A dialogue between mathematics education and special education: ethics, inclusion and differentiation for all

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Ethical issues play an important role in moulding the philosophy of mathematics education. The present study spells out ethical features of mathematical learning in terms of inclusion. We present the OPEN-MATH project that aims at accomplishing inclusive mathematics learning environments and a teaching learning model based in such a framework.

Keywords: Ethics, inclusion, differentiation, theory of objectification, networking.

Introduction

In this contribution, we present a networking of theories that goes beyond the realm of mathematics education. We are interested in the possible connections between mathematics education and special education. The enlarging of the networking space beyond mathematics education has been prompted by the need to realize inclusive learning of mathematics. The study has been carried out within the OPEN MATH project funded by the University of Bolzano[1], which aimed at the design of a teaching model that fosters inclusive learning of mathematics. The project acknowledges the growing interest in foundational issues regarding ethics, equity and the political in Mathematics Education (Ernest, 2018; Radford, 2021). Inclusion is located at the point of intersection of cognition and learning in mathematics, equity, ethics and the political, and we believe that research in the issue of inclusion in mathematics is a breeding ground both for the foundational features of mathematics education and inclusive education. Furthermore, an inclusive outlook on mathematics education, besides addressing ethical issues, fosters further reflections and investigations about the principles behind the learning of mathematics. The OPEN-MATH project addresses the need for research in the field of inclusion, outlined by Roos (2018) that intertwines theoretical framing of this feature of Mathematics Education and its translation into effective school practices. We focus on the dialogue between the theory of objectification (Radford, 2021) and differentiation (Tomlinson, 2014) to frame inclusion in mathematics and thereby design a model implemented to embody inclusion in the reality of the mathematics practices. The aim of our study is creating a conceptual framework for inclusive mathematical teaching–learning activities. The outcome is a consistent definition of inclusion and a teaching model that accomplishes inclusion in mathematics.

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Didactical Differentiation

We refer to a broad definition of inclusion, not focused solely on students with disabilities: inclusion is understood as a process that aims to identify and overcome any barriers that hinder students from accessing education and achieving optimal outcomes, both in learning and socialization (Ainscow, 2016), therefore, it applies to each student, and not only to a specific category.

To promote participation and learning for all, teaching must simultaneously be sensitive to differences and recognize the similarities of all in being part of a shared community. Teachers and educators are required to face a great challenge. They cannot disregard the use of specific measures for some, which risks an unfair flattening of differences, but neither enhance such differences thereby stigmatizing some students.

Based on these premises, the methodological plan of inclusive teaching requires a design that alternates settings aimed at creating a space in which each student can follow his or her own individual and unique path of learning and others that offer an opportunity for collaborative action, in which each can contribute in his or her own way to a common project. The methodology of teaching differentiation, introduced by Tomlinson (2014), attempts to strike this balance. In the OPEN-MATH, the development of classroom environments encourages both plural and decentralized learning processes, and personal paths in which the student is led to choose according to their needs. This allows for differentiation for all pupils. Furthermore, the centrality of freedom of choice on the part of pupils introduces the possibility of "self-determined differentiation": in a learning landscape that offers multiple opportunities, teachers no longer need to design the most appropriate delivery for each student but support subjective and competent choice on the part of girls and boys.

We can say that the vision of the student according to the perspective of differentiation is that which sees them capable of self-determination and of positioning themselves with respect to the community, they belong to, without, however, losing the link with it. In the OPEN-MATH project, this perspective leads the teacher to pay specific attention to the preparation of each student, and their interests and motivations to learn. The design of activities is adapted to the individual because the assignment is broad or because the single task becomes plural, divided into multiple activities that the student can choose autonomously. An example of this approach can be found in *stations* (Demo, 2016), where different learning activities related to a main topic characterized by specific learning objectives are structured and made available in the classroom. Students can move from station to station and choose which ones to complete and which ones not to complete. The activities that characterize each station allow differentiated forms of thinking and action. A final passport serves as a way for the student to make note of the stations completed, the difficulties encountered, and what enabled them to learn in the most effective or enjoyable way, but it also allows the teacher to keep track of and understand individual differences in math learning.

The Theory of Objectification

In this section, we outline to basic features of the theory of objectification that allow us to realize inclusive mathematics inclusion as differentiation for all, described in the previous section. We will discuss learning as a double-sided construct whose elements are *processes of objectification* and *processes of subjectification*.

The theory of objectification, embedded in sociocultural perspectives, stems from a profound intertwining between culture and the individuals, accomplished in activity. Activity is the ontological category of the theory of objectification as it realizes the consubstantiality between individuals and their culture. In the stance of the TO, mathematical thinking and learning are not processes confined in the mind, but they are intertwined with individuals' activity. Signs and artefacts play an important role in the TO, beyond the role of something that stands for something else or as mediators of activity. They are considered an integral part of human thinking and human activity (Radford, 2021).

Learning as objectification

The issue of learning is rooted in the dialectics between the individual and their culture. Learning is a movement pushed by the intrinsic differential between the individual and cultural knowledge. In fact, in attending to knowledge the student has to cope with something that in the beginning is different from him, an alterity that challenges, resists and opposes him. Learning is the process that erases such a difference in order to make sense of cultural knowledge and transform it into something familiar that allows new forms of action, thinking, imagination and feeling. In order to reduce the distance between the individual and cultural knowledge, students engage in processes of objectification, i.e., the sensuous activity interrelated with signs and artifacts that allows them to meaningfully encounter mathematical knowledge (Radford, 2021).

We remark that according to the TO, signs and artefacts are constitutive of the activity (Radford, 2021) that leads students to notice mathematical knowledge. They are bearers of an embodied intelligence and culturally endowed with specific patterns of activity that individuals use in their meaning-making processes.

In view of accomplishing inclusive mathematical learning, objectification has a fundamental feature in realizing inclusion as differentiation. Objectification is a multimodal process deriving from the variety of sensorial channels (sight, touch, movement, imagination etc.) and the diversity of semiotic means of objectification (objects, tools, gestures, linguistic devices, icons, symbolic language etc.) interwoven with mathematical activity. The multimodality of mathematical thinking and learning allows each student to learn according to their specific and unique potentials, difficulties, cognitive style and sensorial channels that suit him best.

The dialectics between the individual and his cultural-historical environment is not fully accomplished by objectification. The TO pays special attention to the production of subjectivities in the learning processes. The basic idea is that humans are unfinished projects of life, subjects in continuous transformation in the creation of a singular and unique cultural and historical subject. Learning is a double-faced construct. We analysed objectification as one of the two facets. Its entangled counterpart is *subjectification.*

Learning as subjectification

"Processes of subjectification are based on the idea that humans are always unfinished projects of life, subjects perpetually in the making". (Radford, 2021, p. 35). Subjectification is related to the production and transformation of subjectivities and to the continuous co-production of singular and unique individuals against the backdrop of their cultural-historical domain. It allows students to critically position themselves in cultural-historical mathematical practice, realizing new ways of thought and action (Radford, 2021).

The notion of subjectification is coherently linked to the basic tenet of the theory of objectification that establishes the intertwining between individuals, their sociocultural environment and activity. The individual and their culture live in a dialectical relationship in which individuals produce reality as much as reality produces them. On the one hand, the student acts, thinks and feels according to the social and cultural reality he encounters in the classroom. On the other hand, in realizing their unique and specific project of life, the student critically and creatively establishes a reflexive and agentic relationship with the world that allows him to change it.

Subjectification accomplishes the ethical, social, and psychological nature of inclusion fostering self-determination, self-realization, attention, and respect of other human beings contributing to the creation of an educational and social environment where teachers and students co-produce themselves in the dialectical relationship with their cultural-historical context.

Networking strategies: a conceptual framework for inclusive mathematics learning

Mathematical learning develops along two complementary plots: social interaction and self-determination. Social interaction is the fabric of the learning of mathematics envisaged as a joint activity that involves teacher and students towards the production of mathematical knowledge. Self-determination nurtures individual's unique distinctive traits, needs, potentials, desires as they creatively engage in mathematical learning. True inclusion cannot be accomplished if we neglect one of the two plots. If we disregard self-determination, the student might not have the possibility to enter the mathematical practices, thus being excluded, or learning mathematics clashes against his needs, desires and individual features. If we disregard social interaction, a meaningful learning of mathematics cannot occur, and we miss the political and ethical issues of mathematics education that call for the production of ethical subjects open to others and their culture. A feature that our conceptual framework should encompass is the multimodal mode of activity students deploy in the learning of mathematics.

In the previous section, we have described the tenets of the TO and differentiated learning featured as open learning. We now sketch how we used the networking strategies devised by mathematics education to construct a conceptual framework for inclusive learning in mathematics.

Lotman's semiosphere

We will carry out networking within Lotman's (1990) semiosphere. A semiosphere is characterized by the following elements: a system of practices, a meta-language, themes, plots that can be developed within this sociocultural space, and the coexistence of multicultural identities, which in our project refer to mathematics and special education.

The semiosphere is extremely effective to study the connection of theories since it is a space that fosters interaction and dialogue between different theories. Networking can be conceived as a dialogue between theories that takes place in the semiosphere using a metalanguage that allows the different theoretical perspectives to communicate and interact. In a networking perspective, the semiosphere blends two important plots: integration that refers to the intertwining between theories and identity that refers to the internal consistency and distinctive traits of a theory. The dialogue between theories overcomes barriers respecting their identity and consistency.

The semiosphere plays a prominent role in our study, in the light of establishing a dialogue between two theoretical perspectives belonging to different disciplines within the educational domain. In fact, we enlarged the semiosphere to include mathematics education and special education in the meta-theoretical dialogue. Within the enlarged semiosphere, we established a specific meta-language, we acknowledged the common system of practices, we allowed the co-existence of the culture of math education and inclusive education in our research, and we developed the plots of integration and identity.

Connecting strategies

Prediger and Bikner-Ahsbahs (2014) outline a "landscape" of possible connecting strategies, which balance the plots of identity and integration. For the scope of our work, we reckoned *coordinating* as an effective networking strategy. Coordinating leads to a conceptual framework built by "fitting together elements from different theories for making sense of an empirical phenomenon" (Bikner-Ahsbahs & Prediger, 2014, p. 120). Combining is an appropriate strategy when networking theories whose systems of principle are complementary to one another.

For the objectives of OPEN-MATH, the outcome of the networking strategy should encompass the following features connected to inclusion as differentiation: i) Outline the notion of inclusion in mathematics ii) Provide learning activities that meet personal needs, potentials, and talents of each student, allowing them to be included in the sociocultural activity. iii) Nurture both the individual's distinctive traits and social interaction. iv) Outline a teaching–learning model to be implemented in everyday mathematics classroom, which in our study involves grade seven students.

Within the enlarged semiosphere of educational sciences, the Theory of Objectification and the Open Learning Approach are not completely compatible. The former rests on sociocultural underpinnings centred on joint labour and being with others, whereas the latter on socio-constructivist underpinnings that stress the role of autonomy and self-determination. Social interaction and the individual's agency are present in both theories but with a different hierarchical position in their system of principles. They are not conflicting theories, and they can be coordinated to get a multi-faceted insight into inclusive mathematical practices and fit together elements from the two theories in view of a conceptual framework for inclusion in mathematics.

A conceptual framework for inclusion in mathematics education

To frame the project, we introduced a tentative framework of inclusion that links the attention to the individual with their specificities and mathematics learning as a process of objectification. We list here the *defining principles* of our conceptual framework.

Inclusion is conceived from the student perspective (subjectification), as "the dialectical and critical positioning of all students in the cultural-historical practice of mathematics, who act, feel and think according to their individual distinctive traits to pursue their project of life." (Demo et al., 2021 p. 8)

Mathematical activity, in its multimodal acceptation, is the meeting point of the social and individual dimension of mathematical learning. Multimodality and sensuous cognition link the individual self-determination and the social interaction. In particular, the semiotic means of objectification, which define the modes of activity of both the single student and the class, can be considered the bridge between the subject and the culture.

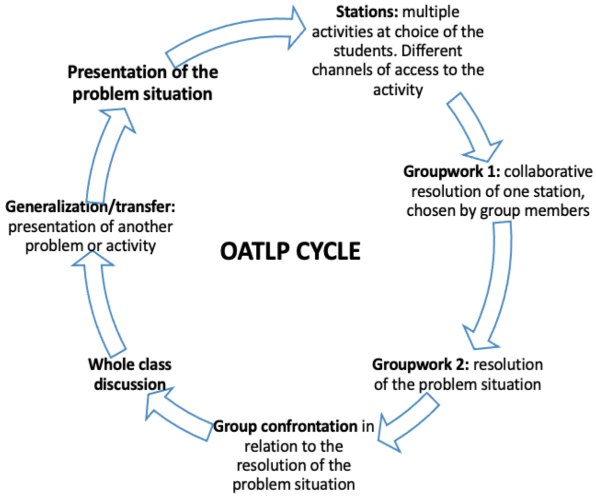


Figure 1: The Open Activity Theory Lesson Plan cycle

Starting from these two principles, we designed a new model of activity that blends self-determination with social interaction. Such a blending is realized by alternating group work with stations. Both group work and stations are informed by the multimodality of objectification and the co-production of subjectivities as they position, according to the notion of subjectification mentioned above, in the sociocultural environment of the classroom. Figure 1 shows the structure of the model that we termed Open Activity Theory Lesson Plan (OATLP) to underline the coordination of activity and open learning that characterize the theory of objectification and differentiation. OATLP is the outcome of the experimentation with a grade 7 class carried out in school year 2020/2021 during the research project.

An example of OATLP

To show how the main phases of the OATLP cycle are built and interact in accordance with the networking of Theory of objectification and Didactical Differentiation we give here an example. The topic of the implemented cycles was chosen in agreement with the mathematics teacher, consistently with the planning of the classroom: in March, we proposed a cycle where the main learning goal could be identified in the resolution of problems related to the estimation of areas. The first individual phase, station work, consisted in several activities that could be chosen by the single student in order to work on the fixed learning goal. In Figure 2, we show as an example two stations: the first, on the left, is more related to a verbal learning approach, while the second, on the right, is more related to a visual learning approach. In the second one, the student is asked to colour the squares in the figure of a tree first considering only the squares that are fully contained within the tree and to count them, and then she is asked to colour the squares that are also partially contained. Then the student is required to estimate the area of the figure as required by the task.

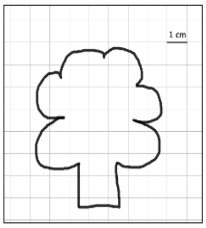
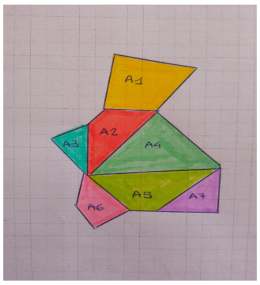
Station 1	Station 2
Invent a problem where you have to measure the extension of a figure or a surface. Try to solve it.	

Figure 2: Two stations out of six from the OATLP cycle of March.



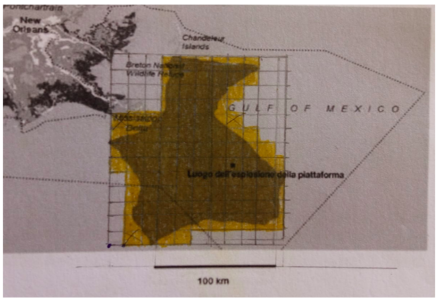


Figure 3: The resolution to the problem situation given by two different groups.

Figure 2 shows two different activities the students were exposed to in two learning stations devoted to the learning of geometry. We can see in this brief example how stations, a didactical methodology that characterizes Open Learning, helps us to exploit the multimodality of mathematics in order to allow each student to choose her path in the learning process, positioning herself with respect to a shared aim. This is also strongly consistent with the principles of Didactical Differentiation: Tomlinson (2014) highlights how differentiation is possible only when the aim of the activity is clear and the essential learning goals are defined. In this example, the aim of the activity is the estimation of areas and the learning goal is defined is a multimodal approach to solving problem.

The individual phase of the OATLP cycle helps the students to access the group work activities according to their individual characteristics and potentialities, and to have a moment to reflect on eventual personal difficulties with the topic, sharing them with a peer or with the teacher before the group work starts. Continuing in the description of the OATLP cycle, after the stations, students were asked to solve a problem, again about area estimation that was more contextualized, and requested the application of a proportion. In Figure 3, we show two solutions of a problem concerning an oil slick in the sea, and the student had to find a method to calculate its area and to justify it.

In addition, within the group work attention was given with respect to the possibility for every student to position herself in the learning goal: in this case, the positioning of the student is made with the other group members in an effort for mutual understanding and joint labour. The phase of group confrontation closes the problem-solving phase allowing each group, and student to justify her work, and to ask for justification to the other: This allows the students to be reciprocal critical friends, and to deepen the understanding of their respective work.

Summarizing, the whole cycle is designed so that there is a balance among the social and the individual dimension of learning, and in order to give every student the possibility to be included, in the sense made explicit above, that is related to the process of becoming a subject that critically position herself with respect to mathematical culture, according to her own characteristics.

Conclusions

Ernest (2018) has singled out ontology and metaphysics, aesthetics, epistemology, learning theory, social and political philosophy, and ethics as the prominent elements of a philosophy of mathematics education. The author deems ethics the *first* philosophy of mathematics education advocating the awareness that we are all the same but different and we are in our present condition (fortunate or unfortunate) only by luck and contingency. He recalls that our subjectivity is formed in and through our subjected-ness to the other, prior to the development of cognition, language, modelling etc. Being-with-others (Radford, 2021) is the true and meaningful motor of learning mathematics, in that the student is engaged in the dialectical interplay between the sociocultural environment of the classroom and their reflexive agency. Resorting to coordinating as a networking strategy, we networked the theory of objectification with differentiation for all, conceived as open learning, to provide a conceptual framework that defines inclusion in mathematics as the intertwining of objectification and subjectification processes. We envisage inclusive mathematics as the production of reflective and ethical subjects who position themselves critically and creatively in mathematical practices, a condition that should be available to all students and respectful of their differences. Inclusion is accomplished via the OATLP teaching model that reflects the coordinating strategy by fostering both social interaction and individual activity in a teaching learning project that involves all students.

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