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Scientific Platform

The effect of COVID-19 in estimating the South African real spot-rate curve

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Abstract: This study investigates the dynamism of different mathematical term-structure models during COVID-19 to estimate the South African real spot-rate curve. The study is based on term-structure models following Nelson-Siegel framework; and further incorporates the recalibration process on static term-structure models over different economic environments. It was found that the recalibration methodology resulted in no improvements on Linear-parametric and Cubic-splines term-structure models. However, the recalibration effect on Nelson-Siegel and Svensson models resulted in an improved fit for South African inflation-indexed spot rates even in periods of heightened market risk.

Keywords: South African inflation-indexed bonds, spot-rate curve, model recalibration, Nelson-Siegel model, COVID-19

1. Introduction

The emergence of the COVID-19 pandemic has been classified as a one-in-hundred-year event that negatively impacted the world's social, political and economic facets. A total closure in the global economy through lockdowns/restricted movements resulted in a dire situation globally. Naidoo et al. (2020) indicated that the South African financial sector demonstrated its resilience and agility against the effect of the COVID-19 pandemic, which tested each country's ability to respond to the crisis. The capacity and availability of instruments to swiftly address this issue from a social, financial and economic point of view were critical to ensuring business continuity for both public and private sectors. As such, a deflationary economic environment's effect might significantly reduce the capabilities of static term-structure models in fitting and forecasting the South African real spot-rate curve. This was proven in both Reid (2009) and Mashoene et al. (2021) that term structure models with static parameters [i.e. Nelson and Siegel (1987) and Svensson (1994) models] were able to capture movements in the South African real spot-rate curve. However, it was argued that these

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models might lose fitting capabilities throughout stressed market conditions. This study seeks a mathematical term-structure model that can withstand time, especially during stressful market conditions and monetary policy regime changes.

In line with suggestions made by the International Monetary Fund (2020) to address government funding liquidity issues in the wake of the effect of COVID-19 on the economy, the South African government made fiscal adjustments as indicated by the National Treasury (2021), which comprised of "drawing down its cash deposits held with the Reserve Bank, increased short-term borrowing (Treasury bills and bridging finance from the Corporation for Public Deposits) and obtained loans from international financial institutions". This was done in line with government risk management strategies of reducing refinancing and currency risk while not compromising efficient operations in the domestic bond market. As a risk management measure during the COVID-19 stressed period, the South African government, as indicated in National Treasury (2020a), has opted for a momentary measure by reviewing its borrowing strategy to concentrate on supplying shorter-dated bonds with a weighted average time-to-maturity of 7 to 10 years, compared to 15 years seen in the previous year. The strategy assisted in balancing available market demand while managing the cost of issuing debt.

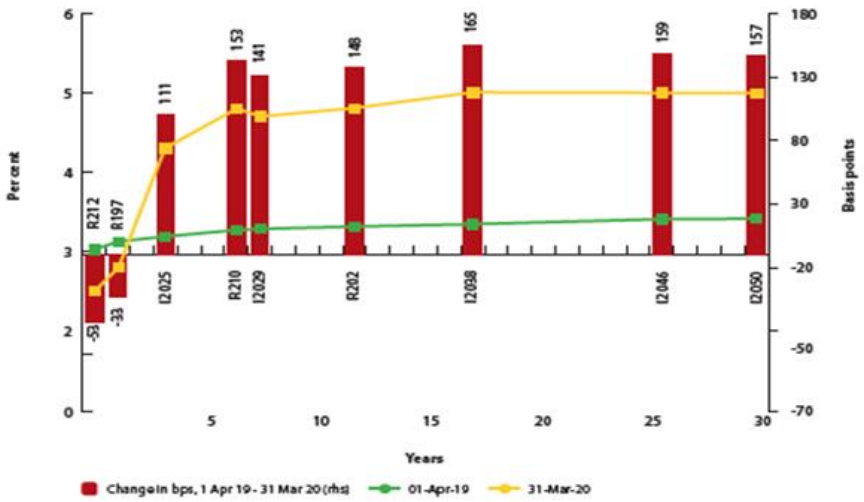
The issuance strategy introduced by the National Treasury to issue short-dated maturities in line with market demand and to manage the cost of raising debt was not apparent in inflation-indexed bonds. Based on Table 1, issuances in the shorter maturities (i.e. R212 and I2025 bonds) have marginally reduced. In comparison, significant reductions were observed in the longer maturities (i.e. I2033 and I2050 bonds). In contrast, significant increases were observed on the I2029 and the I2038 bonds. This is likely attributable to demand issues and availability of issuance space into these maturities, given that the bonds are new in the market. The performance of the inflation-indexed yield curve has a direct impact on the cash received on weekly auctions. This can be observed when issuances into illiquid bonds are made and might result in a lower take-up or costly transaction (i.e. issuance at higher interest rates), thus resulting in lower cash collection from the auction.

Table 1: Nominal issuances into inflation-indexed bonds

Inflation-indexed bonds	Maturity	2019/20 nominal issuance	2020/21 nominal issuance	Absolute difference
R211	31-Jan-22	1,7 %	0,8 %	-0,9 5
I2025	31-Jan-25	19,9 %	19,3 %	-0,6 %
I2029	31-Mar-29	8,1 %	10,9 %	2,8 %
I2033	28-Feb-33	14,8 %	11,4 %	-3,3 %
I2038	31-Jan-38	16,2 %	20,7 %	4,5 %
I2046	31-Mar-46	19,5 %	20,7 %	1,2 %
I2050	31-Dec-50	19,8 %	16,1 %	-3,7 %
Total		**100,0 %**	**100,0 %**	

Source: National Treasury (2021)

Figure 1: Yield curve movement in inflation-indexed bonds, 2019/20



Source: National Treasury (2020b, pp. 21)

It is evident from Figure 1 that the inflation-indexed yield-to-maturity curve has steepened at the end of the 2019/20 fiscal year compared to the beginning of the 2019/20 fiscal year. This could

be attributable to regime switches in monetary policy regimes where significant cuts were observed on the repo rate to caution against the economic stress imposed by the COVID-19 pandemic (National Treasury, 2021). The steeper yield curve could be costly for the government, given that most government funding in inflation-indexed bonds is in the long-to-ultra-long end of the curve. This is in line with South African risk management strategies to lengthen the term-to-maturity of the government debt portfolio (National Treasury, 2020a).

2. Literature review and hypotheses development

As detailed in the work done by Mashoene et al. (2021), it is noted that the National Treasury's current budgeting processes and debt management risk measures are based on the econometric methodology to forecast the short (3-month) and long (10-year) term points. Other points on the yield curve are interpolated based on the output of these two benchmark points. For the pricing of a new bond, the National Treasury depends on the outcome of the Johannesburg Stock Exchange (2012) methodology. It was further indicated that South African inflation-indexed bonds form part of government funding instruments, and proper pricing of these instruments is critical for budgeting processes and fair pricing of newly introduced bonds to minimise the possible gaps in the pricing model, which might result in flawed budget figures.

Given the current global market volatility and monetary policy regime changes brought forth by the outcome of the COVID-19 pandemic, finding a term structure model that can capture these volatilities is mandatory for the South African bond market. It was recently proven by Mashoene et al. (2021) that the Nelson-Siegel and Svensson term structure models could capture movements in the South African real spot-rate curve; however, these models lose fitting capabilities over periods of high market volatilities. This study will examine these models to assess their performance on the South African real spot-rate curve.

The most recent work done by Mashoene et al. (2021) aimed at finding a mathematical term-structure model that could be used to estimate the South African real spot rate. However, the analysis was based on periods of normal market conditions. Mathematical term structure models used in the market were selected to fit the South African real spot-rate curve. To extract real spot rates from coupon-bearing inflation-indexed bond prices, the ordinary least-squares (OLS) methodology was applied. The initial term-structure model to be applied was the Vasicek (1977) one-factor model, which has a mean-reversion property. This suggests that the short rate is pulled back to the long-term average such that interest rates do not persist in being weak/strong over a more extended period. Vasicek's (1977) model allows for negative spot rates, which were assumed appropriate for the South African case given negative coupon-bearing inflation-indexed yields observed in the short-end of the curve. Under the risk-neutral assumption, the spot rate is expressed as follows:

$$\hat{R}(t,\tau) = -\left[\ln\left[\hat{P}(t,\tau)\right]/_{\tau}\right] \tag{1}$$

where

$$\hat{P}(t,\tau) = B_t(T)e^{-A_t(T))r_t} \tag{2}$$

$$r_t = r_0 e^{-kt} + \theta(1 - e^{-kt}) + \sigma \int_0^t e^{-k(t-u)} dW_u \tag{3}$$

$$A_t(T) = \frac{1}{k}(1 - e^{-k.\tau}) \tag{4}$$

$$B_t(T) = e^{\left\{\frac{\left(\frac{\sigma^2}{2} - k^2\theta + \sigma k\lambda(t,r_t)\right)}{k^2}[A_t(T)-\tau] - \frac{\sigma^2}{4k}A_t(T)^2\right\}} \tag{5}$$

Moreover, θ is calculated as the long-term short-rate average of the inflation-indexed short-rate; k is estimated as the log function of the autocovariance/autocorrelation as a proportion of the number of days; and σ is estimated as the instantaneous volatility of the inflation-indexed short-rate. The real short-rate r_t is critical for estimating the entire real spot-rate curve under the Vasicek (1977) one-factor model. However, based on the Johannesburg Stock Exchange (2012) methodology, the real short rate is non-existent in the South African market; as such, more reliance is put on the nominal short rate, which has to be adjusted for inflation to determine the real short rate.

Non-linear parametric term-structure models were also applied to address any drawbacks of arbitrage-free/risk-neutral traditional models. Nelson and Siegel (1987) model is proposed to model the forward rate using three latent factors by employing a relationship from expectation theory. The main advantage of this method is that it ensures a smooth and fairly flexible curve.

$$R(t,\tau) = \beta_0 + \beta_1\left[\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right] + \beta_3\left[\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right] \tag{6}$$

Svensson's (1994) model introduced an extension of Nelson and Siegel (1987) by incorporating the second curvature with the corresponding decay rate. This methodology was considered to address any possible drawbacks of Nelson and Siegel's (1987) model, which are related to the inability to capture movements in the ultra-long end of the curve Christensen et al. (2008).

$$R(t,\tau) = \beta_0 + \beta_1\left[\frac{1-e^{-\lambda_1\tau}}{\lambda_1\tau}\right] + \beta_2\left[\frac{1-e^{-\lambda_1\tau}}{\lambda_1\tau} - e^{-\lambda_1\tau}\right] + \beta_3\left[\frac{1-e^{-\lambda_2\tau}}{\lambda_2\tau} - e^{-\lambda_2\tau}\right] \quad (7)$$

Increased parameters on the Svensson(1994) model helped to capture more complex shapes than the Nelson and Siegel (1987) model; however, Carvalho and Garcia (2019) indicated that the model is unable to produce negative spot rates and has constant parameters over time. As such, Diebold and Li (2006) introduced a time-dependent Nelson and Siegel (1987) model as follows:

$$R(t,\tau) = \beta_{0,t} + \beta_{1,t}\left[\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right] + \beta_{3,t}\left[\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right] \quad (8)$$

Dynamic generalised Nelson-Siegel (DGNS) model, which incorporates a second slope to produce a five-factor loading structure, was applied. This is because the one-slope factor loading setting in the Svensson (1994) model might not be able to produce an arbitrage-free property with a two-curvature factor loading structure.

$$R(t,\tau) = \beta_{0,t} + \beta_{1,t}\left[\frac{1-e^{-\lambda_1\tau}}{\lambda_1\tau}\right] + \beta_{2,t}\left[\frac{1-e^{-\lambda_2\tau}}{\lambda_2\tau}\right] + \beta_{3,t}\left[\frac{1-e^{-\lambda_1\tau}}{\lambda_1\tau} - e^{-\lambda_1\tau}\right]$$
$$+\beta_{4,t}\left[\frac{1-e^{-\lambda_2\tau}}{\lambda_2\tau} - e^{-\lambda_2\tau}\right] \quad (9)$$

However, even though the DGNS factors are chosen to have stochastic dynamics, it is impossible to prevent arbitrage opportunities using the bond prices implied by the resulting Nelson and Siegel (1987) term-structure model. Therefore, this weakness is overcome by introducing the Arbitrage-Free Generalized Nelson-Siegel/ AFGNS model.

$$R(t,\tau) = X_t^{\,1} + X_t^{\,2}\left[\frac{1-e^{-\lambda_1\tau}}{\lambda_1\tau}\right] + X_t^{\,3}\left[\frac{1-e^{-\lambda_2\tau}}{\lambda_2\tau}\right] + X_t^{\,4}\left[\frac{1-e^{-\lambda_1\tau}}{\lambda_1\tau} - e^{-\lambda_1\tau}\right]$$
$$+X_t^{\,5}\left[\frac{1-e^{-\lambda_2\tau}}{\lambda_2\tau} - e^{-\lambda_2\tau}\right] - \frac{C_t(T)}{\tau} \quad (10)$$

where $-\frac{C_t(T)}{\tau} = -\frac{1}{2\tau}\int_t^T \sum_{j=1}^{5}[\Sigma' B_u(T) B_u(T)' \Sigma]_{j,j}\, du.$

Quarterly data of South African inflation-indexed bond prices from April 2010 to April 2018 was used for model development and a two-step forecast covering May 2018 to October 2018. It was observed for model development that there were few variations between the model estimates and actual prices at the short end of the curve. This was attributed to the money-market characteristics embedded in this area of the curve in line with findings made by Reid (2009). Vasicek's (1977) one-factor model priced the South African real zero-curve in deep discount, which is inconsistent with the bond pricing of the real spot-rate curve.

Table 2: RMSE estimates on forecasted values

RMSE	Time step 1	Time step 2
Linear	0.016	0.015
Cubic splines	0.017	0.014
NS	0.018	0.023
DNS	0.056	0.078
Svensson	0.018	0.029
DGNS	0.101	0.334
AFGNS	0.101	0.139

Source: Mashoene et al. (2021, pp.14)

Based on the RMSE estimates, the rotation process on the Rotated Dynamic Nelson-Siegel (2018) model did not improve the original Dynamic Nelson-Siegel (2006) model. Compared to the Ghanaian study by Lartey and Li (2018), it was concluded that the Svensson (1994) model could perfectly fit the illiquid bond market with longer maturities. It was further noted in Table 2 that the static term-structure models performed relatively better than dynamic term-structure models in forecasting the South African real spot-rate curve.

To further review previous studies on Nelson-Siegel framework, the study by Papailias (2022) focused on examining the variation in persistence in the USA yield curve's short-term, medium-term and long-term factors. The study started by investigating the behaviour of the USA's yield curves by applying the Diebold and Li (2006) framework to assess the effect of the COVID-19 pandemic on the fixed-income space. The local linear framework defined in Cai (2007) was further applied to calculate the time-dependent change in the persistence of the yield curve factors. It was indicated that a decrease in persistence was observed towards the end of 2019, resulting in the yield curve factors and, subsequently, the yield curves being more predictable. In contrast, evidence of explosive behaviour was realised in the USA's long-term and short-term yield curve factors during the pandemic.

For this analysis, the study was based on the daily yields for maturities of 1 to 30 years of zero-coupon bonds expressed in months (i.e. 12 to 360 months). A sample of 520 zero-coupon rates was obtained from the Federal Reserve Board (FED) from 2 Jan. 2019 to 29 Jan. 2021. For modelling purposes, sub-samples were defined as follows:

- The incubation phase runs from 2 Jan. 2020 to 17 Jan. 2020, which covers the first COVID-19 announcements in Wuhan and, subsequently, the closing down of the Seafood Wholesale Market.
- The outbreak ran from 20 Jan. 2020 to 21 Feb. 2020, including when the Chinese authorities confirmed the COVID-19 virus transmission and the World Health Organisation (WHO) published its first report.
- The fever phase runs from 24 Feb. 2020 to 30 Mar. 2020, which includes the first lockdown/economic shutdown in Italy.
- The rebound phase, from 23 Mar. 2002 to 30 Apr. 2020, included interventions made by the Federal Reserve (FED) and the stock market rebound.

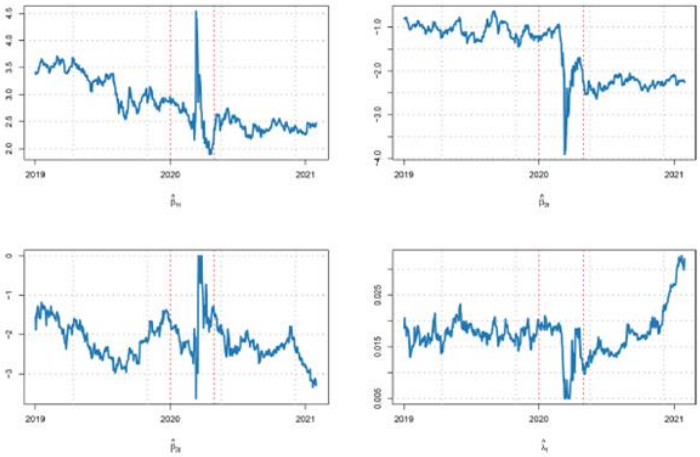
To examine the reaction of the yield curves to the COVID-19 effects, the long-term, short-term and medium-term yield curve factors were extracted following Diebold and Li's (2006) term-structure model:

$$\hat{R}(t,\tau) = a_t + b_t\left[\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right] - c_t e^{\lambda_t.\tau} \tag{11}$$

where $a_t = \beta_{1,t}, b_t = \beta_{2,t} + \beta_{3,t}, c_t = \beta_{3,t}$ are time-varying factors that effect the dynamic structure of the spot-rate curve and $\varepsilon_t \sim N(0,\sigma^2)$. $\hat{R}(t,\tau)$ is the bond yield with maturity τ at time t for $t = 1,2,\dots,T$.

Model parameters $\beta_{1,t}, \beta_{2,t}$ and $\beta_{3,t}$ are the three latent curve factors and λ_t controls for the exponential decay. Small values for the decay factor λ_t provide a better fit for the curve on the long end of the curve, whereas large values provide a better fit on shorter maturities. The three latent factors can be viewed as the short-term, medium-term and long-term factors and can be interpreted as proxies for the yield curve's slope, curvature and level (see Diebold and Li (2006) for more details). A non-linear least squares methodology was applied for each observed day t to estimate a vector of the model. Based on the movements of the model parameters over the COVID period in Figure 2, it is evident that significant volatility was observed during the first wave of COVID-19, as indicated by the red dots. A significant increase in the rate indicates increased volatility in the long-term rates. A decline in the slope factor was observed during the same period, indicating a yield curve inversion. This is reflected by changes in signs (i.e. to negative, thus reflecting a negative sloping yield curve over this period. A sudden decline followed by an increase in the curvature factor was observed during this period.

Figure 2: Nelson Siegel factors on the USA yield curve



Source: Papailias (2022, pp. 5)

In an attempt to proxy the persistence in each model factor and subsequently indirectly proxy the persistence of the yield curve, estimated autoregressive coefficient from the first order autoregressive model AR(1) were applied in line with Diebold and Li (2006) use of AR(1) model in an attempt to capture the underlying dynamics of each yield curve. Papailias (2022) extended the application of the AR(1) model by including a time-dependent framework, which allows for the investigation of how the autoregressive coefficient changes across time, mainly before and during the pandemic. The model is defined as follows:

$$x_t = \varphi_{0,t} + \varphi_{1,t} x_{t-1} + \varepsilon_t \tag{12}$$

with $t = 1,2, \dots, T$ and is estimated using the local linear approach. It was then observed that the persistence of the long-term factor for the US remained relatively stable during 2019.

A gradual decline in persistence is observed towards the end of 2019 since the autoregressive coefficient changes from about 1 to about 0.5. Fluctuations around this level are observed from the end of 2019 to the beginning of 2020, thus indicating that the drop in persistence implies a change in the underlying dynamics, making the long-term factor more predictable during this period. For the slope factor, a decline in persistence is observed towards the end of 2019, a gradual increase during the first two phases of the pandemic with a period of explosive behaviour and a return to the pre-pandemic unit root levels in the last two phases of the pandemic and the whole of 2020. The persistence of the curvature factor goes down towards the last quarter of 2019 and becomes more volatile. After the rebound, the autoregressive coefficient fluctuates between 0.5 and. A gradual decline in persistence was observed before the pandemic for all three model factors. However, in the first two phases of the pandemic, the long-term and short-term factors present explosive behaviour. For an investor, this provides an opportunity to turn to securities of less persistence and, thus, a more predictable yield curve.

In the case of Canada, Severino et al. (2022) indicated that the study focused on unpacking the evolution of Nelson and Siegel (1987) model parameters over different periods that cover the effects of the COVID-19 pandemic on bond markets. Massive corporate bond sales were observed due to a rapid loss of trust in the financial system. As such, disruptions in the debt market were subdued by interventions from the Federal Reserve. Therefore, the study focused more on the changes in the three critical factors of the yield curve in the Canadian government bond market (i.e. level, slope and curvature). Daily data of yields from 1 May 2018 to 30 Oct. 2020 was used with the following sub-samples for modelling purposes:

- The first period runs from 1 May 2018 to 21 Mar. 2019, with 233 daily observations. This is normal with a positive term spread, and the Bank of Canada increased the target rate twice. Even though global growth was solid, tensions were associated with the trade war between the United States of America and China.
- The second period runs from 22 Mar. 2019 to 27 Feb. 2020, with 245 daily observations. The period was considered anomalous with a negative term spread. There were no observed interest rate interventions from the Bank of Canada and sombre economic perspectives anticipated by market actors. It was further noted that trade conflicts between China and the United States of America were still present. There was an observed global slowdown in economic growth in China and the Eurozone due to Brexit concerns.

- The third period runs from 28 Mar. 2020 to 30 Oct. 2020, with 124 daily observations. The period is considered normal with a positive term spread. There were no interest rate interventions from the Bank of Canada; however, economic support was revamped due to sharp contractions in the world economy due to the COVID-19 first wave (i.e., associated with the immense reduction of economic activity to limit the virus spread).

Dynamic Nelson-Siegel (2006) model is defined by a Nelson and Siegel (1987) model with time-dependent parameters applied:

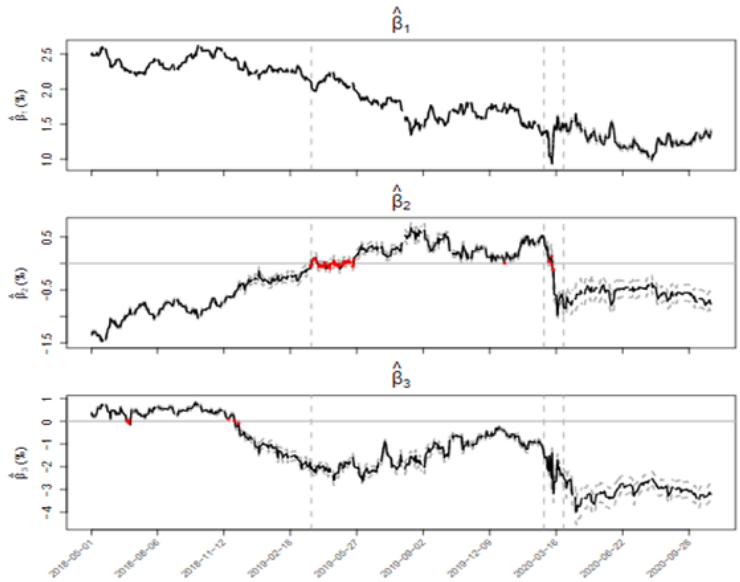
$$\hat{R}(t,\tau) = a_t + b_t\left[\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right] - c_t e^{\lambda_t.\tau} \tag{13}$$

where $a_t = \beta_{1,t}, b_t = \beta_{2,t} + \beta_{3,t}, c_t = \beta_{3,t}$ are time-varying factors that effect a dynamic structure of the spot-rate curve and $\varepsilon_t \sim N(0,\sigma^2)$. $\hat{R}(t,\tau)$ is the yield-to-maturity with the term τ months, and λ_t is a positive parameter that governs the exponential decay. It is noted that $\beta_{1,t}, \beta_{2,t}$ and $\beta_{3,t}$ are three latent dynamic factors that have long-term, short-term and medium-term effects.

The decay parameter was chosen to maximise the medium-term regressor when $\tau = 30$ months, estimated as $\lambda_t = 0.0609$; furthermore, model parameters β were estimated using an ordinary least squares (OLS) methodology for each day t.

The dynamics of the model parameters are presented in Figure 3 to trace their movements over different macroeconomic environments, which included a period of COVID-19 stress. On average, $\beta_{1,t}$ estimates indicated a decline over the estimation period in line with average long-term yields over the three observed periods. Severino et al. (2022) indicated that the Bank of Canada cut its target rate in March 2020, which triggered a decline in $\beta_{1,t}$ from the second to the third period. This reflects relative volatility on longer maturities; however, the analysis of relative volatility measured by the coefficient of variation has indicated lower relative volatility when compared to other model parameters. The proxy for short-term rates $\beta_{2,t}$ reflected a yield curve inversion in the second period. This is reflected by changes in signs (i.e. to positive in the second period and back to negative in the third period). As a result of COVID-19, increasing yields were observed; however, central bank interventions contributed to correcting the curve inversion in the second period. Similarly, the relative volatility on $\beta_{2,t}$ was relatively higher in the second period, echoing the curve inversion observed. The behaviour of $\beta_{3,t}$, a proxy for the curvature factor, showed a steady decline over the three periods. According to Diebold and Li (2006), the curvature factor is closely related to twice the two-year yield minus the sum of the ten-year and three-month yields. A decline in the second period was associated with an inverted hump in the average yield curve. In the third period, a lower value while the yield curve increased was associated with relatively high long-term rates for the two-year yield.

Figure 3: Nelson Siegel factors on the Canadian yield curve



Source: Severino et al. (2022, pp. 487)

3. Research and methodology

For the real spot-rate curve analysis, continuously compounded real spot rates (spot rates) are usually used as a theoretical proxy. The real spot-rate curve is generally preferred for term-structure analysis because of its lesser maturity-specific risk imposed by coupon rates. However, the real spot-rate curve is often unavailable in most countries (i.e. the South African financial market where the zero-coupon bonds are currently not issued), or fewer points are available on the entire maturity profile. The spot-rate curve derived under arbitrage-free traditional one-factor models, which use the short rate to derive the whole spot-rate curve, might not perfectly derive the South African inflation-indexed spot-rate curve (Mashoene et al., 2021). This downside is due to the illiquid nature of these instruments and the shape of the curve that might not be perfectly derived under the one-factor term-structure models, which raises the issue of constructing the real spot-rate curve from the coupon-bearing real YTM curve.

Before the bootstrapping process can be introduced, let the price of a real zero-coupon bond with maturity T at time t be defined as (Hull, 2009):

$$\hat{P}(t,\tau) = P(\tau,\tau)\left(1+\frac{R(t,\tau)}{m}\right)^{-m.\tau} \tag{14}$$

where m is the compounding rate per year, t is time in years, and $R(t,\tau)$ is the real spot rate. To derive a continuously compounding real spot-rate curve, let the compounding rate $m \to \infty$, thus implying that the real spot-rate compounds more frequently with a lower rate of return. Taking the limit of the zero-coupon bond price as $m \to \infty$.

$$\lim_{m\to\infty} \hat{P}(t,\tau) = \lim_{m\to\infty}\left(1+\frac{R(t,\tau)}{m}\right)^{-m.\tau} \qquad \text{assume } \hat{P}(\tau,\tau) = 1$$

As such, given the continuously compounded inflation-indexed spot-rate $R(t,\tau)$, the inflation-indexed zero-coupon bond price at a given point in time t with term-to-maturity τ is defined as (Lyuu, 2004):

$$\hat{P}(t,\tau) = [1+R(t,\tau)]^{-\tau} \qquad \text{in discrete terms, and}$$
$$\hat{P}(t,\tau) = e^{-R(t,\tau)*\tau} \qquad \text{in continuous terms.}$$

To extract the continuously compounded spot-rate curve with an arbitrage-free assumption, the all-in price and discount factors for inflation-indexed coupon-bearing bond should satisfy (Voloshyn, 2015):

$$\hat{B}(t,\tau) = \sum_{t=0.5}^{\tau}[CF(t,\tau).d(\tau)] \tag{15}$$

where

- $d(\tau) = e^{-R(t,\tau).\tau}$ is a discount function using continuously compounded real spot-rates $R(t,\tau)$ for inflation-indexed bonds.
- $R(t,\tau)$ the continuously compounded inflation-indexed spot rate prevailing at time t with term-to-maturity τ. This implies that the spot rate in the document refers to the real spot rate unless indicated otherwise.

3.1. Data analysis

As such, monthly data covering the period between June 2010 and June 2022 was used in this study. To cater for the period of unfavourable market conditions, the development phase covered the period between June 2010 and December 2021 (where the period covering the COVID-19 pandemic is included in the development). The testing phase only covered six months between January 2022 and June 2022. The data used was a sample of over 160 points covering periods of normal and stressed market conditions with monetary policy switches. According to the Central Limit Theorem (CLT), the sample size of 30 data points is deemed sufficient to make a significant conclusion. As such, the data used in this study was sufficient to decide on the model's capability to fit and forecast the South African real spot rate curve over normal and non-normal market conditions.

3.2. Modelling process

To minimise the model risk that could be embedded in using Excel to model different term structures, the work done in this study will be automated in the R software. The modelling process will minimise the error between the actual and model-generated coupon-bearing bond prices. Gujarati and Porter (2009) defined the Ordinary Least-Squares (OLS) methodology as a statistical methodology that analyses the relationship between two variables by minimising the squared

difference between the actual and model-generated variables. Gujarati and Porter (2009) and Verbeek (2017) indicated that OLS estimators should suggest the best linear unbiased estimators (BLUE) characteristic. These characteristics entail that the estimators should not have any bias between the OLS estimator's expected value and the population's true value and should be consistent even for large samples. Given the available sample and the assumptions made, they should be the most accurate estimators and exhibit slight sample variation. As such, the OLS methodology was used to estimate model parameters by minimising the squared error between actual and model-generated coupon-bearing bond prices.

The exact process followed by Mashoene et al. (2021) to extract the continuously compounded real spot-rate curve assuming an arbitrage-free environment, the inflation-indexed coupon-bearing bond all-in price and discount factors should satisfy the following condition:

$$\hat{B}(t,\tau) = \sum_{t=0.5}^{\tau}[CF(t,\tau).d(\tau)] \tag{16}$$

where $d(\tau) = e^{-R(t,\tau).\tau}$ is a discount function using continuously compounded real spot rates for inflation-indexed bonds and $CF(\tau,\tau) = 100 + Coupon_i$ is a cashflow at maturity which comprises of the par value of 100 and the coupon-payment of bond i. This implies that a set of inflation-indexed coupon-bearing bond prices in the absence of arbitrage opportunities can be set up in the following matrix form:

$$B = CF.d + e_t \tag{17}$$

Given that the discount function is exponential, a log transformation is applied to conform to linear regression. Applying the OLS methodology by minimising squared errors, the optimisation function is defined as:

$$\min_{\boldsymbol{\beta}} {e_t}^T e_t = (B - CF.d)^T (B - CF.d) \tag{18}$$

where β is a set of estimated parameters to be optimised.

The analysis of static term-structure models comprises two dimensions: i.e. using initial parameters (i.e., as of June 2010) over the estimations period and using the recalibrated parameters over different periods to account for different economic environments. The recalibration process was applied at the end of 2011 when the last interest rate cut was made by the South African Reserve Bank before stability was maintained with a couple of small increases to tame the volatile local currency; and also in March 2020 to account for the effects of COVID-19 pandemic in the global economy.

A couple of goodness-of-fit measures will be applied to further analyse the goodness of fit in the model output for backtesting analysis. This ensures that a term structure model meets a couple of hypothesis tests to ensure it is significant in capturing movements in the yield curve.

- Root Mean Square Error (RMSE) methodology is the standard deviation of prediction errors to analyse the spread of prediction error:

$$RMSE = \sqrt{\sum_{i=1}^{n} \frac{(P(t,\tau_i) - \hat{P}(t,\tau_i))^2}{n}} \tag{19}$$

  where n is a sample size, $P(t,\tau_i)$ is the actual price of the real zero-coupon bond, and $\hat{P}(t,\tau_i)$ is the model estimated price of the real zero-coupon bond using the selected term structure models. A lower RMSE estimate is preferred as it implies a better model fit to the actual prices. Willmott et al. (2009) indicated that mean absolute error (MAE) is mostly preferred over the RMSE in model valuation statistics citing that RMSE is ambiguous. However, it was proven by Chai and Draxler (2014: 1) that 'RMSE is more appropriate to represent the model performance than the MAE when the error distribution is expected to be Gaussian'. The Gaussian property is in line with assumptions embedded in the OLS methodology. The Akaike and Bayesian information criteria were used to evaluate the model's performance further to supplement RMSE results.
- Akaike information criterion (AIC) methodology is another measure of goodness-of-fit used in this study. Gujarati and Porter (2009) have indicated that the AIC methodology enforces a stricter penalty than the coefficient of variation (R^2) by adding more regressors, and it is more suitable for both the in-sample and out-of-sample performance of the regression model. The AIC methodology is defined as follows:

$$AIC = e^{2k/n} \frac{\sum(P(t,\tau_i)-\hat{P}(t,\tau_i))^2}{n} \tag{20}$$

where k is the number of regressors (including the intercept), and n is the number of observations. The methodology prefers the model with the lowest AIC estimate.

- Bayesian information criterion (BIC) methodology is defined as

$$BIC = n^{k/n} \frac{\sum(P(t,\tau_i)-\hat{P}(t,\tau_i))^2}{n} \tag{21}$$

where k is the number of regressors (including the intercept), and n is the number of observations. Like AIC, the methodology prefers a model with a lower BIC estimate and is suitable for both in-sample and out-of-sample model performance analysis. It provides a stricter penalty than the AIC methodology by penalising more regressors.

4. Results

To assess the performance of the selected models, the spot-rate curve should have the following characteristics as described by Kikuchi and Shintani (2012):

4.1. Abnormal spot-rates

Figure 4: Estimated South African real spot-rates – March 2020

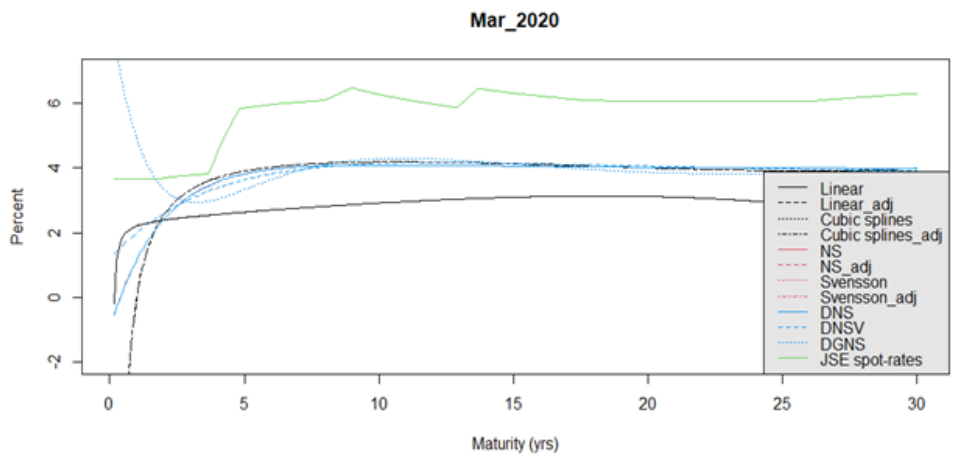


Figure 5: Estimated South African real spot-rates – June 2020

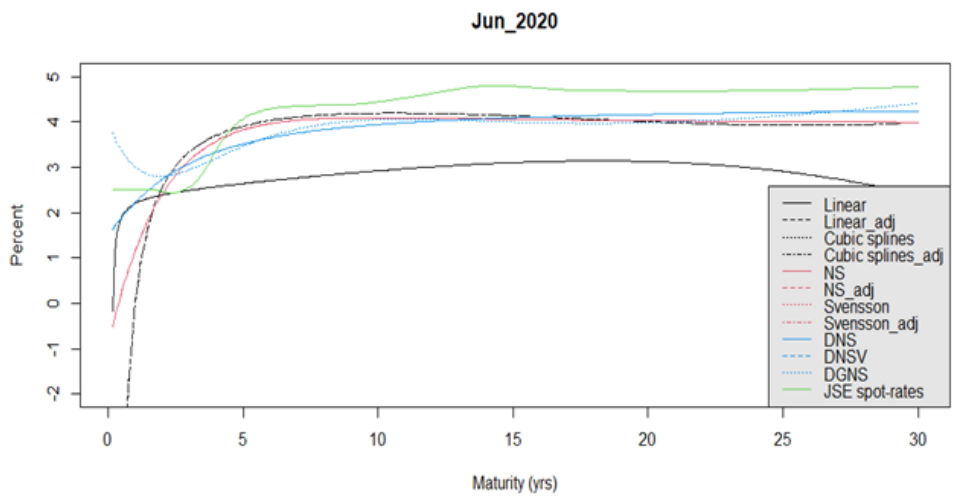
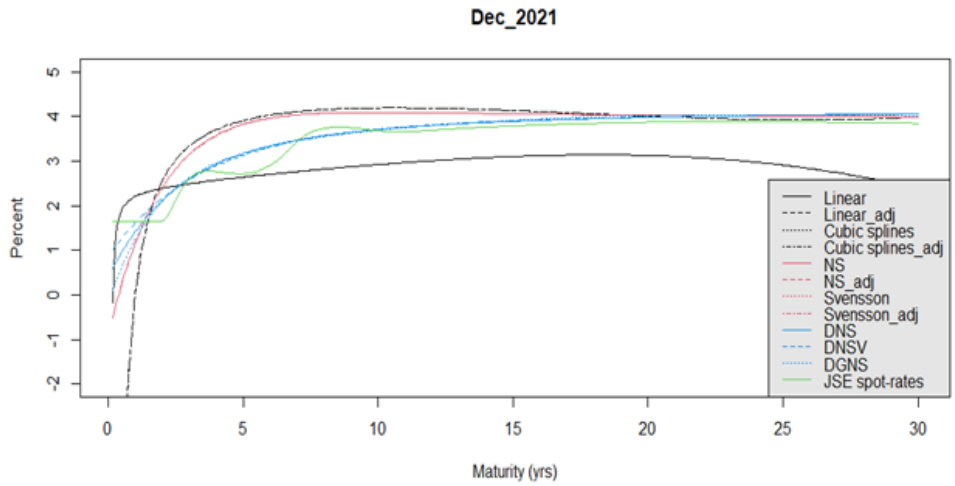


Figure 6: Estimated South African real spot-rates- December 2021



Before the 2020 COVID-19 pandemic, the South African real spot-rate curve has been trading below 4.00 per cent across the curve. An up-pick in real spot rates was realised in the second week of March 2020 when the first case of COVID-19 was realised in the country (National Treasury, 2020a). The cost of borrowing in the inflation-linked bonds (yield-to-maturity) weakened by 240 basis points (bps) on the I2050 (2.5%, 31-Dec-2050) bond between 6 Mar. 2020 and 27 Mar. 2020. This is due to uncertainties brought by the effects of the COVID-19 pandemic, which led to the global economic shutdown. To analyse the effect of the COVID-19 pandemic on the estimation of the South African real spot-rate curve, Figures 4, 5 and 6 show the real spot-rate curve during March 2020 – when the first economic shutdown was introduced in South Africa, June 2020 – post-South African Reserve Bank interventions on the buying of South African bonds and December 2021 when the economy has stabilised.

It was observed that the actual real spot-rate curve from the Johannesburg Stock Exchange (JSE) had painted a very volatile environment, with spot-rates trading at levels around 4 per cent on the short-end of the curve and around 6 per cent on the mid-to-ultra-long end of the curve. None of the selected mathematical term-structure models could perfectly capture this volatility with the estimated rates around 4 per cent on the mid-to-ultra-long end of the curve. However, the estimated real spot rate at the short end of the curve is estimated to be on levels below 2 per cent; and even negative rates on most of the static term-structure models. Post the SARB intervention in bond buying; actual real spot rates were trading at around 5 per cent, which was still 50 bps above where selected mathematical term-structure models are pricing the South African real spot-rate curve. In December 2021, when the economy stabilised, and global economic activities resumed, real spot rates strengthened to levels slightly below 4 per cent on the long end of the curve and around 2 per cent on the short end. During this period, most of the dynamic and recalibrated mathematical term-structure models were able, to some extent, perfectly capture movement in the South African real spot-rate curve.

4.2. A perfect fit

To analyse the perfect fit of selected mathematical term-structure models on the South African real spot-rate curve, a couple of statistical methods will be used to ascertain the performance of the term-structure model in estimating the real spot-rate curve. Firstly, root mean squared error (RMSE) will be applied to analyse squared errors between the actual and estimated bond prices; secondly, Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) to assess the model's performance by enforcing a stricter penalty by adding more regressors. The analysis of RMSE, AIC and BIC estimates of selected term-structure models are presented in Figure 7, Figure 8 and Figure 9, respectively.

Figure 7: RMSE for the estimated term-structure models

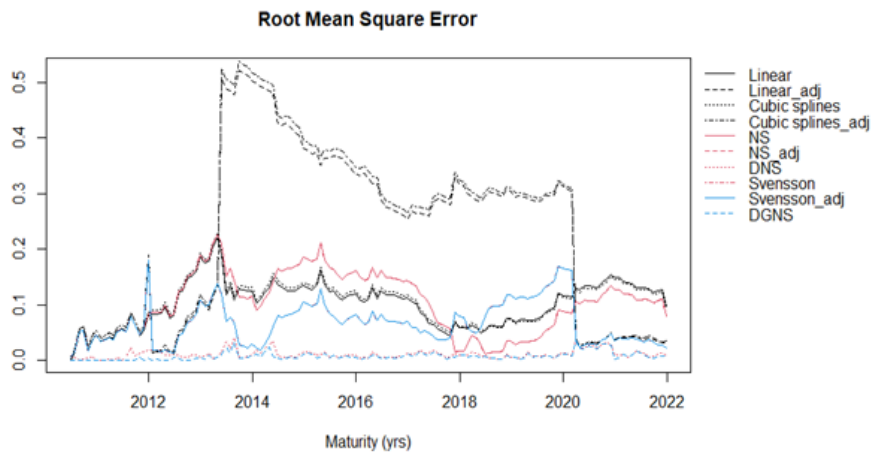


Figure 8: AIC for the estimated term-structure models

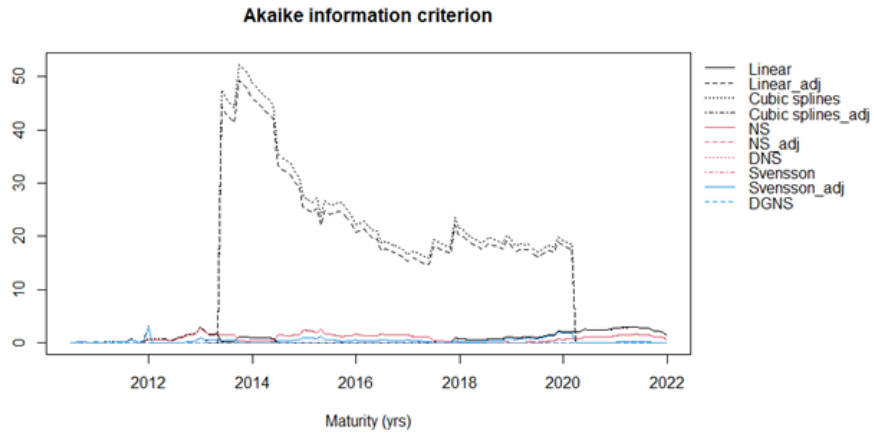
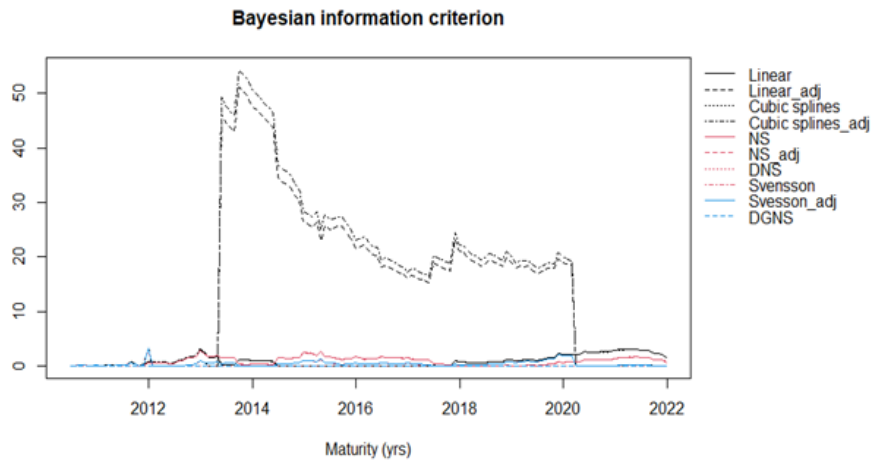


Figure 9: BIC for the estimated term-structure models



A lower estimate is considered a good-performing model in analysing the goodness of fit using RMSE, AIC and BIC (Gujarati and Porter, 2009). It was noted that both recalibrated Linear parametric and Cubic splines models gave a poor performance with significantly higher estimates of closer to 50 on AIC and BIC and closer to 0.5 on the RMSE estimates. Relative to other term-structure models, RMSE estimates are below 0.2, and AIC and BIC estimates of below 5. This implies that the recalibration process on these term-structure models did not give an improved performance; as such, both recalibrated Linear parametric and Cubic splines models are not a good fit for estimating the South African real spot-rate curve.

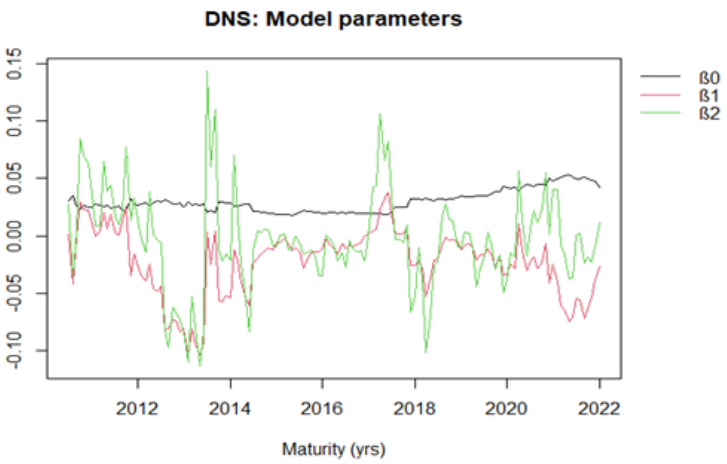
4.3. Movements in the model parameters: Dynamic Nelson-Siegel (2006) model

Given a different economic landscape during the COVID-19 stress period, it will be beneficial to analyse the performance of model parameters for the Dynamic Nelson-Siegel (2006) model over the estimation period. This is because a drastic cut in the repo rate is realised in most economies as part of central banks' stimulus to aid the ailing economy during periods of heightened economic/market volatility.

Table 3: Statistical analysis of Dynamic Nelson-Siegel model parameters

Statistical test	Level ($\beta_{0,t}$)	Slope ($\beta_{1,t}$)	Curvature ($\beta_{2,t}$)
Average	2,99%	-2,27%	-0,40%
Min	1,76%	-10,32%	-11,36%
Max	5,28%	3,82%	14,40%
Coefficient of variation	0,319	-1,291	-10,632
Skewness	0,844	-0,686	0,250
Kurtosis	2,744	3,210	4,275

Figure 10: Movements of Dynamic Nelson-Siegel model parameters



The statistical analysis of model parameters in Table 3 and results in Figure 10 indicates that when the decay rate ($\lambda = 0.5$), there is some stability in the level factor $\beta_{0,t}$, a proxy for a long-term average for the real spot-rate curve. Over the estimation period, covering over a decade on the monthly real spot rate, the average long-term rate was 2.99 per cent with a lower coefficient of variation than other parameters. When analysing the tail risk, it is observed that the kurtosis measure is less than three, which implies that, on average, the frequency of significant values in the tail was relatively lower than what is considered normal distribution. However, some increases were observed during the COVID-19 period while significant cuts in target rates (i.e. a cumulative 275 basis points cut in the repurchase rate) cushion the economy against the effect of COVID-19 shocks. This contradicts the results observed in Severino (2021), where it was indicated that a cut in the target rate by the Bank of Canada triggered a decline in the level factor. As such, this indicates that even though cuts were made, long-term risks remain heightened in real terms.

The slope factor $\beta_{1,t}$, a proxy for the short-term rates, indicates that, on average, the real spot-rate curve has been pricing below the zero per cent mark. A slight increase associated with the curve inversion was observed at the beginning of 2020. This could be associated with the volatilities that followed the first total economic shutdown in South Africa. However, this was short-lived, given interventions made by the central bank to cut target rates. The curvature factor $\beta_{2,t}$, a proxy for the medium-term rates, remained highly volatile over the estimation period, with the highest coefficient of variation and kurtosis estimate of 4.275. A heightened volatility was also observed in 2020 during the peak of COVID-19 stress.

5. Conclusion

It was observed in the study that during the period of the COVID-19 pandemic, drastic monetary policy measures were taken by the South African Reserve Bank to ensure that National Treasury's increased borrowing requirement was not negatively affected by liquidity issues that affected most emerging markets as a massive sell-off of domestic bonds was realised from foreign investors as they look for a safer market. It should also be noted that inflation-linked bonds have lower liquidity due to investors using them to hedge their long-term obligations against inflation. As such, it is critical to have a term-structure methodology to capture movements/changes observed in the cost of borrowing during normal and non-normal market conditions.

In determining a perfect mathematical term-structure methodology that will be able to estimate the South African real spot-rate curve, it was observed that dynamic models had overfitting characteristics by consistently producing negative real spot rates, especially in the 1 to 5-year maturity spectrum. This is expected in the short-term (i.e. most 12 months term-to-maturity) given the money-market characteristics often observed on this area of the curve. As with the analysis by Castello and Resta (2022) on BRICS countries, the effect of the dynamic decay factor on the Dynamic Nelson-Siegel (2006) model yielded no better performance. This could be attributable to the dynamics of the inflation-linked bond market, which is less liquid/less volatile relative to nominal bonds used in Castello and Resta (2022) study for the South African case.

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Competing interests

All authors declare no conflicts of interest in this paper.

Authors' contributions

All authors contributed equally to this work.

Ethical considerations

This article followed all ethical standards for research without direct contact with human or animal subjects. The study is based on the data/information freely available to the public on the institution's website.

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