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Learning trajectory in the material of comparing and ordering fractions using paper folding for elementary school students

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Abstract

Background: Understanding fractions in mathematics often poses greater complexity compared to integral numbers. The primary difficulty lies in students' inadequate grasp of fractional basics, notably in comparing and sequencing fractions.

Aim: This research seeks to create and evaluate a learning pathway's effect on fourth graders' comprehension of fractions, with a focus on their comparison and sequencing.

Method: Employing a design research validation study approach, the research progresses through preliminary design, experimental design, and retrospective analysis. Conducted in a Palembang school involving six fourth graders recommended by their teacher, data collection encompassed essay-based pre and post-tests, Student Activity Sheets, observations, interviews, and documentations. The data analysis was retrospectively carried out, anchored on HLT as the benchmark.

Result: Findings reveal a student learning trajectory encompassing three principal activities. Initially, students used folding and gluing of paper to discern fraction values. Subsequently, they engaged in coloring and illustrating folds for fraction comparison. The final activity involved drawing, coloring boxes, and fraction comparison and sequencing. Overall, students showed proficiency in understanding and determining fraction values and comparing them, yet struggled with ordering certain fractions.

Conclusion: The structured learning path facilitated students' understanding of basic fraction concepts, especially in comparing them. Nevertheless, reinforcing methods or approaches for teaching fraction ordering is essential.

INTRODUCTION

Consider m and n ($n \neq 0$) as two integers, wherein m/n represents a fraction (Marjanović, 2018). In theoretical terms, students often find fractions to be a complex subject (Alkhateeb, 2019). Despite a prior understanding of integers, numerous students encounter obstacles in comprehending fractions (Hariyani et al., 2022). Research has highlighted common errors in dealing with fractions, including their interpretation, comparative analysis, and the arithmetic of adding and subtracting fractions. A tendency is observed where students use rule-driven methods in fraction problems, lacking in-depth comprehension (Alkhateeb, 2019). According to an interview with a math educator who also teaches a class, students exhibit difficulties in both grasping and resolving fraction-related problems, especially in aspects of comparing and

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sequencing them. A prevalent approach among students is to compare fractions with varying denominators by paying attention to these denominators, leading to a misconception that the process is akin to comparing integers (Hariyani et al., 2022).

In the realm of basic and secondary education, understanding fractions is fundamental to learning mathematics (Hariyani et al., 2022). Teaching fractions effectively often involves the use of representational methods (Khuriyati, 2015). Such representation is acknowledged as a key element in the pedagogy of mathematics (Kara & Incikabi, 2018). The application of tangible items, like splitting an item into multiple sections and analyzing these sections, aids in the students' comprehension of fractional concepts (Malla, 2020). These types of manipulative resources effectively connect the tangible experiences with abstract mathematical concepts (Durmus & Karakirik, 2006).

When instructing on fractions in mathematics, teachers must ensure that students thoroughly grasp the material (Parmiti & Arnawa, 2017). This necessity stems from the requirement for students to cultivate an interest in learning, especially given the need for in-depth comprehension of mathematics' abstract concepts (Fang et al., 2023). A variety of tools and environmental contexts can serve as instructional media, including the use of paper folding or origami. Origami, the traditional Japanese art of paper folding, is particularly effective in this regard. This method of folding paper can significantly enhance students' creative learning capabilities (Kahramanoğlu & Alp, 2023). In addition to fostering creativity, paper folding or origami helps to demystify learning concepts, aiding students in grasping the goals of their education (Rivai, 2010). Utilizing paper folding as a tangible learning model is particularly effective in guiding students through the intricacies of fraction concepts, especially in terms of comparison and sequencing.

Research has explored the use of paper folding as an educational aid in math classes, with studies like that of Putri & Azizah (2019) showing an enhancement in students' average performance across various learning cycles. This enthusiasm and enjoyment in learning activities stem from their engagement in playful, artistic activities like drawing, coloring, and arranging vibrant origami paper (Edo & Samo, 2017). Implementing learning trajectories has yielded favorable outcomes. The use of a 'fraction circle' model within these trajectories has been effective in fostering students' understanding of fractional number comparison (Wahyuni, 2017) and grasping the concept of sequential fractions (Zabeta et al, 2015) through the PMRI approach. Furthermore, using a 10x10 grid paper in teaching percentages has been instrumental in aiding students' comprehension, forming a three-part learning trajectory (Ningsih et al., 2017). These findings suggest that learning trajectories generally lead to positive educational results. However, there appears to be a gap in research concerning learning trajectories for comparing and sequencing fractions. Consequently, the researcher is motivated to develop a lesson plan focused on these aspects of fractions, utilizing paper folding as a tool in elementary education.

METHODS

Design

This study utilizes a design research method, particularly a validation study, focusing on crafting educational content for fractions aligned with the PMRI approach. The purpose of design research is to cultivate theories related to the mechanisms and supports of the learning process (Irma et al., 2022). Within this research framework, there is an iterative cycle involving the creation and trial of educational activities and various other components. This iterative nature is rooted in the practice of conducting repeated experimental and instructional trials, continuing until an effective learning trajectory is established, incorporating refinements from previously tested educational methodologies.

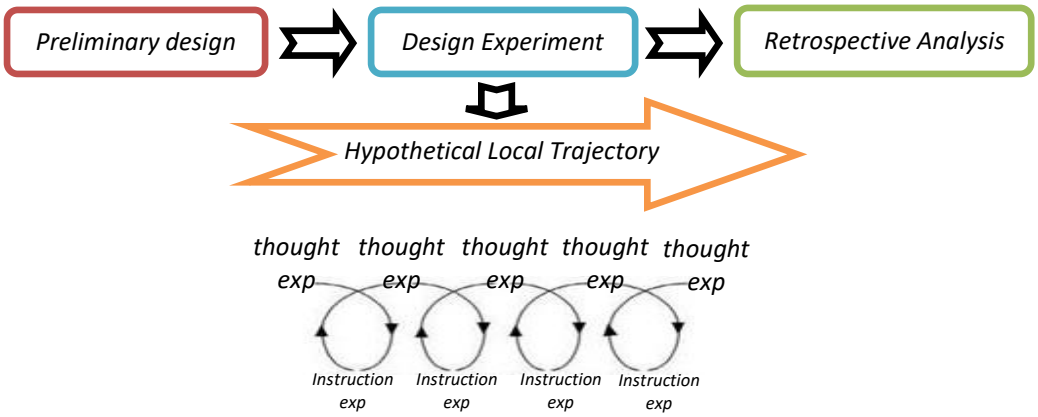


Figure 1. Siklik *design research*

Participants

The research included six students from a Palembang school, chosen for their diverse learning capabilities: two each from low, medium, and high skill levels, following recommendations from a fourth-grade teacher. Among them, one student with lower abilities is identified as a slow learner, a term for children who struggle with learning and assimilating information (Rawani et al., 2023). Such learners typically exhibit slower rates in learning, skill development, and understanding acquired information, often due to intellectual capacities that fall below the usual standards (Prior, 2022).

Instruments

This research involved the collection of data through a range of methodologies, encompassing both pre-tests and post-tests structured as essays, Student Activity Sheets (SAS), as well as documentation, which included video recordings and photographs, alongside observations and interviews.

Data Analysis

The analysis of the data involved a comparison with the Hypothetical Learning Trajectory (HLT), a framework depicting the evolution of students' learning journeys (Baroody et al., 2022). This process begins by assessing students' initial comprehension of the focused concept and proceeds to pinpoint key milestones and potential obstacles they might face in their transition from a basic to an advanced level of understanding (Ivars et al., 2018).

In this study, the pre-test serves as a roadmap for researchers to establish the foundational content of their instructional material. The post-test evaluates how effectively the custom-designed student activity sheets aid in students' grasp of fraction comparison and sequencing. The feedback and insights from students during the pilot experiment stage are crucial for refining these activity sheets. Subsequently, the findings from the teaching experiment stage are instrumental in revising these designs to formulate a Local Instruction Theory (LIT). LIT is a theoretical framework outlining the learning process, specifically mapping the educational trajectory for a particular topic, bolstered by an array of supporting activities (Domu & Mangelep, 2020).

RESULTS AND DISCUSSION

Result

Preliminary Design Stage

In this phase, the researcher analyzed educational materials, including the fourth-grade Math syllabus as per the 2013 Curriculum (K13) and the respective Math textbook, encompassing the Core and Basic Competencies, indicators, and the fundamental fraction content. They conducted observational studies and interviews with fourth-grade math teachers to assess the students' initial mathematical skills, their interest, behavior, and interaction during educational activities, which revealed high student enthusiasm for learning. Subsequently, the researcher collaborated with the class teacher to finalize the research subjects and schedule.

Further, the researcher held discussions with the teacher about the *Hypothetical Learning Trajectory* developed with academic advisors, and these findings were illustrated in Figure 2.

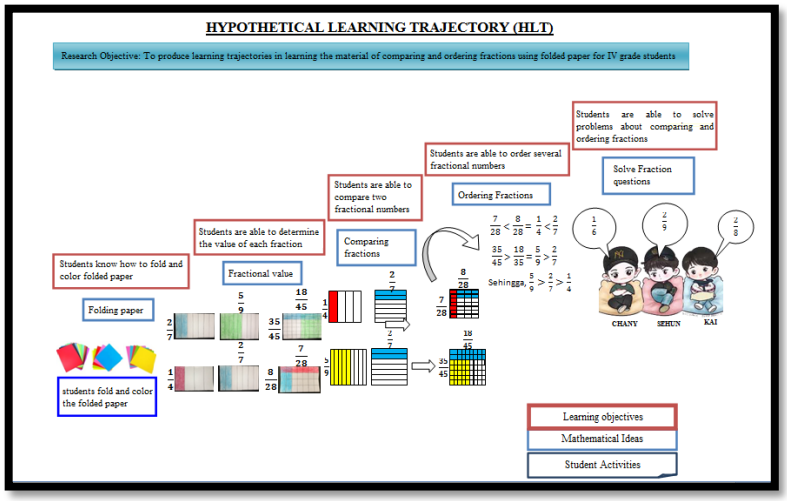


Figure 2. *Hypothetical Learning Trajectory*

This research incorporates a *Hypothetical Learning Trajectory* comprising three activities, each with distinct educational aims:

Activity 1.

1. The first activity is designed to enable students to grasp and ascertain the values of fractions through paper folding exercises. The initial segment, Activity 1.1, focuses on understanding the concept of fractions and their respective values. Here, students are challenged to calculate how many pieces of cake Andi has, utilizing paper folding as an instructional aid.
2. The subsequent segment, Activity 1.2, is aimed at helping students identify specific fractional values. In this task, students are asked to figure out the fractional value represented by a piece of cake. This involves folding and coloring paper to visually represent and segment the fractions. After completing the folding process, students then allocate the cake portions among individuals. To conclude, they independently tackle several questions related to the exercise. Insights into students' thought processes during Activity 1 are outlined in Table 1.

Table 1. Conjecture of Student Thinking in Activity 1

Learning Activities	Students' Thoughts on the Conducted Activities
Engaging with the issue presented in the first student activity sheet	• Students analyze the problem in the first activity sheet and sequentially read the queries.
Utilizing paper folding to segment a square cake from a tray	• For Activity 1.1, students grasp the problem presented on the sheet. • They follow the outlined steps to fold the paper. • The paper's folding process is illustrated by the students. • They successfully use the paper folding technique to segment the cake. • Queries related to the task are addressed by the students.
Assessing the quantity of cake segments belonging to Thoriq and Nizam	• Activity 1.2, students comprehend and read through the problem on the sheet. • Folding paper is applied to figure out the sizes of the cake pieces for Nizam and Thoriq. • They ascertain the number of cake segments for Nizam and Thoriq, responding to the sheet's queries.
Exhibiting the responses	• A student exhibits their responses to the entire class.
Conclusion	• Students are capable of folding paper corresponding to each cake segment. • They can identify the count of Thoriq and Nizam’s cake pieces by folding the paper for each segment and observing the number of pieces.

Activity 2.

1. Activity 2.1, students engage with a scenario involving snack sharing between Father, Mother, Budi, and Edo, focusing on the fractional parts received by each. Budi is curious if he has fewer snacks than Edo. Students begin by folding paper to represent Budi’s

portion, color it, and then repeat the process for Edo's portion. They then ascertain the quantities of snacks for Budi and Edo based on the folded paper. The activity concludes with students responding to queries on the activity sheet.

2. Activity 2.2, students explore a scenario about dividing snacks among Father, Mother, Budi, and Edo. They are given fractions representing each person's share of the snacks. The task is to figure out if Father and Budi receive the same quantity of snacks. The students employ paper folding as a visual aid for this task. Initially, they fold paper to match Budi's portion, adding color for differentiation, and then do the same for Father's share. The next step involves the students evaluating and contrasting the snack shares of Budi and Father based on the folded paper. To conclude the activity, students respond to a set of questions provided on their activity sheet.
3. Activity 2.3, students engage with a scenario about snack sharing between Father, Mother, Budi, and Edo. They receive fractions indicating each individual's snack share. The focus is on assessing whether Mother's snack portion is larger or smaller compared to Budi's. To accomplish this, paper folding is utilized as a supportive tool. Initially, students fold the paper to match the fraction allotted to Budi, adding color for distinction, and then replicate the process for Mother's share. Subsequently, the students evaluate and contrast the snack shares of Budi and Mother based on the paper folds. To conclude, students respond to inquiries presented on their activity sheet.

Table 2. Conjecture of Student Thinking in Activity 2

Learning Activities	Students' Thoughts on the Conducted Activities
Focusing on the problems presented in LAS 2	• Students attentively review the issues detailed in Activity Sheet 2 and methodically read through the questions.
Comparing the amounts of snacks between Budi and Edo with paper folding	• Activity 2.1, students analyze the problem presented in the activity sheet. • hey fold paper to represent portions of Budi and Edo cake. • The folded paper is then illustrated by the students. • Students evaluate and compare Budi and Edo's snack portions by responding to the questions.
Comparing the amounts of snacks between Budi and Father with paper folding	• Activity 2.2, students delve into understanding the problem on the activity sheet • Paper is folded to match portions of Budi and Father cake. • Students create illustrations of the folded paper. • They then analyze and compare the snack portions of Budi and Father by addressing the questions.
Comparing the amounts of snacks between Budi and Mother with paper folding.	• Activity 2.3, students engage with the problem outlined on the activity sheet. • They fold paper according to the portions of Budi and Mother cake. • The students depict these folded paper pieces.. • Finally, they discern and compare the snack portions of Budi and Mother by answering relevant questions.
Presentation of individual student's answer results.	• One of the students takes the lead to present their solutions in front of the class.

Conclusion	• Students are capable of comparing different snack portions using paper folding techniques or by illustrating them.

Activity 3.

The task for the students involves closely examining the problem set out in Activity Sheet 3, which is designed to help them learn how to sequence fractions. The exercise presents a scenario about organizing books in a library. Students, after reviewing the problem, are instructed to tackle the activity sheet by adhering to the provided guidelines for solutions. They are encouraged to seek clarification from the teacher on fraction sequencing methods. The approach suggested for solving the problem is the bar model, and students are expected to work out the solution on their own. Following this, they are to present their solutions to the class and then articulate a summary of the learning experience they have.

Table 3. Conjecture of Student Thinking in Activity 3

Learning Activities	Students' Thoughts on the Conducted Activities
Observing the problems presented in Activity Sheet 3	• Students scrutinize the issues in Activity Sheet 3 and read the questions in sequence.
Comparing the quantities of three types of books	• Activity 3, students focus on the problem of how to arrange books. • They utilize a bar model to determine the fractional value comparison for each type of book.
Determining the order of books	• Activity 3 enables students to establish the sequence of book types, either from smallest to largest or vice versa.
Presentation of student's answer results	• One student showcases their solutions in front of the class.
Kesimpulan	• Activity 3, students conclude that to sequence fractions, they need to ascertain the fractional values, compare them, and then arrange them in order.

Design Experiment

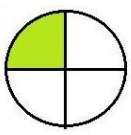
Pilot Experiment Stage

The first cycle starts with giving a preliminary test or pre-test to six students who are part of the pilot experiment. This test aims to evaluate the difficulty level of the designed Hypothetical Learning Trajectory (HLT) and to check its suitability for the student group in the research school. If the HLT is not in line with the students' current capabilities, it may require refinement for better effectiveness. Analysis of the initial test responses provides insights into how students approach problem-solving. This initial assessment includes a total of four questions. Presented here is an example question from this test:

Question 1.

1. Determine the shape of the fraction from the image below!

a. 

b. 

Student answers:

Jawab : $\frac{2 \text{ bagian}}{1}$	Jawab : $\frac{1}{6}$ = Pizza $\frac{1}{3}$ = buat.	Jawab : a). $\frac{1}{7}$ b). $\frac{1}{3}$

Based on the answers of the three students, it is evident that they incorrectly responded to question 1. Interviews with students of lower abilities indicated a lack of understanding of the content, with responses limited to part (a) of question 1. These students mentioned that in the pizza diagram, they calculated the total pizza slices for the numerator, while the denominator represented the single slice taken. Students with medium abilities, as per their interviews, also struggled with the material, attempting both parts (a) and (b) of question 1. Their method involved counting the untouched pizza slices, totaling 6 for the numerator, and considering the taken slice as the denominator. For part (b), they perceived the colored section as the numerator and the uncolored section as the denominator. In contrast, interviews with higher-ability students revealed a misconception in answering part (b) due to a hurried approach.

Retrospective Analysis of Initial Tests (*Pre-test*)

The responses from the preliminary test indicate that the students have yet to fully comprehend the topics of comparing and sequencing fractions. This is attributed to all three students lacking clarity in identifying fractional values and in solving comparative and sequential fraction problems. A particular student described their method as identifying the numerator by the colored section of a diagram and the denominator by the uncolored section, a misunderstanding that reflects their confusion about fraction concepts.

1. **Activity 1: Understand and determine fraction values**

   The students were tasked with a challenge concerning the division of Nizam and Thoriq's cake. Their objective was to grasp the concept of fractions and learn the method of determining fractions through the use of paper folding. They proceeded to fold paper in line with Thoriq's share of the cake, specified as $\frac{2}{7}$. Presented below are the paper folding outcomes from three different students.

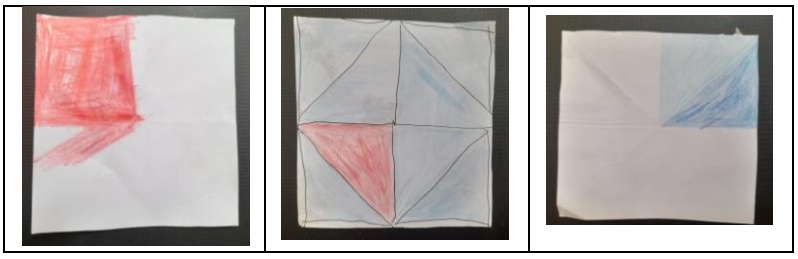


Figure 3. Results of student answers to activity 1 cycle 1

After folding the paper, students then describe it in the column provided on the Student Activity Sheet (LAS) as seen in the picture below.

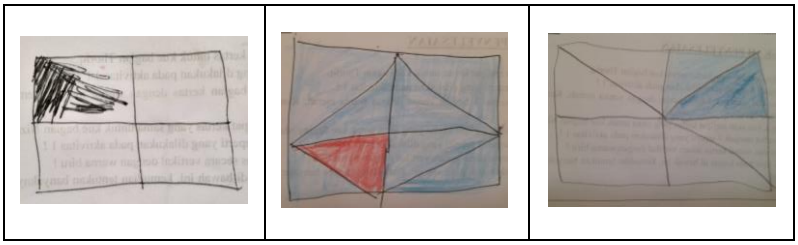


Figure 4. Results of student descriptions of activity 1 cycle 1

Retrospective analysis of activities 1

In line with the established Hypothetical Learning Trajectory (HLT), it was anticipated that students would be able to fold paper to represent specific fractions. Nonetheless, the student responses from Activity 1 suggested a difficulty in folding paper accurately to match the portion of cake allocated to Thoriq. Insights from observations and interviews indicated a lack of understanding among students about the paper folding technique and determining fractions. Consequently, following consultations with mentors, the researchers updated the Activity 1 instructions to include diagrams demonstrating the method of paper folding corresponding to particular fractions.

2. Activity 2: Comparing fractions

Activity 2.1, the task was for students to assess the fractional relationship between the snacks possessed by Budi and Edo. They completed steps outlined in the LAS to facilitate their understanding of fraction comparisons. The expected skill was the ability to fold a single sheet of paper to represent two different fractions. Here are the examples of how the students folded the paper to illustrate the fractions $\frac{1}{4}$ and $\frac{2}{5}$.

Figure 5. Results of student answers to activity 2 cycle 1

Retrospective analysis of activity 2

Based on the results of students' answers in activity 2, it shows that students cannot fold paper. Students are confused and do not understand the instructions for folding paper. From the results of the interviews, students explained that the work instructions made them confused.

3. Activity 3: Ordering fractions

After preliminary activities, students are asked to observe the LAS given by the teacher. Each student is given a LAS which contains student activities in ordering several fractions. The following is a picture of the results of students' answers in completing LAS 3.

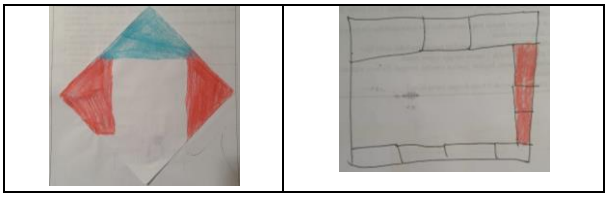


Figure 6. Results of student answers in activity 3 cycle 1

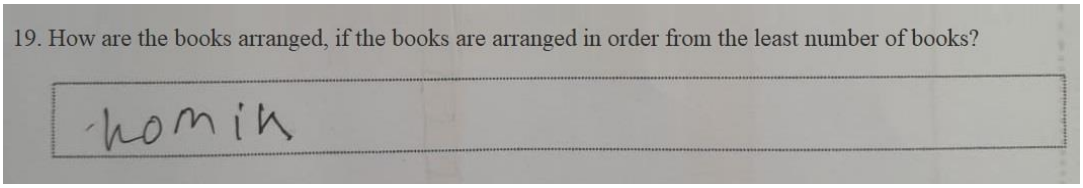


Figure 7. Results of student answers number 19 activity 3

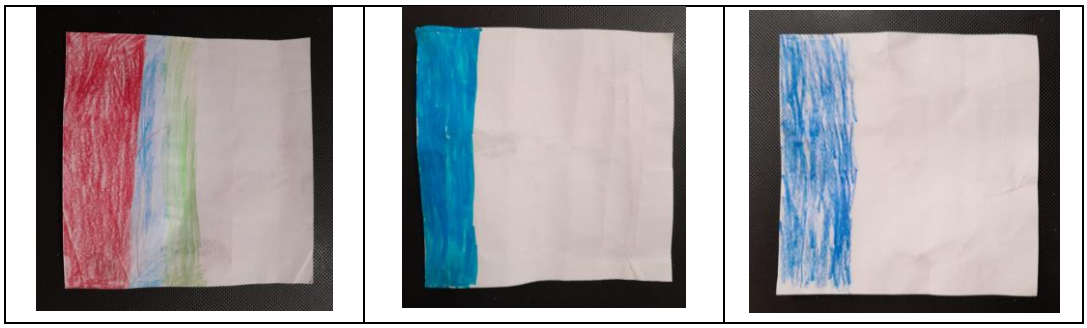
Retrospective analysis of activity 3

Activity 3 aimed to enable students to sequence fractions using their proficiency in folding and drawing folded paper. Instead of using paper folding for fraction comparison, students in this activity represented each fraction using columns and rows. Analysis of the students' work revealed that Student 1 struggled to accurately depict fractions in this format, while Student 2's representation was also imprecise. Additionally, regarding question 19, one of the students provided an incorrect response. Subsequent interviews indicated a lack of comprehension among the students on the method of sequencing fractions.

Teaching Experiment Stage

1. **Activity 1: Understand and determine fraction values**

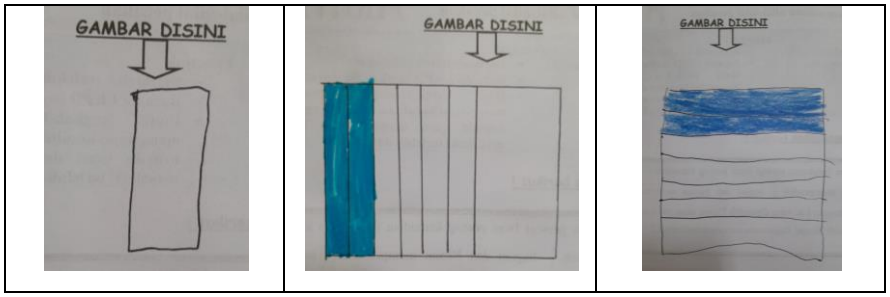
   The tasks undertaken by students in this phase mirrored those of Cycle 1. They engaged with reading and understanding the LAS, aiming to grasp the concept of fractional values. The students were tasked with folding paper to represent the specific fraction of Nizam's cake, quantified as $\frac{2}{7}$. Presented here are the students' solutions from Activity 1.



(a) (b) (c)

Figure 8. Student Answers (a) student 1, (b) student 2, (c) student 3

After folding the paper, students describe the folded results in the column provided in the LAS. The following is a description of the results from 3 students.



(a) (b) (c)

Figure 9. Results of student descriptions (a) student 1, (b) student 2, (c) student 3

Retrospective analysis of activity 1

Based on the results of the students' answers and the results of the students' drawings, it can be seen that only student 1 was unable to fold the folded paper correctly. Meanwhile, student 2 and student 3 were able to understand how to fold and describe columns and rows of a fractional number.

2. Activity 2: Comparing fractions

The objective of Activity 2 is for students to develop the ability to compare multiple fractions through the technique of paper folding. The task involved comparing the snack portions belonging to Budi and Edo. Presented here are the outcomes of the students' paper folding, representing the fractional parts of the snacks for Budi and Edo, specifically $\frac{1}{4}$ and $\frac{2}{5}$.

Figure 10. Student answers to activity 2

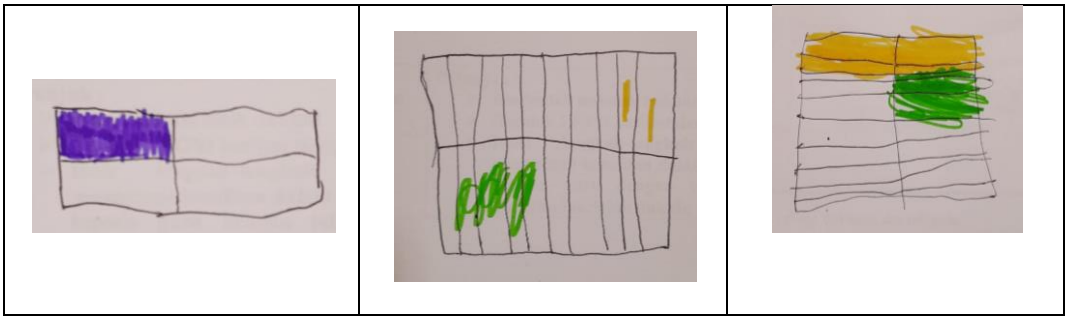


Figure 11. Results of student descriptions

10. Based on the picture in number 9, do Budi get more snacks than Edo gets?

Figure 12. Question number 10 in activity 2

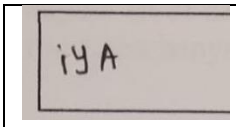
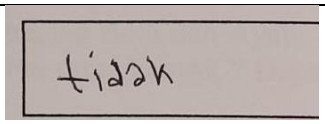
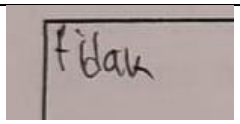
		
Translation: Yes	Translation: No	Translation: No

Figure 13. Results of student answers in activity 2

Retrospective analysis of activity 2

The outcomes of the students' activities and their answers reflect a challenge in folding paper for representing two different fractions. Analysis of their responses to question 10 in Activity 2 suggests they grasped the question's purpose. Further insights gained from interviews shed light on the students' reasoning approach. They primarily considered the denominators in their responses. The reasoning that Edo's denominator was greater than Budi's led to students 2 and 3 answering negatively, while Student 1 appeared to answer arbitrarily.

3. Activity 3: Ordering fractions

The objective of Activity 3 is for students to sequence a set of fractions, drawing upon their ability to depict fractions from folded paper. The process involves comparing two fractions, then relating these comparisons to additional fractions, before ultimately sequencing them. In this third activity, the students encounter a problem that is illustrated in the following image:

The school library receives donations of books from parents. $\frac{3}{10}$ are fiction story books, $\frac{1}{4}$ are educational books and $\frac{9}{10}$ are encyclopedia books. A librarian will arrange the books on shelves in the library. But at the same time, he had to be summoned by the principal. He asked Irene and Joy for help in arranging the books without determining the order in which they were arranged. Irene and Joy have differences of opinion in the order in which the books are arranged. Irene wants to arrange books from the largest number of books, while Joy wants to arrange books from the least number of books. Help Irene and Joy arrange the books according to their wishes!

Figure 14. Problems with activity 3

The following are the results of students' answers to the problems given in activity 3:

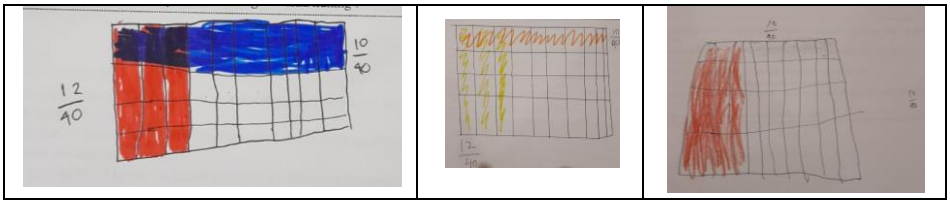


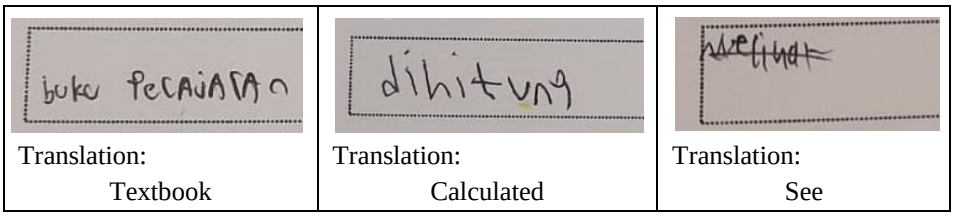
Figure 15. Results of student answers to activity 3

After comparing the parts of each cake, the students then answered the questions. There are several interesting answers for researchers, one of which is the following question.

19. How do you arrange books from the smallest number of books ?

Figure 16. Question number 19 activity 3

From the questions above, here are some student answers:



Translation:	Translation:	Translation:
Textbook	Calculated	See

Figure 17. Student answers number 19 activity 3

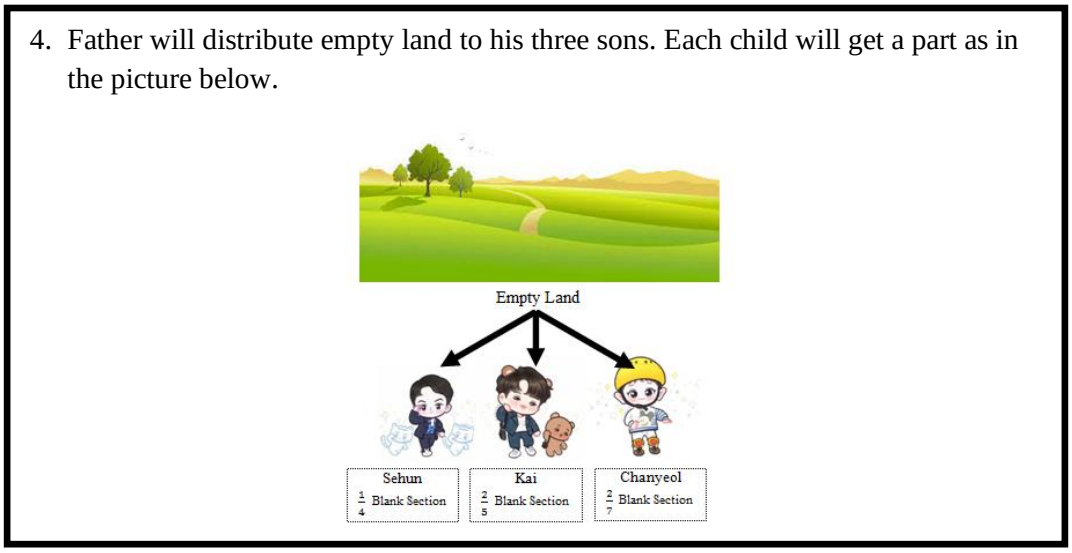
Retrospective analysis of activity 3

The student answers for question 19 indicate a lack of understanding regarding what the question was asking. Subsequent review and analysis of the question by the researchers revealed that its wording led to multiple interpretations and did not clearly convey the intended purpose. As a result, the formulation of question number 19 is slated for revision to enhance clarity.

Retrospective Analysis of Initial Tests (*Post-test*)

Upon the conclusion of the activities in Cycle 2, students were administered a post-test to evaluate their grasp of the fraction concepts that had been instructed. They had a time frame of 35 minutes or the duration of one class period for the test. This post-test included 5 questions, designed to assess all targeted learning outcomes, specifically the skills to compare and order fractions. Presented here are the answers of three students to the fourth question in the post-test:

Question 4.



The question, "Sort the names listed above by the amount of land acquired, beginning with the smallest amount," pertains to the problem presented earlier. Presented here are the answers provided by students to this particular question.

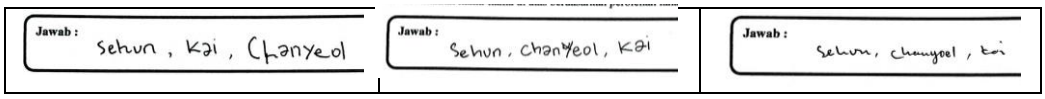


Figure 19. Student answers to post-test questions

Analyzing the responses of a few students, it becomes clear that all three grasped the essence of the question. The student categorized as having lower ability provided an incorrect response, whereas those with medium and higher abilities responded accurately.

Final Test Retrospective Analysis (*Post-test*)

The outcomes of the post-test in this teaching experiment reveal that a majority of the students have grasped the concepts of fractions. This encompasses understanding and calculating fraction values, along with the ability to compare and arrange different fractions. They demonstrated proficiency in identifying fractional values through paper folding, effectively compared fractions using both paper folding and bar models, and were also able to sequence fractions without relying on paper folding as a tool.

Discussion

Based on the research findings from the pre-test phase, Student 1's response indicates that their error was considering all the pizza slices as the numerator and only one taken slice as the denominator. This error in fraction part placement was due to the student's misunderstanding of the question. Further, based on interviews and observational data, the student appeared to hurry through the problem because of a lack of question comprehension, suggesting an incomplete grasp of fractions. These observations are consistent with the findings of Amalia & Unaenah's (2018) research, which highlighted that some students face challenges distinguishing between the numerator and denominator, rooted in a fundamental difficulty in understanding the core concepts essential for fraction learning.

Analysis of student 2's responses, alongside interviews and observations for question 1, revealed a conceptual misunderstanding. The student incorrectly believed that the denominator is only represented by the colored section of a circle or the portion of pizza selected. Conversely, they thought the uncolored sections or the unselected pizza pieces correspond to the denominator. This misinterpretation contradicts the fundamental concept of fractions, where the denominator, the lower number, signifies the total division parts, and the numerator, the upper number, indicates the count of the selected division parts (Suwarto, 2018). Additionally, the student's focus on the colored part for identifying the numerator implies that color significantly aids in engaging them with fraction concepts. They showed more interest in tasks involving colored illustrations, resonating with research findings (Sylviani et al., 2020) that suggest color-centric teaching tools are effective for teaching fractions. Colors also appear to boost student interest and participation in the educational process.

Reflecting on the learning trajectory trial implemented during the pilot experiment, the analysis of students' activities in the LAS, along with interviews and observations, revealed challenges faced by students in folding paper for the second fraction and accurately determining the number of cake slices per fraction, pointing to inaccuracies in paper folding. This issue is contrary to the intended use of paper folding as a teaching tool. Paper folding is recognized as an effective medium in mathematics for teaching basic fraction concepts, particularly because the geometric shapes of colored paper ease manipulation (Nuryani, 2013). Further insights gained from brief student interviews about the lesson revealed their confusion regarding the instructions. However, when developing higher-order thinking skills (HOTS) questions, it is crucial to provide clear instructions (Widana, 2017). This suggests a need to refine the task instructions based on the input and recommendations received.

During the teaching experiment phase, research on Activity 1 indicated that students completed the task effectively and keenly followed guidance from the model teacher. The paper folding work of the student with special needs, identified as a slow learner, was not up to par. Recognizing the student's learning challenges, the model teacher gave extra support during the lesson. It was observed that the student managed to participate in the learning process, though they progressed more slowly compared to their peers. Consequently, the student's response is viewed as their optimal contribution under the circumstances. In this context, the model teacher's role is essentially that of a facilitator, aiming to help students easily grasp and understand the teaching materials, particularly the LAS in this scenario (Yestiani & Zahwa, 2020).

The findings from the trial indicated several issues, such as the students' paper folding not aligning with the anticipated conjectures, and their drawings and colorings deviating from the set conjectures. The students also depicted fractions in a random manner. Student interviews indicated that their drawings were based on their own understanding and the directions provided by the model teacher. Additionally, a response to the query "What is the method for arranging books starting with the least number?" was particularly noteworthy. Students responded with "by counting" and "by looking." This led the researchers to introspect whether the misunderstanding arose from the students' lack of comprehension or the ambiguity of the question. Subsequent independent analysis revealed that question number

19 was indeed open to multiple interpretations. Ambiguous wording is advised against as it can lead to misinterpretation and varied perceptions among respondents (Nuriana et al., 2015).

Implication

he research shows that using paper folding in the learning process can assist students in understanding fractions, determining the value of a fraction, and identifying the numerator and denominator. However, students are not yet able to compare and sequence fractions using paper folding.

Limitation and Suggestion for Further Research

The student post-test results indicate that students can understand and determine fractional values using paper folding but are not able to sequence fractions. This is due to some steps being overlooked by the model teacher during the learning process. Additionally, the material provided was too extensive, especially for slow learners, and there was limited time for the research. Therefore, for future research, it is recommended that model teachers be guided to better understand teacher manuals, select materials that match students' abilities and the school's context, and estimate time more precisely to avoid data collection time constraints.

CONCLUSIONS

The findings from this research indicate that the developed learning trajectory consisted of a series of educational experiences, involving three distinct activities. The initial activity focused on paper folding, attaching paper, and identifying fraction values. The subsequent activity entailed paper folding, pasting, coloring, illustrating the folds, and fraction comparison. The final activity involved drawing and coloring boxes, alongside comparing and ordering fractions.

The implementation of paper folding in teaching fraction comparison and sequencing proved beneficial, as it helped most students grasp the concept of fractions and learn to calculate their values. While paper folding was effective for comparing fractions, it was observed that students still struggled with sequencing certain fractions.

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AUTHOR CONTRIBUTIONS STATEMENT

RF served as the main researcher, contributing ideas and designing the learning trajectory for comparing and sequencing fractions using paper folding. ES and H acted as supervising professors, assisting the researcher in completing the study.

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