Mathematical modeling the heat balance of a solar pond device

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Abstract: This article describes the method of accumulation and use of solar energy in the development of heat energy necessary for industrial processes and building heat supply in the southern climate of Uzbekistan. The thermal and technical calculation of the experimental solar pond built in the city of Karshi (size is 0.7m and 1.5m, height is 1.5m) is presented in the research paper. The energy storage efficiency of the solar pond layers was calculated analytically at 25% salt concentration of pond water, in values of 700 W/m^2, 800 W/m^2, 900 W/m^2 and 1000 W/m^2 of solar radiation, in values of 20 °C, 25 °C, 30 °C, 35 °C of ambient temperature in the non-stationary mode of solar pond device. Mathematical model of the time variation of layer temperatures was developed (on the basis of boundary conditions) during the propagation and absorption of solar radiation along the depth of the solar pond.

1 Introduction

In the development of the green economy in the world, the use of environmentally friendly methods, the production of electricity and thermal energy, and the increase of the share of renewable energy sources in the energy supply systems are urgent tasks. [1]. Among the renewable energy sources, solar energy is taking the leading place in meeting the energy needs of various industries. Among the solar devices, the solar pond is less studied by scientists, and it is possible to accumulate solar thermal energy and use its heat in various processes [2-5].

Solar ponds are used in the production of electricity and supply of industrial processes with thermal energy [6-7]. Research results show that the solar pond is a deep salt water pond with three different layers, which can be considered as a water helioaccumulator that collects solar heat. Solar ponds are one of the simplest and cheapest technologies for collecting and storing solar energy, and meet this demand through energy storage installed in the lower convective zone (LCZ) of the pond. [8].

There are four main types of solar ponds:

- Solar ponds with salt gradient.
- Non-salt convective ponds.
- Shallow sun pond.

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- Gel and viscosity stabilized pond.

The most commonly used type of solar ponds are salt gradient solar ponds [9-10].

Many studies have been conducted by scientists in the field of increasing the efficiency of solar ponds and obtaining energy from them. In particular, the method of obtaining electricity from a solar pond was carried out by Thailand researchers Tundee and other scientists [11]. The use of solar pond for heat pump operation was theoretically studied by Date and Akbarzoda [12]. Small power generation of electricity from solar pond using thermoelectric generators was researched by Singh et al [13].

The analysis of the conducted scientific research works and published literature and scientific articles shows that our Republic is one of the countries rich in solar energy sources, and until today, not enough research has been conducted on obtaining alternative energy from solar devices based on the solar pond or its combination. The purpose of this study is to develop a combined solar pond-heat pump heating system and apply it to swimming pool heating systems, taking into account the results of the experiment and the climatic conditions of Karshi city.

2 Materials and methods

The efficiency of a solar pond depends mainly on the amount of solar energy inputted into the solar pond together with the heat losses of the pond. Solar radiation falls on the upper convective surface of the pond and is converted into useful heat in the salt water of the solar pond. Solar radiation falling on the top surface of the solar pond passes through each layer and the radiant energy is absorbed by the pond layers. The incident, return and refraction of sunlight on the water surface is shown in figure 1. Long-wavelength rays of solar radiation are mainly absorbed by the upper convective zone, and short-wavelength rays are absorbed by the lower convective zone.

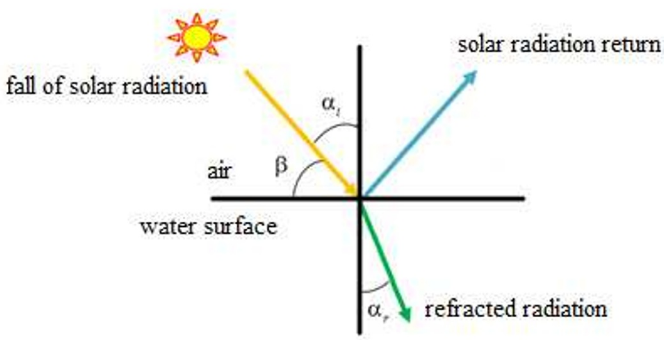

Figure 1. Scheme of reflection and refraction of solar radiation.

The formula proposed by British scientists Bryant and Colbeck can be used to calculate the radiation flux falling on the surface of the solar pond and the radiation flux absorbed by the pond [14]:

$$\frac{Q_x}{Q_o} = \{0,36 - 0,08\ln(\frac{x}{\cos\alpha_r})\} \tag{1}$$

Where, Q_o is insolation on the horizontal surface, W/m^2; and Q_x is x, the radiation flux falling to depth. α_r - the angle of refraction of the radiation falling on the pond.

Such refraction of solar radiation can be as follows for the air-water interface [15]:

$$\frac{\sin\alpha_i}{\sin\alpha_r}=\frac{n_r}{n}=1,333 \qquad (2)$$

Where, n - air refractive index, n_r - water refractive index, α_i - angle of direct incidence of radiation to the normal horizontal plane.

The efficiency of sunlight absorption in solar pond layers depends on the clarity of the salt water and the transparency of the salt used. Therefore, it is necessary to keep the water clean.

The middle layer of the solar pond acts as a heat insulator and prevents the loss of energy collected in the lower layer. The economic efficiency of the solar pond depends on the size of its thermal energy accumulation and construction costs. Therefore, it is important to accurately analyze its heat balance. By developing a mathematical model for solar pond layers, the heat balance of the pond is constructed and solved (figure 2).

The loss of energy from each layer and the accumulation of energy in the layer over time depend on the amount of incident energy and the radiant energy absorbed by the layer. According to this, during the time change, the energy balance is made for each of the layers of the solar pond model, UCZ, NCZ and LCZ.

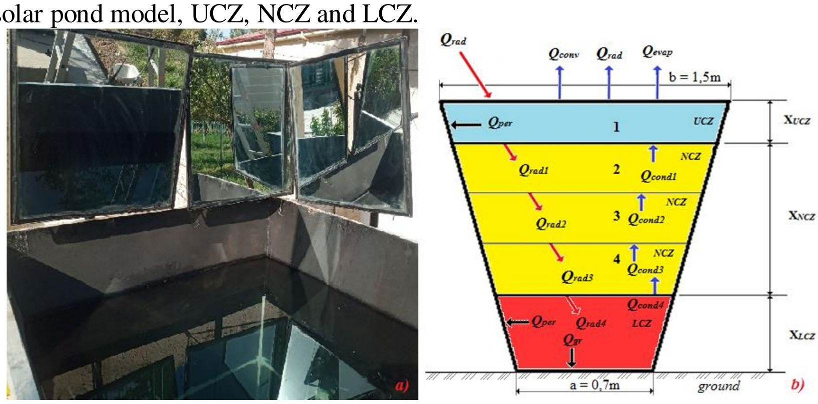

Figure 2. Solar pond device (a) and its heat balance diagram (b).

Heat losses from the surface of the solar pond significantly affect its performance. Heat losses are observed as a result of evaporation, radiation, conduction and convection processes. Although the solar pond is a source of heat, it has been found in research that its surface temperature is 3-5% lower than the ambient temperature [1].

In solar pond zones, heat is absorbed by solar radiation, but some of this heat is lost through the 4 processes mentioned above. The values of heat absorption and heat loss in the solar pond zones are summarized briefly in figure 2. The heat balance equation of the solar pond zones can be expressed mathematically as follows:

$$\begin{cases} Q_{UCZ}=Q_{rad1}+Q_{cond1}-Q_{total}-Q_{per_{UCZ}} \\ Q_{NCZ}=Q_{rad2}+Q_{cond3}-Q_{cond1}-Q_{per_{NCZ}} \\ Q_{LCZ}=Q_{rad3}-Q_{cond3}-Q_{per_{LCZ}}-Q_{gr} \end{cases} \qquad (3)$$

Here: $Q_{rad1}, Q_{rad2}, Q_{rad3}$ - absorbed heat of solar radiation in solar pond zones, W; Q_{cond1} - heat transferred from the middle layer to the upper layer, W; Q_{cond3} - heat transferred from the lower layer to the middle layer, W; Q_{total} - total heat loss from the surface of the solar pond water to the environment, W; $Q_{per_{UCZ}}, Q_{per_{NCZ}}, Q_{per_{LCZ}}$ - heat loss from the side wall of the pool (by pool zones), W; Q_{gr} - heat lost from the pond to the ground, W.

The absorbed heat of solar radiation falling on the solar pond zones can be calculated using the following formulas:

$$Q_{rad1} = \alpha\tau F_{UCZ} q_{rad} , \tag{4}$$

$$Q_{rad2} = \alpha\tau F_{NCZ} q_{rad} , \tag{5}$$

$$Q_{rad3} = \alpha\tau F_{LCZ} q_{rad} , \tag{6}$$

Here: α, τ - coefficients of solar pond water transmittance and absorption; $F_{UCZ}; F_{NCZ}; F_{LCZ}$ - the surface of the upper, middle and lower layers of the solar pond, m^2 ; q_{rad} - solar radiation intensity, W/m^2.

The amount of heat given by the solar pond from the middle layer to the upper layer and from the lower layer to the middle layer can be calculated using formulas (7) and (8):

$$Q_{cond1} = \frac{\lambda \cdot F_{NCZ}}{L_{UCZ}} (t_{NCZ} - t_{UCZ}) , \tag{7}$$

$$Q_{cond3} = \frac{\lambda \cdot F_{LCZ}}{L_{NCZ}} (t_{LCZ} - t_{NCZ}) , \tag{8}$$

Where, t_{NCZ} is the temperature of the middle layer of the solar pond, $°C$; t_{UCZ} - upper layer temperature, $°C$; t_{LCZ} - lower layer temperature, $°C$; λ - average heat transfer coefficient of solar pond water, $W/m \cdot K$; L_{UCZ}, L_{NCZ} - upper and middle layer thickness, *m*.

The total heat loss energy from the surface of the upper layer of the solar pond to the environment by evaporation, convective and radiation can be calculated using the following formula:

$$Q_{total} = Q_{conv} + Q_{rad} + Q_{evop} , \tag{9}$$

This formula can be simplified as follows:

$$Q_{total} = (\alpha_{conv} + \alpha_{rad}) F_{UCZ} \cdot (t_{UCZ} - t_a) , \tag{10}$$

Where, α_{conv} - convective heat transfer coefficient, $W/m \cdot K$; α_{rad} - heat transfer coefficient of water in solar radiation, $W/m \cdot K$; t_a - ambient temperature, $°C$.

The following equation can be used to calculate the convective heat transfer coefficient [16-19]:

$$\alpha_{conv} = 2,8 + 3,0 w_{air}, \tag{11}$$

w_{air} is the speed of air movement above the water level of the solar pond, m/s.

Heat loss from the sidewalls of each layer is important in calculating the energy balance of solar pond layers. It can be calculated using the following formulas:

$$Q_{per_{UCZ}} = \frac{F_{wall_{UCZ}}}{R}(t_{UCZ} - t_a), \tag{12}$$

$$Q_{per_{NCZ}} = \frac{F_{wall_{NCZ}}}{R}(t_{NCZ} - t_a), \tag{13}$$

$$Q_{per_{LCZ}} = \frac{F_{wall_{LCZ}}}{R}(t_{LCZ} - t_a), \tag{14}$$

Where, $F_{wall_{UCZ}}, F_{wall_{NCZ}}, F_{wall_{LCZ}}$ - the surface of the side walls of the upper, middle, lower layer of the solar pond, m^2; R - Thermal resistance of the solar pond wall, $m^2 {}^\circ C$.

The amount of heat lost from the base of the solar pond to the ground can be calculated using the following formula:

$$Q_{gr} = \frac{F_{basis}}{R}(t_{LCZ} - t_{gr}), \tag{15}$$

Where, F_{basis} - the surface of the base of the solar pond, m^2; R - thermal resistance of the base of the solar pond, $m^2 {}^\circ C$; t_{gr} - ground temperature, ${}^\circ C$.

Summarizing these equations, the heat balance of the solar pond zones (3) can be expressed as a system of equations as follows:

$$\begin{cases} \rho_w C_w V_{UCZ} \frac{dt_{UCZ}}{d\tau} = \alpha\tau F_{UCZ} q_{rad} + \frac{\lambda \cdot F_{NCZ}}{L_{UCZ}}(t_{NCZ} - t_{UCZ}) - (\alpha_{conv} + \alpha_{rad}) F_{UCZ} \cdot (t_{UCZ} - t_a) - \frac{F_{wall_{UCZ}}}{R}(t_{UCZ} - t_a) \\ \rho_w C_w V_{NCZ} \frac{dt_{NCZ}}{d\tau} = \alpha\tau F_{NCZ} q_{rad} + \frac{\lambda \cdot F_{LCZ}}{L_{NCZ}}(t_{LCZ} - t_{NCZ}) - \frac{\lambda \cdot F_{NCZ}}{L_{UCZ}}(t_{NCZ} - t_{UCZ}) - \frac{F_{wall_{NCZ}}}{R}(t_{NCZ} - t_a) \\ \rho_w C_w V_{LCZ} \frac{dt_{LCZ}}{d\tau} = \alpha\tau F_{LCZ} q_{rad} - \frac{\lambda \cdot F_{LCZ}}{L_{NCZ}}(t_{LCZ} - t_{NCZ}) - \frac{F_{wall_{LCZ}}}{R}(t_{LCZ} - t_a) - \frac{F_{basis}}{R}(t_{LCZ} - t_{gr}) \end{cases}, \tag{16}$$

Where, ρ_w - water density, kg/m^3; C_c - specific heat capacity of water, $J/kg \cdot K$; $VV_{UCZ,} V_{NCZ,} V_{LCZ}$ - the size of the upper, middle and lower zones of solar pond, m^3.

The temperatures of the solar pond zones vary depending on the amount of solar radiation. The temperatures of these zones are related to each other depending on the thickness of the solar pond zones and the depth of the pond. The temperatures of the solar pond zones that we are researching can be related as follows [20-26] (figure 3).

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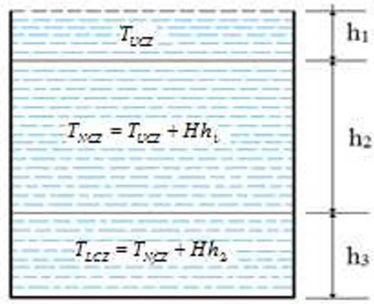

Figure 3. Correlation of zone temperatures.

Using the above calculations (15), we simplify the system of equations to the following form:

$$\begin{cases} \dfrac{dt_{UCZ}}{d\tau} = -\left(\dfrac{\alpha_{conv}F_{UCZ} + \alpha_{rad}F_{UCZ} + \dfrac{F_{wall_{UCZ}}}{R}}{\rho_w C_w V_{UCZ}}\right)t_{юкз} + \dfrac{\alpha_{conv}F_{UCZ} + \alpha_{rad}F_{UCZ}}{\rho_w C_w V_{UCZ}}t_a + \dfrac{\alpha\tau F_{UCZ}q_{rad}}{\rho_w C_w V_{UCZ}} + \dfrac{5,4\lambda F_{NCZ}}{\rho_w C_w V_{UCZ} L_{UCZ}} \\ \dfrac{dt_{NCZ}}{d\tau} = -\dfrac{F_{wall_{NCZ}}}{R\rho_w C_w V_{NCZ}}t_{NCZ} + \dfrac{F_{wall_{NCZ}}}{R\rho_w C_w V_{NCZ}}t_a + \dfrac{\alpha\tau F_{NCZ}q_{rad}}{\rho_w C_w V_{NCZ}} + \dfrac{11,57\lambda\cdot F_{LCZ}}{L_{NCZ}\rho_w C_w V_{NCZ}} - \dfrac{5,4\lambda\cdot F_{ўкз}}{L_{UCZ}\rho_w C_w V_{NCZ}} \\ \dfrac{dt_{LCZ}}{d\tau} = -\left(\dfrac{F_{wall_{LCZ}} + F_{basis}}{R\rho_w C_w V_{LCZ}}\right)t_{LCZ} + \dfrac{F_{wall_{LCZ}}}{R\rho_w C_w V_{LCZ}}t_a + \dfrac{F_{basis}}{R\rho_w C_w V_{LCZ}}t_{gr} + \dfrac{\alpha\tau F_{LCZ}q_{rad}}{\rho_w C_w V_{LCZ}} - \dfrac{11,57\lambda\cdot F_{LCZ}}{L_{NCZ}\rho_w C_w V_{LCZ}} \end{cases} \quad (17)$$

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3 Results

MATLAB/Simulink package block diagram of calculation of temperature changes of solar pond device zones (figure 4).

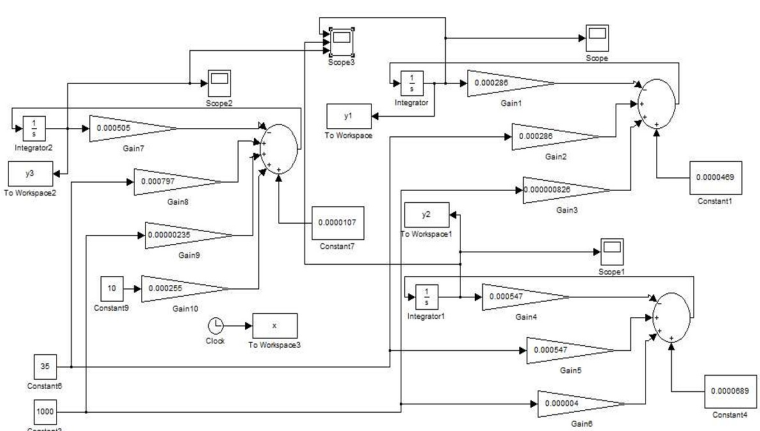

Figure 4. Block diagram of a solar pond device.

Table 1 presents the necessary data for the calculation of the heat balance of the experimental solar pond. Based on these data, taking into account the climatic parameters of Karshi city in non-stationary mode, when the concentration of water in the solar pond is 25%, the graphs of the temperature of the pond zones over time were obtained through the given block diagram at different values of solar radiation and ambient temperature (5-6-7-8- pictures).

The system is called upper convective zone (UCZ), middle non-convective zone (NCZ) and lower convective zone (LCZ), which clearly shows the distribution of salt according to its concentration in water [8].

Table 1. Calculation table of heat balance of solar pond experimental device.

No.	Parameters	Assignment	Unit of measurement	Value
1	Water density	ρ_w	kg / m^3	1000
2	Heat capacity of water	C_w	$J/(kg \cdot K)$	4200
3	Upper convective zone capacity of solar pond	V_{UCZ}	m^3	0.325
4	Middle non-convective zone capacity of solar pond	V_{NCZ}	m^3	1.266
5	Lower convective zone capacity of solar pond	V_{LCZ}	m^3	0.302
6	UCZ temperature of solar pond	t_{UCZ}	$°C$	Calculated
7	NCZ temperature of solar pond	t_{NCZ}	$°C$	Calculated
8	LCZ temperature of solar pond	$t_{пкз}$	$°C$	Calculated
9	Ambient temperature	t_a	$°C$	20
10	Ground temperature under the solar pond	t_{gr}	$°C$	7
11	The surface of the UCZ water of solar pond	F_{UCZ}	m^2	2.25
12	The upper surface of the NCZ of solar pond	F_{NCZ}	m^2	2.01
13	The upper surface of the LCZ of solar pond	F_{LCZ}	m^2	0.88
14	The surface of the side walls of the UCZ of solar pond	$F_{wall_{UCZ}}$	m^2	0.906
15	The surface of the side walls of the MNCZ of solar pond	$F_{wall_{NCZ}}$	m^2	4.396
16	The surface of the side walls of the LCZ of solar pond	$F_{wall_{LCZ}}$	m^2	1.527
17	The surface of the base of the solar pond	F_{basis}	m^2	0.49
18	Solar radiation	q_{rad}	W / m^2	$700 \div 1000$
19	Absorption coefficients of solar pond water	τ	m^{-1}	0.377
20	Transmittance coefficients of solar pond water	α	-	0.9
21	Convective heat transfer coefficient	α_{conv}	$W/(m^2 \cdot K)$	5.8
22	Heat transfer coefficient of water in solar radiation	α_{rad}	$W/(m^2 \cdot K)$	5.7

23	Average heat transfer coefficient of water	λ	$W/(m \cdot K)$	0.6
24	Solar pond UCZ thickness	L_{UCZ}	m	0.15
25	Solar pond NCZ thickness	L_{NCZ}	m	0.9
26	Thermal resistance of solar pond wall	R	$(m^2 \cdot °C)/W$	1.51

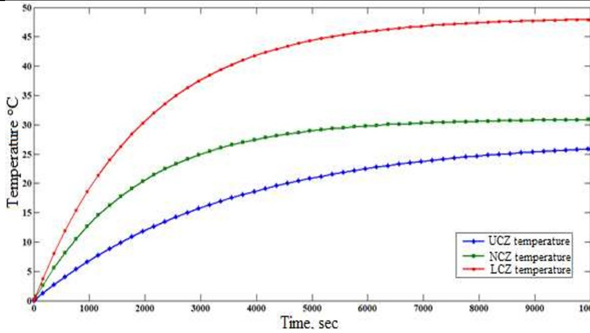

Figure 5. The graph of changes in the temperature of the pond zones over time at an ambient temperature of 20 °C and a solar radiation at a value of 700 W/m^2.

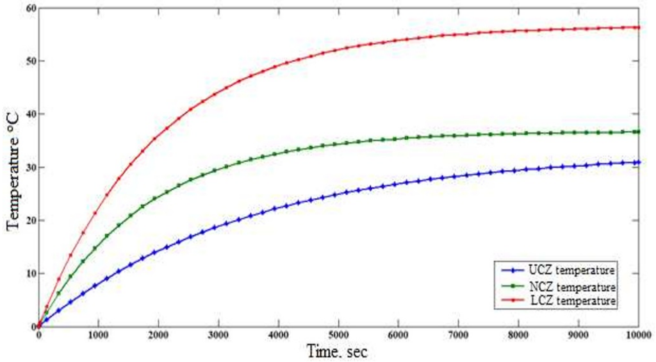

Figure 6. The graph of changes in the temperature of the pond zones over time at an ambient temperature of 30 °C and a solar radiation at a value of 900 W/m^2.

4 Conclusions

The heat balance of the solar pool was studied based on the results of thermodynamic laws and mathematical modeling calculations. Based on the analysis of the research results, the following conclusions were made:

Water turbidity is an important parameter in calculating the efficiency of solar ponds, as the turbidity level increases, the efficiency of the pond decreases.

We can reduce the consumption of fuel and energy resources by collecting solar radiation heat in the pond on sunny days and using it for heat supply of industrial processes.

At a value of water concentration between 25-30%, the temperature and heat storage efficiency of the solar pond device are observed to increase maximally with increasing solar radiation.

When the solar radiation is 1000 W/m^2, in a solar pond device in a non-stationary mode the accumulation of heat to $Q \approx 2kJ$ has been scientifically proven.

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