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A mixed-integer convex approximation for optimal load redistribution in bipolar DC networks with multiple constant power terminals

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ABSTRACT

This paper proposes a mixed-integer convex model for optimal load-balancing in bipolar DC networks while considering multiple constant power terminals. The proposed convex model combines the Branch and Cut method with interior point optimization to solve the problem of optimal load balancing in bipolar DC networks. Additionally, the proposed convex model guarantees that global optimum of the problem is found, which ensures minimal power losses in the bipolar DC distribution grid branches, as the total monopolar load consumption has been balanced at the substation's terminals. In addition, an optimal load balancing improves the voltage profiles due to current redistribution between the positive and negative poles. Numerical results in the 21- and 85-bus test feeders and a comparison with three metaheuristic techniques show the effectiveness of the proposed convex model in reducing the total grid imbalance while minimizing the power losses and improving the voltage profiles.

1. Introduction

The constant growth of electrical consumption has caused technological advances, which improve the quality of life in urban and rural areas, in addition to the continuous adoption of Industry 4.0 to automate production processes. This makes it necessary to transform the electrical distribution sector [1,2]. Such transformation requires adopting new distribution technologies such as hybrid AC/DC networks or DC networks with monopolar and/or bipolar configurations [3]. The use of DC distribution technologies over classical AC networks has two main advantages: (i) improved the electrical efficiency regarding the number of energy losses caused during the distribution process due to nonexistent reactive power in the grids [4]; and (ii) simplified control design, given that DC networks through AC/DC converters only require controlling the voltage magnitude since the frequency control associated with rotating machines in AC networks is not present in DC grids [5,6].

Electrical distribution networks using DC technology can have a monopolar or bipolar structure. In Fig. 1, both configurations are illustrated for a four-node DC network.

The monopolar topology corresponds to a grid with a positive pole and a return cable (neutral wire) that allows serving users through the voltage difference between these terminals [7]. The bipolar configuration corresponds to a grid configuration composed of three conductors (positive and negative poles and the neutral wire), which allows connecting two times the number of loads supported by a monopolar configuration. Hence, the main advantage of bipolar networks is that some loads can work with double the voltage magnitude (bipolar loads) being connected between the positive and negative poles [8].

One of the main differences between monopolar and bipolar DC networks is the latter's capability to work under unbalanced operation conditions, *i.e.*, the positive pole can consume more power than the negative pole (or *vice versa*), which increases the power losses and also deteriorates the voltages in the pole with the highest load [9,10]. Due to the complexities of the bipolar DC networks, given their likely unbalanced operation to analyze them under steady-state conditions, it is necessary to use efficient power flow methodologies [11,12]. Research regarding bipolar DC networks can be classified into two main groups in the current literature. The first group proposes efficient power flow algorithms, while the second group focuses on grid optimization. Some of the most relevant research works on bipolar DC networks are presented below.

The authors of [11] developed a new power flow methodology for strictly radial distribution networks working with floating or solidly

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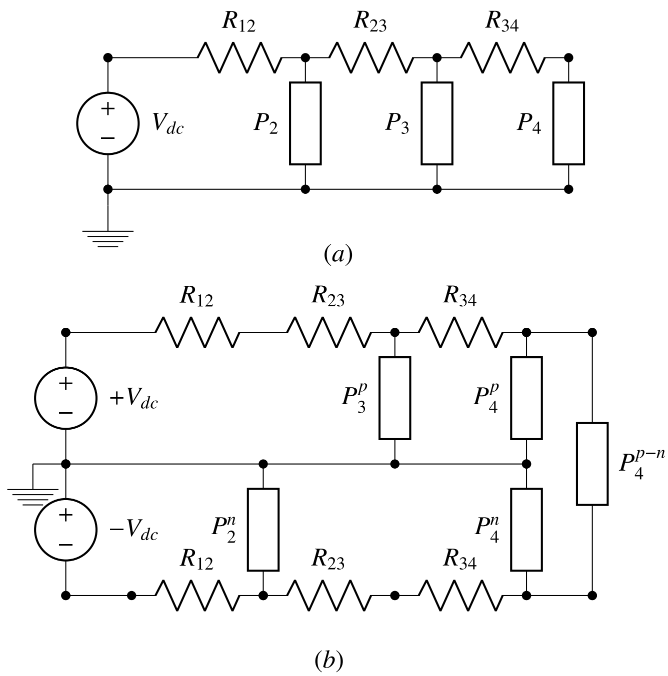

Fig. 1. Possible grid topologies for a DC distribution network with the neutral wire solidly grounded at each node: (a) monopolar configuration and (b) bipolar connection.

grounded neutral wire in all the nodes of the network. The main advantage of this power flow formulation is that it is derivative-free and can be easily formulated using graph theory based on the upper triangular matrix that relates branch and nodal currents. The computational validation employed two test feeders composed of 21 and 85 buses. In Ref. [13], the authors proposed a numerical method to solve the power flow problem in bipolar DC networks with multiple asymmetric constant power terminals. The main advantage of this solution method is its applicability to radial and meshed configurations using a recursive power flow formula that ensures convergence via the Banach fixed-point theorem. Numerical results in a 21-bus grid demonstrated the effectiveness of this method in dealing with the power flow problem in general bipolar DC networks. The authors of [9] presented a generic version of the Newton-Raphson method applied to the power flow solution in bipolar DC grids. The authors' main contribution lies in the inclusion of the droop control constant for dispersed generators, as well as the effect of the neutral grounding through a non-ideal impedance. The numerical validation involved two test feeders composed of 6 and 33 nodes, respectively. All the proposed Newton-Raphson method results were implemented in PSCAD/MTDC, demonstrating the effectiveness of the proposed generic power flow solution. The power flow problem for bipolar DC networks with multiple constant power terminals was addressed in Ref. [14]. The authors presented the application of the nodal circuit method to determine the power balance equation in a small test feeder composed of 4 nodes. However, the authors did not present any contribution regarding the power flow problem –such as an efficient algorithm– and only focused on validating the power flow equations in the PSCAD/MTDC software. The authors of [15] presented a solution strategy to address the optimal power flow problem in bipolar DC networks considering unbalanced loads. The proposed solution strategy was based on the current injection method and considered the complete bipolar grid structure. The authors determined the optimal power injections in dispersed generators to minimize the total grid voltage unbalance.

The authors of [16] implemented a genetic algorithm to determine the optimal connection of monopolar loads in bipolar DC networks. This optimal connection reduces the total grid voltage unbalance and system energy losses. The authors proposed combining the genetic algorithm with an automatic commutation switching scheme to perform load switching. This methodology is based on predefined control diagrams that determine the sequence of switching movements required to reconfigure loads. All the numerical validations were performed in the MATLAB/Simulink software. In Ref. [17], the authors proposed a multi-objective optimization method to solve the optimal phase-balancing problem in bipolar DC networks. Even though the proposed model belongs to the family of mixed-integer linear programming, which ensures that the global optimum is reached, the authors avoided the inclusion of nonlinear loads. They only worked with linear loads, simplifying the real behavior of DC networks with multiple constant power terminals. In Ref. [18], the authors proposed a new solution methodology for the optimal power flow problem in bipolar DC networks while considering voltage imbalances. The authors proposed linearizing the constant power loads to determine the effect of power flow congestion in the distribution lines on nodal prices. Two test feeders were employed to validate the proposed optimization method. However, the authors did not provide any comparison with additional optimal power flow solvers [19]. solved the optimal power flow problem in unbalanced bipolar DC networks using the current injection method. The authors employed the Jacobian matrix of the power flow problem as a sensitive factor to determine the costs of the voltage imbalance. Their numerical results using the current-based injection method were corroborated via the PSCAD/EMTDC software.

Having reviewed state of the art, this research corroborated that there is no work in the current literature regarding the optimal load redistribution for DC bipolar networks with multiple monopolar and bipolar constant power terminals. The only existing approximation corresponds to the mixed-integer linear programming model proposed by Ref. [17] for phase-balancing. However, the authors neglected the constant power terminals and assumed the load terminals to have a linear behavior. Unlike this work, the main contribution of our research can be summarized as follows:

✓ A solution for the optimal load redistribution problem in terminals of the substation for bipolar DC grids with a mixed-integer convex formulation which ensures that the global optimum is found. The optimal solution is reached via the Branch and Cut method, combined with the interior point approach available in the Gurobi solver of the CVX optimization package in the MATLAB programming environment.
✓ A comparison of the proposed convex formulation against metaheuristic optimizers has been included. The results show the proposed convex formulation does not contain the problem of the metaheuristic optimizers, which do not ensure that the global optimum as the system increases in variables these optimizers are not able to reach the optimum of the problem
✓ A technical evaluation of the initial and final grid operation conditions in the bipolar DC network through implementing a specialized power flow solver based on the upper triangular-based matrix. This evaluation allows for determining the total grid power losses and the voltage profiles, which are variables of interest under steady-state operation conditions in electrical networks.

It is important to highlight that the numerical validations were performed in 21- and 85-bus grids, initially used in Ref. [11] to evaluate the triangular-based power flow formulation.

The remainder of this document is structured as follows: Section 2 presents the proposed mixed-integer convex formulation to represent the optimal load balancing problem in bipolar DC networks with multiple constant power terminals; Section 3 describes the main characteristics of the power flow solver, which is based on the upper triangular formulation; Section 4 summarizes the main aspects of the solution methodology, which is composed of the initial evaluation of the electrical state of the network (benchmark case), followed by the solution of the MIC model and its evaluation in the power flow formulation; Section

5 presents the main characteristics of the electrical distribution networks under analysis, *i.e.*, the 21- and 85-bus grids; Section 6 presents the main numerical results, as well as their analysis and discussion; and Section 7 presents the main concluding remarks derived from this research, as well as some proposals for future work.

2. Mixed-integer convex formulation

To balance the loads at the terminals of the substation, a mixed-integer convex model can be implemented [20]. The objective function of this problem corresponds to the minimization of the total deviation in the positive and negative poles concerning the ideal balanced case. This function can be formulated as presented in Equation (1).

$$\min T_{\text{unb}} = \frac{100\%}{2P_{\text{ave}}}\left(\left|p_T^p - P_{\text{ave}}\right| + \left|p_T^n - P_{\text{ave}}\right|\right), \tag{1}$$

where T_{umb} represents the average grid imbalance in percentage; P_{ave} is the ideal power consumption per pole; and p_T^p and p_T^n correspond to the total power consumptions at the terminals of the substation for the positive and negative poles, respectively.

The main advantage of the objective function (1) is that it is a convex objective function with a minimum value of 0 when the grid is perfectly balanced.

To illustrate the structure of the objective function (1), an analogical function $w = |x| + |y|$ is plotted as depicted in Fig. 2.

To ensure the correct operation of the bipolar DC network, the following set of linear constraints is implemented:

$$p_k^p = x_k^p P_k^p + x_k^n P_k^n, \ \{\forall k \in \mathcal{N}\} \tag{2}$$

$$p_k^n = x_k^n P_k^p + x_k^p P_k^n, \ \{\forall k \in \mathcal{N}\} \tag{3}$$

$$p_T^p = \sum_{k \in \mathcal{N}} p_k^p, \tag{4}$$

$$p_T^n = \sum_{k \in \mathcal{N}} p_k^n, \tag{5}$$

$$x_k^p + x_k^n = 1, \ \{\forall k \in \mathcal{N}\} \tag{6}$$

$$\{x_k^p, x_k^n\} \in \{0, 1\}, \ \{\forall k \in \mathcal{N}\} \tag{7}$$

where p_k^p is the final power assigned to the positive pole at node k; p_k^n is the final power assigned to the negative pole at node k; x_k^p is a binary

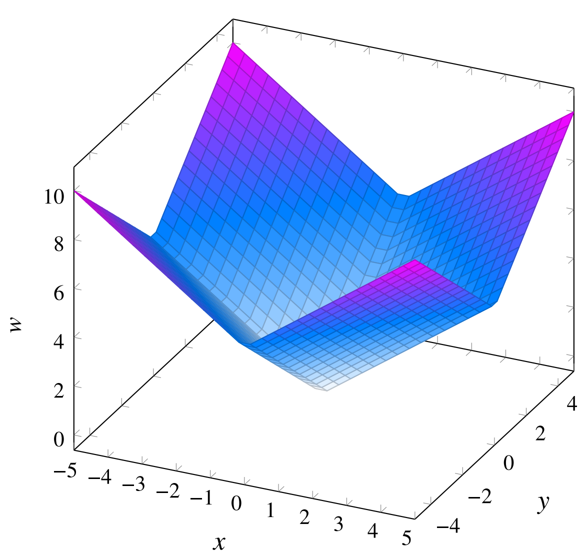

Fig. 2. Graphical representation of the convex function (1).

variable that defines whether the original load connection at node k is maintained; x_k^n is a binary variable that determines that the loads at node k must be interchanged between positive and negative poles, respectively; and P_k^p and P_k^n are the initial load connections for the positive and negative poles at node k.

Note that Equations (2) and (3) define the final load connection at each node k per pole; Equations (4) and (5) quantify the total load connection in the positive and negative poles seen in terminals of the substation, respectively; Equation (6) defines that only one connection for the constant power terminals is possible at each node, *i.e.*, (i) to maintain the initial load connection or (ii) to interchange the loads between poles; and, finally, Equation (7) defines the binary nature of the decision variables x_k^p and x_k^n, respectively.

The main feature of the optimization model (1)–(7) is that it is a mixed-integer convex model, which implies that it is possible to ensure its global optimal solution through specialized optimization tools [21].

It is important to mention that the number of variables for a grid with m nodes is distributed as follows: the number of continuous variables in the MIC model (1)–(7) is $2m + 3$. While the number of integers (binary) variables is $2m$ and the size of the solution space regarding the binary variables is 2^{m+1}.

3. Power flow solution

A specialized power flow tool is required to determine the electrical performance of any bipolar DC network under steady-state conditions. Here, we adopt the triangular-based power flow method, which has been recently developed in Ref. [11] for bipolar DC networks with a solidly grounded or floating neutral wire. This research focuses on a neutral wire that is only grounded at the substation bus and floating at the remaining nodes since this operation scenario has a higher power loss level compared to the solidly grounded simulation scenario [11].

The general power flow formula for the triangular-based power flow method is defined in Equation (8). The main characteristic of this iterative formula is that it can be derived using graph-based theory, as presented in Ref. [22] for AC networks.

$$V_d^{pon,m+1} = \mathbf{A}_{ds} V_s^{pon} - \mathbf{R}_{dd}^{pon} I_d^{pon,m}, \tag{8}$$

where $V_d^{pon,m+1}$ is the vector containing all the demand voltages at iteration $m + 1$ for the positive, neutral, and negative poles, respectively, which is a nonlinear function of the demanded currents $I_d^{pon,m}$. In addition, V_s^{pon} represents the voltage outputs at the substation, which are defined as $V_s^{pon} = [1, 0, -1]^\top V_{\text{nom}}$. Note that $\mathbf{A}_{ds}$ and $\mathbf{R}_{dd,pon}$ are matrices that depend on the grid topology under analysis [22]. In addition, the demanded currents $I_d^{pon,m}$ are hyperbolic relations between voltages and powers, which also depend on the binary variables x_k^p and x_k^n.

It is important to highlight that the demanded current $I_{d,k}^{pon,m}$ depends on the load connection at terminals of the node k, *i.e.*, it is a function of the binary variables x_k^p and x_k^n. The general form of $I_{d,k}^{pon,m}$ is presented in Equation (9).

$$I_{dk}^{pon,m} = \mathbf{H}\text{diag}^{-1}\left(\Delta v_{dk}^{pon,m}\right)\left(x_k^p P_{dk}^{pon} + x_k^n P_{dk}^{nop}\right), \tag{9}$$

where

$$I_{d,k}^{pon,m} = \begin{bmatrix} i_{d,k}^p \\ i_{d,k}^o \\ i_{d,k}^n \end{bmatrix}, \mathbf{H} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & -1 & 1 \end{bmatrix}, P_{dk}^{pon} = \begin{bmatrix} p_{d,k}^p \\ p_{d,k}^{p-n} \\ p_{d,k}^n \end{bmatrix}.$$

$$P_{dk}^{nop} = \begin{bmatrix} p_{d,k}^n \\ p_{d,k}^{p-n} \\ p_{d,k}^p \end{bmatrix}, \ \Delta v_{dk}^{pon,m} = \begin{bmatrix} v_{d,k}^p - v_{d,k}^o \\ v_{d,k}^p - v_{d,k}^n \\ v_{d,k}^n - v_{d,k}^o \end{bmatrix}$$

To define the convergence of the power flow formula (8) to the power flow solution, the maximum difference between two consecutive iterations is tested, *i.e.*, $\max\left\{\left\||V_d^{pon,m+1}| - |V_d^{pon,m}|\right\|\right\} \leq \gamma$, with γ being the maximum convergence error, typically chosen as 1×10^{-10} [11].

Once the power flow problem has been solved, the final grid power losses are assigned to the optimal solution obtained via the MIC model (1)–(7). The total grid power losses are calculated as follows:

$$p_{\text{loss}} = J_l^{pon,\top} \mathbf{R}_{ll}^{pon} J_l^{pon}, \tag{10}$$

where $J_l^{pon,\top}$ represents the vector with all currents passing through the lines, and $\mathbf{R}_{ll}^{pon}$ is the primitive resistance matrix associated with all the branches [11].

4. Summary of the solution methodology

The optimization methodology to redistribute loads at terminals of the substation of bipolar DC networks with a radial structure while also reducing the amount of grid power losses is presented in Algorithm 1.

Algorithm 1
General evaluation of the grid power.

Data: Select the bipolar DC network to be analyzed
Find the per-unit equivalent representation of the network;
Define the number of iterations $m_{\max}$ for the power flow analysis;
Construct the matrices $\mathbf{R}_{ll}^{pon}$, $\mathbf{A}_{ds}$, and $\mathbf{R}_{dd}^{pon}$;
Set γ as 1×10^{-10};
Make $m = 0$;
Define the voltages at each node as $V_{d,k}^{pon,m} = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}^\top$;
Select $O_c = 0$ for the original configuration;
Select $O_c = 1$ for the final solution of the MIC model (1)–(7);
if $Oc == 0$ **then**
    Make $x_k^p = 1$, and $x_k^n = 0$ (original grid configuration);
else
    Solve the MIC model (1)-(7) with the Gurobi solver using CVX in MATLAB;
    Set the final values for x_k^p, and x_k^n;
end
for $m = 1 : m_{\max}$ **do**
    Use formula (9) to define the bipolar current at node k, *i.e.*, the values of $I_{dk}^{pon,m}$;
    Evaluate the power flow formula (8);
    if $\max\left\{\||V_d^{pon,m+1}| - |V_d^{pon,m}|\|\right\} \leq \gamma$ **then**
        Update all the demanded nodal currents $I_{dk}^{pon,m}$;
        Calculate the branch currents $J_l^{pon,\top}$ using the triangular matrix;
        Determine the grid power losses p_{loss};
    else
        Make $V_{d,k}^{pon,m} = V_d^{pon,m+1}$;
    end
end
Result: Report the total grid power losses

It is worth mentioning that the information provided in Algorithm 1 is general for any solution methodology regarding load redistribution in bipolar DC networks with multiple constant power terminals. In addition, it is easily extensible to the optimal pole-swapping problem with the objective of minimizing the total grid power losses using a discrete metaheuristic optimizer that defines the values of the binary variables x_k^p and x_k^n.

5. Test feeders

Two radial test feeders with 21 and 85 buses were employed to validate the proposed optimization model for redistributing loads in bipolar DC distribution networks. The information on the test feeders is presented below.

5.1. 21-Bus system

The 21-bus grid corresponds to the bipolar extension of the original monopolar DC network presented in Ref. [23] to demonstrate the convergence of the Newton-Raphson power flow method using the Kantarovich theorem. The bipolar DC network was initially proposed for validating the effectiveness of the triangular-based power flow method in Ref. [11]. The electrical configuration of this grid is presented in Fig. 3.

The 21-bus grid is characterized by the following aspects:

✓ It is a radial bipolar DC network with multiple monopolar and bipolar constant power terminals;

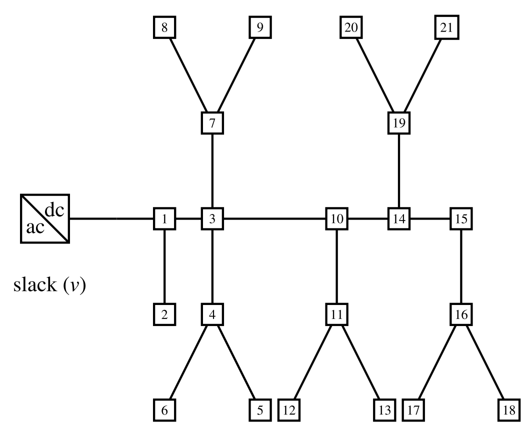

Fig. 3. Electrical configuration of the 21-bus system.

Table 1
Electrical data of the loads and branches for the 21-bus grid (all powers in kW).

Node j	Node k	R_{jk} (Ω)	$P_{d,k}^p$	$P_{d,k}^n$	$P_{d,k}^{p-n}$
1	2	0.053	70	100	0
1	3	0.054	0	0	0
3	4	0.054	36	40	120
4	5	0.063	4	0	0
4	6	0.051	36	0	0
3	7	0.037	0	0	0
7	8	0.079	32	50	0
7	9	0.072	80	0	100
3	10	0.053	0	10	0
10	11	0.038	45	30	0
11	12	0.079	68	70	0
11	13	0.078	10	0	75
10	14	0.083	0	0	0
14	15	0.065	22	30	0
15	16	0.064	23	10	0
16	17	0.074	43	0	60
16	18	0.081	34	60	0
14	19	0.078	9	15	0
19	20	0.084	21	10	50
19	21	0.082	21	20	0

✓ The substation operates with ±1 kV for the positive and negative poles at node 1. While its neutral pole is assigned 0 V.
✓ The total monopolar loads in the positive pole amount to 554 kW and 445 kW in the negative pole.

The complete parametric information of this test feeder is listed in Table 1.

5.2. 85-Bus system

The 85-bus grid is a bipolar DC network originally proposed in Ref. [11] to analyze the power flow problem in unbalanced, strictly radial bipolar DC networks with multiple monopolar and constant bipolar power loads. The substation of this system is assigned at node 1, which is operated with ±11 kV in the positive and negative poles for the neutral wire, which is assigned 0 kV. The electrical diagram of the 85-bus grid is presented in Fig. 4.

The complete parametric information of the 85-bus grid with bipolar structure is presented in Table 2.

It is important to mention that the total monopolar consumption in the positive pole is 1745.48 kW, and the negative pole absorbs 2682.19 kW.

6. Numerical results

To validate the proposed MIC model for redistributing loads in bipolar DC networks with constant power terminals, CVX and the Gurobi solver in the MATLAB 2021*b* programming environment were used. This implementation was made on a PC with an AMD Ryzen 7 3700 2.3-GHz processor and 16.0 GB RAM, which was running a 64-bit version of Microsoft Windows 10 Single Language.

6.1. Initial and final load redistribution behavior

Fig. 5 presents the initial and final load imbalances in the 21- and 85-bus grid before and after solving the MIC model (1)–(7). Note that these imbalances are calculated per pole, as presented below:

$$I_p = \frac{100\%}{P_{\text{ave}}}\left(\left|p_T^p - P_{\text{ave}}\right|\right), \tag{11}$$

$$I_n = \frac{100\%}{P_{\text{ave}}}\left(\left|p_T^n - P_{\text{ave}}\right|\right), \tag{12}$$

where the ideal power consumptions for the 21-bus and 85-bus grids are 499.5 kW and 2213.835 kW, respectively.

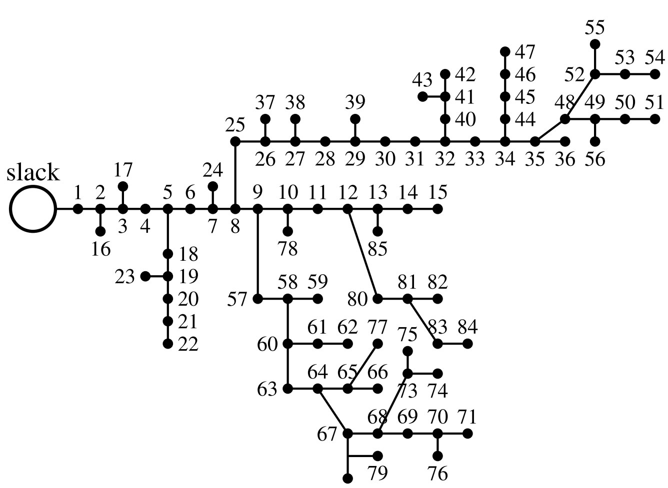

Fig. 4. Electrical configuration of the 85-bus system.

Table 2
Electrical data of the loads and branches for the 85-bus grid (all powers in kW).

Node j	Node k	$R_{jk}(\Omega)$	P^p_{dk}	P^p_{dk}	P^{p-n}_{dk}
1	2	0.108	0	0	10.075
2	3	0.163	50	0	40.35
3	4	0.217	28	28.565	0
4	5	0.108	100	50	0
5	6	0.435	17.64	17.995	25.18
6	7	0.272	0	8.625	0
7	8	1.197	17.64	17.995	30.29
8	9	0.108	17.8	350	40.46
9	10	0.598	0	100	0
10	11	0.544	28	28.565	0
11	12	0.544	0	40	45
12	13	0.598	45	40	22.5
13	14	0.272	17.64	17.995	35.13
14	15	0.326	17.64	17.995	20.175
2	16	0.728	17.64	67.5	33.49
3	17	0.455	56.1	57.15	50.25
5	18	0.820	28	28.565	200
18	19	0.637	28	28.565	10
19	20	0.455	17.64	17.995	150
20	21	0.819	17.64	70	152.5
21	22	1.548	17.64	17.995	30
19	23	0.182	28	75	28.565
7	24	0.910	0	17.64	17.995
8	25	0.455	17.64	17.995	50
25	26	0.364	0	28	28.565
26	27	0.546	110	75	175
27	28	0.273	28	125	28.565
28	29	0.546	0	50	75
29	30	0.546	17.64	0	17.995
30	31	0.273	17.64	17.995	0
31	32	0.182	0	175	0
32	33	0.182	7	7.14	12.5
33	34	0.819	0	0	0
34	35	0.637	0	0	50
35	36	0.182	17.64	0	17.995
26	37	0.364	28	30	28.565
27	38	1.002	28	28.565	25
29	39	0.546	0	28	28.565
32	40	0.455	17.64	0	17.995
40	41	1.002	10	0	0
41	42	0.273	17.64	25	17.995
41	43	0.455	17.64	17.995	0
34	44	1.002	17.64	17.995	0
44	45	0.911	50	17.64	17.995
45	46	0.911	25	17.64	17.995
46	47	0.546	7	7.14	10
35	48	0.637	0	10	0
48	49	0.182	0	0	25
49	50	0.364	18.14	0	18.505
50	51	0.455	28	28.565	0
48	52	1.366	30	0	15
52	53	0.455	17.64	35	17.995
53	54	0.546	28	30	28.565
52	55	0.546	38	0	48.565
49	56	0.546	7	40	32.14
9	57	0.273	48	35.065	10
57	58	0.819	0	50	0
58	59	0.182	18	28.565	25
58	60	0.546	28	43.565	0
60	61	0.728	18	28.565	30
61	62	1.002	12.5	29.065	0
60	63	0.182	7	7.14	5
63	64	0.728	0	0	50
64	65	0.182	12.5	25	37.5
65	66	0.182	40	48.565	33
64	67	0.455	0	0	0
67	68	0.910	0	0	0
68	69	1.092	13	18.565	25
69	70	0.455	0	20	0
70	71	0.546	17.64	38.275	17.995
67	72	0.182	28	13.565	0
68	73	1.184	30	0	0
73	74	0.273	28	50	28.565
73	75	1.002	17.64	6.23	17.995

(continued on next page)

Table 2 (*continued*)

Node j	Node k	$R_{jk}(\Omega)$	P_{dk}^{p}	P_{dk}^{p}	P_{dk}^{p-n}
70	76	0.546	38	48.565	0
65	77	0.091	7	17.14	25
10	78	0.637	28	6	28.565
67	79	0.546	17.64	42.995	0
12	80	0.728	28	28.565	30
80	81	0.364	45	0	75
81	82	0.091	28	53.75	0
81	83	1.092	12.64	32.995	62.5
83	84	1.002	62	72.2	0
13	85	0.819	10	10	10

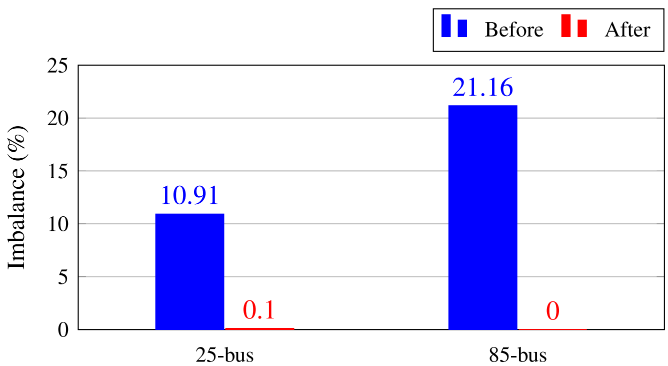

Fig. 5. Behavior of the imbalances before and after solving the MIC model.

In Fig. 5 can be noted that the deviation of the 21-bus system for the ideal consumption is about 10.91%, whereas, for the 85-bus grid, it is about 21.16%.When the MIC model is solving for the 21-bus grid, the final imbalance is reduced to 0.10%. This implies that the total consumption for the positive pole is 500 kW and 499 kW for the negative pole. In the case of the 85-bus system, the final imbalance is 0%, which means that the positive and negative poles are perfectly balanced, with a final consumption equal to the expected value, *i.e.*, 2213.835 kW.

6.2. Comparative analysis with metaheuristic optimizers

To evaluate the effectiveness of our MIC model at finding the global optimal solution for the optimal load balancing problem, its efficiency is contrasted with three well-known optimization algorithms recently proposed in Ref. [24]. These algorithms correspond to: (i) the Chu and Beasley Genetic Algorithm (CBGA), (ii) the black-hole optimizer (BHO), and (iii) the sine-cosine algorithm (SCA). Table 3 presents the numerical performance of each algorithm after 100 consecutive runs. Note that these algorithms were parameterized with 10 individuals in the initial population and 1000 iterations per evaluation. These parameters were taken from Ref. [24].

The numerical results in Table 3 show that:

✓ All the numerical methods reach the global optimal solution for the 21-bus grid, *i.e.*, 0.1001%. The CBGA and the BHO are 100% effective, while the SCA exhibits a repeatability of 85% for this system after 100 consecutive evaluations. Note that the SCA, even though it is efficient at redistributing loads in the 21-bus system, it sometimes gets stuck in local optima.

✓ For the 85-bus grid, the CBGA and the BHO reach the global optimal solution (*i.e.*, 0% of imbalance). However, the CBGA only found this value five times after 100 consecutive evaluations, while the BHO found it only once. These values confirm that, once the solution space size increases, the efficiency of metaheuristic optimizers decreases drastically. Hence, for the 85 bus grid, the CBGA is 95% likely to get stuck in a local optimum, which increases to 99% in the case of the BHO. Note that, in the case of the SCA for the 85-bus system, after 100 consecutive evaluations, the global optimal solution is not found, which confirms that this algorithm only ensures good quality local optimums.

✓ The numerical results obtained with the MIC model confirm the effectiveness of the proposed formulation at solving the optimal load redistribution problem, as its mixed-integer convex structure ensures that the global optimum is found with an appropriate solution tool. The main advantage of this formulation is that each MIC model is solved, and the solution will be always the same (the same load redistribution scenario) for the same power inputs (*i.e.*, the global optimum).

As for the processing times, note that the metaheuristic optimizers take less than 1 s to solve the MIC formulation. However, a statistical analysis is required to determine the average behavior and the efficiency of each algorithm. The above implies that, after 100 executions, all of them are between 12 and 63 s, while the proposed MIC model does not require this evaluation, as it is more efficient with respect to the studied problem.

Table 3
Comparative analysis between the proposed MIC model and the CBGA, the BHO, and the SCA.

Method	Min. (%)	Max. (%)	Mean (%)	Std. Dev. (%)	Time (s)	Rep. (%)
21-bus system						
CBGA	0.1001	0.1001	0.1001	0	0.1247	100
BHO	0.1001	0.1001	0.1001	0	0.3574	100
SCA	0.1001	0.7007	0.1341	0.0902	0.1656	85
MIC	0.1001	0.1001	0.1001	0	0.8095	100
85-bus system						
CBGA	0	0.0045	0.0013	8.9500×10^{-04}	0.1260	5
BHO	0	0.0425	0.0075	6.9000×10^{-03}	0.6213	1
SCA	2.2585×10^{-04}	0.3006	0.0386	0.0567	0.2676	0
MIC	0	0	0	0	1.3545	100

6.3. Electrical performance

Two main indicators are presented to determine the positive changes in the 21- and the 85-bus grids when the optimal load redistribution is implemented. The first indicator corresponds to the initial and final power losses and the second to the voltage profile behavior in the neutral wire. The optimal solution obtained with the proposed MIC model to redistribute loads in bipolar DC networks is presented in Table 4.

Table 4
Optimal solutions reached with CVX and the Gurobi solver for the MIC model (1)–(7) for the 21- and 85-bus grids.

Method	Nodes with mod.	p_{loss} (kW)	Red. (%)	Time (s)
21-bus system				
Ben. case	–	95.4237	–	–
CVX	$\begin{bmatrix} 2,4,5,8,9,10,11, \\ 15,16,17,18,19,21 \end{bmatrix}$	92.0798	3.5043	0.8095
85-bus system				
Ben. case	–	489.5759	–	–
CVX	$\begin{bmatrix} 3,4,5,6,7,8,9,10,11, \\ 13,14,15,17,18,20,21, \\ 22,25,27,30,31,32,36, \\ 38,40,42,43,44,45,46, \\ 48,50,52,53,55,56,57, \\ 59,61,62,65,66,69,70, \\ 72,73,74,75,76,77,80, \\ 81,83,84,85 \end{bmatrix}$	441.3573	9.8491	1.3545

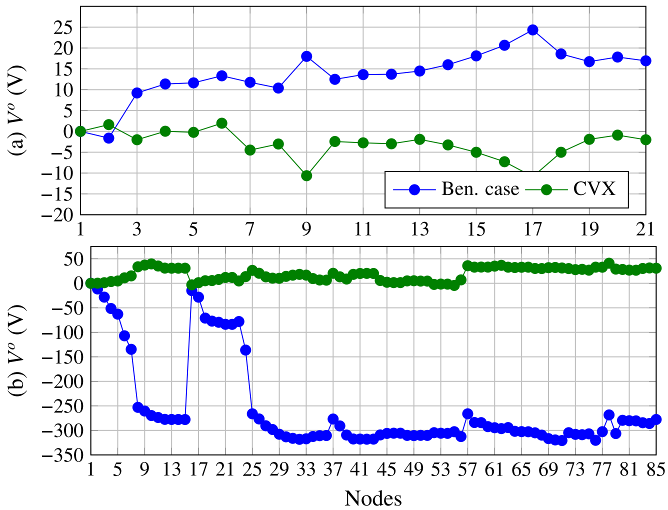

Fig. 6. Voltage behavior in the neutral wire: (a) 21-bus grid, (b) 85-bus grid.

The results in Table 4 show that:

i. The proposed optimization model finds that the optimal solution for the 21-bus grid requires load movements in 13 nodes, whereas the optimal solution for the 85-bus grid requires load interchange in 55 nodes.
ii. The minimization of the total load imbalance at terminals of the substation has positive effects on the total power loss minimization indeed. The 21-bus grid goes from 95.4237 kW to 92.0798 kW, which implies a reduction of 3.5043%. The improvement in the 85-bus grid is about 9.8491%, *i.e.*, from 489.5759 kW to 441.3573 kW.
iii. Regarding the processing times, it is important to note that the solution of the MIC model with 2^{22} possible solutions in the 21-bus grid only takes 0.8095 s. While for the 85-bus grid with a dimension of the solution space of 2^{86}, the total processing time increases to 1.3545 s. These results confirm the scalability of the MIC model with minimal increments in the total processing times.

To confirm the positive effect on the voltage profile after the load redistribution, Fig. 6 presents the behavior of the voltage in the neutral wire before and after implementing the MIC solution.

The voltage profiles in Fig. 6 show that:

i. For the 21-bus grid, the voltage magnitude in the neutral wire has a peak value of 24.3408 V at node 17, which is reduced after implementing the MIC solution to a magnitude of 10.8798 V (the negative sign is neglected since it is not important for this analysis). For the 85-bus grid, a maximum peak in the voltage of the wire conductor occurred at node 71, with a magnitude of 320.6733 V. However, after implementing the solution reached with the MIC model in Table 4, the maximum peak was 40.9263 V at node 78. These results demonstrate that the load redistribution effectively reduced the peak of the voltage drops in the neutral wire.
ii. The average voltage value before the load redistribution had a magnitude of 13.6938 V for the 21-bus grid and 254.2004 V for the 85-bus grid. After implementing the proposed load balancing strategy, these values decreased to 3.0055 V and 18.7630 *V*, which indicates averaged improvements of 78.0521 and 92.6188% for the benchmark cases in both test feeders. This confirms the positive impact of the load redistribution on the neutral voltage profile.

Finally, it is important to highlight that the solution reported with CVX and the Gurobi solver is indeed the global optimal solution for the load balancing problem, as it is solved with a combination between the Branch and Cut method and the interior point approach, which ensures that the global optimum is reached [20]. In addition, the Mosek solver in the CVX tool was also used to confirm this result, and the numerical results (Table 4) were the same.

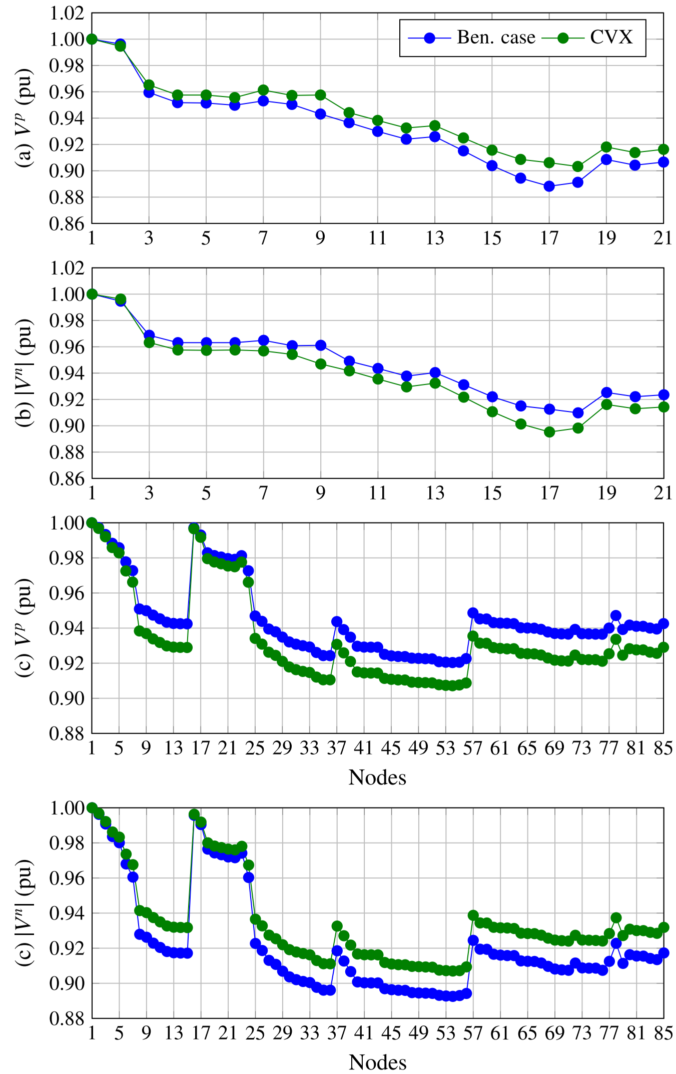

Fig. 7. Voltage behavior in the positive and negative poles: (a) 21-bus grid, (b) 85-bus grid.

Fig. 7 shows the positive and negative voltage profiles before and after optimal load balancing at the substation terminals of both test systems. This figure demonstrates the positive effect of optimal load balancing in bipolar DC networks with regard to the voltage profiles.

Fig. 7 shows that the positive and negative voltage profiles can improve or deteriorate when optimal load balancing is implemented. This behavior is normal since properly distributing the loads in the bipolar DC network is required in order to reduce its losses.

6.4. Economical assessment

To validate the economical feasibility of the proposed load balancing methodology. The benefit of implementing the proposed load balancing solution in the bipolar DC network was evaluated, *i.e.*, the reduction in the costs of the energy losses, by including the costs of sending a working group to physically implement the optimal solution in the distribution grid. The analysis presented herein is based on the economical assessment proposed in Ref. [25] for the phase balancing problem in

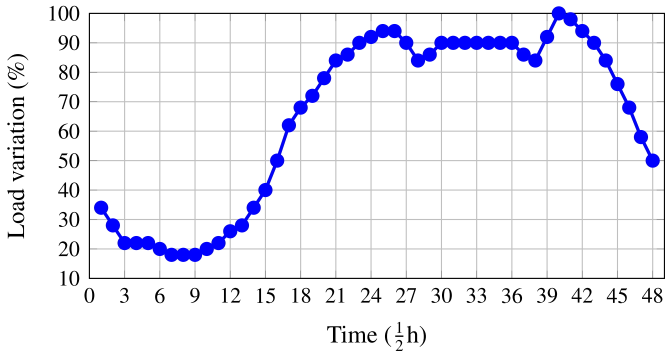

Fig. 8. Typical demand behavior in an ordinary day (working day) in Colombia [25].

Table 5
Economical effect of the load balancing problem in bipolar DC networks.

Moment	Losses costs (USD/kWh-year)	Intervention costs (USD)	Total costs (USD)
21-bus grid			
Before	57042.7470	0	57042.7470
After	55214.3514	1300	56514.3514
85-bus grid			
Before	291486.2114	0	291486.2114
After	264971.8805	5500	270471.8805

three-phase electrical networks. Fig. 8 depicts the average demand curve of all the nodes of the network.

To evaluate the annual expected costs of the energy losses, it was considered that the average energy costs was 0.1390 USD/kWh. This value corresponded to the energy costs in Bogotá, Colombia in May 2019 [26]. The number of days is considered to be 365 for an ordinary year, and the length of the power flow period, Δ_t, is 0.5 h. Note that the cost of the load balancing of the working group is specified as USD 100 per node that requires modifying its load connection.

Table 5 reports the economical evaluation of the proposed methodology in both test feeders before and after implementing the load balancing solution in the bipolar DC network.

The numerical results in Table 5 show that: (i) the implementation of the load balancing long the distribution grid has a positive effect on both test feeders, as the energy losses costs during the operation horizon are reduced by 1828.3956 dollars per year of operation in the case of the 21-bus grid, and by 26514.3308 dollars per year of operation in the case of the 85-bus grid; (ii) the investment required to implement the load balancing solution in the 21-bus grid was about 1300 dollars, which increases to 5500 dollars for the 85-bus grid (however, in both cases, the net profits during the first year of operation were USD 528.3956 and 21014.3308, respectively); and (iii) the percentage of improvement in the 21-bus grid was 0.9623%, which increases to 7.2093% in the case of the 85-bus grid.

7. Conclusions and future work

The problem regarding the optimal load redistribution in bipolar DC networks with multiple monopolar and bipolar constant power terminals was addressed in this research through a mixed-integer convex formulation. It was ensured that the proposed MIC model found the global optimum by combining the Branch and Cut method and the interior point approach available via the MATLAB programming environment CVX tool, namely with the Gurobi solver. Numerical results in 21-and 85-bus grids showed reductions in the total load grid imbalances of 91.66 and 100%, respectively, compared with the benchmark case. Additionally, the proposed convex model was compared to three metaheuristic techniques, which reached the optimal solution shown by the proposed model in the 21-bus grid. However, in the 85-bus grid, they did not always achieve the optimal solution.

When the initial and final electrical performances of the 21- and 85-bus grids with the upper triangular power flow approach were compared, it was observed that: (i) in the 21-bus grid, the power losses were reduced by about 3.5043% (*i.e.*, 3.3439 kW), whereas the reduction in the 85-bus grid was about 9.8491% (*i.e.*, 48.2186 kW), respectively; (ii) regarding the processing times, it was observed that the total computation times were lower than 1.5 s, which includes two power flows (initial and final evaluations) and the solution of the MIC model with a dimension of the solution space of 2^{m+1} (where m is the number of nodes of the network); and (iii) the behavior of the voltage in the neutral wire showed the positive effect of the load balancing in the network: for the 21-bus grid, the average voltage improvement was 78.0521%, while, for the 85-bus grid, it was about 92.6188% when compared to the mean grid voltages for the benchmark case and the solution provided with the MIC model.

When comparing the voltage profiles before and after implementing the load redistribution strategy in the 21- and 85-bus grids, the voltage regulation improved from 11.1740 to 10.4718% for the 21-bus grid, and from 10.7452 to 9.2999% for the 85-bus grid. In addition, once the load redistribution strategy had been implemented while considering a year of continuous operation, the 21-bus grid had a positive net profit of USD 528.3956 and the 85-bus grid gained USD 21014.3308. These results demonstrate the effectiveness of the proposed load balancing strategy at improving the electrical performance of unbalanced bipolar DC networks.

Some proposals for future work are (i) improving the proposed MIC model by including the current and voltage variables to formulate a new optimization model that reduces the total grid power losses and (ii) proposing a sequential quadratic convex formulation to solve the optimal power flow problem in bipolar DC networks with high penetration of dispersed generators.

Credit author statement

O.D. Montoya: conceptualization, methodology, software, validation, formal analysis, investigation, resources, data curation, writing (original draft, review, and editing), visualization, supervision, project management, acquisition of funds. **A. Molina-Cabrera:** conceptualization, methodology, writing (review and editing). **W. Gil-González:** conceptualization, methodology, software, validation, formal analysis, investigation, resources, data curation, writing (original draft, review and editing, visualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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