МАТЕМАТИЧНІ МЕТОДИ, МОДЕЛІ,
ПРОБЛЕМИ І ТЕХНОЛОГІЇ ДОСЛІДЖЕННЯ
СКЛАДНИХ СИСТЕМ

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ALGORITHM FOR SIMULATING MELTING POLAR ICE, EARTH INTERNAL MOVEMENT AND VOLCANO ERUPTION WITH 3-DIMENSIONAL INERTIA TENSOR

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Abstract. This paper reports the result of an investigation of a hypothesis that the melting polar ice of Earth flowing down to the equatorial region causes volcano eruptions. We assumed a cube inside the spherical body of Earth, formulated a 3-dimensional inertia tensor of the cube, and then simulated the redistribution of the mass that is to be caused by the movement of melted ice on the Earth's surface. Such mass distribution changes the inertia tensor of the cube. Then, the cube's rotation inside Earth was simulated by multiplying the Euler angle matrix by the inertia tensor. Then, changes in the energy intensity and the angular momentum of the cube were calculated as coefficients of Hamiltonian equations of motion, which are made of the inertia tensor and sine and cosine curves of the rotation angles. The calculations show that the melted ice increases Earth's internal energy intensity and angular momentum, possibly increasing volcano eruptions.

Keywords: inertia tensor, volcano eruption, mass distribution, Hamiltonian equation of motion.

INTRODUCTION

The polar ice melts and flows to the Equatorial region by Earth's centrifugal force by its rotation (from Fig. 1 to Fig. 2), then Earth swells along the Equatorial region (Fig. 3). The mass balance of Earth changes, then its inertia tensor changes. We assume that the change of the inertia tensor, which is caused by the redistribution of mass from the polar region to the equatorial region, excites the Earth's internal energy and the angular momentum, causing volcano eruptions.

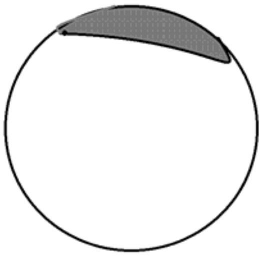

Fig. 1. Polar ice

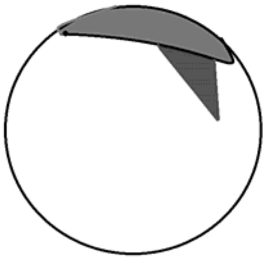

Fig. 2. Melting ice

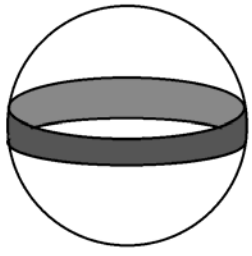

Fig. 3. Swell around Equator

ALGORITHM OF THE SIMULATION

Equation (1) shows how to calculate inertia tensor of a solid body, r is a distance between the centre of mass and each point of the body, V is the volume of the body, α and β are suffixes that represent coordinates' axis, and δ is Kronecker's delta:

$$I_{\alpha\beta} = \int_V \rho(r)(\delta_{\alpha\beta} r^2 - r_\alpha r_\beta) dV . \quad (1)$$

Fig. 4 shows a spherical body in a 3-dimensional flat space. Inside of the sphere, we put a cube, 3 sides of which are on x, y and z axis of a flat 3-dimensional coordinate system. The cube's mass is M, each side's length is a, and we set:

$$b = Ma^2 .$$

Applying (1) to this cube, the inertia tensor becomes:

$$I = \begin{bmatrix} \frac{2}{3}b & -\frac{1}{4}b & -\frac{1}{4}b \\ -\frac{1}{4}b & -\frac{2}{3}b & -\frac{1}{4}b \\ -\frac{1}{4}b & -\frac{1}{4}b & -\frac{2}{3}b \end{bmatrix} . \quad (2)$$

The inertia tensor (2) of this cube is taken from the example shown at end of the chapter 5.3 "The inertia tensor and the moment of inertia" in page 94 of [1].

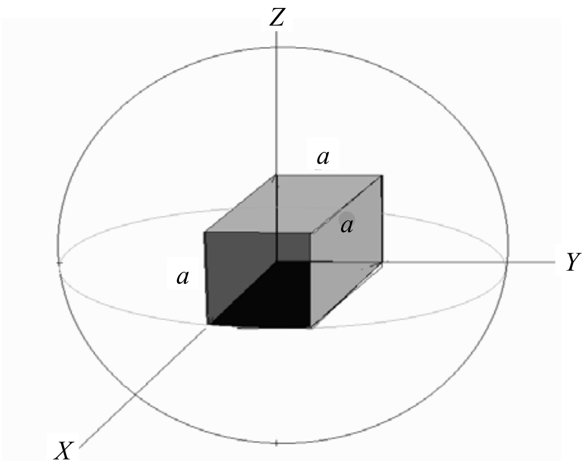

Fig. 4. A spherical body in a 3-dimensional flat space

By rotating the spherical body around z-axis, (2) is multiplied by Euler angle D:

$$D = \begin{bmatrix} \cos & \sin & 0 \\ -\sin & \cos & 0 \\ 0 & 0 & 1 \end{bmatrix} .$$

Then we get:

$$ID = \begin{bmatrix} \frac{2}{3}b\cos & -\frac{1}{4}b\sin & \frac{2}{3}b\sin & -\frac{1}{4}b\cos & -\frac{1}{4}b \\ -\frac{1}{4}b\cos & -\frac{2}{3}b\sin & -\frac{1}{4}b\sin & +\frac{2}{3}b\cos & -\frac{1}{4}b \\ -\frac{1}{4}b\cos & +\frac{1}{4}b\sin & -\frac{1}{4}b\sin & -\frac{1}{4}b\cos & \frac{2}{3}b \end{bmatrix}. \tag{3}$$

For calculating the energy intensity of the rotation, we take the diagonal components of (3) to make a vector

$$X = \begin{bmatrix} (2/3)b\cos \quad +(1/4)b\sin \\ -(1/4)b\sin \quad +(2/3)b\cos \\ (2/3)b \end{bmatrix}, \tag{4}$$

while, for calculating the angular momentum of the rotation, we take non-diagonal (y, x) and (x, y) components of (3) to make a vector

$$X = \begin{bmatrix} -\frac{1}{4}\cos & +\frac{2}{3}b\sin \\ -\frac{1}{4}\cos & -\frac{2}{3}b\sin \end{bmatrix}. \tag{5}$$

This re-formulation of the matrix to the vectors for energy intensity and for angular momentum is explained from page 14 to page 21 of [2].

Then we make the Hamiltonian equation of motion:

$$H = kT - Xc\,. \tag{6}$$

Here, kT is a stress energy that reflects the energy intensity and angular momentum of the rotating body. In this simulation, we set it as a unity vector. And c is a coefficient vector, which is to be calculated as energy intensity of the rotating body or the angular momentum; Xc for energy intensity is

$$Xc = C_0\left(\frac{2}{3}b\cos \quad +\frac{1}{4}\sin\right) + C_1\left(-\frac{1}{4}b\sin \quad +\frac{2}{3}b\cos\right) + C_2\left(\frac{2}{3}b\right),$$

and for angular momentum it is

$$Xc = C_0\left(-\frac{1}{4}b\cos \quad +\frac{2}{3}b\sin\right) + C_1\left(-\frac{1}{4}b\cos \quad -\frac{2}{3}b\sin\right).$$

Then, X' is multiplied from the left side of (6), and we set it to be zero as the boundary condition to make $X'H = X'(kT - Xc) = 0$. Then, c will be calculated by transforming $X'H = X'(kT - Xc) = 0$ to $X'Xc = X'kt$, and then to $X'Xc = X'kt$.

Here, X' is a transposed vector of X. $(X'X)^{-1}$ is an inverse matrix of $(X'X)$.

INPUT DATA FOR THE NUMERIC SIMULATION

We assign unity for b, therefore M and a become unity in (2) in order to simulate the relative values and their changes of the energy intensity and angular momentum, not the absolute values. Then we deduct dx, dy and dz from M in each of 3 directions in x, y and z-axis, as shown in (7) for the energy level and (8) for angular momentum:

$$X=\begin{bmatrix} \frac{2}{3}b\cos \quad +\frac{1}{4}b\sin \quad +dx \\ -\frac{1}{4}b\sin \quad +\frac{2}{3}b\cos \quad +dy \\ \frac{2}{3}b-dz \end{bmatrix}; \qquad (7)$$

$$X=\begin{bmatrix} -\frac{1}{4}\cos \quad +\frac{2}{3}b\sin \quad +dx \\ -\frac{1}{4}\cos \quad -\frac{2}{3}b\sin \quad +dy \end{bmatrix}. \qquad (8)$$

First, we assign the value for d_z, then calculate d_x and d_y by $dx=dy=\sqrt{\frac{1}{a-dz}}-1=\sqrt{\frac{1}{1-dz}}-1$, so that these can make the volume of the cube to be unity. Then we simulate 4 cases by assigning 4 different sets of the values of d_x, d_y and d_z, which are shown in Table 1:

$$dx=dy=\left(\sqrt{\frac{1}{a-dz}}-1=\sqrt{\frac{1}{1-dz}}-1\right).$$

Table 1. Marginal changes, d_x, d_y and d_z, of Earth's mass, which are to be reflected to the cubic

Marginal mass change	Case 1	Case 2	Case 3	Case 4
d_x	0	0.00503	0.0541	0.118
d_y	0	0.00503	0.0541	0.118
d_z	0	0.01	0.1	0.2

Here, the image of Case 1 is shown in Fig. 1, and Cases 2, 3 and 4 are in Fig. 3.

For (4), (5), (7) and (8), we use sine and cosine curves shown in Fig. 5.

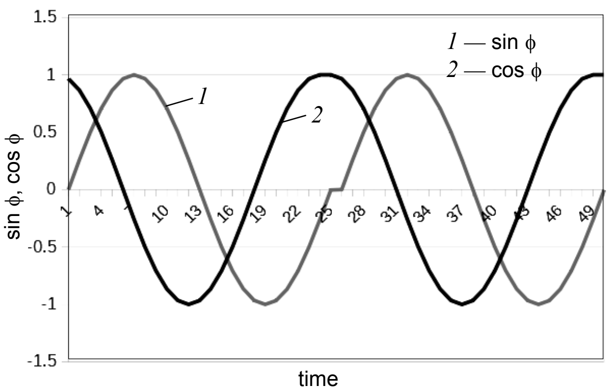

Fig. 5. Sin ϕ and cos ϕ

RESULT

The result of the calculation of the coefficient vectors are shown in Table 2 and Fig. 6 for the energy intensity and in Table 3 and Fig. 7 for the angular momentum. As the mass moves from the North Pole to the Equatorial region the energy intensity becomes larger on z-axis and the angular momentum also becomes larger on x-y plane. Fig. 8 shows the caricatured images of these calculated results.

Table 2. Calculated coefficient vector for energy intensity

Energy intensity in	Case 1, d_z=0	Case 2, d_z=0.01	Case 3, d_z=0.1	Case 4, d_z=0.2
x	$3.053 \cdot 10^{-16}$	$- 6.94 \cdot 10^{-17}$	$- 2.08 \cdot 10^{-16}$	$- 7.49 \cdot 10^{-16}$
y	$- 2.220 \cdot 10^{-16}$	$- 5.03 \cdot 10^{-17}$	$- 2.03 \cdot 10^{-16}$	$- 8.67 \cdot 10^{-18}$
z	1.50	1.52	1.76	2.14

Table 3. Calculated coefficient vector for angular momentum

Angular momentum in	Case 1, d_z=0	Case 2, d_z=0.01	Case 3, d_z=0.1	Case 4, d_z=0.2
x	- 0.146	$- 7.19 \cdot 10^{-2}$	0.617	1.16
y	0.178	$- 8.72 \cdot 10^{-2}$	0.750	1.41

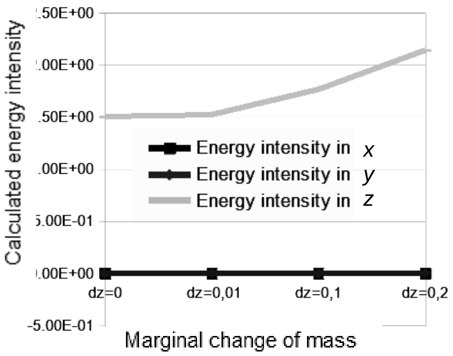

Fig. 6. Calculated energy intensity

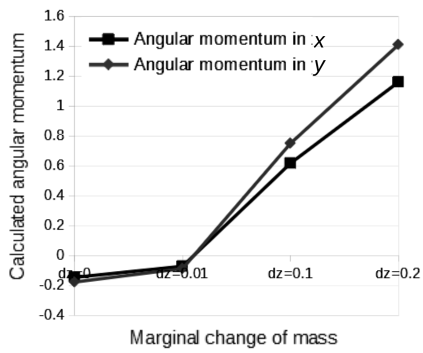

Fig. 7. Calculated angular momentum

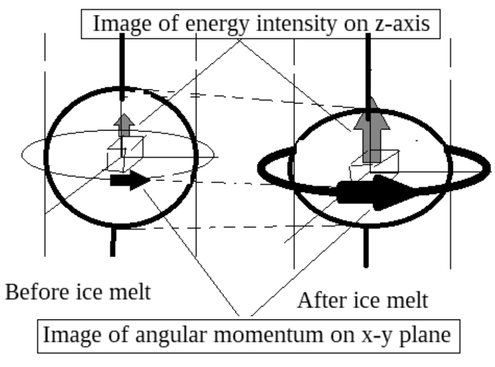

Fig. 8. Images of the simulation results

CONCLUSIONS AND RECOMMENDATIONS

After the polar ice melting, the internal energy intensity of Earth becomes larger on z-axis, and the angular momentum also becomes larger on x-y plane. This result suggests that the polar ice melting influences the Earth's internal energy intensity as well as the internal angular momentum, and larger. This result means a possibility of volcano eruptions.

The simulated result should be compared to the observations on Earth, and appropriate methodologies need to be developed for the comparison.

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АЛГОРИТМ МОДЕЛЮВАННЯ ТАНЕННЯ ПОЛЯРНОГО ЛЬОДУ, ВНУТРІШНЬОГО РУХУ ЗЕМЛІ ТА ВИВЕРЖЕННЯ ВУЛКАНУ З 3-ВИМІРНИМ ТЕНЗОРОМ ІНЕРЦІЇ / Й. Мацукі, П.І. Бідюк

Анотація. Подано результати дослідження гіпотези про те, що танення полярного льоду Землі, що стікає до екваторіальної ділянки, викликає виверження вулканів. Припустили, що всередині сферичного тіла Землі є куб, сформовано тривимірний тензор інерції куба, потім змодельовано перерозподіл маси, який буде спричинений рухом талого льоду на поверхні Землі. Такий розподіл мас змінює тензор інерції куба. Змодельовано обертання куба всередині Землі шляхом множення матриці кута Ейлера на тензор інерції. Зміни енергоємності та моменту імпульсу куба розраховано як коефіцієнти гамільтонових рівнянь руху, які складаються з тензора інерції та кривих синусів і косинусів кутів повороту. Результати розрахунків показують, що талий лід збільшує інтенсивність внутрішньої енергії Землі та кутовий момент, що означає можливе збільшення вивержень вулканів.

Ключові слова: тензор інерції, виверження вулкана, розподіл маси, гамільтонове рівняння руху.