\partial \Phi}{\partial x} + V\frac{\partial \Phi}{\partial r} = \mathrm{E}^\Phi + \Pi^\Phi.$$

$$\Phi = \begin{Bmatrix} U \\ V \\ U_p \\ V_p \end{Bmatrix}, \quad \mathrm{E}^\Phi = \begin{Bmatrix} \dfrac{\partial}{\mathrm{Re}\,\partial x}\left(\dfrac{\partial U}{\partial x}\right) + \dfrac{\partial}{\mathrm{Re}\,\partial r}\left(\dfrac{\partial U}{\partial r}\right) \\[2mm] \dfrac{\partial}{\mathrm{Re}\,\partial x}\left(\dfrac{\partial V}{\partial x}\right) + \dfrac{\partial}{\mathrm{Re}\,\partial r}\left(\dfrac{\partial V}{\partial r}\right) \\[2mm] 0 \\[1mm] 0 \end{Bmatrix},$$

$$\Pi^{\Phi} = \begin{cases} -\dfrac{\partial p}{\partial x} + \mathrm{K}\left(U - U_p\right) \\ -\dfrac{\partial p}{\partial r} + \mathrm{K}\left(V - V_p\right) - \overline{\mathrm{F}}_{\mathrm{g}} \\ K(U - U_p) \\ K(V - V_p) - \overline{\mathrm{F}}_{\mathrm{g}} \end{cases}$$

To numerically solve the system of equations, the SIMPLE method was used. [4] In this case, the system of equations was solved by dividing the physical variables velocity-pressure. The solution of displacement equations in the volume control method, expressions for hybrid differentials, and chessboard nodes have been implemented. In this method, solving a system of equations written in new coordinates includes two stages. [2]

$$\frac{\tilde{\Phi} - \Phi^n}{\Delta t} + \frac{\partial U \Phi^n}{\partial x} + \frac{\partial V \Phi^n}{\partial r} = \mathrm{E}^{\Phi} + \Pi^{\Phi}$$

The McCormack method was used to numerically solve this system of equations (2). The McCormack method provides second-order accuracy in time and step.

Predictor

$$\Phi_{ij}^{n+1} = \Phi_{ij}^n - U \frac{\Delta t}{\Delta x}(\Phi_{i,j}^n - \Phi_{i-1,j}^n) - V \frac{\Delta t}{\Delta r}(\Phi_{i,j}^n - \Phi_{i,j-1}^n) + \mathrm{E}^{\Phi} + \Pi^{\Phi}$$

Corrector

$$\Phi_{ij}^{n+1} = \frac{1}{2}[\Phi_{ij}^n + \Phi_{ij}^{n+1} - U \frac{\Delta t}{\Delta x}(\Phi_{i+1,j}^{n+1} - \Phi_{i,j}^{n+1}) - V \frac{\Delta t}{\Delta r}(\Phi_{i,j+1}^{n+1} - \Phi_{i,j}^{n+1}) + \mathrm{E}^{\Phi} + \Pi^{\Phi}$$

3 Initial and boundary conditions

Initial boundary

$$U = U_p = 1, \quad V = V_p = 0.$$

An extrapolation condition was used for all velocities in the outlet

$$\frac{\partial^2 U}{\partial x^2} = \frac{\partial^2 U_p}{\partial x^2} = \frac{\partial^2 V}{\partial x^2} = \frac{\partial^2 V_p}{\partial x^2} = \frac{\partial^2 p}{\partial x^2} = 0.$$

An extrapolation condition was used for all velocities at the wall
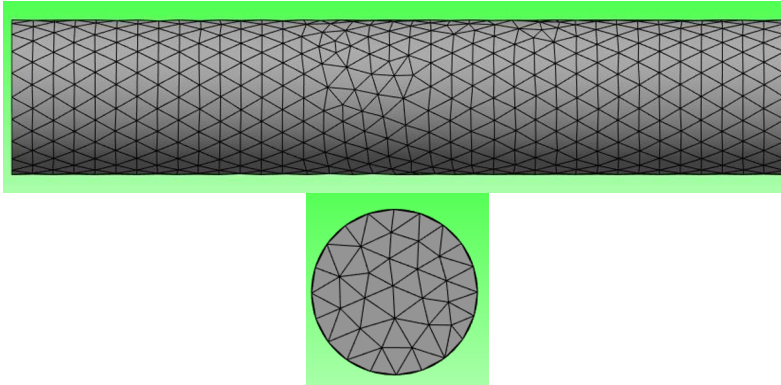
$$U = U_p = V = V_p = 0.$$

Fig.2. Mesh of flow

4 Solution method

Numerical solutions of the presented systems of equations were carried out in the physical variables velocity - pressure by physically splitting the fields of velocity and pressure. In this case, a checkerboard difference grid using the control volume method is used for the transport equations.

$$
\begin{cases}
\dfrac{\tilde{U}_{i,j} - U_{i,j}^n}{\Delta t} + U_{i,j}^n \dfrac{\partial U_{i,j}^n}{\partial x} + V_{i,j}^n \dfrac{\partial U_{i,j}^n}{\partial r} + \dfrac{\partial p_{i,j}^n}{\partial x} = \dfrac{1}{\text{Re}}\left(\dfrac{\partial^2 \tilde{U}_{i,j}}{\partial x^2} + \dfrac{\partial^2 U_{i,j}^n}{\partial r^2} + \dfrac{\partial U}{r\partial r} \right) \\[4mm]
\dfrac{\tilde{V}_{i,j} - V_{i,j}^n}{\Delta t} + U_{i,j}^n \dfrac{\partial V_{i,j}^n}{\partial x} + V_{i,j}^n \dfrac{\partial V_{i,j}^n}{\partial r} + \dfrac{\partial p_{i,j}}{\partial r} = \dfrac{1}{\text{Re}}\left(\dfrac{\partial^2 \tilde{V}_{i,j}}{\partial r^2} + \dfrac{\partial^2 V_{i,j}^n}{\partial x^2} + \dfrac{\partial V}{r\partial r} - \dfrac{V}{r^2} \right).
\end{cases}
\tag{2}
$$

$$
\begin{cases}
U^{n+1} = \tilde{U} - \Delta t \dfrac{\partial \delta p}{\partial x} \\[3mm]
V^{n+1} = \tilde{V} - \Delta t \dfrac{\partial \delta p}{\partial r}
\end{cases}
\tag{3}
$$

Equations (2) The superscript "$\tilde{U}$"denotes the intermediate grid function for the velocity vector; $\delta p = p^{n+1} - p^n$ pressure correction. Multiplying equation (2) by the gradient and taking into account the solenoidal nature of the velocity vector at the th time layer, $(n+1)$ we obtain the Poisson equation for determining the correction to pressure:

$$
\Delta t\left(\frac{\partial^2 \delta p}{\partial x^2} + \frac{\partial^2 \delta p}{\partial r^2} \right) = \frac{\partial \tilde{U}}{\partial x} + \frac{\partial \tilde{V}r}{r\partial r}.
$$

Equation (4) was solved using the iteration method, for which equation (4) was reduced to parabolic form

$$
\frac{\partial \delta p}{\partial t_0} - \Delta t\left(\frac{\partial^2 \delta p}{\partial x^2} + \frac{\partial^2 \delta p}{\partial y^2} \right) = \frac{\partial \tilde{U}}{\partial x} + \frac{\partial \tilde{V}r}{r\partial r}
$$

a) $x/r = 30$

b) $x/r = 30$
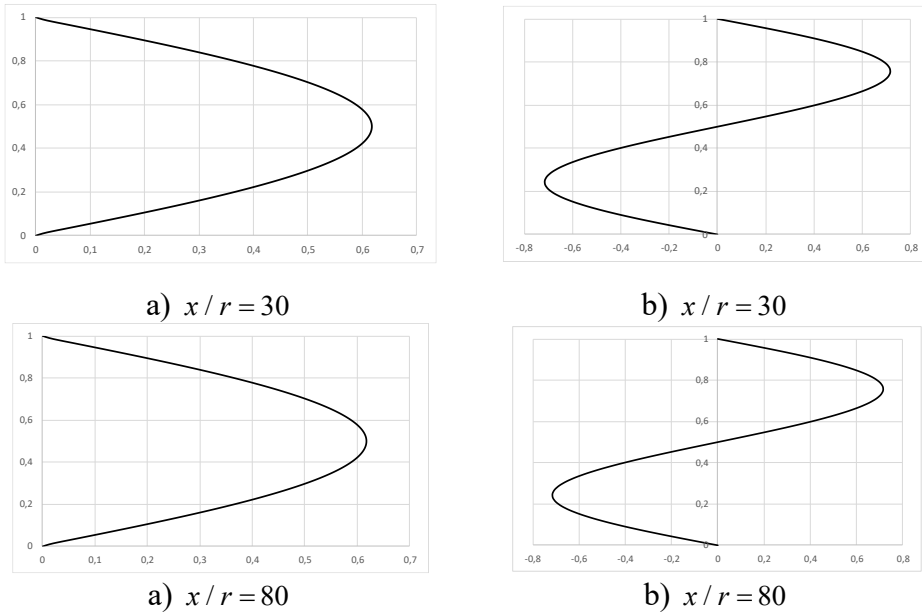

a) $x/r = 80$

b) $x/r = 80$

Fig. 3. shows graphs of a) radial velocity U and b) axial velocity V.

It is easy to show that the calculation scheme has an order of accuracy $O(\Delta t, \Delta x, \Delta r^2)$. This means that the velocity and pressure components are determined at different nodes. This approach is similar to the SIMPLE methods and provides certain advantages when calculating the pressure field [7–11]. The layout of cells and nodes is similar to the layout of the SIMPLE method.

5 Calculation results and their discussion

In Fig. Figure 3 shows comparison graphs of calculated data. The figures show profiles of the longitudinal velocity U in various measured sections
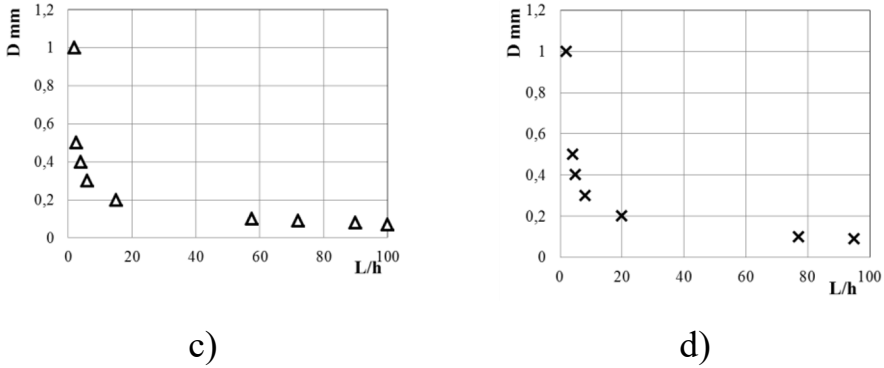

a)

b)

c)   d)
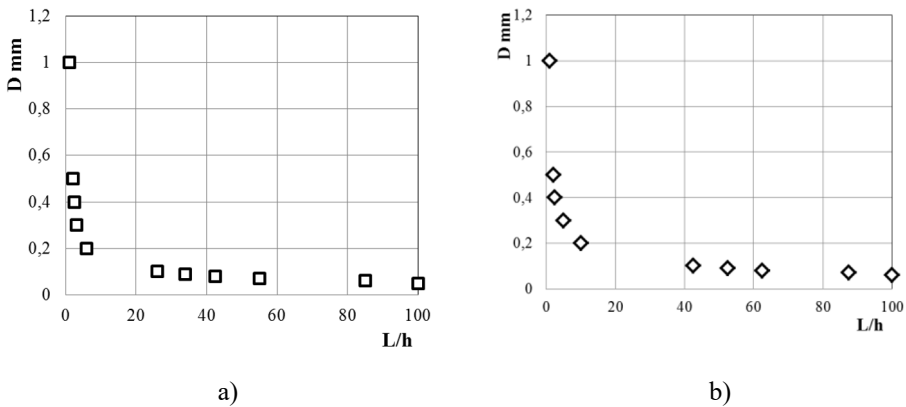
Fig 4. Particle settling distance as a function of flow velocity

 a)U=0.35 m/s, b) U=0.55m/s, c) U=0.75m/s, d) U=1 m/s.
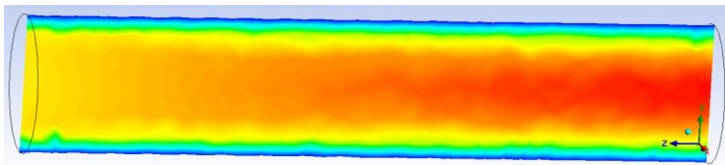

Fig 5. Result of isoline velocity

6 Conclusion

In this article, the results were obtained at the Reynolds number equal to Re=1000. This article examines the movement of dispersed particles. The Navier-Stokes equation was used to determine flow rates. The Lagrangian approach was used to determine the particle trajectory. In numerical calculations, it was determined that if the size and dimensions of the particle are large, then it will settle very quickly, if the particle is light, then it will travel a long distance, which is shown in the graphs.

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