Journal for Research in Applied Sciences and Biotechnology

www.jrasb.com

ISSN: 2583-4053

Volume-2 Issue-4 || August 2023 || PP. 176-181

https://doi.org/10.55544/jrasb.2.4.25

An Estimation of Underground Economy in Afghanistan Using Mathematical Fuzzy Model Based on Mean and Standard Deviation

Wali Mohammad Azizi[1], Athiqullah Hayat[2], Shamsullah Shams[3] and Mohammad Izat Emir Zulkifly[4]

[1]Department of Mathematics, Faculty of Education, Kandahar University, AFGHANISTAN.

[2]Civil Department, Faculty of Engineering, Kandahar University, AFGHANISTAN.

[3]Department of Business & Administration, Faculty of Economics, Kandahar University, AFGHANISTAN.

[4]Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, MALAYSIA.

[1]Corresponding Author: wali.azizi321@gmail.com

ORCID

https://orcid.org/0009-0006-8673-2721

www.jrasb.com || Vol. 2 No. 4 (2023): August Issue

Received: 14-07-2023	**Revised:** 18-08-2023	**Accepted:** 15-09-2023

ABSTRACT

The underground economy (UE) briefly comprises services, activities, and transactions, which could be legal or illegal. In this paper the size of UE is estimated through mathematical fuzzy model based on fuzzy set, fuzzy logic and constructed a yearly time-series for UE over the period 2001 to 2020 in Afghanistan. Two input variables are used; unemployment rate (UR) and the government regulations (REG). Fuzzification, fuzzy inference and defuzzification; the three steps that are considered for estimating UE in the country, based on mean and standard deviation (SD) for each variable individually. The result indicates four cycles for time series and shows that people were more involved in UE activities over the first and third cycles and less involved over the second and fourth cycles.

***Keywords*-** Fuzzy Logic, Fuzzy Set, Fuzzy Modelling, Index Value, Underground Economy.

I. INTRODUCTION

The growth of the underground economy (UE) has led to various concerns in the last few decades. It has led to a problem that challenged most of the countries where the informal transactions are not recoreded in the reported data [10]. The developments of the UE activities have drawn much consideration in the last few years since most of researchers believe that the development of the unseen sector of economic activities is too much faster than the official economic activities and could become the most significant sector of economy [2]. Generally, there are several factors affecting the size of UE which include effective tax rate, the degree of government regulation, unemployment rate, corruption and so on[5]. Therefore, in many countries researchers have applied numerous model approaches to estimate the size, trend and cycles of UE for several input variables affecting on. The fuzzy model approach is also applied for this purpose which has avoided complicated mathematical calculations and is quite easily understandable [4]. This approach is defined based on the mathematical concept of fuzzy logic and fuzzy sets which is a generalized form of the classic set theory. The fuzzy logic concept is introduced by Lotfi Zadeh in1965[1]. A fuzzy logic system modelling is comprised of three steps, fuzzification, fuzzy inference and defuzzification based on a fuzzy set and fuzzy logic [2].

Accordingly, there are a lack of findings derived from a model measuring the size of UE in Afghanistan. Therefore, in this study we aim to estimate the size, trend and cycle of the UE in Afghanistan based on UR and REG

ISSN: 2583-4053

Volume-2 Issue-4 || August 2023 || PP. 176-181

https://doi.org/10.55544/jrasb.2.4.25

input variables by applying fuzzification, fuzzy inference and defuzzification three steps of fuzzy logic modeling approach based on mean and standard deviation over the period of 2001 to 2020. The information on UE is useful to the related organization for instance; government, it can utilize these information as a references to amend a regulation system and work for decreasing UE [9]. Numerous researches have done for estimating size of UE for several variables applying fuzzy model which are extremely discussed in the second section. Contents of the paper are arranged in five sections. The first section includes introduction, aim and problem statement of the study, the second section is allocated to review previous studies and, the third section is for materials and methods which including fuzzification, fuzzy inference and defuzzification three basic steps of fuzzy modelling. Section four includes numerical results with respect to whole time series and estimated the indexed values, cycles and trend of UE. Lastly, the study is concluded in section five.

II. LITERATURE REVIEW

Most of the researchers believe that fuzzy logic approach is a suitable and best fit for estimating UE. It is believed that the calculations are easy for human comprehension based on fuzzy logic rules and linguistic expressions [10].

Therefore, there are several researches which estimated size of UE using fuzzy model. Yu *et al*. [2] applies fuzzy set theory and fuzzy logic system for modeling Taiwan's UE from 1960 to 2003, and has used two major factors, the effective tax rate and the degree of government regulations, affecting the UE size associated with some others models. The benefit of this approach is believed to avoid the difficult computations compared to conventional economical models. Moreover, the fuzzy logic rules and linguistic terms are easy for human understanding. Pourkazemi *et al*. [7] applied fuzzy model approach to find the UE size for Iran during the period of 1978-2010, and used three input variables, such as per capita GDP, direct taxes to GDP ratio and business environment index for considering the influence of institutional structures. The UE size is approximately estimated 20 percent during the whole period.

Draeseke and Giles [4] used fuzzy model to create a yearly time-series for New Zealand UE from 1968-1994, and applied for two input variables, first the effective tax rate and second, an index of government regulations. They have compared fuzzy model with Multiple Indicator Multiple Cause (MIMIC) model. The result of two approaches has yielded sensible, but a bit different and shown fuzzy logic approach involves several subjective judgments for measurement the problem. The results are quite strong and meaningful to these choices. Kayalvizhi *et al*. [6] applied fuzzy logic system to construct an annual time-series for the Indian UE during the period 2010 to 2016, with two input variables namely, the effective tax rate and an index of the degree of regulation compared with (MIMIC) model. The result yield sensible, but a bit different and expressive. ENE and HURDUC (2010) have applied fuzzy model method to find the UE size in Romania over 1990 to 2007. The included variables were the share of direct taxes to GDP, per capita production, UR and index of corruptions. The outcomes yielded 33.76 percent for the studied period.

Maršić and Oreški [5] have applied fuzzy model method to investigate and compare the UE size of Croatian to the UE of European Union countries from 2004 - 2012.

They have done in three steps. First, reviewed the size of UE based on existing estimates. Second, applied a unique calculation technique for estimation based on fuzzy logic. Third, measured and compared UE index of 25 EU countries. The results showed that Croatia is set at 13th largest UE among EU members list. Wahab *et al*. [3] used IT2 FLS (interval type-2 FLS) model approach for estimating the index value of Malaysian UE during the period of time from 2001 to 2010. The effectiveness of IT2 FLS model approach, the trend and flow of UE is discussed in Malaysia. Zulkifly *et al*. [9] applied fuzzy model to investigate the size of UE for Malaysia. The model was developed on the data collected over the period 2002 - 2018.

III. MATERIALS AND METHODS

The data is obtained from World Bank and Statistics Databases as shown in the Table 1. The unemployment rate (UR) is measured based on total (% of total labor force) and government regulations (REG) is measured as a representative of the government spending ratio to % GDP.

Table 1: Initial Data of UR and REG in Afghanistan (World Bank Data Base)

Years	UR	REG	Years	UR	REG
2001	11.513	0.00	2011	11.488	21.94
2002	11.596	6.94	2012	11.508	25.03
2003	11.604	11.93	2013	11.534	24.98
2004	11.528	15.07	2014	11.448	25.40
2005	11.429	15.70	2015	11.387	25.95
2006	11.224	18.30	2016	11.313	28.03
2007	10.046	21.48	2017	11.184	27.73
2008	11.985	20.92	2018	11.057	28.94
2009	11.387	21.17	2019	11.118	28.00
2010	11.484	20.80	2020	11.164	27.94

Fuzzy model normally includes three common steps: fuzzification, fuzzy inference and defuzzification. Figure 1 illustrates these steps and how they are related.

Journal for Research in Applied Sciences and Biotechnology
www.jrasb.com
ISSN: 2583-4053
Volume-2 Issue-4 || August 2023 || PP. 176-181
https://doi.org/10.55544/jrasb.2.4.25

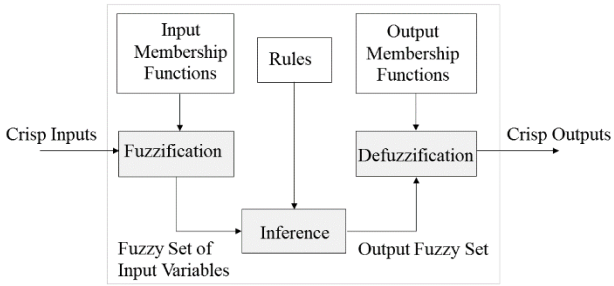

Figure 1. Fuzzy Logic System

1. Fuzzification

There are five LTs (Linguistic Terms) of the UR and REG: VL (Very Low), L (Low), N (Normal), H (High) and VH (Very High) with the associated linguistic terms of UE as: VS (Very Small), S (Small), A (Average), B (Big) and VB (Very Big). To transform crisp data to fuzzy data, we need to calculate mean and SD (Standard Deviation) of initial data in several periods of time. For instance, the calculations performed for 2010. Mean and SD are calculated based on five preceding years up until to the year 2010 for both UR and REG. In Table 2 for 2010, this process is demonstrated for UR and REG as follows; to compute mean and SD value for UR and REG in 2010, the crisp data of UR and REG from 2006 to 2010 are required. For this period of time the data and its mean and SD are calculated in Table 2.

Next, the linguistic terms would be assigned to UR and REG in 2010 (UR_{2010} and REG_{2010}). Now we need to calculate the associated value with the pick of MFs (Membership Functions) of each linguistic terms as seen in Table 3.

Table 2: Mean and Standard Deviation for 2010

Year	UR	REG
2006	11.224	18.30
2007	10.046	21.48
2008	11.985	20.92
2009	11.387	21.17
2010	11.484	20.80
Mean	11.2252	20.534
SD	0.6420	1.141

Table 3. Fuzzification of Initial Data for UR and REG for 2010.

UR	REG	Mean	SD	VL	L	N	H	VH
				Mean − 2SD	Mean − SD	Mean	Mean + SD	Mean + 2SD
$UR_{2010} = 11.484$		11.23	0.64	9.94	10.58	11.225	11.867	12.509
						N	H	
	$REG_{2010} = 20.800$	20.53	1.14	18.25	19.39	20.534	21.675	22.816
						N	H	

Then, as we have the linguistic terms for both of the UR and REG, the associated linguistic terms could be found by using Table 4.

The value of UR ($UR_{2010} = 11.484$) is located between Mean and Mean+SD, therefore, the MFs $\mu_N(x)$ and $\mu_H(x)$ are computed based on equations 4 and 5 in Table 4. Also, the value of REG ($REG_{2010} = 20.80$) is located between Mean and Mean+SD, therefore, the MFs $\mu_N(x)$ and $\mu_H(x)$ are similar computed based on equations 4 and 5 in Table 4. The numerical values are shown in Table 5 which are now fuzzy values. Figure 2 represents the MF of UR in 2010 and Figure 3 represents the MF of REG in 2010.

Table 4: Membership Functions

$\mu_{VL}(x) = 1.0$	if $x \leq$ Mean $-$ 2SD
$\mu_{VL}(x) = (\text{Mean} - \text{SD} - x)/\text{SD}$	if Mean $-$ 2 SD $\leq x \leq$ Mean $-$ SD
$\mu_{L}(x) = (x - \text{Mean} + 2\text{SD})/\text{SD}$	if Mean $-$ 2 SD $\leq x \leq$ Mean $-$ SD
$\mu_{L}(x) = (\text{Mean} - x)/\text{SD}$	if Mean $-$ SD $\leq x \leq$ Mean
$\mu_{N}(x) = (x - \text{Mean} + \text{SD})/\text{SD}$	if Mean $-$ SD $\leq x \leq$ Mean
$\mu_{N}(x) = (\text{Mean} + \text{SD} - x)/\text{SD}$	if Mean $\leq x \leq$ Mean $+$ SD
$\mu_{H}(x) = (x - \text{Mean})/\text{SD}$	if Mean $\leq x \leq$ Mean $+$ SD
$\mu_{H}(x) = (\text{Mean} + 2\text{SD} - x)/\text{SD}$	if Mean + SD $\leq x \leq$ Mean + 2SD
$\mu_{VH}(x) = (x - \text{Mean} - \text{SD})/\text{SD}$	if Mean + SD $\leq x \leq$ Mean + 2SD
$\mu_{VH}(x) = 1.0$	if $x \geq$ Mean $+$ 2SD

Journal for Research in Applied Sciences and Biotechnology
www.jrasb.com

ISSN: 2583-4053
Volume-2 Issue-4 || August 2023 || PP. 176-181
https://doi.org/10.55544/jrasb.2.4.25

Table 5: Membership Functions and Linguistic Terms of UR and REG

Year	UR	REG	VL	L	N	H	VH
2010	11.484				0.5966	0.4034	
		20.800			0.7669	0.2331	

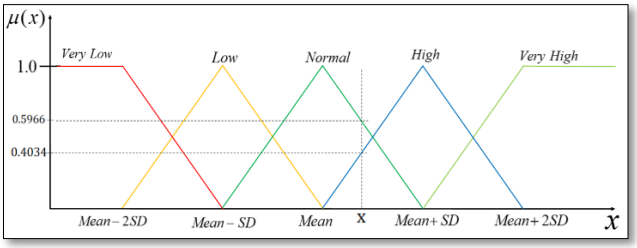

Figure 2: The Associated Membership Function of UR.

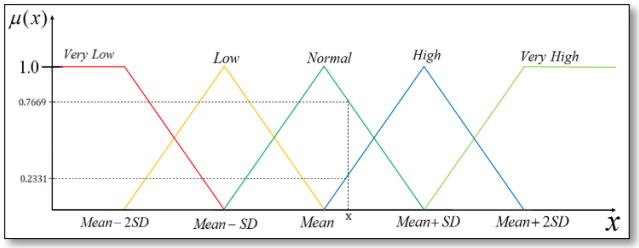

Figure 3: The Associated Membership Function of REG.

2. Fuzzy Inference

The real data are fuzzified for both UR and REG. Table 6 shows fuzzy rules of UE and there are 25 fuzzy rules with their related degrees. We have two linguistic terms for the UE and REG one by one. Therefore, at most $2 \times 2 = 4$ combinations we have to apply the fuzzy inference for empirical analysis. These combinations are shown in Table 7. For each rule, we use AND to connect two conditions, min operator to calculate the degree of membership and max operator for the degree of membership among more rules:

$$\mu_{UR\ AND\ REG} = \text{MIN}\big(\mu(UR), \mu(REG)\big) d_i ,$$

where, d_i is the membership degree.

$$\mu_{UE} = \text{MAX}\big(\mu(UE_1), \mu(UE_2)\big) ,$$

where, $\mu(UE_1)$ and $\mu(UE_2)$ are of the same linguistic terms.

Table 6: Fuzzy Rules for UE.

Rule	REG	UR	UE	d_i	Rule	REG	UR	UE	d_i
1	VH	VH	VB	1.0	14	N	L	S	0.8
2	VH	H	VB	0.8	15	N	VL	S	1.0
3	VH	N	S	1.0	16	L	VH	B	1.0
4	VH	L	S	0.8	17	L	H	A	1.0
5	VH	VL	A	0.8	18	L	N	S	0.8
6	H	VH	VB	1.0	19	L	L	S	1.0
7	H	H	B	1.0	20	L	VL	VS	1.0
8	H	N	B	0.8	21	VL	VH	A	0.8
9	H	L	A	1.0	22	VL	H	S	0.8
10	H	VL	S	1.0	23	VL	N	S	1.0
11	N	VH	B	1.0	24	VL	L	VS	0.8
12	N	H	B	0.8	25	VL	VL	VS	1.0
13	N	N	A	1.0					

Table 7: Final Results of Fuzzy Inference for the UR and REG

Order	UR	REG	Rule	UE level MIN(UR,REG)	UE association (MAX value for each level)
1	N	N	13	A : [1.0 × 0.5966]	A, 0.5966
2	N	H	12	B : [0.8 × 0.2331]	B,0.03008 (drop)
3	H	N	8	B : [0.8 × 0.4034]	B,0.3227
4	H	H	7	B : [1.0 × 0.2331]	B,0.23310 (drop)

3. Defuzzification

Now we have the fuzzy inference results which are now fuzzy data and should be transformed again to crisp data for measuring the size of UE in 2010 through defuzzification step. Therefore, we use center of area (COA) for defuzzification as follows:

$$UE\ index = \frac{\sum(\mu_{UE} \times y_i)}{\sum(\mu_{UE})}$$

Journal for Research in Applied Sciences and Biotechnology

www.jrasb.com

ISSN: 2583-4053

Volume-2 Issue-4 || August 2023 || PP. 176-181

https://doi.org/10.55544/jrasb.2.4.25

$$= \frac{(0.5966 \times 0.5) + (0.3227 \times 0.75)}{0.5966 + 0.3227} = 0.58776$$

$$= \text{UE index value in 2010}$$

where, y_i are the values $0, 0.25, 0.5, 0.75$ and 1 correspondingly associated with the linguistic terms VS, S, A, B and VB for UE as shown in Table 8.

Table 8: Results of Defuzzification

Level	Value	Weight
A	0.5966	0.5
B	0.3227	0.75

IV. FINAL RESULTS

Based on the Table 9 and Figure 4, the time series can divided by four cycles. The indexed values of UE are highest or on its peak on year 2001 to 2004, indexed values of UE are small on year 2005 to 2007, indexed values of UE are high on year 2008 to 2013 and the indexed values of UE are small on year 2014 to 2020.

Figure 4 shows that on the first and third period people were tending towards working 'underground' rather than above-board. And also shows that on the second and fourth period people were tending towards working openly and above-board. In general the trend and flow of indexed values are decreased during the whole period of time. The leading phenomena of the fuzzy index values demonstrate that the application of fuzzy logic can capture the UE patterns better. Hence, the concerning organization can use the fuzzy rules to identify the UE situations earlier and then apply proper strategies and plans to control the UE.

Table 9: Final Defuzzification Results of the UE

Years	Yearly UE Index	Years	Yearly UE Index
2001	0.7500	2011	0.8331
2002	0.8942	2012	0.7292
2003	0.8625	2013	0.7967
2004	0.5962	2014	0.3730
2005	0.4036	2015	0.3444
2006	0.3992	2016	0.5957
2007	0.4090	2017	0.3918
2008	0.7513	2018	0.4697
2009	0.6743	2019	0.4051
2010	0.5877	2020	0.4015

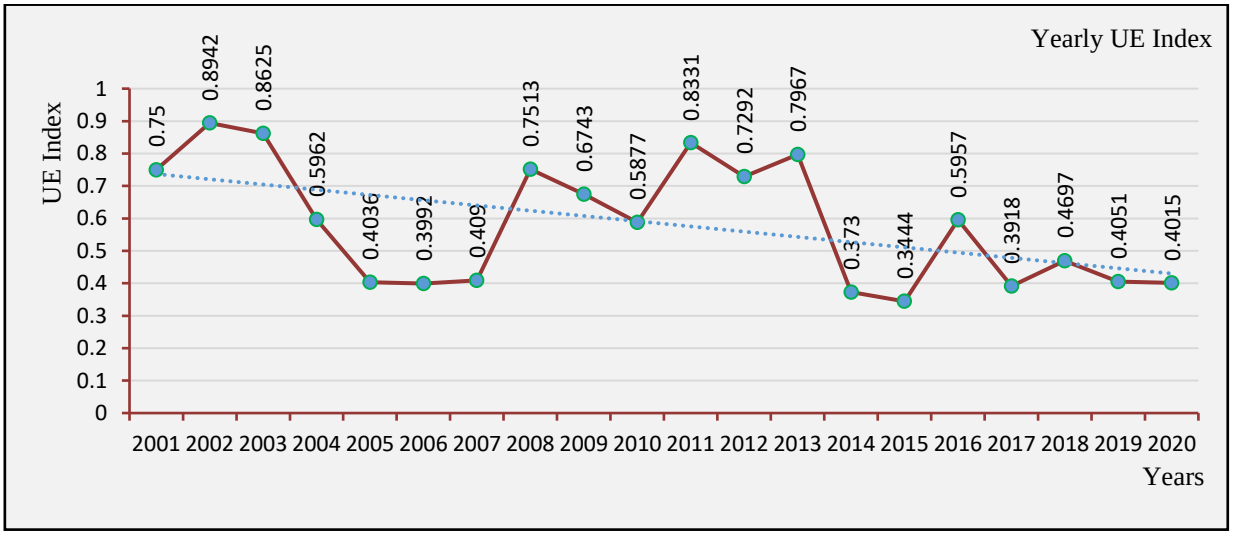

Figure 4: Graphically Representation of Yearly UE Index.

V. CONCLUSION

This study is the application of fuzzy model approach to estimate the size, trend, and cycles of UE in Afghanistan on the period 2001 to 2020 considering two input variables UR and REG. A fuzzy model approach is applied based on mean and standard deviation and has got two major advantages. First, the model avoids complicated mathematical calculations and second, linguistic terms and fuzzy rules are easily understandable. Based on this approach the time series is divided into four cycles. Years related to the first cycle, the index of the UE

ISSN: 2583-4053

Volume-2 Issue-4 || August 2023 || PP. 176-181

https://doi.org/10.55544/jrasb.2.4.25

is approximately 0.7757, related to the second cycle, the index of the UE is approximately 0.4039, related to the third cycle, the index of the UE is approximately 0.7287, and related fourth cycle the index of the UE is approximately 0.4259. However, the index of the UE for the whole period is approximately 0.5834 which means the people are involved in UE activities.

This information are useful to the related organization for instance; government, it could use it as a references to build a better REG system and should prevent unemployment to reduce size of UE. This demonstrates that fuzzy model approach is a powerful tool for answering this kind of problems when the information are uncertain and hidden.

REFERENCES

[1] Zadeh, L. (1965). Fuzzy sets. *Information and Control* 8(3),338-353.

[2] Hui-Kuang Yu, T., Wang, D.H.M. and Chen, S.J. (2005). "A fuzzy logic approach to modeling the underground economy in Taiwan" , *Physica A* , No.362 , 471-479.

[3] Wahab , A., Zulkifly , M. E., A. Rahim , H., and Zakaria, R. (2013). Interval Type-2 Fuzzy Logic System Model in Measuring the Index Value of Underground Economy in Malaysia. *Applied Mathematical Sciences, Vol. 7, no. 102*, pp. 5071 - 5084.

[4] Draeseke, R., and Giles, D. (2002). A Fuzzy Logic Approach to Modelling the Underground Economy. *Math. Comput. Simulat*, 115–123.

[5] Maršić , K., and Oreški , D. (2016). Estimation and Comparison of Underground Economy in Croatia and European Union Countries: Fuzzy Logic Approach. *Journal of Information and Organizational Sciences* , 82-104.

[6] Kayalvizhi, S., Gunasekar, T., and Thenmozhi, S. (2021). A Fuzzy Logic Approach to Modelling the Indian Underground Economy. *International Journal of Mathematics Trends and Technology (IJMTT) – V. 39 N. 2*, 94-98.

[7] Pourkazemi, M., Sherafat, M., and Azari, Z. D. (2015). Modeling Iran`s Underground Economy: A Fuzzy Logic Approach. *Iran. Econ. Rev. Vol.19, No.1,*, 91-106.

[8] ENE , C.-M., and HURDUC, N. (2010). A Fuzzy Model To Estimate Romanian Underground Economy. *Internal Auditing and Risk Management* , 29-38.

[9] Zulkifly, M. E., Wahab, A., Abd Rahim, S. A., and Shah, M. M. (2020). Fuzzy Logic System Model in Measuring the Size underground in malaysia. *Advances and Applications in Discrete Mathematics Volume 25, Number 2*, pp.303-313.

[10] Tahmasebi, M., and Rocca, M. (2015). A fuzzy model to estimate the size of the underground economy applying structural equation modeling. *Journal of Applied Economics*, Volume XVIII, Number 2.