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# Some results on *l*-fuzzy soft groups under bounded sum and bounded difference operations

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## Abstract

*Lattice ordered group has been developed by J.Vimala. After that J. Arockia Reeta and J. Vimala introduced Lattice ordered fuzzy soft groups(l-FSG) and obtained real life applications. In this present work, we have attained some results on l-FSGs under the operations of bounded sum and bounded difference.*

**Keywords:** Fuzzy soft groups, *l*-FSG, operations of bounded sum and bounded difference in *l*-FSG.

## 1. Introduction

The notion of lattice theory was proposed by Gratzer [6]. Fuzzy set theory has been propounded by Zadeh [10] and developed by Zimmerman [11]. Soft set theory has been originated by Molodsov [8]. Fuzzy soft set [7] has been developed and used in recent years to solve real life problems [5]. Fuzzy soft group was defined by Abdulkadir Aygunoglu and Halis Aygun [1]. Lattice ordered group has been investigated by J.Vimala [9]. A lattice structure on fuzzy soft group was constructed and some of its properties were investigated by J. Arockia reeta et al [2]. After that, anti-lattice ordered fuzzy soft groups [3] and its matrix operations have been studied and implemented in deciding process. Further, Algebraic relations over l-Fuzzy soft groups has been examined [4]. In this present work, we have attained some results on l-FSGs under the operations of bounded sum and bounded difference.

## 2. Preliminaries

**Definition 2.1[9]** Let X be a non-empty set, then a fuzzy set $\mu$ over X is a function from X into $I = [0,1]$. ie., $\mu : X \to$ I.

**Definition 2.2 [5]** Let X be an initial universe set and E a set of parameters with respect to X. Let P(X) denote the power set of X and A $\subseteq$ E. A pair (F,A) is called a soft set over X, where F is a mapping given by F:A $\to$P(X).

A soft set over X is a parameterized family of subsets of the universe X.

**Definition 2.3 [5]** Let $I^X$ denote the set of all fuzzy sets on X and A$\subset$ E. A pair $(f, A)$ is called a fuzzy soft set over X, where $f$ is a mapping from A into $I^X$. That is, for each $a \in A$ , $f(a) = f_a : X \to I,$ is a fuzzy set on X.

**Definition 2.4** *[10]* The bounded sum of two fuzzy sets $\tilde{A}$ and $\tilde{B}$ is denoted by $\tilde{C} = \tilde{A} \oplus \tilde{B} = \{(x, \mu_{\tilde{A}\oplus\tilde{B}}(x)); x \in X\}$, *where* $\mu_{\tilde{A}\oplus\tilde{B}}(x) = min\{1, \mu_{\tilde{A}}(x) + \mu_{\tilde{B}}(x)\}$.

**Definition 2.5** *[10]* The bounded difference of two fuzzy sets $\widetilde{A}$ and $\widetilde{B}$ is denoted by $\tilde{C} = \widetilde{A} \ominus \widetilde{B} = \{(x, \mu_{\widetilde{A}\ominus\widetilde{B}}(x)); x \in X\}$, where $\mu_{\widetilde{A}\ominus\widetilde{B}}(x) = \mathrm{Max}\{0, \mu_{\widetilde{A}}(x) + \mu_{\widetilde{B}}(x) - 1\}$.

**Definition 2.6 [9]**The complement of a fuzzy soft set $(f, A)$ is denoted by $(f, A)^c$ and is defined by $(f, A)^c = (f^c, \neg A)$, where $f^c: \neg A \to P(U)$ is a mapping given by $f^c(\sigma) = (f(\neg\sigma))^c$ for all $\sigma \in \neg A$.

**Definition 2.7[5]** Let X be a group and $(f, A)$ be a FSG over X. A FSG $(f, A)$ is said to be an l - FSG over X. If for the mapping $f: A \to I^X$, $a \leq b$ implies $f_a \subseteq f_b$, for all $a, b \in A$.

**Example 2.8** Let $\mathbb{N}$ be the set of all natural numbers and $(\mathbb{N}, \leq)$ be a lattice. Let $\mathbb{R}$ be the set of all real numbers. Define $f: \mathbb{N} \to I^{\mathbb{R}}$ by $f(n) = f_n: \mathbb{R} \to I$ for each $n \in \mathbb{N}$, where

$$f(x) = \begin{cases} 1 - \dfrac{1}{n+1}, & if\ x = k5^n, \exists\ k\epsilon Z \\ 0, & otherwise \end{cases}$$

Then the pair $((f, \mathbb{N}), \vee, \wedge, \subseteq)$ forms an $l$ - FSG over $\mathbb{R}$.

**3. Some results on *l*-fuzzy soft groups under bounded sum and bounded difference operations**

In this section, we develop some results on $l$-FSGs under the operations of bounded sum and bounded difference.

[1]Throughout this work, let $X$ be a group and $P(X)$ be the power set of $X$. If the set of parameters $E$ is also a lattice with respect to certain binary operations or partial order, then a non-empty subset $A$ of $E$ also inherits the partial order from the set $E$ and we use $\vee$ for *maximum* and $\wedge$ for *minimum*.

**Proposition 4.1** *Let $(f, A)$ be an l- FSG over a group X. Then we have the following results for all $a_1, a_2, a_3 \in A$ and $a_1 \leq a_2 \leq a_3$,*

1. $f_{a_1} \vee (f_{a_2} \oplus f_{a_3}) = (f_{a_1} \vee f_{a_2}) \oplus (f_{a_1} \vee f_{a_3})$,
2. $f_{a_1} \wedge (f_{a_2} \oplus f_{a_3}) = f_{a_1}$,
3. $(f_{a_1} \ominus f_{a_2}) \vee f_{a_3} = f_{a_3}$,
4. $f_{a_1} \oplus (f_{a_2} \vee f_{a_3}) = f_{a_1} \oplus f_{a_3}$,
5. $f_{a_1} \oplus (f_{a_2} \wedge f_{a_3}) = f_{a_1} \oplus f_{a_2}$.

**Proof**:

**1. Let X be a group and $(f, A)$ be an $l$- FSG over X. Then we have for all $a_1, a_2, a_3 \in A$,** $a_1 \leq a_2 \leq a_3 \Rightarrow f_{a_1} \subseteq f_{a_2} \subseteq f_{a_3}$.

Then, we get $f_{a_1} \vee f_{a_2} = f_{a_2}$, $f_{a_1} \vee f_{a_3} = f_{a_3}$,

$f_{a_1} \subseteq f_{a_2} \oplus f_{a_3} \Rightarrow f_{a_1} \vee (f_{a_2} \oplus f_{a_3}) = f_{a_2} \oplus f_{a_3}$ and

$(f_{a_1} \vee f_{a_2}) \oplus (f_{a_1} \vee f_{a_3}) = f_{a_2} \oplus f_{a_3}$.

Therefore $f_{a_1} \vee (f_{a_2} \oplus f_{a_3}) = (f_{a_1} \vee f_{a_2}) \oplus (f_{a_1} \vee f_{a_3})$.

**2.** $\boldsymbol{f_{a_2} \oplus f_{a_3} = \{(x, 1 \wedge [\mu_{f_{a_2}}(x) + \mu_{f_{a_3}}(x)])/x \in X\}}$

We know that $a_1 \leq a_2 \leq a_3 \Rightarrow \mu_{f_{a_1}}(x) \leq \mu_{f_{a_2}}(x) \leq \mu_{f_{a_3}}(x)$, for each $x \in X$.

$\Rightarrow \mu_{f_{a_1}}(x) < \mu_{f_{a_2}}(x) + \mu_{f_{a_3}}(x)$ and $\mu_{f_{a_1}}(x) \leq 1$, for each $x \in X$.

$\Rightarrow \mu_{f_{a_1}}(x) < 1 \wedge (\mu_{f_{a_2}}(x) + \mu_{f_{a_3}}(x))$, for each $x \in X$.

$\Rightarrow \mu_{f_{a_1}}(x) \wedge [1 \wedge (\mu_{f_{a_2}}(x) + \mu_{f_{a_3}}(x))] = \mu_{f_{a_1}}(x)$, for each $x \in X$.

Then $\{(x, \mu_{f_{a_1}}(x))/x \in X\} \wedge \{(x, 1 \wedge [\mu_{f_{a_2}}(x) + \mu_{f_{a_3}}(x)])/x \in X\} = \{(x, \mu_{f_{a_1}})/x \in X\}$.

Hence we get $f_{a_1} \wedge (f_{a_2} \oplus f_{a_3}) = f_{a_1}$.

**3.** $\boldsymbol{f_{a_1} \ominus f_{a_2} = \left\{(x, 0 \vee [\mu_{f_{a_1}}(x) + \mu_{f_{a_2}}(x) - 1])/x \in X\right\}}$.

We know that $a_1 \leq a_2 \leq a_3 \Rightarrow \mu_{f_{a_1}}(x) \leq \mu_{f_{a_2}}(x) \leq \mu_{f_{a_3}}(x)$, for each $x \in X$.

$\Rightarrow 1 + \mu_{f_{a_3}}(x) \geq \mu_{f_{a_2}}(x) + \mu_{f_{a_1}}(x)$, for each $x \in X$.

$\Rightarrow \mu_{f_{a_1}}(x) + \mu_{f_{a_2}}(x) - 1 \leq \mu_{f_{a_3}}(x)$, for each $x \in X$.

$\Rightarrow 0 \vee (\mu_{f_{a_1}}(x) + \mu_{f_{a_2}}(x) - 1) \vee \mu_{f_{a_3}}(x) = \mu_{f_{a_3}}(x)$, for each $x \in X$.

Therefore

$$\{x, 0 \vee [\mu_{f_{a_1}}(x) + \mu_{f_{a_2}}(x) - 1])/x \in X\} \vee \{(x, \mu_{f_{a_3}}(x)/x \in X\} = \{(x, \mu_{f_{a_3}}(x))/x \in X\}.$$

Then we get $(f_{a_1} \ominus f_{a_2}) \vee f_{a_3} = f_{a_3}$.

(iv) and (v) Proof is obvious.

**Example 4.1** *From the example 2.8, consider the l-FSG* $(f, A) = \{f_2, f_5, f_7, f_8, f_{10}\}$

$$ie., (f, A) = \left\{\frac{\text{k}5^{-2}}{0.7}, \frac{\text{k}5^{-5}}{0.8}, \frac{\text{k}5^{-7}}{0.88}, \frac{\text{k}5^{-8}}{0.89}, \frac{\text{k}5^{-10}}{0.9}\right\}.$$

Clearly it shows that $f_2 \subseteq f_5 \subseteq f_7 \subseteq f_8 \subseteq f_{10}$.

$$\begin{aligned} f_2 \vee (f_5 \oplus f_7) &= \left\{\frac{k5^{-2}}{0.7} \vee \left(\frac{k5^{-5}}{0.8} \oplus \frac{k5^{-7}}{0.88}\right)\right\}. \\ &= \left\{\frac{k5^{-2}}{0.7} \vee \frac{k5^{-5}}{1}\right\}. \qquad (1) \\ &= \left\{\frac{k5^{-5}}{1}\right\}. \end{aligned}$$

$$\begin{aligned} (f_2 \vee f_5) \oplus (f_2 \vee f_7) &= \left\{\left(\frac{\text{k}5^{-2}}{0.7} \vee \frac{\text{k}5^{-5}}{0.8}\right) \oplus \left(\frac{\text{k}5^{-2}}{0.7} \vee \frac{\text{k}5^{-7}}{0.88}\right)\right\}. \\ &= \left\{\frac{\text{k}5^{-5}}{0.8} \oplus \frac{\text{k}5^{-7}}{0.88}\right\}. \qquad (2) \\ &= \left\{\frac{\text{k}5^{-5}}{1}\right\}. \end{aligned}$$

From (1) and (2), we get $f_2 \vee (f_5 \oplus f_7) = (f_2 \vee f_5) \oplus (f_2 \vee f_7)$.

**Proposition 4.2** *Let* $(f, A)$ *be an l- FSG over a group X. Then*

1. $(f_{a_1} \oplus f_{a_2})^c = f_{a_1}^c \ominus f_{a_2}^c$,
2. $(f_{a_1} \ominus f_{a_2})^c = f_{a_1}^c \oplus f_{a_2}^c$,
3. $(f_{a_1}^c \oplus f_{a_2}^c)^c = f_{a_1} \ominus f_{a_2}$,
4. $(f_{a_1}^c \ominus f_{a_2}^c)^c = f_{a_1} \oplus f_{a_2}$,

**Proof:**

1. $f_{a_1} \oplus f_{a_2} = \{(x, \min\{1, \mu_{f_{a_1}}(x) + \mu_{f_{a_2}}(x))/x \in X\}$.

Then $(f_{a_1} \oplus f_{a_2})^c = \{(x, 1 - \{1 \wedge \{\mu_{f_{a_1}}(x) + \mu_{f_{a_2}}(x)\}\})/x \in X\}$.

$= \{(x, \{1 - 1\} \vee \{1 - \mu_{f_{a_1}}(x) - \mu_{f_{a_2}}(x)\})/x \in X\}$.

$= \{(x, 0 \vee \{1 - \mu_{f_{a_1}}(x) + 1 - \mu_{f_{a_2}}(x) - 1\})/x \in X\}$.

$= f_{a_1}^c \ominus f_{a_2}^c.$

2. The proof is similar to (1).
3. $f_{a_1}^c \oplus f_{a_2}^c = \{(x, min\{1,1-\mu_{f_{a_1}}(x)+1-\mu_{f_{a_2}}(x)\})/x \in X\}$

$$(f_{a_1}^c \oplus f_{a_2}^c)^c = \{(x, 1-\{1 \wedge \{1-\mu_{f_{a_1}}(x)+1-\mu_{f_{a_2}}(x)\}\})/x \in X\}$$
$$= \{(x, \{1-1\} \vee \{1-1+\mu_{f_{a_1}}(x)-1+\mu_{f_{a_2}}(x)\})/x \in X\}$$
$$= \{(x, 0 \vee \{\mu_{f_{a_1}}(x)+\mu_{f_{a_2}}(x)-1\})/x \in X\}$$
$$= f_{a_1} \ominus f_{a_2}$$

4. The proof is similar to (3).

**Proposition 4.3** *Let $(f, A)$ and (f, B) be an l- FSG over a group X. Then*
(1)$((f,A) \cup (f,B))^c = (f^c, \neg A) \cup (f^c, \neg B) = (f, A \cup B)^c$.
(2)$((f,A) \cap (f,B))^c = (f^c, \neg A) \cap (f^c, \neg B) = (f, A \cap B)^c$.

Proof:(1)
Let $(f,A) = \{f_{a_1}, f_{a_2}, f_{a_3}, f_{a_4}, f_{a_5}\}$ and $(f,B) = \{f_{b_1}, f_{b_2}, f_{b_3}, f_{b_4}, f_{b_5}\}$ .
Therefore $(f,A) \cup (f,B) = \{f_{a_1}, f_{a_2}, f_{a_3}, f_{a_4}, f_{a_5}, f_{b_1}, f_{b_2}, f_{b_3}, f_{b_4}, f_{b_5}\}$.

$$((f,A) \cup (f,B))^c = \{f_{\neg a_1}, f_{\neg a_2}, f_{\neg a_3}, f_{\neg a_4}, f_{\neg a_5}, f_{\neg b_1}, f_{\neg b_2}, f_{\neg b_3}, f_{\neg b_4}, f_{\neg b_5}\}.$$
$$= \{f_{\neg a_1}, f_{\neg a_2}, f_{\neg a_3}, f_{\neg a_4}, f_{\neg a_5}\} \cup \{f_{\neg b_1}, f_{\neg b_2}, f_{\neg b_3}, f_{\neg b_4}, f_{\neg b_5}\},$$
$$= (f^c, \neg A) \cup (f^c, \neg B),$$
$$= (f^c, \neg A \cup \neg B),$$
$$= (f, A \cup B)^c.$$

Hence $((f,A) \cup (f,B))^c = (f^c, \neg A) \cup (f^c, \neg B) = (f, A \cup B)^c$.
Proof of (2) is similar to (1).

**Proposition 4.4** *Let $(f, A)$ and (f, B) be an l- FSG over a group X. Then*
(1)$(f, A \cup B)^c = (f,A)^c \cup (f,B)^c$ (2)$(f, A \cap B)^c = (f,A)^c \cap (f,B)^c$.
Proof :
(1) $(f, A \cup B)^c = (f^c, \neg(A \cup B))$,

$$= (f^c, \neg A \cup \neg B)),$$
$$= (f^c, \neg A) \cup (f^c, \neg B)),$$
$$= (f,A)^c \cup (f,B)^c.$$

Proof of (2) is similar to (1).

**Proposition 4.5** *Let $(f, A)$ be an l- FSG over a group X. Then it holds the following results:*
$\forall\, a_1, a_2, a_3 \in A$,

1. $f_{a_1} \oplus (f_{a_2} \ominus f_{a_1}) = f_{a_1}$, when $\mu_{f_{a_1}}, \mu_{f_{a_2}} \leq 0.5$,
2. $f_{a_1} \ominus (f_{a_2} \oplus f_{a_1}) = f_{a_1}$, when $\mu_{f_{a_1}}, \mu_{f_{a_2}} \geq 0.5$.

Proof:
Let X be a group and $(f, A)$ be an $l$- FSG over X. Then we have for all $a_1, a_2, a_3 \in A$,
$a_1 \leq a_2 \leq a_3 \Rightarrow f_{a_1} \subseteq f_{a_2} \subseteq f_{a_3}$.

$$f_{a_2} \ominus f_{a_1} = \left\{(x, 0 \vee [\mu_{f_{a_2}}(x) + \mu_{f_{a_1}}(x) - 1])/x \in X\right\} = \{(x,0)/x \in X\},$$

Then $f_{a_1} \oplus (f_{a_2} \ominus f_{a_1}) = \{(x, \min\{1, \mu_{f_{a_1}}(x) + 0/x \in X\}\}$,

$$= \{x, \mu_{f_{a_1}}(x)\},$$
$$= f_{a_1}(x).$$

Proof of (2) is similar to (1).

## 5. Conclusion

In this present work, we have obtained some results on $l$-FSGs under the operations of bounded sum and bounded difference. In future, application of $l$-FSGs will be established and analysed.

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