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Hydrodynamic analysis of an unsteady pressureless filtration flow in earth cofferdams

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Abstract. Non-pressure filtration flows with a free surface, on which the fluid pressure is a constant and equal to the external atmospheric pressure, are essential characteristic of groundwater filtration through such hydraulic structures as, dams, water drawdowns, drains, foundations, and pits during their drain. The problems of fluid filtration in porous media are distinguished by a variety of boundary conditions for the desired complex filtration potential, geometric and physical characteristics of the filtration flow. Solving such problems by analytical methods becomes significantly more complicated due to the nonlinearity of the equation describing the filtration movement, the presence of a free surface and the geometry of the slopes of the structure. An alternative to their solution is the use of numerical methods for estimating unsteady free-flow filtration flows. This research is dedicated to developing a hydrodynamic analysis approach of the process of unsteady filtration by the methods of computational fluid dynamics on the example of a rectangular cofferdam of various configurations to apply the results in the design of hydraulic structures. Numerical modeling of an unsteady free-flow filtration in a rectangular cofferdam using the finite volume URANS method (ANSYS FLUENT) and finite-element method (PLAXIS 2D) was carried out. The depression curve evolution through time was obtained. Also, numerical results have been compared both with the experimental results and classical theoretical assumptions. Was found that the constructed models both for finite-volume and finite-element methods are consistent enough with the experimental data, and on the other hand, theoretical assumptions don't agree with experimental and numerical data. Further, the filtration patterns in rectangular cofferdams with different drain positions were obtained using the developed calculation model, which allows to choose the most effective drain position for different purposes.

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1. *Introduction*

Non-pressure filtration flows with a free surface, on which the fluid pressure is a constant and equal to the external atmospheric pressure, are essential characteristic of groundwater filtration through such hydraulic structures as, dams, water drawdowns, drains, foundations, and pits during their drain. These characteristics occupy more than 80% of all designed and built groundwater hydraulic structures in the world.

The filtration theory for such water management, transport and environmental hydraulic structures was developed by N.N. Pavlovsky [1]. It is noted in [2] that in the exact formulation, the study of non-pressure motion presents significant mathematical difficulties. Therefore, the researches of P.Y. Polubarinova-Kochina, who obtained some exact solutions to the problem of an unpressurized filtration flow through a rectangular cofferdam [3–5].

The existing methods of filtration calculation of dams and barrier dams are based on the use of a model with a linear filtration law (Darcy's law) [4], which establishes a relationship between the filtration velocity vector *u* and the pressure field *p,* which causes a filtration flow

$$u = -k \cdot \nabla H; H = y + \frac{p}{\rho g}, \tag{1}$$

where g is the gravity acceleration, ρ is the density of the filtering liquid, k is the filtration coefficient of the porous medium, y is the height (parallel to the direction of gravity).

It should be noted that the analytical solutions to the problems of non-pressure filtration flows are obtained under the assumption of stationarity of fluid motion in a porous medium. The assumption of motion stationarity is fundamentally important [2–6], since the classical methods for calculating seepage flows use the apparatus of the theory of analytic holomorphic functions.

Subsequent studies showed the need to improve Darcy's law. The researches [7–9] indicate the reasons for such a correction. The first is that the application of the macroscopic law of motion when filtering sufficiently fast flows inside a porous medium (Darcy's law) must consider the volume force of viscous resistance acting on the fluid from the side of the porous skeleton. Such conditions are typical for filtration through gravel, pebbles, granular media in industrial process plants. The second is the difference between the rheological relationships of the filtered liquid from the Navier-Stokes law. In [10], a review of the calculation approaches that considers the nonlinear, nonstationary regime of liquid filtration through a porous medium. In [11, 12], a criterion for the nonlinear regime of liquid filtration through a porous medium was proposed. In the papers of [13–15], numerical methods for solving the fluid flow through a porous medium with a nonlinear filtration is proposed.

The application of analytical and numerical methods of filtration for a classical object such as a rectangular cofferdam and objects with a complex hydrogeological structure is considered in [16–19]. The filtration methods for liquids in modern medical and building materials with a regular and random structure are considered in [20–22].

In the context of the current literature on unsteady pressureless filtration flow, it is important to highlight St. Petersburg Polytechnic university contribution to the study of the impact of unsteady flow conditions on filtration processes in porous media. The scientific group of M.R. Petrichenko proceeded both theoretical studies [28–29], in which Crocco equation is employed to describe filtration, and experimental [27, 30–31], where filtration process was modelled with special experimental rig and also some numerical work was proceeded.

In many engineering filtration problems, it is required to determine the instantaneous characteristics of filtration flows [15–18], which leads to the need to study unsteady motion modes. For example, the instantaneous positions of the depression curve and the height of the seepage interval characterize the drain efficiency of the construction pit and the dimensions of the sheet pile wall fixing the slope. The instantaneous positions of the depression curve in the body of the earth cofferdam must be considered when the water level changes in the pools.

The unsteady mode of the filtration flow interaction with a porous medium affects the strength and stability of soil hydraulic structures. Thus, the location of the depression curve determines the boundary of the area occupied by saturated and unsaturated media, thereby determining the area suitable for construction work. The exit of the depression curve to the tailwater slope is accompanied by the formation of a seepage gap. The value of the seepage interval must be considered to prevent frost heaving of the alluvial soil and failure of the slope. At the same time, the problems of fluid filtration in porous media are distinguished by a variety of boundary conditions for the desired complex filtration potential, geometric and physical characteristics of the filtration flow.

Solving such problems by analytical methods becomes significantly more complicated due to the nonlinearity of the equation describing the filtration movement, the presence of a free surface and the geometry of the slopes of the structure.

An alternative to their solution is the use of numerical methods for estimating unsteady free-flow filtration flows.

This research is dedicated to developing a hydrodynamic analysis approach of the process of unsteady filtration by the methods of computational fluid dynamics on the example of a rectangular cofferdam of various configurations to apply the results in the design of hydraulic structures. So, this cofferdam is a research object and the depression curve in this cofferdam that obtained through filtration is the research subject. To achieve the aims of the research, the following tasks were formulated and solved:

- The simulation of the problem of establishing a flow in a rectangular cofferdam was performed using the PLAXIS and ANSYS software packages FLUENT.
- Verification of the obtained numerical solutions by comparing them with experimentally based analytical solutions.
- A study of the efficiency of the cofferdam in the presence of drain was carried out and the characteristic features of the transient process for one of the selected solutions were described.

2. Methods

With non-pressure fluid filtration through the body of a hydraulic structure (dam), caused by the difference in the hydrostatic pressure of the fluid on opposite walls of the dam, the area of fluid movement is limited from above by a free surface, called the depression surface, at each point of which a constant pressure act. The section of the surface of the depression along the movement of fluid through the body of the structure is a depression curve. In the works of [23–26], it is shown that the exit of the free surface to the wall of the structure is carried out not at the level of the tailwater, but above. The resulting gap between the outlet of the depression curve and the tailwater is the seepage surface.

The filtration calculation scheme (Figure 1) assumes a non-stationary nature of filtration in a porous medium. Stationary filtration modes are treated as limiting (for "large" times) states of the filtration flow. The starting identity is the continuity condition for unsteady motion: $\partial h / \partial t + \partial q / \partial x = 0$, moreover, due to the Dupuis condition $q = -kh\frac{\partial h}{\partial x}$, where k is the filtration coefficient. Then to distribute $h\,(t,x)$ the Boussinesq equation is achieved:

$$m\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(kh\frac{\partial h}{\partial x}\right), \qquad (2)$$

where t is time, x is coordinate, $h=h\,(t,x)$ is seepage flow depth, $h_e \le h_0 < h \le H$, m is porosity coefficient, $0 < m < 1$.

The change of variables $t' := \frac{kt}{mH} > 0,\ x' = \frac{x}{H},\ 0 < x' < \lambda := l/H,\ \lambda \le \Lambda := L/H,\ u := h/H,$ $u_0 < u < 1, u_0 = h_0 / H,$ brings the Boussinesq equation to a dimensionless form:

$$\frac{\partial u}{\partial t'} = \frac{\partial}{\partial x'}\left(u\frac{\partial u}{\partial x'}\right), \qquad (3)$$

The solution of the Boussinesq equation for unsteady flows is associated with significant difficulties and requires the search for approaches to its solution, one of which is the use of numerical simulation.

The problem of establishing a filtration flow in a vertical plane in a rectangular cofferdam is considered. For setting the tasks variants, a linear drain is used, located on the lower face of the cofferdam. The distance from the tailwater to the beginning of the drain varies (l_0 = 0H, 1H, 4.5H, 8H). The length of the drain is H. The problem is formulated in a dimensionless setting (Figure 1), the height of the cofferdam is H, equal to 1 meter, is chosen as the length scale. The time scale is introduced – $\tau = kt/mH$.

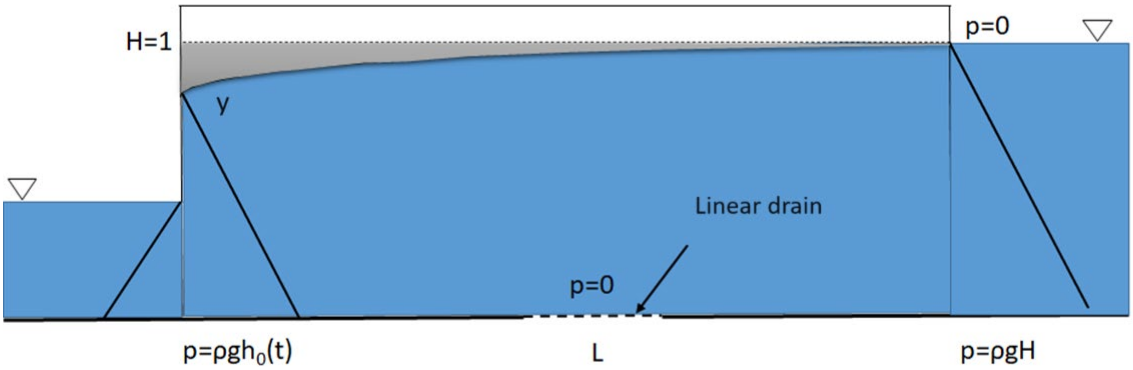


Figure 1 Statement of the problem.

3. Results and Discussions

3.1. Conditions for modeling the non-pressure filtration process

A constant hydrostatic pressure and a free boundary condition is set on the tailpipe, where the pressure is extrapolated from within the computational domain. On the lower boundary of the computational domain, except for the drain area, the no-slip condition is set. At the lower boundary in the drain area and at the upper boundary, a constant overpressure $p = 0$ is set.

The convergence criterion for the class of problems under consideration is the constancy of the flow rate through the tailwater and drain; for a numerical experiment, the problem can be considered converged if, at the next time step, the flow rate has changed by less than 0.5% relative to the previous one, as well as the position of the depression curve, which does not change over time.

To solve the filtration problem, a two-dimensional finite-volume method was used. Non-stationary Navier-Stokes equations are solved numerically together with the Volume of Fluid method for tracking the position of the interface between media. Since the velocity of the liquid during filtration is small, it is considered that filtration is laminar. The cofferdam is modeled by a porous body with constant isotropic resistance (porosity is 0.2, absolute permeability – 10^{-6} m^2). For each geometry of the cofferdam, a structured finite-volume mesh with sizes from 3,000 to 7,000 elements was constructed. The problem was solved with a time step of 0.1 s. Water with a density of 998.2 kg/m^3 and a viscosity of $1.003 \cdot 10^{-3}$ Pa*s and air with a density of 1.22 kg/m^3 and a viscosity of $1.7 \cdot 10^{-5}$ Pa*s were employed as working fluids. At the interface in the VOF model, a surface tension of 0.072 N/m is established.

3.2. Numerical simulation of filtration flow in a rectangular cofferdam in the ANSYS software package FLUENT: Data verification

The calculation of a rectangular cofferdam was performed without drain at a ratio L/H equal to 10. Since the fields of quantities at any time can be obtained as a result of the solution, the field of the liquid volume fraction was reduced during its evolution in time, Figure 2. The position of the interface zone, where the volume fraction is 50%, is essentially the depression curve. The flow pattern changes most actively in the period from $\tau = 0$ to $\tau = 50$, then from $\tau = 50$ to $\tau = 200$, where the flow pattern changes slightly, however the depression curve still changes its shape, Figure 3. The flow after $\tau = 200$ is considered to be steady.

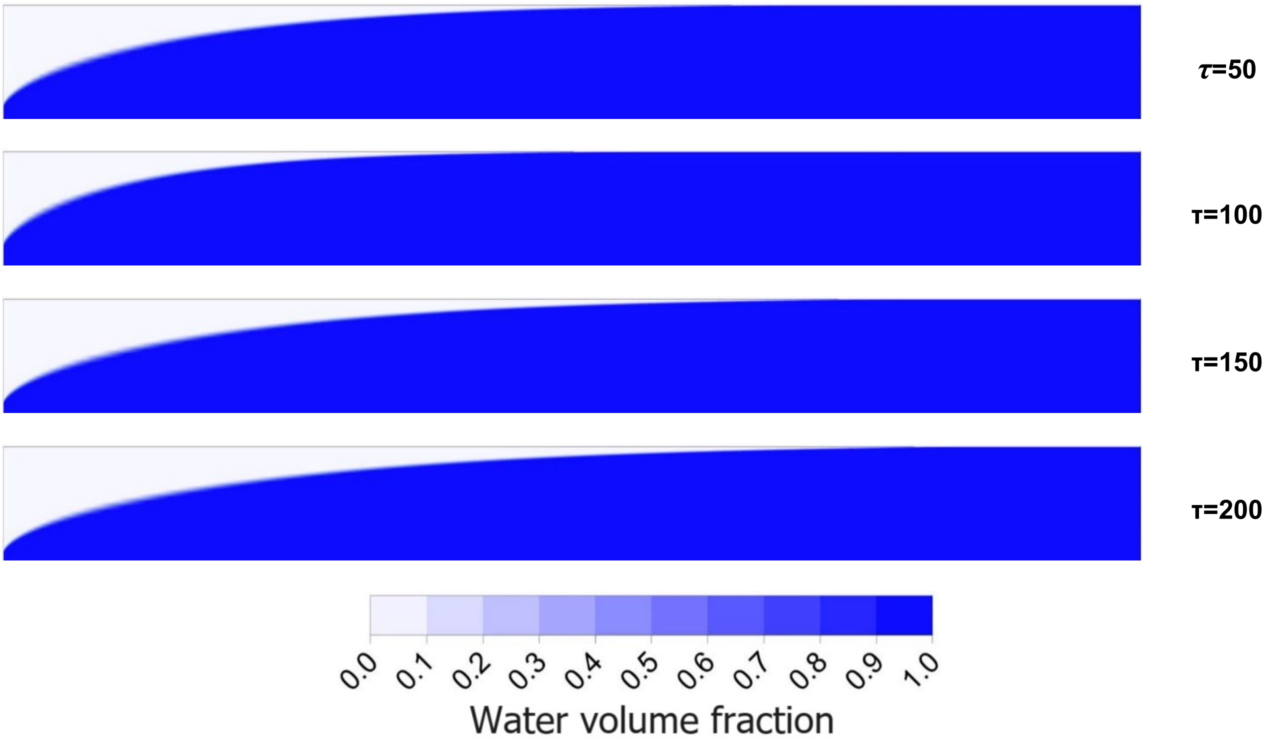


Figure 2. Volumetric concentration of liquid in the calculation area at different moments of the given time for setting without drainage.

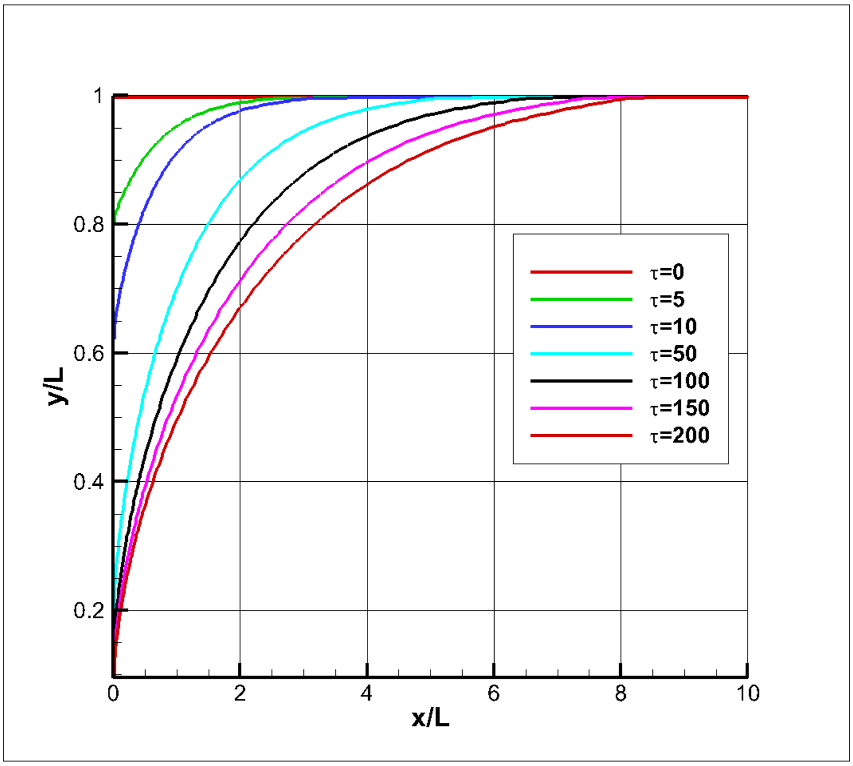


Figure 1Depression curves at different time points.

An important characteristic of the transient seepage process is the tailwater flow. In the case of calculations in the ANSYS software package, it is possible to compare the results with experimental data and, thus, verify the calculation model. The flow rate is reduced to a dimensional form for ease of comparison with the results of experimental research carried out by D.D. Zaborova [27] as shown in Figure 4. The flow mismatch at the initial moment of time is due to a sharp increase in the flow rate in the calculation, the flow rate increases from 0 to some unsteady level. The observed flow peak is due to the possibility of instantaneous "opening" of the damper in the calculation, which is difficult to achieve on an experimental stand. In general, the consumptions rate at $t/t_{set} > 0.3$ differ by 5% or less, which indicates a good agreement between the calculation model and experimental results. Based on the gained results, the ANSYS non-stationary computational model has been verified experimentally.

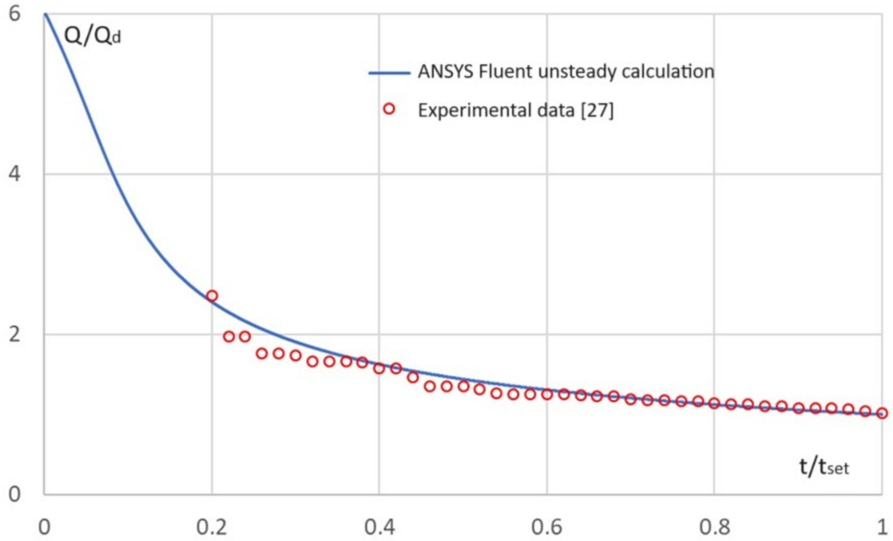
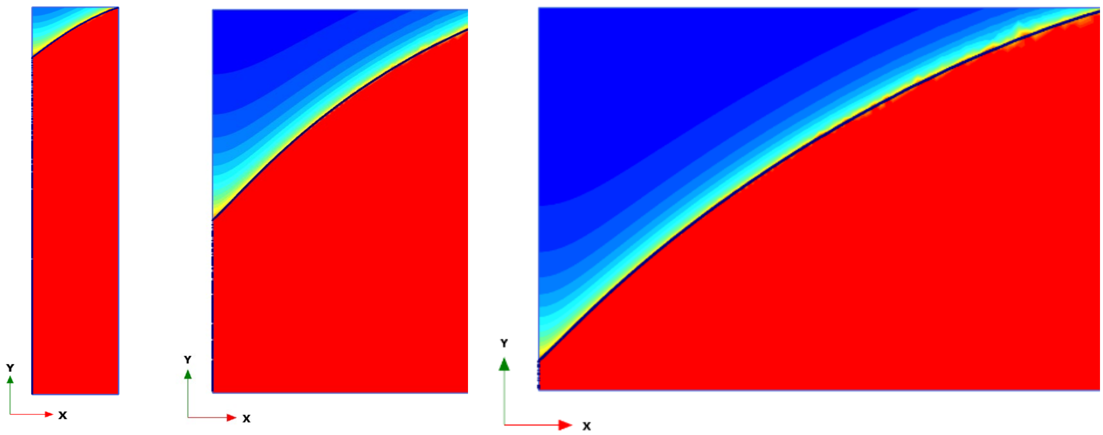


Figure 4. Comparison of reduced discharges across the tailwater.

3.3. Numerical simulation of filtration flow in a rectangular cofferdam in the PLAXIS 2D software package

Similar calculations of rectangular lintels in an identical formulation were performed by the PLAXIS 2D software package.

The calculations were carried out for lintels with linear dimensions L/H=0.2, 0.4, 0.6, 0.8, 1.5, 2. The results were obtained on computational grids that ensure the convergence of the solution. Figure 5 shows the volumetric concentration fields of liquid in the cofferdams with a linear size L/H=0.2, 0.8, 1.5 after the establishment of the filtration flow.



(a) L/H**=0.2** **(b)** L/H**=0.8** **(in)** L/H**=1.5**

Figure 5. The volumetric concentration field of liquid in the cofferdams of various lengths after the establishment of the filtration flow.

Since is impossible to compare the instantaneous positions of the drawdown curves due to the difference in the applied approaches to the solution implemented in the considered software packages, the comparison of the drawdown curves was carried after the establishment of the filtration flow. Figure 6 shows the values of seepage gaps depending on the longitudinal size of the cofferdam, obtained analytically and numerically.

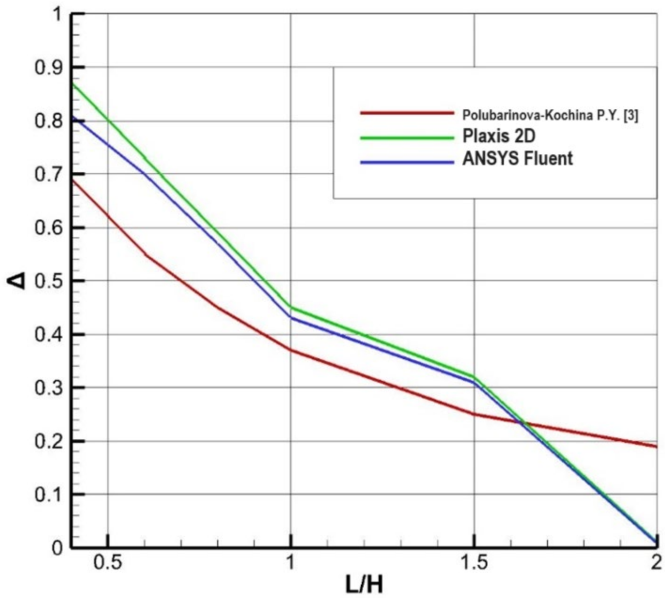


Figure 6. A comparison of the seepage intervals values obtained analytically and numerically.

The seepage intervals obtained using the ANSYS and PLAXIS 2D software packages practically coincide. The mismatch is observed for the cofferdams of L/H≤1. Seepage gaps obtained according to the theory of P.Y. Polubarinova-Kochina [3] are significantly smaller for lintels with L/H ≤1.7 in relation to the solutions obtained using numerical analysis. For cofferdams with L/H≥ 1.7 analytical solution overestimates the value of the seepage interval.

3.4. Time evolution of the depression curve for the problem with drain

A similar problem to the solved task 1, however with the presence of drain, is considered. The problem without drain is referred to by the number "1", while the problem with drain is referred to by "l_0" and indicated at l_0 = 0 – 2, at l_0 = 1 - 3, at l_0 = 4.5 – 4, and at l_0 = 8 – 5. To track the evolution of depression curves, a comparison between the tasks of tasks 1–5 were carried out as shown in Figure 7.

In the absence of drain or its location near the tailwater (tasks 1–3), the shape of the steady depression curves between the drain (if any) and the upstream is identical. However, the depression curves at the initial moments of time (up to τ = 10 inclusive) have the greatest difference in shape and characterize the structure of the filtration flow. For tasks 2 and 3, a soft slope "descent" of the depression curve (than in task 1) is observed in the drain area at this time stage.

For the formulated problems 3–5, after τ = 50 the depression curve is divided into two sections, the gap is located at the site of drain, the maximum of the left part of the depression curves is located exactly between the drain and the tailwater, and the left part of the depression curve itself is symmetrical. At the same time, the liquid remains between the runoff and the tailwater remain even in problem 3.

In the case of tasks 4 and 5, at the initial moments of time (up to τ = 10 inclusive), the depression curve, on the contrary, has a steeper "descent" in the drain area. The position of the lower point of the depression curve for tasks 4 and 5 coincides. It should be noted that in the case of problem 5, due to the extremely small distance between the upstream and the runoff, the depression curve on the right side is established faster than on the left.

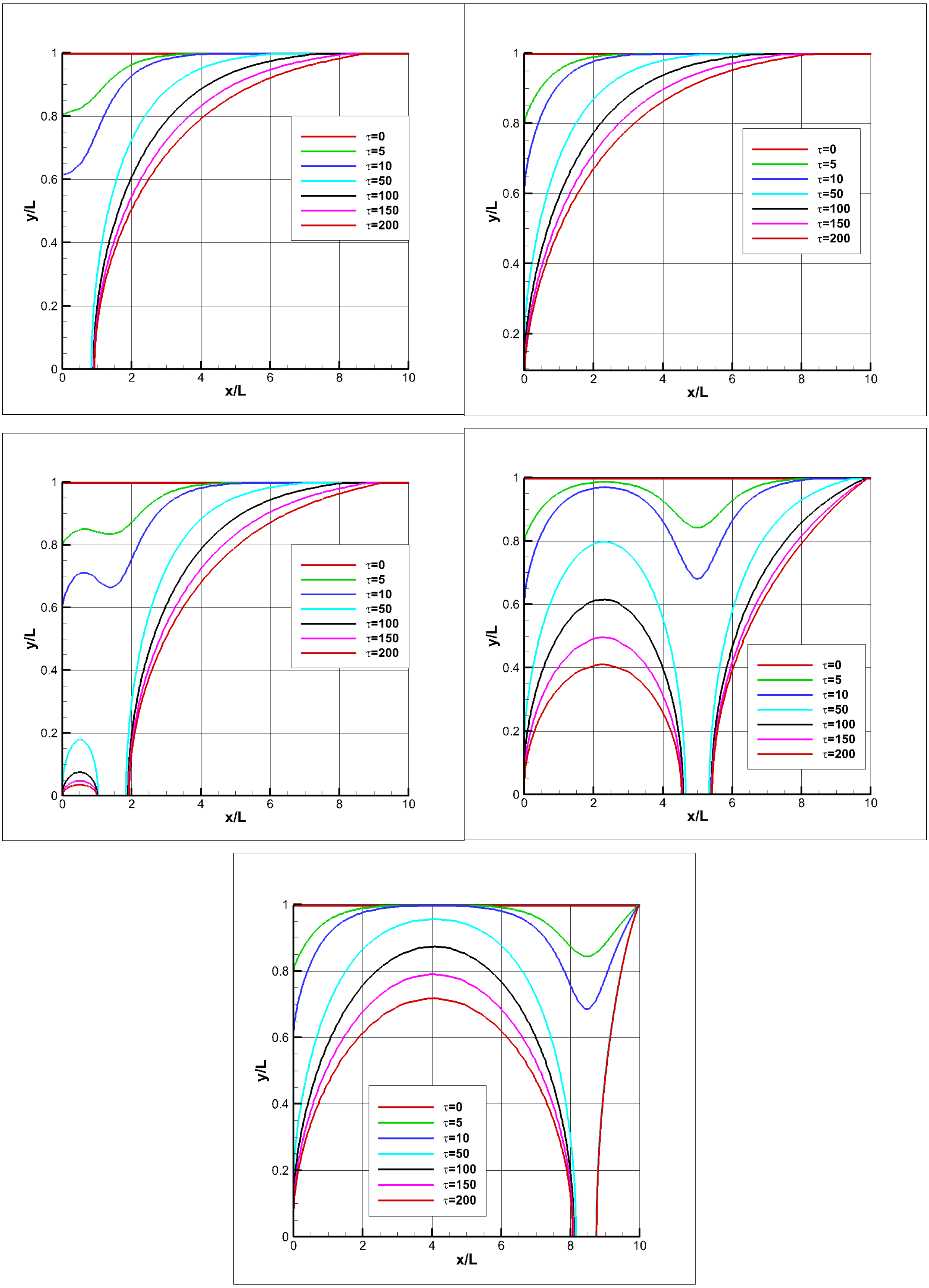


Figure 7. Evolution of depression curves for tasks 1-5 (numbering from left to right, top to bottom).

3.5. Study of the work of a cofferdam with drain depending on its position

For a more visual demonstration of the operation of the drain from its position, a comparison was made of the steady depression curves for tasks 1–5 as shown in Figure 8. The shape of the left side of the curve for tasks 3–5 is identical, the shape of the right side of the curve is also identical for tasks 1–5. The left parts of the curves are similar to each other and can be obtained from each other by scaling along the X and Y axes, while the right parts of the curves are also similar but can be obtained from each other by scaling along the X axis.

According to the obtained depression curves, it can be concluded that it is rational to place the drain one caliber away to the right of the tailwater to accelerate the drain of saturated masses in a rectangular cofferdam of a fixed height. This approach allows to drain a larger volume of the porous body to the tailwater slope.

The introduction of linear drain at some distance to the right of the tailwater is equivalent to reducing the length of the cofferdam under consideration by the value of this distance, however, the time for the complete establishment of such a problem will be slightly longer, since in the interval from the tailwater to the location of the drain, a hilly area saturated with liquid is formed, the drain of which occurs exclusively due to the forces of gravity.

The result obtained is of significant practical importance, since if linear drain is located close to the upstream, a reservoir with a depth of about $0.3H$ and a width of $9H$ can be drained in a time equal to the time of setting the problem without drain.

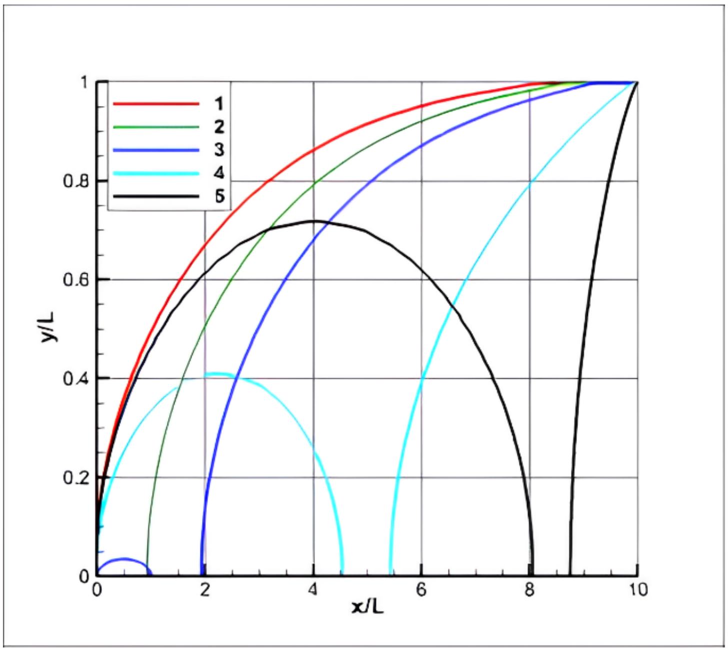


Figure 8. Steady-state depression curves for tasks 1–5.

The efficiency of drain is an essential practical task in the researched area. To evaluate the effectiveness, the given consumptions rates for tasks 2–5 are compared as shown in Figure 9.

The discharge through the drain is significantly higher than the discharge through the tailwater for all considered problems. When the drain is removed from the tailwater, the flow through it increases. This is justified by the fact that the size of the hilly area saturated with liquid increases, the liquid filtration in this area occurs solely due to the action of gravity. At the same time, at the initial stage, the configuration from task 3 is the most effective, in this case, both drain and tailwater show the largest reduced flow rate in the time interval from 0 to 0.2 t/t set. For tasks 2 and 3, almost the entire flow rate, when the current is established, goes through the drain; in the case of tasks 4 and 5, a small part of the flow goes through the tailwater.

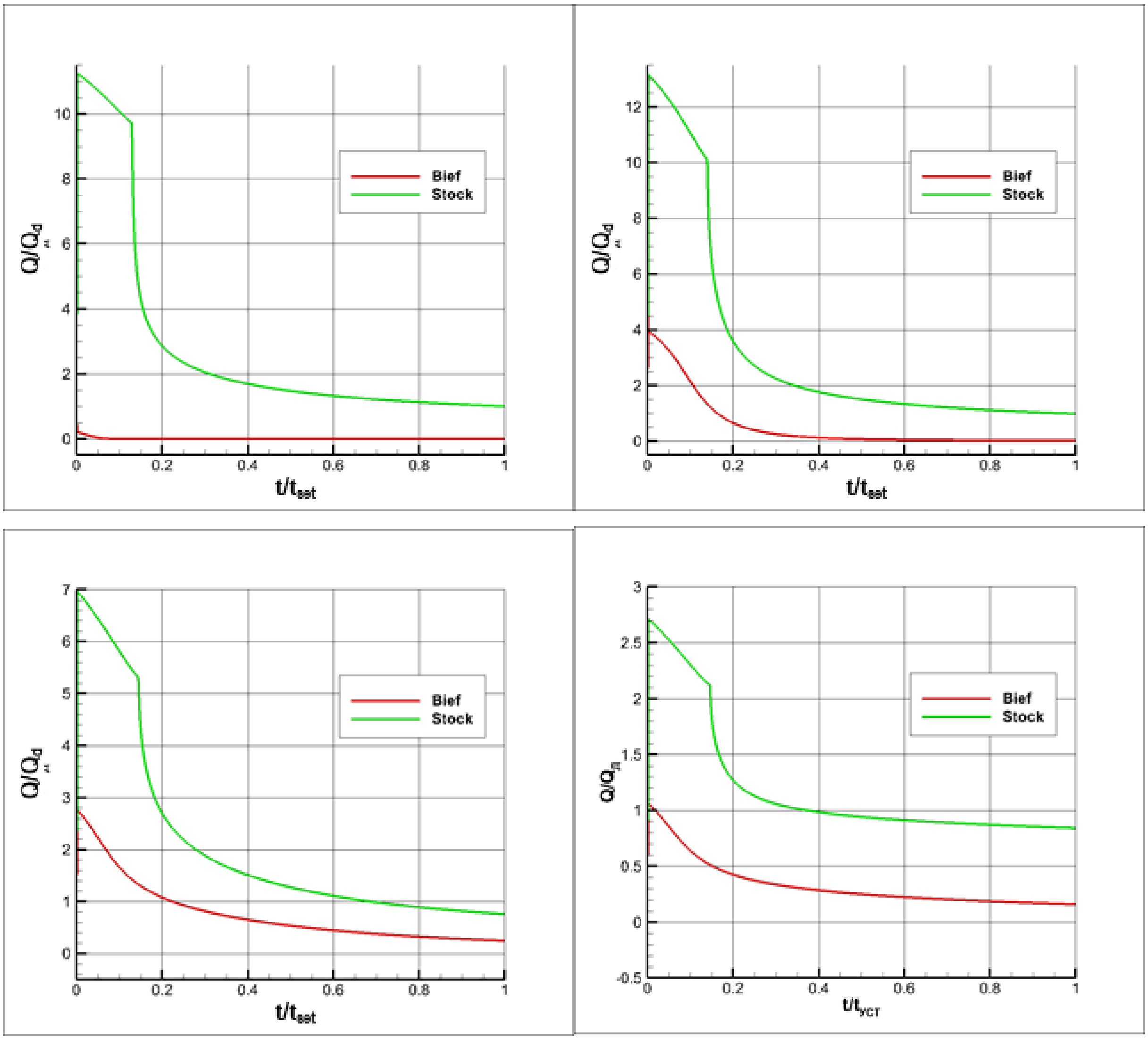


Figure 9. Comparison of the given consumption rates for tasks 2-5 (numbering from left to right, top to bottom).

4. Conclusion

According to the analysis of experimentally verified proposed numerical models of an unsteady free-flow filtration in a rectangular cofferdam using the ANSYS FLUENT and PLAXIS 2D software packages, the following are concluded:

1. Numerical modeling of an unsteady free-flow filtration flow in a rectangular cofferdam using ANSYS software packages FLUENT and PLAXIS 2D allows obtaining instantaneous values of the coordinates of the depression curve and instantaneous values of the seepage interval. The proposed models are consistent with the theoretical solutions.
2. A verification of the proposed models was carried out through a comparison of the gained results using the numerical developed model built in ANSYS FLUENT with the existing experimental data in the literature. It was found that the numerical gained results show a good consistency with the existing in the literature experimental results, thereby, the proposed models were verified.
3. With the help of the verified models, the filtration patterns in rectangular dams with different drain positions were obtained. It was found that, in the case of a sufficient removal of the drain from the tailwater, the established drawdown curve will be divided into two parts by the drain.
4. According to the criterion of the reduced flow rate, the most effective is a cofferdam with drain removed from the tailwater by one height of the cofferdam.

The obtained results can be applied in the design of hydrotechnical construction objects.

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