

## Elementenmethode I

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Elementenmethode I

Uitwerkingen van vraagstukken

Studiemateriaal bij de P.a.t.o.-cursus

Methode der eindige elementen - fundamenten

*Uitwerking Opgaven 3,4 en 9.*

Voorjaar 1979

Vakgroep Technische Mechanica

Afdeling der Werktuigbouwkunde

Technische Hogeschool Eindhoven

Code: V.4 aanhangsel.

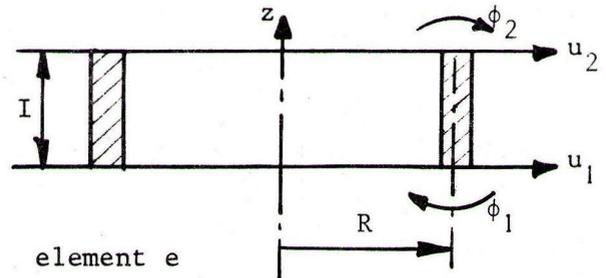
Uitwerking opgave 3

a. De bepaling van de stijfheidsmatrix

Berekening van de stijfheidsmatrix  $Q_e$  voor het pipelement  $e$

De stijfheidsmatrix is:

$$Q_e = A_e^{-1} \cdot K_e \cdot A_e^{-1} \quad (1)$$



We hebben dus de matrices  $A_e^{-1}$  en  $K_e$  nodig.

Voor het verplaatsingsveld  $\hat{u}_e$  nemen we een derdegraads polynoom.

$$\text{Stel } \hat{u}_e = \alpha_0 + \alpha_1 \cdot z + \alpha_2 \cdot z^2 + \alpha_3 \cdot z^3 = \alpha_e \cdot \hat{\Phi}_e = \hat{\Phi}_e \cdot \alpha_e \quad (2)$$

Hierin zijn  $\alpha_e$  en  $\hat{\Phi}_e$  de kolomvectoren gedefinieerd door:

$$\alpha_e = (\alpha_0 \quad \alpha_1 \quad \alpha_2 \quad \alpha_3) ; \hat{\Phi}_e = (1 \quad z \quad z^2 \quad z^3) \quad (3)$$

De onbekende grootheden  $\alpha_0, \alpha_1, \alpha_2$  en  $\alpha_3$  kunnen worden uitgedrukt in de verplaatsingen en verdraaiingen  $u_e^{(1)}, \phi_e^{(1)}, u_e^{(2)}$  en  $\phi_e^{(2)}$  aan de randen van het element  $e$ .

$$\text{Hierbij geldt: } \hat{\Phi}_e = \frac{d\hat{u}_e}{dz} = \alpha_1 + z \cdot \alpha_2 + 3\alpha_3 \cdot z^2 \quad (4)$$

De uit (3), (4) en (5) volgende vier vergelijkingen in matrixvorm geschreven zijn:

$$\begin{bmatrix} u_e^{(1)} \\ \phi_e^{(1)} \\ u_e^{(2)} \\ \phi_e^{(2)} \end{bmatrix} = \begin{bmatrix} 1 & z_1 & z_1^2 & z_1^3 \\ 0 & 1 & 2z_1 & 3z_1^2 \\ 1 & z_2 & z_2^2 & z_2^3 \\ 0 & 1 & 2z_2 & 3z_2^2 \end{bmatrix} \cdot \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} \quad (5)$$

$u_e = A_e \alpha_e$

Omdat de matrix  $A_e$  regulier is kan  $\alpha_e$  worden opgelost volgens

$$\alpha_e = A_e^{-1} \cdot u_e \quad (6)$$

Omdat de stijfheidsmatrix voor ieder element identiek is mogen we voor ieder element  $z_1 = 0$  en  $z_2 = H = \frac{l}{ne}$  nemen. Hierin is H de hoogte van elk element bij ne elementen.

De matrix  $A_e$  wordt dan:

$$A_e = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & H & H^2 & H^3 \\ 0 & 1 & 2H & 3H^2 \end{bmatrix} \quad (7)$$

Inverse van  $A_e$

$$A_e^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -3H^{-2} & -2H^{-1} & 3H^{-2} & -H^{-1} \\ 2H^{-3} & H^{-2} & -2H^{-3} & H^{-2} \end{bmatrix} \quad (8)$$

Ter controle:  $A_e \cdot A_e^{-1} = I$

De elastische energie voor element e kan geschreven worden als

$$U_e = \frac{1}{2} \alpha_e^T K_e \alpha_e \quad (9)$$

Met behulp van  $U_e$  kan matrix  $K_e$  bepaald worden

$$U_e = \frac{\pi RE}{(1-\nu^2)} \int_{z_1=0}^{z_2=H} \left\{ \frac{1}{12} t^3 (u'')^2 + t(1-\nu^2) \left( \frac{u}{R} \right)^2 \right\} dz \quad (10)$$

Met  $\hat{u}_e = \alpha_e^T \hat{\phi}_e$

$$K_e = \frac{2\pi RE}{1-\nu^2} \int_0^H \left\{ \frac{1}{12} t^3 \frac{d^2 \phi_e}{dz_e^2} \frac{d^2 \phi_e}{dz_e^2} + \frac{t}{R^2} (1-\nu^2) \phi_e \cdot \phi_e \right\} dz_e \quad (11)$$

Met  $\nabla_e = [1 \quad z \quad z^2 \quad z^3]$  en  $\frac{d^2 \nabla}{dz^2} = [0 \quad 0 \quad 2 \quad 6z]$  en

substitutie van  $C = \frac{t^2 R^2}{(1-v^2)}$  geeft dit na integratie van (11):

$$K_e = \frac{2\pi E t}{R} \begin{bmatrix} H & \frac{1}{2}H^2 & \frac{1}{3}H^3 & \frac{1}{4}H^4 \\ \frac{1}{2}H^2 & \frac{1}{3}H^3 & \frac{1}{4}H^4 & \frac{1}{5}H^5 \\ \frac{1}{3}H^3 & \frac{1}{4}H^4 & \frac{1}{5}H^5 + \frac{1}{3}CH & \frac{1}{6}H^6 + \frac{1}{2}CH^2 \\ \frac{1}{4}H^4 & \frac{1}{5}H^5 & \frac{1}{6}H^6 + \frac{1}{2}CH^2 & \frac{1}{7}H^7 + CH^3 \end{bmatrix}$$

$$Q_e = A_e^{-1} \cdot K_e \cdot A_e^{-1}$$

Uitgewerkt:

$$Q_e = \frac{2\pi E t}{R} \begin{bmatrix} \frac{13}{35}H + CH^{-3} & \frac{11}{120}H^2 + \frac{1}{2}CH^{-2} & \frac{9}{70}H - CH^{-3} & \frac{-13}{420}H^2 + \frac{1}{2}CH^{-2} \\ \frac{11}{210}H^2 + \frac{1}{2}CH^{-2} & \frac{1}{105}H^3 + \frac{1}{3}CH^{-1} & \frac{13}{420}H^2 - \frac{1}{2}CH^{-2} & \frac{-1}{140}H^3 + \frac{1}{6}CH^{-1} \\ \frac{9}{70}H - CH^{-3} & \frac{13}{420}H^2 - \frac{1}{2}CH^{-2} & \frac{13}{35}H + CH^{-3} & \frac{-11}{210}H^2 - \frac{1}{2}CH^{-2} \\ \frac{-13}{420}H^2 + \frac{1}{2}CH^2 & \frac{-1}{140}H^3 + \frac{1}{6}CH^{-1} & \frac{-11}{210}H^2 - \frac{1}{2}CH^{-2} & \frac{1}{105}H^3 + \frac{1}{3}CH^{-1} \end{bmatrix}$$

$$b. \quad \Omega_{ce} = -\nabla_e f_{ce} = - \int_G \nabla_e \hat{q}_e^0 dG$$

$$\text{en met } \hat{u}_e = \nabla_e \hat{\phi}_e \alpha_e \text{ en } u_e = A_e \alpha_e$$

$$\hat{u}_e = \nabla_e A_e^{-1} u_e \text{ dus}$$

$$\Omega_{ce} = - \int_G \nabla_e A_e^{-1} \hat{\phi}_e \hat{q}_e^0 dG$$

$$f_{ce} = A_e^{-1} \int_G \hat{\phi}_e \hat{q}_e^0 dG = A_e^{-1} \cdot 2\pi R \int_{z=0}^H \hat{\phi}_e \hat{q}_e^0 dz$$

$$\hat{\phi}_e = \begin{bmatrix} 1 \\ z \\ z^2 \\ z^3 \\ z \end{bmatrix} \quad \hat{q}_e^0 = q_1 + \frac{z}{H} (q_2 - q_1)$$

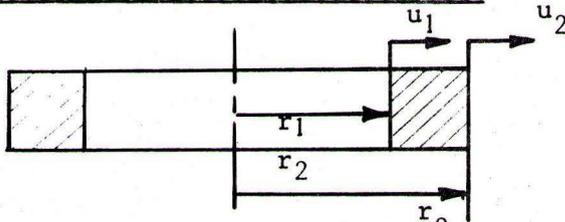
$$f_{ce} = 2\pi R A_e^{-1} \int_{z=0}^H \left\{ q_1 + \frac{z}{H} (q_2 - q_1) \right\} \begin{bmatrix} 1 \\ z \\ z^2 \\ z^3 \\ z \end{bmatrix} dz$$

$$= 2\pi R A_e^{-1} \begin{bmatrix} q_1 H + \frac{H}{2} (q_2 - q_1) \\ \frac{1}{2} q_1 H^2 + \frac{H^2}{3} (q_2 - q_1) \\ \frac{1}{3} q_1 H^3 + \frac{H^3}{4} (q_2 - q_1) \\ \frac{1}{4} q_1 H^4 + \frac{H^4}{5} (q_2 - q_1) \\ \frac{1}{2} q_1 H + \frac{H}{2} (q_2 - q_1) \end{bmatrix}$$

$$f_{ce} = 2\pi R \begin{bmatrix} \frac{1}{2} q_1 H + \frac{3}{20} H (q_2 - q_1) \\ \frac{1}{12} q_1 H^2 + \frac{1}{30} H^2 (q_2 - q_1) \\ \frac{1}{2} q_1 H + \frac{7}{20} H (q_2 - q_1) \\ \frac{1}{12} q_1 H - \frac{1}{20} H^2 (q_2 - q_1) \end{bmatrix}$$

Uitwerking opgave 4

b. Bepaling van de stijfheidsmatrix



$$u = C_1 + C_2 r$$

Elastische energie: 
$$U_e = \frac{1}{2} h \int_{r_1}^{r_2} (\sigma_r \epsilon_r + \sigma_\theta \epsilon_\theta) 2\pi r dr$$

$$\sigma_r = \frac{E}{1-\nu^2} \{\epsilon_r + \nu \epsilon_\theta\} \quad \sigma_\theta = \frac{E}{1-\nu^2} \{\epsilon_\theta + \nu \epsilon_r\}$$

$$U_e = \frac{1}{2} \int_{r_1}^{r_2} \frac{Eh}{1-\nu^2} \{\epsilon_r^2 + \epsilon_\theta^2 + 2\nu \epsilon_r \epsilon_\theta\} 2\pi r dr$$

$$\epsilon_r = u' \quad \epsilon_\theta = \frac{u}{r}$$

$$U_e = \frac{1}{2} \frac{Eh}{1-\nu^2} \int_{r_1}^{r_2} \{u'^2 + \left(\frac{u}{r}\right)^2 + 2\nu u' \frac{u}{r}\} 2\pi r dr$$

$$u = C_1 + C_2 r = \hat{\Phi}_e \alpha_e \quad \hat{\Phi}_e = \begin{bmatrix} 1 \\ r \end{bmatrix} \quad \alpha_e = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$$

$$U_e = \frac{1}{2} \frac{Eh}{1-\nu^2} \int_{r_1}^{r_2} \{(\hat{\Phi}_e' \alpha_e)^2 + \left(\frac{1}{r} \hat{\Phi}_e \alpha_e\right)^2 + 2\nu (\hat{\Phi}_e' \alpha_e)' \left(\frac{1}{r} \hat{\Phi}_e \alpha_e\right)\} 2\pi r dr$$

$$= \frac{1}{2} \frac{Eh}{1-\nu^2} \int_{r_1}^{r_2} \{\alpha_e' \hat{\Phi}_e' \hat{\Phi}_e' \alpha_e + \alpha_e' \frac{1}{r^2} \hat{\Phi}_e \hat{\Phi}_e \alpha_e + \alpha_e' \frac{\nu}{r} \hat{\Phi}_e' \hat{\Phi}_e \alpha_e +$$

$$+ \alpha_e' \frac{\nu}{r} \hat{\Phi}_e \hat{\Phi}_e' \alpha_e\} 2\pi r dr$$

$$= \frac{1}{2} \nabla_e \left( \frac{Eh}{1-\nu^2} \int_{r_1}^{r_2} \{ \hat{\phi}'_e \nabla'_e + \frac{1}{r^2} \hat{\phi}_e \nabla_e + \frac{\nu}{r} \hat{\phi}'_e \nabla_e + \frac{\nu}{r} \hat{\phi}_e \nabla'_e \} 2\pi r \, dr \right) \alpha_e$$

$$= \frac{1}{2} \nabla_e K_e \alpha_e$$

$$K_e = \frac{Eh}{1-\nu^2} \int_{r_1}^{r_2} \left\{ \begin{pmatrix} 0 \\ 1 \end{pmatrix} [0 \ 1] + \frac{1}{r^2} \begin{pmatrix} 1 \\ r \end{pmatrix} [1 \ r] + \frac{\nu}{r} \begin{pmatrix} 0 \\ 1 \end{pmatrix} [1 \ r] + \frac{\nu}{r} \begin{pmatrix} 1 \\ r \end{pmatrix} [0 \ 1] \right\} 2\pi r \, dr$$

$$= \frac{Eh}{1-\nu^2} \int_{r_1}^{r_2} \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{r^2} \begin{bmatrix} 1 & r \\ r & r^2 \end{bmatrix} + \frac{\nu}{r} \begin{pmatrix} 0 & 0 \\ 1 & r \end{pmatrix} + \frac{\nu}{r} \begin{pmatrix} 0 & 1 \\ 0 & r \end{pmatrix} \right\} 2\pi r \, dr$$

$$= \frac{Eh}{1-\nu^2} \int_{r_1}^{r_2} \begin{bmatrix} \frac{1}{r^2} & \frac{1+\nu}{r} \\ \frac{1+\nu}{r} & 2+2\nu \end{bmatrix} 2\pi r \, dr = \frac{E2\pi h}{1-\nu^2} \int_{r_1}^{r_2} \begin{bmatrix} \frac{1}{r} & 1+\nu \\ 1+\nu & (2+2\nu)r \end{bmatrix} dr$$

$$= 2\pi \frac{Eh}{1-\nu^2} \begin{bmatrix} \ln \frac{r_2}{r_1} & (1+\nu)(r_2 - r_1) \\ (1+\nu)(r_2 - r_1) & (1+\nu)(r_2^2 - r_1^2) \end{bmatrix}$$

$$u_e = A_e \alpha_e \quad \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 & r_1 \\ 1 & r_2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \rightarrow \begin{cases} u_1 = c_1 + c_2 r_1 \\ u_2 = c_1 + c_2 r_2 \end{cases} \rightarrow c_1 = u_1 - c_2 r_1$$

$$u_2 = u_1 - c_2 r_1 + c_2 r_2 \rightarrow u_2 - u_1 = c_2 (r_2 - r_1) \quad c_2 = \frac{u_2 - u_1}{r_2 - r_1}$$

$$\begin{aligned} c_1 &= u_1 - c_2 r_1 = u_1 - r_1 \frac{u_2 - u_1}{r_2 - r_1} + r_1 \frac{u_1}{r_2 - r_1} = u_1 \left\{ 1 + \frac{r_1}{r_2 - r_1} \right\} + \\ &\quad + u_2 \left\{ -\frac{r_1}{r_2 - r_1} \right\} \\ &= u_1 \left\{ \frac{r_2}{r_2 - r_1} \right\} + u_2 \left\{ -\frac{r_1}{r_2 - r_1} \right\} \end{aligned}$$

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} \frac{r_2}{r_2-r_1} & -\frac{r_1}{r_2-r_1} \\ \frac{-1}{r_2-r_1} & \frac{1}{r_2-r_1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad \alpha_e = A_e^{-1} \cdot K_e$$

$$A_e^{-1} = \begin{bmatrix} \frac{r_2}{r_2-r_1} & -\frac{r_1}{r_2-r_1} \\ -\frac{1}{r_2-r_1} & \frac{1}{r_2-r_1} \end{bmatrix} \quad \nabla_{A_e}^{-1} = \begin{bmatrix} \frac{r_2}{r_2-r_1} & -\frac{1}{r_2-r_1} \\ \frac{r_1}{r_2-r_1} & \frac{1}{r_2-r_1} \end{bmatrix}$$

$$U_e = \frac{1}{2} \nabla_{A_e}^{-1} K_e \alpha_e = \frac{1}{2} \nabla_{A_e}^{-1} Q_e u_e$$

$$Q_e = A_e^{-1} K_e A_e^{-1}$$

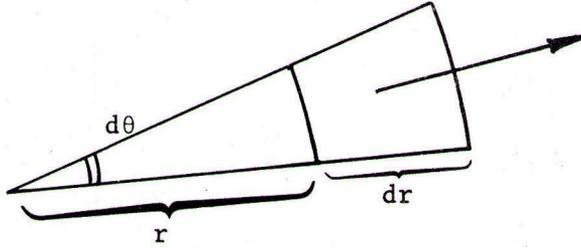
$$\text{met } \alpha_e = A_e^{-1} u_e$$

$$Q_e = 2\pi \frac{Eh}{1-\nu^2} \begin{bmatrix} \frac{r_2}{r_2-r_1} & -\frac{1}{r_2-r_1} \\ -\frac{r_1}{r_2-r_1} & \frac{1}{r_2-r_1} \end{bmatrix} \begin{bmatrix} \ln \frac{r_2}{r_1} & (1+\nu)(r_2-r_1) \\ (1+\nu)(r_2-r_1) & (1+\nu)(r_2-r_1)^2 \end{bmatrix}$$

$$\cdot \begin{bmatrix} \frac{r_2}{r_2-r_1} & -\frac{r_1}{r_2-r_1} \\ -\frac{1}{r_2-r_1} & \frac{1}{r_2-r_1} \end{bmatrix}$$

$$= 2\pi \frac{Eh}{1-\nu^2} \frac{1}{(r_2-r_1)^2} \begin{bmatrix} r_2^2 \ln \frac{r_2}{r_1} - (1+\nu)(r_2-r_1)^2 & -r_1 r_2 \ln \frac{r_2}{r_1} \\ -r_1 r_2 \ln \frac{r_2}{r_1} & r_1^2 \ln \frac{r_2}{r_1} + (1+\nu)(r_2-r_1)^2 \end{bmatrix}$$

Bepaling van de kinematisch consistente belastingvector  $f_{ce}$



$$\rho \omega^2 h r^2 dr d\theta = \hat{q}_e^0$$

potentiaal van de uitw. belasting:  $\Omega_{ce} = - \int_{r=r_1}^2 \int_{\theta=0}^{2\pi} \nabla_e \hat{u}_e \hat{q}_e^0 =$

$$= - \nabla_e \int_{r=r_1}^2 \int_{\theta=0}^{2\pi} \Phi_e \hat{q}_e^0$$

$$= - \nabla_e \int_{r_1}^2 \int_0^{2\pi} \left( \frac{1}{r} \right) \rho \omega^2 h r^2 dr d\theta = - \nabla_e \nabla_e^{-1} \left\{ 2\pi \rho \omega^2 h \int_{r_1}^2 \begin{bmatrix} r^2 \\ r^3 \end{bmatrix} dr \right\}$$

$$= - \nabla_e \nabla_e^{-1} 2\pi \rho \omega^2 h \left[ \begin{array}{l} \frac{1}{3} r^3 \\ \frac{1}{4} r^4 \end{array} \right] \Big|_{r_1}^{r_2} = - \nabla_e \nabla_e^{-1} 2\pi \rho \omega^2 h \left[ \begin{array}{l} \frac{1}{3} (r_2^3 - r_1^3) \\ \frac{1}{4} (r_2^4 - r_1^4) \end{array} \right] =$$

$$= - \nabla_e f_{ce}$$

$$f_{ce} = 2\pi \rho \omega^2 h \nabla_e^{-1} \left[ \begin{array}{l} \frac{1}{3} (r_2^3 - r_1^3) \\ \frac{1}{4} (r_2^4 - r_1^4) \end{array} \right] = 2\pi \rho \omega^2 h \left[ \begin{array}{cc} \frac{r_2}{r_2 - r_1} & - \frac{1}{r_2 - r_1} \\ - \frac{r_1}{r_2 - r_1} & \frac{1}{r_2 - r_1} \end{array} \right]$$

$$f_{ce} = \frac{2\pi \rho \omega^2 h}{r_2 - r_1} \left[ \begin{array}{l} \frac{1}{3} r_2 (r_2^3 - r_1^3) - \frac{1}{4} (r_2^4 - r_1^4) \\ - \frac{1}{3} r_1 (r_2^3 - r_1^3) + \frac{1}{4} (r_2^4 - r_1^4) \end{array} \right]$$

Uitwerking opgave 9

Evenwichtrelaties:  $\Sigma H = 0 \quad \longrightarrow H_2 = -(H_1 + H_3) \quad (1)$

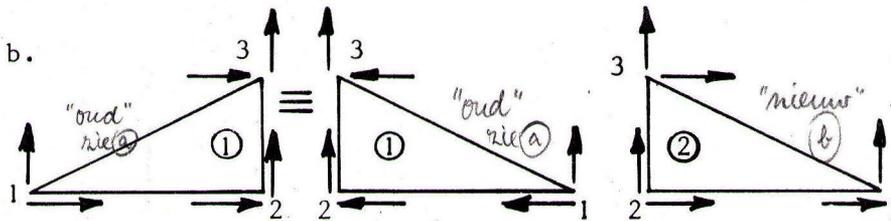
$\Sigma M_{t.o.v.2} = 0 \quad \longrightarrow V_1 = -\frac{1}{2} H_3 \quad (2)$

$\Sigma V = 0 \quad \longrightarrow V_2 = -(V_1 + V_3) \quad (3)$

Dan:

$$\begin{matrix} H_1 \\ H_2 \\ H_3 \\ V_1 \\ V_2 \\ V_3 \end{matrix} = \begin{bmatrix} 8 & -8 & 0 & 0 & 4 & -4 \\ -8 & 20 & -12 & 6 & -10 & 4 \\ 0 & -12 & 12 & -6 & 6 & 0 \\ 0 & 6 & -6 & 3 & -3 & 0 \\ 4 & -10 & 6 & -3 & 35 & -32 \\ -4 & 4 & 0 & 0 & -32 & 32 \end{bmatrix} \begin{matrix} u_1 \\ u_2 \\ u_3 \\ v_1 \\ v_2 \\ v_3 \end{matrix}$$

$f_e = \quad \quad \quad Q_e \quad \quad \quad u_e$



$\longrightarrow Q_2$  ontstaat uit  $Q_1$  door zowel de rijen als de kolommen 1, 2 en 3 met  $-1$  te vermenigvuldigen.

Dus  $Q_2 = \begin{bmatrix} 8 & -8 & 0 & 0 & -4 & 4 \\ -8 & 20 & -12 & -6 & 10 & -4 \\ 0 & -12 & 12 & 6 & -6 & 0 \\ 0 & -6 & 6 & 3 & -3 & 0 \\ -4 & 10 & -6 & -3 & 35 & -32 \\ -4 & -4 & 0 & 0 & -32 & 32 \end{bmatrix}$

*Hier is lokale nummering gebruikt.*

c. 
$$\mathbf{v}_t = \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \underbrace{[u_2 \ u_4 \ v_4]}_{u_l} & \underbrace{[u_1 \ u_3 \ v_1 \ v_2 \ v_3]}_{u_s} \end{matrix}$$

$$\mathbf{f}_t = \left[ \underbrace{[H_2 \ H_4]}_{f_l} \quad \underbrace{[V_4]}_{f_p} \quad \underbrace{[H_1 \ H_3 \ V_1 \ V_2 \ V_3]}_{f_s} \right] ; \quad \mathbf{f}_e = [0 \ 0]$$

