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## A SPIN 1/2 PARTICLE WITH ANOMALOUS MAGNETIC AND ELECTRIC DIPOLE MOMENTS

**Abstract.** It is Petras who first developed the  $P$ -symmetric theory for a spin 1/2 particle with an anomalous magnetic moment within the general Gel'fand – Yaglom approach. Recently, similarly it was introduced a  $P$ -asymmetric wave equation for a spin 1/2 particle which describes a particle with an electric dipole moment. In this paper, we study solutions of the equation for the  $P$ -asymmetric particle in presence of external magnetic fields. It turns out that the energy spectra are the same for  $P$ -asymmetric and  $P$ -symmetric particles. To clarify this coincidence, we demonstrate that there exists a simple transformation relating these two models, by which one wave equation can be reduced to the form of the other. Meanwhile, expressions for wave functions and  $P$ -reflection operators are different in these two theories. We extend this approach to the model in which both  $P$ -symmetric and  $P$ -asymmetric sectors are presented. The main result is the same, namely, there exists a simple, more general as compared with the mentioned above transformation relating the  $P$ -symmetric model and the model with two sectors, and expressions for wave functions and  $P$ -reflection operators are different in these two bases. We demonstrate that in the presence of an external uniform magnetic field, the energy spectra in the model with two sectors coincide with those in the  $P$ -symmetric theory. Thus, we develop a general theory for the  $P$ -asymmetric model and the model with two sectors within the Petras approach.

**Keywords:** Gel'fand – Yaglom approach, spin 1/2, anomalous magnetic moment, electric dipole moment, magnetic field, energy spectrum

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## ЧАСТИЦА СО СПИНОМ 1/2 С АНОМАЛЬНЫМ МАГНИТНЫМ И ЭЛЕКТРИЧЕСКИМ ДИПОЛЬНЫМ МОМЕНТАМИ

**Аннотация.**  $P$ -симметричная теория частицы с аномальным магнитным моментом была развита Петрашем в рамках общего подхода Гельфанда – Яглома. Недавно в рамках аналогичного подхода было введено  $P$ -асимметричное волновое уравнение для частицы со спином 1/2, которое описывает частицу с электрическим дипольным моментом. В настоящей работе исследуются решения уравнения для  $P$ -асимметричной частицы в присутствии внешних магнитных полей. Оказалось, что энергетический спектр  $P$ -асимметричной частицы такой же, как у  $P$ -симметричной частицы. Для обоснования этого совпадения показано, что существует простое преобразование, с помощью которого одно волновое уравнение может быть приведено к виду другого, при этом выражения для волновых функций и операторов  $P$ -отражения различны в этих двух теориях. Данный подход обобщен на модель, в которой представлены оба сектора:  $P$ -симметричный и  $P$ -асимметричный. Основным результатом оказался тем же: существует простое преобразование, более общее, чем указанное выше, связывающее  $P$ -симметричную модель и модель с двумя секторами, выражения для волновых функций и операторов  $P$ -отражения различны в этих двух базисах. Показано, что при наличии внешнего однородного магнитного поля энергетические спектры в модели с двумя секторами совпадают с таковыми в  $P$ -симметричной теории. Таким образом, развита общая теория  $P$ -асимметричной модели и модели с двумя секторами в рамках подхода Петраша.

**Ключевые слова:** подход Гельфанда – Яглома, спин 1/2, аномальный магнитный момент, электрический дипольный момент, магнитное поле, энергетический спектр

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**Introduction.** In paper [1], from the general formalism by Gel'fand – Yaglom [2], a  $P$ -asymmetric wave equation for a spin 1/2 particle was introduced. This theory describes the particle with an electric dipole moment. In [1], this equation was studied in the presence of an external Coulomb field; however, for simplicity, additional interaction due to the electric dipole moment was removed, so only the possible manifestation of  $P$ -asymmetry was tested. The theory of the  $P$ -symmetric equation for a particle with an anomalous magnetic moment is represented, for example, in [3–8]. Petras firstly developed this theory within the general Gel'fand – Yaglom approach [9].

The present paper is organized as follows. In section II, we study solutions of the equation for the  $P$ -asymmetric particle in the presence of an external uniform magnetic field. It turns out that in this case the energy spectra are the same as for the  $P$ -symmetric particle, referring to the anomalous magnetic moment. To clarify this coincidence, in section III we demonstrate that there exists a simple transformation relating these two models, by which one wave equation can be transformed to the form of the other, correspondingly the function  $\Psi$  transforms to a new one  $\Psi'$ , expressions for the  $P$ -reflection operator are different in these two theories. In section IV, we extend this approach to the model, in which both  $P$ -symmetric and  $P$ -asymmetric sectors are presented. The main result is the same: there exists a simple transformation (more general than the mentioned above) relating the  $P$ -symmetric model and the model with two sectors. We demonstrate that in the presence of an external uniform magnetic field, the energy spectrum in the model with two sectors, coincides with those in the  $P$ -symmetric one. In section V, we develop a general theory for the  $P$ -asymmetric model within the Petras approach; in section VI, within the Petras approach we develop a general theory for the model with two sectors.

**A  $P$ -asymmetric particle in external magnetic fields.** Let us use the tetrad formalism and take into account the pseudo-Riemannian space-time structure [10]. Then the  $P$ -asymmetric equation has the following form

$$\left\{ i\gamma^\alpha(x) [\nabla_\alpha + \Gamma_\alpha(x) + ieA_\alpha(x)] + \frac{\lambda}{iM} \gamma^5 \left[ -ieF_{\mu\nu}(x) \sigma^{\mu\nu}(x) + \frac{R(x)}{4} \right] + i\gamma^5 M \right\} \Psi(x) = 0, \quad (1)$$

where  $\sigma^{\mu\nu}(x) = \frac{1}{4}(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu)$ ;  $\lambda$  is an arbitrary parameter which determines the value of the electric dipole moment;  $R(x)$  is the Ricci scalar. The presence of the matrix  $\gamma^5$  means the  $P$ -asymmetry of equation (1). The physical dimensions of the quantities are  $[M] = L^{-1}$ ,  $[e'A] = L^{-1}$ ,  $[e'F] = L^{-2}$ ;  $e' = e / \hbar c$ ; the parameter  $\lambda$  is dimensionless.

In the presence of a uniform magnetic field with the 4-potential  $A_\phi = -\frac{Br^2}{2}$ , eq. (1) takes the following form

$$\left[ i\gamma^0 \partial_0 + i\gamma^1 \partial_r + \gamma^2 \left( \frac{i\partial_\phi}{r} + \frac{e'Br}{2} \right) + i\gamma^3 \partial_z - i\lambda \frac{2e'B}{M} \gamma^5 \Sigma_3 + i\gamma^5 M \right] \Psi = 0, \quad (2)$$

where  $\Sigma_3 = i\sigma^{12} = \frac{i}{2} \gamma^1 \gamma^2$ . Meanwhile, we apply the known cylindrical coordinates and diagonal tetrad. It is convenient to use a parameter  $\Gamma$  with the dimension of the inverse length  $-i\lambda \frac{e'B}{M} = -i\Gamma$ ,  $[\Gamma] = L^{-1}$ , then eq. (2) can be rewritten

$$\left[ i\gamma^0 \partial_0 + i\gamma^1 \partial_r + \gamma^2 \left( \frac{i\partial_\phi}{r} + \frac{e'Br}{2} \right) + i\gamma^3 \partial_z - i\Gamma \gamma^5 \Sigma_3 + i\gamma^5 M \right] \Psi = 0.$$

Let us find the solutions in the form

$$\Psi = e^{-icx^0} e^{im\phi} e^{ikz} \begin{pmatrix} f_1(r) \\ f_2(r) \\ f_3(r) \\ f_4(r) \end{pmatrix}, \quad \epsilon = \frac{E}{\hbar c}, \quad x_0 = ct,$$

where the quantity  $m$  stands for the eigenvalues of the third projection of the total angular momentum, and takes the half integer values:  $m = \pm 1/2, \pm 3/2, \dots$ . Using expressions for the Dirac matrices in the spinor basis, we obtain four equations for functions  $f_i(r)$ :

$$\begin{aligned} -i\left(\frac{d}{dr} + \mu\right)f_4 + (\epsilon + k)f_3 + (-i\Gamma + iM)f_1 &= 0, \\ -i\left(\frac{d}{dr} - \mu\right)f_3 + (\epsilon - k)f_4 + (+i\Gamma + iM)f_2 &= 0, \\ i\left(\frac{d}{dr} + \mu\right)f_2 + (\epsilon - k)f_1 + (+i\Gamma - iM)f_3 &= 0, \\ i\left(\frac{d}{dr} - \mu\right)f_1 + (\epsilon + k)f_2 + (-i\Gamma - iM)f_4 &= 0; \end{aligned} \quad (3)$$

where  $\mu(r) = \frac{m}{r} - \frac{e'Br}{2}$ . Equations in (3) may be considered as two linear subsystems

$$\begin{aligned} (-i\Gamma + iM)f_1 + (\epsilon + k)f_3 &= iD_+f_4, & (\epsilon - k)f_1 + (+i\Gamma - iM)f_3 &= -iD_+f_2; \\ (+i\Gamma + iM)f_2 + (\epsilon - k)f_4 &= iD_-f_3, & (\epsilon + k)f_2 + (-i\Gamma - iM)f_4 &= -iD_-f_1, \end{aligned} \quad (4)$$

where  $D_+ = d/dr + \mu$ ,  $D_- = d/dr - \mu$ . Whence we get

$$\begin{aligned} f_1 &= +i \frac{(\epsilon + k)D_+f_2 + (i\Gamma - iM)D_+f_4}{(\Gamma - M)^2 - (\epsilon^2 - k^2)}, & f_3 &= -i \frac{(-i\Gamma + iM)D_+f_2 + (\epsilon - k)D_+f_4}{(\Gamma - M)^2 - (\epsilon^2 - k^2)}; \\ f_2 &= +i \frac{(\epsilon - k)D_-f_1 - (i\Gamma + iM)D_-f_3}{(\Gamma + M)^2 - (\epsilon^2 - k^2)}, & f_4 &= -i \frac{(i\Gamma + iM)D_-f_1 + (\epsilon + k)D_-f_3}{(\Gamma + M)^2 - (\epsilon^2 - k^2)}. \end{aligned} \quad (5)$$

Let us take into account eqs. (5) in eqs. (4):

$$\begin{aligned} -\frac{\Gamma + M}{\Gamma - M}f_2 - \frac{\epsilon - k}{i\Gamma - iM}f_4 &= \frac{D_-D_+f_2}{(\Gamma - M)^2 - (\epsilon^2 - k^2)} - \frac{\epsilon - k}{i\Gamma - iM} \frac{D_-D_+f_4}{(\Gamma - M)^2 - (\epsilon^2 - k^2)}, \\ f_2 - \frac{i\Gamma + iM}{\epsilon + k}f_4 &= \frac{D_-D_+f_2}{(\Gamma - M)^2 - (\epsilon^2 - k^2)} + \frac{i\Gamma - iM}{\epsilon + k} \frac{D_-D_+f_4}{(\Gamma - M)^2 - (\epsilon^2 - k^2)}; \end{aligned} \quad (6)$$

Let us take into account the following identities

$$D_+D_- = \frac{d^2}{dr^2} - \frac{d\mu}{dr} - \mu^2, \quad D_-D_+ = \frac{d^2}{dr^2} + \frac{d\mu}{dr} - \mu^2.$$

In (6), let us subtract the second equation from the first:

$$f_2 = \frac{i}{2\Gamma(\epsilon + k)} \left[ \Gamma^2 - M^2 - k^2 + \epsilon^2 + \frac{d^2}{dr^2} + \mu' - \mu^2 \right] f_4,$$

and substitute  $f_2$  into the first equation in (6):

$$\begin{aligned} \frac{d^4 f_4}{dr^4} + \left[ -\frac{1}{2}e'^2 B^2 r^2 - \frac{2m(1+m)}{r^2} + 2\Gamma^2 - 2M^2 + \right. \\ \left. + e'B(2m-1) + 2(\epsilon^2 - k^2) \right] \frac{d^2 f_4}{dr^2} + \left[ -e'^2 B^2 r + \frac{4m(1+m)}{r^3} \right] \frac{df_4}{dr} + \end{aligned}$$

$$\begin{aligned}
 & + \left[ \frac{1}{16} e'^4 B^4 r^4 - \frac{1}{4} (e' B (2m - 1) + 2(\Gamma^2 - M^2) - 2(k^2 - \epsilon^2)) e'^2 B^2 r^2 - \right. \\
 & \quad \left. - \frac{2(\Gamma^2 - M^2 - 1/2 e' B - k^2 + m e' B + \epsilon^2)(1+m)m}{r^2} + \right. \\
 & \quad \left. + \frac{(m-2)(m+3)(1+m)m}{r^4} + \frac{3}{2} \left( -\frac{1}{6} + m^2 - \frac{1}{3} m \right) e'^2 B^2 + \right. \\
 & \quad \left. + (2m-1) e' B (\Gamma^2 - M^2 - k^2 + \epsilon^2) + (\Gamma + M)^2 - \epsilon^2 + k^2 + (\Gamma - M)^2 - \epsilon^2 + k^2 \right] f_4 = 0. \quad (7)
 \end{aligned}$$

In a similar manner, we may derive the 4-th order equation for the function  $f_2$ :

$$\begin{aligned}
 & \frac{d^4 f_2}{dr^4} + \left[ -\frac{1}{2} e'^2 B^2 r^2 - \frac{2m(1+m)}{r^2} + 2\Gamma^2 - 2M^2 + \right. \\
 & \quad \left. + e' B (2m - 1) + 2(\epsilon^2 - k^2) \right] \frac{d^2 f_2}{dr^2} + \left[ -e'^2 B^2 r + \frac{4m(1+m)}{r^3} \right] \frac{df_2}{dr} + \\
 & + \left[ \frac{1}{16} e'^4 B^4 r^4 - \frac{1}{4} (e' B (2m - 1) + 2(\Gamma^2 - M^2) - 2(k^2 - \epsilon^2)) e'^2 B^2 r^2 - \right. \\
 & \quad \left. - \frac{2(\Gamma^2 - M^2 - 1/2 e' B - k^2 + m e' B + \epsilon^2)(1+m)m}{r^2} + \right. \\
 & \quad \left. + \frac{(m-2)(m+3)(1+m)m}{r^4} + \frac{3}{2} \left( -\frac{1}{6} + m^2 - \frac{1}{3} m \right) e'^2 B^2 + \right. \\
 & \quad \left. + (2m-1) e' B (\Gamma^2 - M^2 - k^2 + \epsilon^2) + ((\Gamma + M)^2 - \epsilon^2 + k^2) + ((\Gamma - M)^2 - \epsilon^2 + k^2) \right] f_2 = 0. \quad (8)
 \end{aligned}$$

Equations (7) and (8) for  $f_2$  and  $f_4$  coincide. It suffices to study only one of them, taking in mind two constraints:

$$\begin{aligned}
 f_2 & = + \frac{i}{2\Gamma(\epsilon + k)} \left( \Gamma^2 - M^2 - k^2 + \epsilon^2 + \frac{d^2}{dr^2} + \mu' - \mu^2 \right) f_4, \\
 f_4 & = - \frac{i}{2\Gamma(\epsilon - k)} \left( \Gamma^2 - M^2 - k^2 + \epsilon^2 + \frac{d^2}{dr^2} + \mu' - \mu^2 \right) f_2,
 \end{aligned}$$

which permits to find a relative coefficient between the functions  $f_2$  and  $f_4$ . We will apply the factorization method:

$$\begin{aligned}
 \hat{F}_4(r) f(r) & = \hat{f}_2(r) \hat{g}_2(r) f(r) = 0, \\
 \hat{f}_2(r) & = \frac{d^2}{dr^2} + P_0 r^2 + P_1 + \frac{P_2}{r^2}, \quad \hat{g}_2(r) = \frac{d^2}{dr^2} + Q_0 r^2 + Q_1 + \frac{Q_2}{r^2}.
 \end{aligned}$$

Comparing the result of multiplying two 2-nd order operators  $\hat{F}_4(r)$ :

$$\hat{F}_4 = \left( \frac{d^2}{dr^2} + P_0 r^2 + P_1 + \frac{P_2}{r^2} \right) \left( \frac{d^2}{dr^2} + Q_0 r^2 + Q_1 + \frac{Q_2}{r^2} \right)$$

with the 4-th order operator, we get two sets of relations:

$$1) \quad P_0 = -\frac{1}{4} B^2 e'^2, \quad P_2 = -m(m+1),$$

$$\begin{aligned}
 P_1 &= e'B \left( m - \frac{1}{2} \right) + \Gamma^2 - M^2 - k^2 + \epsilon^2 + 2\Gamma\sqrt{\epsilon^2 - k^2}, \\
 Q_0 &= -\frac{1}{4}B^2e'^2, \quad Q_2 = -m(m+1), \\
 Q_1 &= e'B \left( m - \frac{1}{2} \right) + \Gamma^2 - M^2 - k^2 + \epsilon^2 - 2\Gamma\sqrt{\epsilon^2 - k^2}; \\
 2) \quad P_0 &= -\frac{1}{4}B^2e'^2, \quad P_2 = -m(m+1), \\
 P_1 &= e'B \left( m - \frac{1}{2} \right) + \Gamma^2 - M^2 - k^2 + \epsilon^2 - 2\Gamma\sqrt{\epsilon^2 - k^2}, \\
 Q_0 &= -\frac{1}{4}B^2e'^2, \quad Q_2 = -m(m+1), \\
 Q_1 &= e'B \left( m - \frac{1}{2} \right) + \Gamma^2 - M^2 - k^2 + \epsilon^2 + 2\Gamma\sqrt{\epsilon^2 - k^2}.
 \end{aligned}$$

These expressions for coefficients coincide with those arising in the theory of the  $P$ -invariant particle, therefore the energy spectra should be the same. Let us detail the procedure of obtaining these spectra. We solve two 2-nd order equations (to avoid confusion, let us designate the corresponding functions as  $f$  and  $g$ ):

$$\left[ \frac{d^2}{dr^2} - \frac{B^2e'^2r^2}{4} + e'B \left( m - \frac{1}{2} \right) + \Gamma^2 - M^2 - k^2 + \epsilon^2 + 2\Gamma\sqrt{\epsilon^2 - k^2} - \frac{m(m+1)}{r^2} \right] f = 0, \quad (9)$$

$$\left[ \frac{d^2}{dr^2} - \frac{B^2e'^2r^2}{4} + e'B \left( m - \frac{1}{2} \right) + \Gamma^2 - M^2 - k^2 + \epsilon^2 - 2\Gamma\sqrt{\epsilon^2 - k^2} - \frac{m(m+1)}{r^2} \right] g = 0. \quad (10)$$

Meanwhile, they differ only in the sign at  $\Gamma$ . For definiteness let us follow eq. (9). Assuming  $e'B > 0$  we find in the variable  $x = e'Br^2 / 2$

$$x \frac{d^2f}{dx^2} + \frac{1}{2} \frac{df}{dx} + \left[ -\frac{x}{4} - \frac{1}{4} \frac{m(m+1)}{x} + \frac{4\Gamma\sqrt{\epsilon^2 - k^2} + (2m-1)e'B + 2(\Gamma^2 - M^2 - k^2 + \epsilon^2)}{4e'B} \right] f = 0.$$

Let us find the solutions in the form  $f(x) = x^a e^{bx} F(x)$ :

$$\begin{aligned}
 &x \frac{d^2F}{dx^2} + \left( \frac{1}{2} + 2a + 2bx \right) \frac{dF}{dx} + \left[ \left( b^2 - \frac{1}{4} \right) x + \right. \\
 &\left. + \frac{2e'Bb(4a+1) + 4\Gamma\sqrt{\epsilon^2 - k^2} + (2m-1)e'B + 2(\Gamma^2 - M^2 - k^2 + \epsilon^2)}{4eB} + \frac{1}{4} \frac{(2a+m)(2a-m-1)}{x} \right] F = 0.
 \end{aligned}$$

If the parameters are fixed as  $a = -\frac{1}{2}m$ ,  $\frac{1}{2} + \frac{1}{2}m$ ,  $b = -\frac{1}{2}$ , the above equation for  $F(x)$  simplifies

$$x \frac{d^2F}{dx^2} + \left( \frac{1}{2} + 2a - x \right) \frac{dF}{dx} - \frac{e'B(4a+1) - 4\Gamma\sqrt{\epsilon^2 - k^2} - (2m-1)e'B - 2(\Gamma^2 - M^2 - k^2 + \epsilon^2)}{4eB} F = 0,$$

which is the confluent hypergeometric equation with the parameters

$$\alpha = \frac{e'B(4a+1) - 4\Gamma\sqrt{\epsilon^2 - k^2} - (2m-1)e'B - 2(\Gamma^2 - M^2 - k^2 + \epsilon^2)}{4e'B}, \quad \gamma = \frac{1}{2} + 2a.$$

To get solutions referring to the bound states, we should use the positive values for  $a$ :

$$a = -\frac{m}{2} \quad (m < 0); \quad a = \frac{m}{2} + \frac{1}{2} > 0 \quad (m \geq 0). \tag{11}$$

The series become polynomials if  $\epsilon^2 - k^2 = \lambda > 0$ . This provides us with an algebraic equation

$$a + \frac{1}{2} - \frac{m}{2} + \frac{M^2 - \Gamma^2}{2e'B} + n = \frac{\Gamma\sqrt{\lambda}}{e'B} + \frac{\lambda}{2e'B}.$$

Further we obtain the following formula for energy values

$$\epsilon^2 - k^2 = \left( \sqrt{M^2 + 2e'B(a + 1/2 - m/2 + n)} - \Gamma \right)^2.$$

Depending on the values for  $a$  (see (11)), we obtain two different formulas

$$m < 0, \quad \epsilon^2 - k^2 = \left( \sqrt{M^2 + 2e'B(1/2 - m + n)} - \Gamma \right)^2;$$

$$m \geq 0, \quad \epsilon^2 - k^2 = \left( \sqrt{M^2 + 2e'B(1 + n)} - \Gamma \right)^2.$$

Taking in mind  $m = -1/2, -3/2, \dots; 1/2 - m = 1, 2, 3, \dots = n'$ , we can join these two formulas into the following one

$$\epsilon^2 - k^2 = \left( \sqrt{M^2 + 2e'BN} - \Gamma \right)^2, \quad N = 1 + \frac{-m + |m|}{2} + n.$$

Other possibility for the energy spectrum (see (10)) differs only in the sign at the parameter  $\Gamma$ :

$$\epsilon^2 - k^2 = \left( \sqrt{M^2 + 2e'BN} + \Gamma \right)^2, \quad N = 1 + \frac{-m + |m|}{2} + n.$$

**Symmetry properties.** Let us for simplicity restrict ourselves to the case of the Minkowski space. Moreover, for brevity we will omit the prime at the parameter  $e$ . Thus, the initial  $P$ -asymmetric equation may be rewritten as follows

$$\left[ -\gamma^5 \gamma^a D_a + \frac{\lambda}{M} (-ieF_{ab} \sigma^{ab}) - M \right] \Psi = 0. \tag{12}$$

Let us transform this equation to a new function

$$\Psi' = S\Psi, \quad S = \frac{1+i}{2} + \frac{1-i}{2} \gamma^5, \quad S^{-1} = \frac{1-i}{2} + \frac{1+i}{2} \gamma^5.$$

Taking in mind the properties of the Dirac matrices, we derive the rules

$$S\gamma^5\gamma^a S^{-1} = -i\gamma^a, \quad S\gamma^a\gamma^a S^{-1} = \gamma^a\gamma^b.$$

Therefore eq. (12) transforms to a new form in the variable  $\Psi'$ :

$$\left[ i\gamma^a D_a + \lambda(-ieF_{ab}\sigma^{ab}) - M \right] \Psi' = 0,$$

which formally coincides with the ordinary  $P$ -invariant equation for a spin 1/2 particle with an anomalous magnetic moment. However, we should take into account that the transformation properties of the new wave function  $\Psi'$  are different from those for the initial function  $\Psi$ . Indeed, we have explicit expressions for  $S$  and  $S^{-1}$  in the spinor basis

$$S = \begin{vmatrix} I & 0 \\ 0 & iI \end{vmatrix}, \quad S^{-1} = \begin{vmatrix} I & 0 \\ 0 & -iI \end{vmatrix}.$$

Therefore, the continuous Lorentz transformations preserve their form

$$U = \begin{vmatrix} B & 0 \\ 0 & (B^+)^{-1} \end{vmatrix}, \quad \bar{\Psi}' = S \begin{vmatrix} B & 0 \\ 0 & (B^+)^{-1} \end{vmatrix} S^{-1} \Psi' = \begin{vmatrix} B & 0 \\ 0 & (B^+)^{-1} \end{vmatrix} \Psi';$$

however, the  $P$ -reflection operator changes substantially

$$\bar{\Psi} = \gamma^0 \Psi = \begin{vmatrix} 0 & I \\ I & 0 \end{vmatrix} \Psi \Rightarrow \bar{\Psi}' = S \gamma^0 S^{-1} \Psi = S \begin{vmatrix} 0 & I \\ I & 0 \end{vmatrix} S^{-1} \Psi' = P' \Psi',$$

where the new operation  $P'$  is determined as

$$P' = \begin{vmatrix} 0 & -iI \\ +iI & 0 \end{vmatrix}, \quad P' = S \gamma^0 S^{-1} = i\gamma^0 \gamma^5 = -i\gamma^5 P.$$

**Extension to the theory with two sectors.** The results can be generalized to the theory with the presence of both  $P$ -symmetric and  $P$ -asymmetric sectors

$$\left\{ i \left( \frac{a+b}{2} \gamma^\alpha(x) + \frac{a-b}{2} \gamma^5 \gamma^\alpha(x) \right) (\nabla_\alpha + \Gamma_\alpha(x) + ieA_\alpha(x)) + \frac{\lambda}{M} \left( -ieF_{\mu\nu}(x) \sigma^{\mu\nu}(x) + \frac{R(x)}{4} \right) - M \right\} \Psi(x) = 0. \tag{13}$$

This equation may be easily verified by symmetry considerations. Indeed, let us introduce the following transformation in the bispinor space

$$S = \frac{1+a}{2} + \frac{1-a}{2} \gamma^5, \quad S^{-1} = \frac{1+b}{2} + \frac{1-b}{2} \gamma^5, \quad ab=1, \tag{14}$$

and calculate the combination  $S^{-1}i\gamma^\mu S$ :

$$S^{-1}i\gamma^\mu S = i \left( \frac{a+b}{2} \gamma^\mu + \frac{a-b}{2} \gamma^5 \gamma^\mu \right).$$

So, we can conclude that eq. (13) is formally derived from the ordinary  $P$ -invariant equation by means of transformation (14).

Let us consider eq. (13) in the presence of a uniform magnetic field. We start with the equation specified for the cylindric tetrad:

$$\left[ i \left( \frac{a+b}{2} \gamma^0 + \frac{a-b}{2} \gamma^5 \gamma^0 \right) \partial_0 + i \left( \frac{a+b}{2} \gamma^1 + \frac{a-b}{2} \gamma^5 \gamma^1 \right) \partial_r + \left( \frac{a+b}{2} \gamma^2 + \frac{a-b}{2} \gamma^5 \gamma^2 \right) \left( \frac{i\partial_\phi}{r} + \frac{e'Br}{2} \right) + i \left( \frac{a+b}{2} \gamma^3 + \frac{a-b}{2} \gamma^5 \gamma^3 \right) \partial_z + \lambda \frac{2e'B}{M} \Sigma_3 - M \right] \Psi = 0,$$

where  $\Sigma_3 = i\sigma^{12} = \frac{i}{2}\gamma^1\gamma^2$ . As above we use the parameter  $\lambda (e'B / M) = \Gamma$ ,  $[\Gamma] = L^{-1}$ ; so the previous equation can be rewritten

$$\left[ i\left(\frac{a+b}{2}\gamma^0 + \frac{a-b}{2}\gamma^5\gamma^0\right)\partial_0 + i\left(\frac{a+b}{2}\gamma^1 + \frac{a-b}{2}\gamma^5\gamma^1\right)\partial_r + \left(\frac{a+b}{2}\gamma^2 + \frac{a-b}{2}\gamma^5\gamma^2\right)\left(\frac{i\partial_\phi}{r} + \frac{e'Br}{2}\right) + i\left(\frac{a+b}{2}\gamma^3 + \frac{a-b}{2}\gamma^5\gamma^3\right)\partial_z + \Gamma\Sigma_3 - M \right] \Psi = 0.$$

With the use of the known substitution for the wave function, we get

$$\left[ \epsilon\left(\frac{a+b}{2}\gamma^0 + \frac{a-b}{2}\gamma^5\gamma^0\right) + i\left(\frac{a+b}{2}\gamma^1 + \frac{a-b}{2}\gamma^5\gamma^1\right)\frac{d}{dr} - \left(\frac{a+b}{2}\gamma^2 + \frac{a-b}{2}\gamma^5\gamma^2\right)\mu(r) - k\left(\frac{a+b}{2}\gamma^3 + \frac{a-b}{2}\gamma^5\gamma^3\right) + \Gamma\Sigma_3 - M \right] \psi = 0.$$

After separating the variables, applying the Dirac matrices in the spinor basis

$$\gamma^0 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & +i \\ 0 & 0 & -i & 0 \\ 0 & -i & 0 & 0 \\ +i & 0 & 0 & 0 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix},$$

we derive the system

$$\begin{aligned} -ia\left(\frac{d}{dr} + \mu\right)f_4 + a(\epsilon + k)f_3 + (\Gamma - M)f_1 &= 0, \\ -ia\left(\frac{d}{dr} - \mu\right)f_3 + a(\epsilon - k)f_4 - (\Gamma + M)f_2 &= 0, \\ ib\left(\frac{d}{dr} + \mu\right)f_2 + b(\epsilon - k)f_1 + (\Gamma - M)f_3 &= 0, \\ ib\left(\frac{d}{dr} - \mu\right)f_1 + b(\epsilon + k)f_2 - (\Gamma + M)f_4 &= 0. \end{aligned} \tag{15}$$

Equations in (15) may be considered as two subsystems:

$$\begin{aligned} (\Gamma - M)f_1 + a(\epsilon + k)f_3 &= iaD_+f_4, & b(\epsilon - k)f_1 + (\Gamma - M)f_3 &= -ibD_+f_2; \\ -(\Gamma + M)f_2 + a(\epsilon - k)f_4 &= iaD_-f_3, & b(\epsilon + k)f_2 - (\Gamma + M)f_4 &= -ibD_-f_1, \end{aligned} \tag{16}$$

where  $D_+ = \frac{d}{dr} + \mu$ ,  $D_- = \frac{d}{dr} - \mu$ . If we take into account that  $ab = 1$ , we obtain

$$\begin{aligned} f_1 &= +i \frac{(\epsilon + k)D_+f_2 + a(\Gamma - M)D_+f_4}{(\Gamma - M)^2 - (\epsilon^2 - k^2)}, & f_3 &= -i \frac{b(\Gamma - M)D_+f_2 + (\epsilon - k)D_+f_4}{(\Gamma - M)^2 - (\epsilon^2 - k^2)}; \\ f_2 &= +i \frac{(\epsilon - k)D_-f_1 - a(\Gamma + M)D_-f_3}{(\Gamma + M)^2 - (\epsilon^2 - k^2)}, & f_4 &= -i \frac{-b(\Gamma + M)D_-f_1 + (\epsilon + k)D_-f_3}{(\Gamma + M)^2 - (\epsilon^2 - k^2)}. \end{aligned} \tag{17}$$

Let us take into account (17) in eqs. (16):

$$-\frac{\Gamma + M}{\Gamma - M}bf_2 + \frac{\epsilon - k}{\Gamma - M}f_4 = \frac{bD_-D_+f_2}{(\Gamma - M)^2 - (\epsilon^2 - k^2)} + \frac{\epsilon - k}{\Gamma - M} \frac{D_-D_+f_4}{(\Gamma - M)^2 - (\epsilon^2 - k^2)},$$

$$bf_2 - \frac{\Gamma + M}{\epsilon + k} f_4 = \frac{bD_- D_+ f_2}{(\Gamma - M)^2 - (\epsilon^2 - k^2)} + \frac{\Gamma - M}{\epsilon + k} \frac{D_- D_+ f_4}{(\Gamma - M)^2 - (\epsilon^2 - k^2)}. \quad (18)$$

Let us subtract the second equation from the first one in (18):

$$f_2 = \frac{1}{2b\Gamma(\epsilon + k)} \left[ \Gamma^2 - M^2 - k^2 + \epsilon^2 + \frac{d^2}{dr^2} + \mu' - \mu^2 \right] f_4.$$

We substitute this expression for  $f_2$  into the first equation in (18), so we obtain the 4-th order equation for  $f_4$ :

$$\begin{aligned} & \frac{d^4 f_4}{dr^4} + \left[ -\frac{1}{2} e'^2 B^2 r^2 - \frac{2m(1+m)}{r^2} + 2\Gamma^2 - 2M^2 + \right. \\ & \left. + e'B(2m-1) + 2(\epsilon^2 - k^2) \right] \frac{d^2 f_4}{dr^2} + \left[ -e'^2 B^2 r + \frac{4m(1+m)}{r^3} \right] \frac{df_4}{dr} + \\ & + \left[ \frac{1}{16} e'^4 B^4 r^4 - \frac{1}{4} (e'B(2m-1) + 2(\Gamma^2 - M^2) - 2(k^2 - \epsilon^2)) e'^2 B^2 r^2 - \right. \\ & \left. - \frac{2(\Gamma^2 - M^2 - 1/2e'B - k^2 + me'B + \epsilon^2)(1+m)m}{r^2} + \frac{(m-2)(m+3)(1+m)m}{r^4} + \right. \\ & \left. + \frac{3}{2} \left( -\frac{1}{6} + m^2 - \frac{1}{3}m \right) e'^2 B^2 + (2m-1)e'B(\Gamma^2 - M^2 - k^2 + \epsilon^2) + \right. \\ & \left. + ((\Gamma + M)^2 - \epsilon^2 + k^2)((\Gamma - M)^2 - \epsilon^2 + k^2) \right] f_4 = 0. \end{aligned}$$

This equation coincides with equation (7). Similarly, we may derive the 4-th order equation for  $f_2$ , which coincides with the equation for  $f_4$ . Besides, we have two constraints

$$\begin{aligned} f_2 &= \frac{1}{2b\Gamma(\epsilon + k)} \left[ \Gamma^2 - M^2 - k^2 + \epsilon^2 + \frac{d^2}{dr^2} + \mu' - \mu^2 \right] f_4, \\ f_4 &= \frac{1}{2a\Gamma(\epsilon - k)} \left[ \Gamma^2 - M^2 - k^2 + \epsilon^2 + \frac{d^2}{dr^2} + \mu' - \mu^2 \right] f_2. \end{aligned}$$

Evidently, the spectra in this generalized theory will be the same as in the ordinary  $P$ -invariant theory. However, the presence in all formulas of the parameters  $a$  and  $b = a^{-1}$  modifies the explicit form of the wave function.

In the two next sections, starting from the general Gel'fand – Yaglom approach, we will develop generalized theories for the  $P$ -asymmetric particle and a more complicated theory with two sectors.

**The  $P$ -asymmetric theory, the Gel'fand – Yaglom approach.** In the  $P$ -asymmetric 20-component Petras theory, let us start with definitions for the space reflection operator and generators. In the two last sections we will use the *ict*-metric and the formalism of elements of the complete matrix algebra:

$$\Pi = \gamma_4 \otimes (2\beta_4^2 - 1), \quad J_{[\mu\nu]} = J_{[\mu\nu]}^{(D)} \otimes I_5 + I_4 \otimes (\beta_\mu \beta_\nu - \beta_\nu \beta_\mu),$$

where  $\beta_\mu = e^{0,\mu} + e^{\mu,0}$  stands for the Duffin – Kemmer (5×5)-matrices, and the bispinor generators  $J_{[\mu\nu]}^{(D)} = \frac{1}{4}(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)$ .

The matrices  $\Gamma_\mu$  of the  $P$ -asymmetric equation are defined by the formula in which the complete wave function consists of bispinor and vector-bispinor:

$$\Gamma_\mu = \frac{1}{\sqrt{2}}(1 + i\gamma_5)\gamma_\mu \otimes e^{0,0} + 2c_1 \left( \frac{\sqrt{3}}{2} I_4 \otimes \beta_\mu + I_{[\mu\nu]}^{(D)} \otimes \beta_\nu \right).$$

Let us describe the properties of the matrices  $\Gamma_\mu$  under the space reflection. To this end, firstly we calculate the product

$$\begin{aligned} \Pi\Gamma_4 &= \left\{ \gamma_4 \otimes [e^{0,0} - e^{a,a} + e^{4,4}] \right\} \left\{ \frac{1}{\sqrt{2}}(1 + i\gamma_5)\gamma_4 \otimes e^{0,0} + 2c \left[ \frac{\sqrt{3}}{2} I_4 \otimes \beta_4 + \frac{1}{2} \gamma_4 \gamma_6 \otimes \beta_6 \right] \right\} = \\ &= \frac{1}{\sqrt{2}}(1 - i\gamma_5) \otimes [e^{0,0}] + c\sqrt{3}\gamma_4 \otimes [e^{0,4} + e^{4,0}] + c\gamma_b \otimes [e^{0,b} - e^{b,0}], \end{aligned}$$

and the inverse product

$$\begin{aligned} \Gamma_4\Pi &= \left\{ \frac{1}{\sqrt{2}}(1 + i\gamma_5)\gamma_4 \otimes e^{0,0} + 2c \left[ \frac{\sqrt{3}}{2} I_4 \otimes \beta_4 + \frac{1}{2} \gamma_4 \gamma_b \otimes \beta_b \right] \right\} \left\{ \gamma_4 \otimes [e^{0,0} - e^{a,a} + e^{4,4}] \right\} = \\ &= \frac{1}{\sqrt{2}}(1 + i\gamma_5) \otimes [e^{0,0}] + c\sqrt{3}\gamma_4 \otimes [e^{4,0} + e^{0,4}] + c\gamma_b \otimes [e^{0,b} - e^{b,0}]. \end{aligned}$$

Therefore, two products are different  $\Pi\Gamma_4 \neq \Gamma_4\Pi$ , which means the non-invariance of this equation with respect to the spatial reflection. We can derive similar relations for remaining three matrices, getting the relation  $\Pi\Gamma_c \neq \Gamma_c\Pi$ .

Let us introduce the transformation to the other basis (it turns out to be of Petras type [9])

$$S^{-1} = \frac{1}{2\sqrt{2}} \left\{ (1 + \sqrt{2} - i) - (1 - \sqrt{2} - i)\gamma_5 \right\} \otimes I_5, \quad S = \frac{1}{2\sqrt{2}} \left\{ (1 + \sqrt{2} - i) - (1 - \sqrt{2} + i)\gamma_5 \right\} \otimes I_5.$$

By a simple calculation, we derive the rule

$$\Gamma_4 = S^{-1} \Gamma_4^{\text{Petras}} S, \quad \Gamma_\mu^{\text{Petras}} = \gamma_\mu \otimes e^{0,0} + 2c \left[ \frac{\sqrt{3}}{2} I_4 \otimes \beta_\mu + I_{[\mu\nu]}^{(D)} \otimes \beta_\nu \right].$$

In a more detailed form, the  $P$ -asymmetric equation  $(\partial_\mu \Gamma_\mu + M)\Phi = 0$  is

$$\begin{aligned} &\partial_\mu \frac{1}{\sqrt{2}}(1 + i\gamma_5)\gamma_\mu \delta_{A0} \Phi_0 + c\sqrt{3} \partial_\mu [\delta_{0A} \Phi_\mu + \delta_{\mu A} \Phi_0] + \\ &+ \frac{c}{2} \partial_\mu (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu) [\delta_{0A} \Phi_\nu + \delta_{\nu A} \Phi_0] + M(\delta_{\nu A} \Phi_\nu + \delta_{0A} \Phi_0) = 0. \end{aligned}$$

It may be divided into two subcases:

$$\begin{aligned} A = 0, \quad &\frac{1}{\sqrt{2}}(1 + i\gamma_5)\gamma_\mu \partial_\mu \Phi_0 + c\sqrt{3} \partial_\mu \Phi_\mu + \frac{c}{2} \partial_\mu (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu) \Phi_\nu + M\Phi_0 = 0; \\ A = \rho, \quad &c\sqrt{3} \partial_\rho \Phi_0 + \frac{c}{2} \partial_\mu (\gamma_\mu \gamma_\rho - \gamma_\rho \gamma_\mu) \Phi_0 + M\Phi_\rho = 0. \end{aligned} \tag{19}$$

Let us exclude the vector-bispinor with the help of (19), so we get

$$\begin{aligned} &\frac{1}{\sqrt{2}}(1 + i\gamma_5)\gamma_\mu \partial_\mu \Phi_0 + c\sqrt{3} \partial_\mu \left( -\frac{c}{M} \right) \left\{ \sqrt{3} \partial_\mu + \frac{1}{2} \partial_\lambda (\gamma_\lambda \gamma_\mu - \gamma_\mu \gamma_\lambda) \right\} \Phi_0 + \\ &+ \frac{c}{2} \partial_\mu (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu) \left( -\frac{c}{M} \right) \left\{ \sqrt{3} \partial_\nu + \frac{1}{2} \partial_\lambda (\gamma_\lambda \gamma_\nu - \gamma_\nu \gamma_\lambda) \right\} \Phi_0 + M\Phi_0 = 0 \end{aligned}$$

or

$$\frac{1}{\sqrt{2}}(1+i\gamma_5)\gamma_\mu\partial_\mu\Phi_0 - \frac{3c^2}{M}\partial_\mu\partial_\mu\Phi_0 - \frac{c^2}{4M}\partial_\mu\partial_\lambda\{\gamma_\mu\gamma_\nu\gamma_\lambda\gamma_\nu - \gamma_\mu\gamma_\nu\gamma_\nu\gamma_\lambda - \gamma_\nu\gamma_\mu\gamma_\lambda\gamma_\nu + \gamma_\nu\gamma_\mu\gamma_\nu\gamma_\lambda\}\Phi_0 + M\Phi_0 = 0.$$

Thus, we obtain

$$\frac{1}{\sqrt{2}}(1+i\gamma_5)\gamma_\mu\partial_\mu\Phi_0 + M\Phi_0 = 0.$$

Now let us take into account the presence of external electromagnetic fields,  $D_\mu = \partial_\mu - ieA_\mu$ ; so we should start with the modified equations

$$\frac{1}{\sqrt{2}}(1+i\gamma_5)\partial_\mu D_\mu\Phi_0 + c\sqrt{3}D_\mu\Phi_\mu + \frac{c}{2}D_\mu(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)\Phi_\nu + M\Phi_0 = 0,$$

$$c\sqrt{3}D_p\Phi_0 + \frac{c}{2}D_\mu(\gamma_\mu\gamma_p - \gamma_p\gamma_\mu)\Phi_0 + M\Phi_p = 0.$$

Excluding the vector-bispinor  $\Phi_p$ , we derive the equation for the main bispinor (all details are omitted)

$$\frac{1}{\sqrt{2}}(1+i\gamma_5)\gamma_\mu D_\mu\Phi_0 + c\sqrt{3}\left(-\frac{c}{M}\right)D_p\left\{\sqrt{3}D_p\Phi_0 + \frac{1}{2}D_\mu(\gamma_\mu\gamma_p - \gamma_p\gamma_\mu)\Phi_0\right\} + \frac{c}{2}D_\mu(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)\left(-\frac{c}{M}\right)\left\{\sqrt{3}D_\nu\Phi_0 + \frac{1}{2}D_\lambda(\gamma_\lambda\gamma_\nu - \gamma_\nu\gamma_\lambda)\Phi_0\right\} + M\Phi_0 = 0,$$

so we arrive at

$$\frac{1}{\sqrt{2}}(1+i\gamma_5)\gamma_\mu D_\mu\Phi_0 - ie3c^2 F_{[\mu\lambda]}I_{[\mu\lambda]}\Phi_0 + M\Phi_0 = 0. \tag{20}$$

It should be noted that due to the identity  $A\frac{1}{\sqrt{2}}(1+i\gamma_5)\gamma_\mu A^{-1} = \gamma_\mu$ , where

$$A = \frac{1}{2\sqrt{2}}\{(1+\sqrt{2}+i) - (1-\sqrt{2}+i)\gamma_5\}, \quad A^{-1} = \frac{1}{2\sqrt{2}}\{(1+\sqrt{2}-i) - (1-\sqrt{2}-i)\gamma_5\},$$

eq. (20) can be transformed to the ordinary Petras equation for the new function  $\Phi^{\text{Petras}} = A\Phi_0$ .

**The  $P$ -noninvariant equation with two sectors.** Starting with the ordinary  $P$ -invariant Petras equation

$$\Gamma_\mu^{\text{Petras}} = \gamma_\mu \otimes e^{0,0} + k\sqrt{3}I \otimes (e^{0,\mu} + e^{\mu,0}) + 2I_{[\mu\nu]} \otimes (e^{0,\nu} + e^{\nu,0}),$$

where  $k$  is the Petras parameter, we apply the transformation of the following form

$$A = S \otimes I_5, \quad A^{-1} = S^{-1} \otimes I_5,$$

$$S = \frac{1}{2}\{(1+b) + (1-b)\gamma_5\}, \quad S^{-1} = \frac{1}{2}\{(1+a) + (1-a)\gamma_5\}, \quad ab = 1.$$

With the use of the explicit form for Petras matrices, we derive the identity

$$A^{-1}\Gamma_\mu^{\text{Petras}}A = S^{-1}\gamma_\mu S \otimes I + k\sqrt{3}I \otimes (e^{0,\mu} + e^{\mu,0}) + 2I_{[\mu\nu]} \otimes (e^{0,\nu} + e^{\nu,0}).$$

Also we need detailed expressions for  $S$  and  $S^{-1}$ :

$$S^{-1} = \frac{1+a}{2} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} + \frac{1-a}{2} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & a \end{vmatrix},$$

$$S = \frac{1+b}{2} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} + \frac{1-b}{2} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & b \end{vmatrix},$$

and easily derive the identities  $S^{-1}\gamma_\mu S = \frac{1}{2}\{(a+b)-(a-b)\gamma_5\}\gamma_\mu$ ; further we obtain

$$\Gamma_\mu = \frac{1}{2}\{(a+b)-(a-b)\gamma_5\}\gamma_\mu \otimes e^{0,0} + k\sqrt{3}I \otimes (e^{0,\mu} + e^{\mu,0}) + 2kI_{[\mu\nu]} \otimes (e^{0,\nu} + e^{\nu,0}),$$

$$S^{-1}\gamma_\mu\gamma_\nu S = S^{-1}S\gamma_\mu\gamma_\nu = \gamma_\mu\gamma_\nu, \quad S^{-1}I_{[\mu\nu]}^{(D)}S = I_{[\mu\nu]}^{(D)} = \frac{1}{4}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu).$$

Therefore, from the ordinary Petras  $P$ -invariant equation by means of a simple transformation we can formally derive a  $P$ -noninvariant Petras equation with two sectors (till the present time we considered only the case of a free particle). Now let us take into account the presence of electromagnetic fields:

$$D_\mu \frac{1}{2}\{(a+b)-(a-b)\gamma_5\}\gamma_\mu \delta_{A,0}\Psi_0 + k\sqrt{3}D_\mu(\delta_{A,0}\Psi_\mu + \partial_{A,\mu}\Psi_0) + 2kD_\mu I_{[\mu\nu]}(\delta_{A,0}\Psi_\nu + \delta_{A,\nu}\Psi_0) + M(\delta_{A,0}\Psi_0 + \delta_{A,\nu}\Psi_\nu) = 0.$$

Divide this equation into two subcases:

$$A = 0, \quad \frac{1}{2}\{(a+b)-(a-b)\gamma_5\}\gamma_\mu D_\mu \Psi_0 + k\sqrt{3}D_\mu \Psi_\mu + 2kD_\mu I_{[\mu\nu]}\Psi_\nu + M\Psi_0 = 0;$$

$$A = \rho, \quad k\sqrt{3}D_\rho \Psi_0 + 2kD_\mu I_{[\mu\rho]}\Psi_0 + M\Psi_\rho = 0.$$

Excluding the vector-bispinor  $\Psi_\rho$ , we obtain

$$\frac{1}{2}\{(a+b)-(a-b)\gamma_5\}\gamma_\mu D_\mu \Psi_0 - \frac{k^2\sqrt{3}}{M}\{\sqrt{3}D_\mu D_\mu \Psi_0 + 2D_\mu D_\lambda I_{[\lambda\mu]}\Psi_0\} - \frac{k^2}{M}\{2\sqrt{3}I_{[\mu\nu]}D_\mu D_\nu \Psi_0 + 4D_\mu D_\lambda I_{[\mu\nu]}I_{[\lambda\nu]}\Psi_0\} + M\Psi_0 = 0.$$

Let us consider the term proportional to  $k^2$ :

$$\begin{aligned} & -\frac{k^2}{M}\{3D_\mu D_\mu - 2\sqrt{3}D_\mu D_\lambda I_{[\mu\lambda]} + 2\sqrt{3}D_\mu D_\nu I_{[\mu\nu]} + 4D_\mu D_\lambda I_{[\mu\nu]}I_{[\lambda\nu]}\}\Psi_0 = \\ & = -\frac{k^2}{M}\left\{3D_\mu D_\mu + \frac{1}{4}D_\mu D_\lambda[-2\gamma_\mu\gamma_\lambda - 4\gamma_\mu\gamma_\lambda - 4\delta_{\mu\lambda} - 2\gamma_\mu\gamma_\lambda]\right\}\Psi_0 = \\ & = -\frac{k^2}{M}\{2D_\mu D_\mu - D_\mu D_\lambda(\gamma_\mu\gamma_\lambda + \gamma_\lambda\gamma_\mu) - D_\mu D_\lambda(\gamma_\mu\gamma_\lambda - \gamma_\lambda\gamma_\mu)\}\Psi_0 = \\ & = -\frac{k^2}{M}D_\mu D_\lambda(\gamma_\mu\gamma_\lambda - \gamma_\lambda\gamma_\mu)\Psi_0 = -ie\frac{k^2}{2M}F_{[\mu\lambda]}(\gamma_\mu\gamma_\lambda - \gamma_\lambda\gamma_\mu)\Psi_0. \end{aligned}$$

Thus, we arrive at the equation

$$\left(\frac{a+b}{2} - \frac{a-b}{2}\gamma_5\right)\gamma_\mu D_\mu \Psi_0 - \frac{2iek^2}{M} F_{[\mu\lambda]} I_{[\mu\lambda]}^D \Psi_0 + M\Psi_0 = 0;$$

it is the generalized Petras equation, in which the  $P$ -symmetric and  $P$ -asymmetric sectors are presented. Let us rewrite this equation as

$$(A + B\gamma_5)\gamma_\mu D_\mu \Psi_0 - \frac{2iek^2}{M} F_{[\mu\lambda]} I_{[\mu\lambda]}^D \Psi_0 + M\Psi_0 = 0; \quad (21)$$

then multiply eq. (21) by

$$(A + B\gamma_5)^{-1} = \frac{A}{A^2 - B^2} - \frac{B}{A^2 - B^2}\gamma_5$$

in this way we obtain

$$\gamma_\mu D_\mu \Psi_0 - \frac{2iek^2}{M} \frac{A - B\gamma_5}{A^2 - B^2} F_{[\mu\lambda]} I_{[\mu\lambda]}^D \Psi_0 + \frac{A - B\gamma_5}{A^2 - B^2} M\Psi_0 = 0.$$

Allowing for the identity  $A^2 - B^2 = 1$ , we arrive at

$$\gamma_\mu D_\mu \Psi_0 - \frac{2iek^2}{M} \left(\frac{a+b}{2} + \frac{a-b}{2}\gamma_5\right) F_{[\mu\lambda]} I_{[\mu\lambda]}^D \Psi_0 + \left(\frac{a+b}{2} + \frac{a-b}{2}\gamma_5\right) M\Psi_0 = 0.$$

**Conclusions.** The theory for the spin 1/2 particle that includes both anomalous magnetic and electric dipole moments is herein developed. It is shown that it suffices to solve the  $P$ -invariant equation (referring to the anomalous magnetic moment) in the presence of an electromagnetic field, then solutions of the  $P$ -asymmetric equation (referring to the electric dipole moment) or the wave equation including both  $P$ -symmetric and  $P$ -asymmetric sectors may be obtained with the use of the simple linear transformation over wave functions. At this transformation, the energy spectra preserve their form.

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