UNIMORE
UNIVERSITÀ DEGLI STUDI DI
MODENA E REGGIO EMILIA

Dipartimento di Economia
Marco Biagi

DEMB Working Paper Series

N. 232

Unveiling Sentiment Dynamics and Forecasting Future Economic Sentiment in the Eurozone using Option-Implied Asymmetry Measures

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November 2023

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ISSN: 2281-440X online

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Abstract:

In this paper, we introduced several asymmetry indices based on option prices for the Eurozone. The aim is to investigate the ability of option-implied asymmetry measures to explain sentiment dynamics and forecast future market sentiment. To achieve our objectives, we measured asymmetry in two ways. Firstly, we decomposed the SKEW index into its positive and negative components. Secondly, we introduced the Risk-Asymmetry (RAX) index as an alternative measure of asymmetry. Our findings suggest that asymmetry indices play a significant role in explaining the level of economic sentiment indicators. Additionally, the asymmetry index obtained from the left tail of the risk-neutral distribution (put prices) contains useful information for predicting the level of sentiment in the following month.

Keywords: sentiment indices, economic sentiment, asymmetry, tail risk

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1. Introduction

Investor sentiment encompasses the expectations and perceptions of future cash flow and investment risks that may not necessarily be based on available facts. The sentiment of investors reflects the overall "level of confidence" in the market towards a particular asset associated with a specific situation (Baker and Wurgler, 2006; 2007). Correctly identifying the level of sentiment allows us to unveil critical insights into the underlying market dynamics, empowering both investors and regulators to assess market behaviour adeptly and anticipate potential shifts in demand and asset valuations. The manifestation of investors' irrationality constitutes the foundation of every speculative bubble. Influenced by emotions, investors drive prices excessively, overlooking the true value of the financial instrument and pushing prices to significantly distant levels from what would be considered reasonable.

Therefore, sentiment provides a nuanced perspective for comprehending short-term market fluctuations and long-term trends, exerting influence over financial decisions with far-reaching implications for asset prices (Benhabib et al., 2016; Piccione and Spiegler, 2014). Notably, sentiment has proven instrumental in elucidating financial market movements during periods marked by irrational panic or unwarranted optimism (Reis and Pinho, 2020a). Additionally, investor sentiment emerges as a primary driving force behind the co-movements of stock returns, offering explanatory power for the level, variance, and covariance of the non-fundamental component of returns (Frijns et al., 2017). Consequently, the study of sentiment has evolved into an indispensable tool for navigating the intricacies of financial markets, facilitating well-informed decisions that align with prevailing investor sentiment.

In a recent study conducted by Bevilacqua and Tunaru (2021), the relationship between sentiment in the US market and the SKEW index is explored, both by exploiting the index in its entirety and by breaking it down into its components of negative and positive skewness. The rationale underlying the link between sentiment and option-implied measures may reside in the fact that option-implied measures, such as skewness, reflect market participants' expectations of future price movements and uncertainty. Moreover, Seo and Wachter (2019) find that option prices reflect the risk of rare economic events, such as consumption disasters, providing additional evidence for their importance in capturing future economic uncertainty. In line with this insight, Bevilacqua and Tunaru (2021) find that option-implied indices are able to explain current fluctuations in sentiment indicators. Moreover, they also find that the positive skewness index is able to predict four out of the six sentiment measures adopted for the US market.

Despite the key role assigned to option-implied skewness as a measure of risk and as a tool to monitor and forecast investor sentiment, the literature on the relationship between asymmetry indices and sentiment is scant and limited to the US (e.g. Stambaugh et al., 2012; Buraschi and Jiltsov, 2006; Han, 2008; Garleanu et al., 2009; Friesen et al., 2012; Lemmon and Ni, 2014; Bevilacqua and Tunaru, 2021). For most of the European markets, a measure of the asymmetry in the return distribution and tail risk has yet to be introduced (Gambarelli and Muzzioli, 2023); consequently, the relationship between asymmetry indices and sentiment in the European context is far from being clarified.

To fill the existing gap, the objective of this paper is to understand whether there is a relationship of option-implied asymmetry indices with market sentiment in the Eurozone context, initially examining the association with investor sentiment and subsequently analysing the ability of these indices to predict future changes in investor sentiment. As the Eurozone market lacks measures capturing the asymmetry of the risk-neutral distribution, we firstly introduce a skewness index computed on Eurostoxx 50 options. In particular, we introduce both a measure of asymmetry based on the *CBOE SKEW* index method (to serve as a benchmark for measuring risk-neutral skewness) and on the risk-asymmetry index (*RAX*) developed by Elyasiani et al. (2018). To deal with the limited availability of option-based data for European countries that represents the main obstacle for construction of such indices in the EU (see, Elyasiani et al. 2021), we adopt a specific procedure that involves interpolation among the existing strike prices and extrapolation outside of them. In this way, we obtain the series of the two asymmetry measures for the standard 30-day maturity. Moreover, we resort to the methodology suggested by Bevilacqua and Tunaru (2021) to decompose the SKEW index into its positive and negative components: the *CALL* index, obtained by applying the SKEW formula to call options, and the *PUT*, index, obtained by considering only put options.

As sentiment measures for the Eurozone, we adopt three widely used indicators: i) the *ESI*, obtained from surveys addressed to representatives of the manufacturing industry, services, retail trade, construction, and consumers of the Euro area economies; the European Sentix Investor Confidence (*SENTIX*), based on information processed through a monthly survey conducted with around 5,500 investors and analysts; ii) the Eurozone *ZEW* Economic Sentiment (*ZEW*), based on monthly interviews addressed to 300 experts from banks, insurance companies, and financial departments of selected companies about their expectations for the next six months on important international financial market data.

Our results indicate that asymmetry indices play a vital role in explaining the level of economic sentiment indicators. Out of the four asymmetry indices, those based on the entire option-implied distribution such as *RAX* and *SKEW* are more appropriate for capturing the fluctuation of the *ESI* index. On the other hand, the index obtained from call (put) options is better suited for capturing the evolution of the *SENTIX* (*ZEW*) index. Moreover, the asymmetry index obtained from the left tail of the risk-neutral distribution (put prices) provides useful information to forecast the level of sentiment in the following month. The *PUT* index has the highest forecasting power among the option-implied asymmetry indices, and it remains a significant or marginally significant predictor of future sentiment levels for both the *ESI* and the *SENTIX* indices even after controlling for their lagged levels.

The remainder of the paper is structured as follows. In Section 2, we review the methodologies to obtain asymmetry measures from a cross-section of option prices. In Section 3, we introduce the dataset and the interpolation-extrapolation method adopted for mitigating the low number of strike prices traded in the European markets. Section 4 investigates the relationship between the developed asymmetry indices and contemporaneous and future sentiment levels. Finally, Section 5 concludes and provides policy implications.

2. Asymmetry measures based on option prices

2.1 The standard skewness measure for financial markets

The standard market practice to compute risk-neutral skewness is to use the model-free skewness formula due to Bakshi et al. (2003). The CBOE SKEW index is designed to complement the information provided by the CBOE volatility index (CBOE VIX), which measures the overall risk in the 30-day S&P500 log-returns, by indicating the asymmetry of the return distribution. The following formula is commonly used in line with the CBOE procedure:

$$SK(\mathrm{t},\tau) \equiv \frac{E_t^q\left\{\left(R(\mathrm{t},\ \tau)-E_t^q\left[R(\mathrm{t},\ \tau)\right]\right)^3\right\}}{\left\{\left(E_t^q(R(\mathrm{t},\ \tau)-E_t^q\left[R(\mathrm{t},\ \tau)\right]\right)^2\right\}^{3/2}} = \frac{e^{r\tau}W(\mathrm{t},\tau)-3e^{r\tau}\mu(\mathrm{t},\tau)V(\mathrm{t},\tau)+2\mu(\mathrm{t},\tau)^3}{\left[e^{r\tau}V(\mathrm{t},\tau)-\mu(\mathrm{t},\tau)^2\right]^{3/2}} \tag{1}$$

where $R(\mathrm{t},\ \tau)$, $\left[R(\mathrm{t},\ \tau)\right]^2$ and $\left[R(\mathrm{t},\ \tau)\right]^3$ are the payoffs of the contracts at time *t* with maturity τ, based on the first, second and third moment of the distribution, respectively. Accordingly, $\mu(\mathrm{t},\tau)$, $V(\mathrm{t},\tau)$, $W(\mathrm{t},\tau)$ and $X(t,\tau)$ are the prices of the contracts, at time *t*, with maturity τ, based on the first, second, third and, fourth moment of the distribution, respectively. The prices of these contracts are obtained under the risk-neutral expectation (E_t^q). For a more detailed discussion of the contracts, see Appendix A.

2.2 The decomposition of the SKEW index

Despite its importance in describing the return distribution, the CBOE SKEW index has not gained the same level of recognition as the CBOE VIX index (Elyasiani et al., 2021). This may be partly due to the fact that changes in the CBOE SKEW index are positively correlated with changes in market returns (as shown by Liu and Faff, 2017), meaning that an increase in the CBOE SKEW index is associated with a simultaneous increase in market returns. Additionally, while the volatility index (CBOE VIX) spikes during periods of market downturn, the CBOE SKEW has been observed to increase during both calm and turbulent periods. This raises doubts about the effectiveness of the CBOE SKEW index as an indicator of fear in the US market. According to studies by Liu and Faff (2017) and Elyasiani et al. (2021), the SKEW index is not a reliable barometer of market fear, as it does not necessarily spike during periods of high volatility and market downturn. Therefore, existing studies have called for alternative measures of asymmetry that can better capture market fear.

A proposal in this direction is attributed to Bevilacqua and Tunaru (2021), who show that a more refined directional construction of the implied skewness enhances information extracted from US equity index option prices. More specifically, the total *SKEW* index can be decomposed into two components: a positive *SKEW* obtained only from S&P 500 calls, and a negative *SKEW* index computed only from S&P 500 puts. To elaborate, in the formulas (A7)–(A9) in the Appendix A, for the computation of *CALL* (*PUT*) index, we exploit only call (put) options when $K_i \geq K_0$ $(K_i \leq K_0)$.

2.3 An alternative measure of asymmetry obtained from option prices

The decomposition of the *SKEW* index into its positive and negative components is not the only way to assess the risk in a specific part of the option-implied distribution. In Elyasiani et al. (2018), to account for the fact that investors like positive spikes while they dislike negative spikes in the returns, the concept of upside and downside corridor implied volatility measures is exploited to obtain an alternative indicator of asymmetry risk: the risk-asymmetry index (*RAX*). The RAX index is a metric used to measure the asymmetry of the risk-neutral distribution. It aims to capture the pricing asymmetry of investors towards gains and losses. To calculate the RAX index, we follow the method used by Elyasiani et al. (2018) and combine the corridor implied volatilities for upside and downside.

The *RAX* index is then derived by measuring the difference between the upside and downside corridor implied volatilities, divided by the total volatility (for a more detailed explanation of how the RAX index is derived, please refer to Appendix B). In particular, the numerator is standardised by total volatility, so that the *RAX* index is not influenced by the level of volatility in bullish or bearish market periods:

$$RAX\left(t,\tau\right)=\frac{\sigma_{UP}\left(t,\tau\right)-\sigma_{DW}\left(t,\tau\right)}{\sigma_{TOT}\left(t,\tau\right)} \tag{2}$$

where $\sigma_{TOT}\left(t,\tau\right)$ is the sum of the upside and downside corridor implied volatilities and it is equal to model-free implied volatility. Downside (σ_{DW}) and upside (σ_{UP}) corridor implied volatility are the square root of downside and upside corridor implied variance and can be obtained as:

$$\sigma_{UP}\left(t,\tau\right)=\sqrt{\frac{2e^{r\tau}}{\tau}\int_{F_t}^{\infty}\frac{M\left(K,\tau\right)}{K^2}dK} \tag{3}$$

$$\sigma_{DW}\left(t,\tau\right)=\sqrt{\frac{2e^{r\tau}}{\tau}\int_{t}^{F_t}\frac{M\left(K,\tau\right)}{K^2}dK} \tag{4}$$

where $F_t = K^* e^{r\tau} * difference$, and K^* is the reference strike price (i.e. the strike at which the $difference$ in absolute value between the at-the money call and put prices is the smallest).[1]

Computing the RAX index will allow us to compare and contrast the results of measuring asymmetry based on SKEW with those based on corridor-implied volatilities. This will help us to better understand the relationship between economic sentiment and option-implied asymmetry measures. Additionally, the measures of risk based on corridor implied volatility have already proven to be useful in forecasting future market returns in the European market, as shown in Elyasiani et al. (2017, 2018).

3. Data and methodology

Our data set includes various sources of information such as the option data required to compute the asymmetry measures and economic sentiment indicators for the Eurozone. The option dataset and the procedure adopted to obtain the asymmetry indices are outlined in Sections 3.1 and 3.2, respectively. Section 3.3 provides further details on the economic sentiment measures adopted in our study.

3.1 The option dataset

The option dataset consists of closing prices of Eurostoxx 50 index options, recorded from January 2007 to December 2022 based on availability. The options data set, the dividend yield and the Euribor rates are collected from OptionMetrics (IvyDB Europe). As for the underlying assets, the time series of the Eurostoxx 50 was obtained from Bloomberg. Options are of the European type, in the sense that they can be exercised only at the expiration date. Given that the underlying asset (S_t) of the option series pays dividend, following Elyasiani et al. (2021), we compute its adjusted value ($\hat{S}_t$) at time *t* as:

$$\hat{S}_t = S_t e^{-\delta_t \Delta t} \tag{5}$$

where δ_t is the dividend yield, and Δt is the time to maturity of the option. As a proxy for the risk-free rate, Euribor rates with maturities of one week, one month, two months, and three months are used. The appropriate yield to maturity is computed through linear interpolation.

We applied to the options dataset specific filters to remove any irregularities and opportunities for arbitrage in the prices. First, we exclude options that have a time-to-maturity of less than eight days, which is in line with the computational method used in constructing other indices, such as the CBOE SKEW. Second, we retain only at-the-money and out-of-the-money options, following the method used by Ait-Sahalia and Lo (1998). In-the-money options are rarely traded, and their prices can be affected by the illiquidity of the option contracts.

As per Elyasiani et al. (2018), we measure the value of options in terms of moneyness, which is calculated as *K/S*, where *K* represents the strike price and S represents the index value. We consider put options with moneyness values lower than 1.03 (i.e., $K/S < 1.03$) and call options with moneyness values greater than 0.97 (i.e., $K/S > 0.97$). Further, to establish a one-to-one relationship between strikes and implied volatilities, we take the average of the implied volatilities of options corresponding to the same strike price. Finally, we eliminate option prices that violate the standard no-arbitrage constraints and those whose closing price is less than 1 Euro, since they are frequently non-traded deep-out-of-the-money options.

3.2 The interpolation-extrapolation method

Limited availability of option-based data for European countries remains the main obstacle to constructing indices based on option prices (Elyasiani et al., 2021). The assumption of a continuum of strike prices ranging from zero to infinity, which is required for Eq. (1) and Eqs. (3-4), is not fulfilled in the reality of the options market. While this assumption can be mitigated for the US market due to the high number of option prices traded (usually more than 100 per day), it can represent a significant issue for European markets, which are characterised by a limited number of strike prices traded (Elyasiani et al., 2021), leading to truncation and discretization errors.

To overcome the limitations of infrequent option trades and assess the reliability of the SKEW index estimate, we also resort to a procedure that allows us to nearly eliminate truncation and discretization errors and greatly improves the precision of the skewness estimate. The procedure involves the following steps. After applying the filters described above, we create a table of available strike prices and implied volatilities, which serves as our initial input. To achieve a sufficient number of strike prices, we follow an interpolation-extrapolation method (as described in Jiang and Tian, 2005). Implied volatilities are interpolated between two adjacent knots using cubic splines to keep the function smooth in the knots and extrapolated outside the traded domain of strike prices. Specifically, we assume constant volatility for strike prices higher than the maximum strike price traded and lower than the minimum strike price traded. The constant volatility used in the left (right) part of the extended smile is set to be equal to the volatility of the lowest (highest) strike price traded. This ensures that we avoid negative implied volatilities (as recommended by Muzzioli et al., 2018). Finally, we compute missing implied volatility and strike prices from the interpolated-extrapolated smile by using a specific space interval ΔK to ensure insignificant truncation errors. On the other hand, truncation errors are mitigated by computing a matrix of strike prices and implied volatility in the interval $S/(1+u) \le K \le S(1+u)$, where S is the underlying asset value, and u is a parameter equal to 2, in line with Gambarelli and Muzzioli (2019). As a last step, after obtaining the implied volatilities, we convert them into option prices and by applying equations (A7-A9) we obtain: i) the *SKEW* index by considering all the options; the *CALL* index by exploiting only call options, and the *PUT* index by using only *PUT* options.

Therefore, our procedure is designed to follow the CBOE method as closely as possible, with the exception of the interpolation-extrapolation step. The interpolation-extrapolation procedure will be used also for the construction of the risk-asymmetry index. In particular, following Elyasiani et al. (2018), corridor implied volatility are computed as a discrete version of Eqs. (3)-(4) with integration domain equal to [$K_{\min}$, F] and [F, $K_{\max}$], where $K_{\min}$ and $K_{\max}$ correspond to the minimum and maximum strike price of our interpolated-extrapolated volatilities, thus ensuring an insignificant truncation error (for more details see Muzzioli, 2015).

3.2 The final skewness index

Following the CBOE procedure (CBOE, 2010), we compute a measure with a fixed 30-day maturity by exploiting two option series. Specifically, a first option series with a maturity of less than 30 days and a second option series with time to maturity greater than 30 days, are used:

$$M_{30} = wM_{near} + \left(1-w\right)M_{next} \tag{6}$$

with $w=(T_{next}-30)/(T_{next}-T_{near})$, where T_{near} (T_{next}) is the time to expiration of the near (next term) options, and M_{near} and M_{next} are the estimated measures of asymmetry, which refer to the near and next term options, respectively.

According to the CBOE procedure (CBOE, 2010), we calculate the final value of the asymmetry indices as:

$$SKEW = 100 - 10 \times M_{30} \tag{7}$$

where M_{30} is obtained in Eq. (6). As the risk-neutral skewness attains typically negative values for equity indices, formula (7) enhances the interpretation of a skewness index. For symmetric distributions, risk-neutral skewness is equal to zero, and the skewness index will be equal to 100. On the other hand, values higher (lower) than 100 for the skewness index point to a left (right) skewed risk-neutral distribution. The higher the risk, the higher the perceived risk related to negative returns will be. Moreover, a high value of the *SKEW* index indicates that buying protection against market downturns (put options) is more expensive.

To obtain a constant 30-day measure for the skewness index, following Elyasiani et al. (2018) the *RAX* index is constructed by using 30-day volatility measures obtained with the same linear interpolation procedure of the near- and next-term options adopted for the *SKEW* (equation 6). Moreover, the transformation in equation (7) is applied to the daily values of the *RAX* index for ease of comparison with the *SKEW* index. As a result, a value of the *RAX* higher than 100 indicates that the volatility of the left side of the distribution (σ_{DW}) is higher than the one of the right side (σ_{UP}), indicating that investors attach a higher (risk-neutral) probability to negative returns.

Finally, in order to make the asymmetry indices comparable to the economic sentiment indices (which are computed on a monthly frequency), we average the daily estimates for each month to obtain the monthly series that will form our final dataset.

3.3 Economic sentiment indicators

The first indicator is the Economic Sentiment Indicator (hereafter, ESI). It is a composite index measuring the level of confidence in the Euro area. The index is computed monthly by the European Commission's Directorate-General for Economic and Financial Affairs, which regularly conducts harmonised surveys for various sectors of the European Union and candidate countries' economies. The ESI is obtained from surveys addressed to representatives of the manufacturing industry, services, retail trade, construction, and consumers. These surveys allow for the comparison of economic cycles across different countries and have become an indispensable tool for monitoring the evolution of the EU and euro area economies, as well as developments in candidate countries. The index is generated by calculating the weighted average of the scores from each survey, which is subsequently normalised to ensure a long-term average of 100 and a standard deviation of 10. Values above 100 indicate economic sentiment above average, while values below 100 indicate a position below the average. Assuming an approximately normal distribution, setting the standard deviation to 10 implies that, in about 68% of cases, the ESI falls within the 90-110 range.

The second indicator is the European Sentix Investor Confidence, also known as SENTIX. It is an index that evaluates the economic situation and prospects for the euro area for the next six months. The index calculation is based on information processed through a monthly survey conducted with around 5,500 investors and analysts. These participants are interviewed about their estimates regarding the 14 financial markets under analysis. If the reading is above zero, it indicates optimism, while a reading below zero indicates pessimism.

The Eurozone ZEW Economic Sentiment (referred to as ZEW) is the third indicator that is used to assess the economic outlook for the Eurozone over the next six months. It is a monthly index that is based on the ZEW Financial Market Test, where 300 experts from banks, insurance companies, and financial departments of selected companies are interviewed every month. These experts are questioned about their assessments and forecasts on important international financial market data, including the economy, inflation rates, interest rates, stock markets, and exchange rates. Their expectations for the next six months are recorded and used to form the ZEW index.

To facilitate the comparison between the ESI and the asymmetry indices (characterised by a baseline level of 100), we have added 100 to both the SENTIX and the ZEW. This adjustment was made to align the sentiment index with the asymmetry indices without changing its essential meaning. It is crucial to note that an index above 100 still indicates a positive sentiment, while an index below 100 still connotes a negative sentiment.

4. Empirical results: the relationship between economic sentiment and asymmetry indices

4.1 Descriptive statistics

The descriptive statistics for the economic sentiment indicators and the asymmetry indices are reported in Table 1. Among the asymmetry measures, three of the four indices show average values higher than 100, indicating that the option implied distribution for the Eurostoxx 50 index is skewed to the left, i.e., the expected and realised probability of negative returns is greater than that of positive returns. The only exception is the *CALL* index, which is computed using only call option prices. A *CALL* index average value lower than 100 indicates that the distribution is asymmetrical to the right if we consider only call options. This result is somewhat expected, given the well-known V-shape attained by the volatility smile.[2]

Moreover, it appears that the *CALL* index follows a normal distribution, as the null hypotheses cannot be rejected according to the Jarque-Bera test. On the other hand, the *PUT*, *SKEW*, and *RAX* indices exhibit positive asymmetry and are not normally distributed. This suggests that the left tail of their distribution presents sharp changes, which is likely due to investors protecting themselves

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[2] The "smile" is a graphical representation of implied volatility, which is calculated by inverting the Black-Scholes formula, plotted against the strike price. The curve of the smile can either resemble an upward smile (when the implied volatility is higher for out-of-the-money options than it is for at-the-money options) or a smirk (when the implied volatility is higher for put prices and lower for call prices).

against market downturns and taking advantage of put options, thus impacting their prices. On the other hand, economic sentiment indices tend to have a negative skewness, suggesting that the likelihood of a significant decrease in sentiment is higher than the likelihood of a significant increase. However, the hypotheses of a normal distribution cannot be rejected for those indices.

The correlation coefficients between the indices in our sample are presented in Table 2. Surprisingly, we found that there is a positive relationship between all the indices. This suggests that a high level of sentiment is generally associated with a high level of asymmetry in the Eurostoxx 50 risk-neutral distribution. This result was unexpected and will be further investigated in Sections 4.2 and 4.3. Additionally, we observed a low correlation between the *CALL* and *SKEW* indices, while the association between the *PUT* and *SKEW* indices was high. This result indicates that fluctuations in the *SKEW* index are mainly associated with fluctuations in the asymmetry measured by the *PUT* option. Moreover, the two asymmetry indices measuring asymmetry in the whole distribution (*SKEW* and *RAX*) are highly associated. The two indices will be further compared and contrasted in their ability to assess and forecast economic sentiment indices in Section 4.2 and 4.3, respectively.

4.2 The relationship between sentiment and asymmetry indices

In this section, we aim to explore the relationship between sentiment indices and asymmetry indices based on option prices. Understanding fluctuations in sentiment will enable investors and regulators to gain critical insights into the underlying market dynamics. This will empower them to assess market behaviour accurately and anticipate potential shifts in demand and asset valuations. Existing studies have investigated the relationship between sentiment and risk-neutral skewness only in the US market (e.g. Stambaugh et al., 2012; Buraschi and Jiltsov, 2006; Han, 2008; Garleanu et al., 2009; Friesen et al., 2012; Lemmon and Ni, 2014; Bevilacqua and Tunaru, 2021). On the other hand, the European market has been little investigated both in terms of option implied asymmetry measures and sentiment indices and the results between the European and US stock markets may differ due to distinct institutional characteristics. On average, the European market is characterised by a more dominant financial sector and a less dominant technological sector, whereas the opposite holds true for the US market.

To investigate the relationship between sentiment and asymmetry indices, following Bevilacqua and Tunaru (2021), we estimate the following regression model:

$$SENT_t = \alpha + \beta\, asymmetry_t + \varepsilon_t \tag{8}$$

where $SENT_t$ is the monthly level of the sentiment index and is proxied alternatively by *ESI*, *SENTIX* and *ZEW*, and $asymmetry_t$ is the value of option-implied asymmetry measured alternatively by the *CALL*, *PUT*, *SKEW*, and *RAX* indices. The output of the regression model is reported in Table 3.

The output of the model indicates a strong and positive association between asymmetry and sentiment indices in almost all cases, with the exception of the relationships between the *CALL* and *RAX* index on one hand, and the *ZEW* economic sentiment indicator on the other. It is important to note that no single asymmetry index is found to be better than others in explaining sentiment indices in all cases. In fact, the *RAX* index is the most effective in explaining the *ESI* sentiment index. Meanwhile, *CALL* and *PUT* are better suited to explain the *SENTIX* and *ZEW* economic sentiment indices, respectively. The reason for the dissimilarity could be related to the different measurement of sentiment provided by the three indicators. The *ESI* is obtained from surveys addressed to representatives of the manufacturing industry, services, retail trade, construction, and consumers. On the other hand, *SENTIX* and the *ZEW* indices consider sentiment obtained from investors and analysts, and experts from financial intermediaries, respectively.

The different type of sentiment indicators adopted in our analysis could be at the basis of the difference between our results and those obtained in Bevilaqua and Tunaru (2021). According to their findings, the *SKEW* and the asymmetry index obtained using put options were characterised by a negative relationship with sentiment indices. On the other hand, a positive association with sentiment indices was detected only for asymmetry obtained from call option prices. Moreover, it is noting that the sentiment indices used in their study are mainly market-based sentiment indicators, often obtained by combining financial variables. On the other hand, sentiment indicators available for the Eurozone are based on surveys, thus convey different information content. The comparison with Bevilaqua and Tunaru (2021) may be more appropriate by looking at the results for the University of Michigan Consumer Sentiment index, which is a monthly survey of consumer confidence levels in the United States conducted by the University of Michigan. However, their results on the association between asymmetry indices and the University of Michigan Consumer Sentiment index point to an overall weak relationship (r-squared around 1%), significant only for the index obtained using call option prices.

In summary, our findings suggest that all the asymmetry indices play a significant role in explaining the level of economic sentiment indicators. In terms of index selection, those that measure asymmetry based on the entire option-implied distribution (such as *RAX* and *SKEW*) are more appropriate for representing economic sentiment of the manufacturing industry, services, retail trade, construction and consumers. On the other hand, the index obtained from call (put) options is better suited to capture the sentiment of investors and analysts (financial intermediaries).

4.3 The predictive power of asymmetry indices on sentiment indicators

Among the different methodologies adopted in the literature to construct sentiment indices (see González-Sánchez and Morales de Vega, 2021), some indices could be more responsive to incorporating changes in sentiment than others. In particular, indices based on option prices, could incorporate investors’ expectation about future sentiment fluctuations. In order to further investigate the relationship between asymmetry indices and sentiment, we test whether indices based on option prices have some predictive power on sentiment indicators based on surveys by running the following predictive equation:

$$SENT_t = \alpha + \beta\, asymmetry_{t-1} + \varepsilon_t \tag{9}$$

where $SENT_t$ is the monthly level of the sentiment index at month t and is proxied alternatively by *ESI*, *SENTIX* and *ZEW*, and $asymmetry_t$ is the value of option-implied asymmetry measured alternatively by the *CALL*, *PUT*, *SKEW*, and *RAX* indices, and recorded in the previous month (t-1). The output of the regression model is reported in Table 4, Panel A.

The findings presented in Panel A reveal that there is a strong and positive correlation between option implied asymmetry indices and future sentiment levels. This implies that when the asymmetry of the option-implied distribution is high, the future levels of sentiment indices are also high. However, it is worth noting that the *RAX* and *CALL* indices are not significant in predicting the *ZEW* sentiment index. Additionally, for all sentiment indices, the *PUT* index has the highest forecasting power among the option-implied asymmetry indices. Moreover, our findings differ from those presented in Bevilaqua and Tunaru (2021) research. They found a strong positive correlation between the asymmetry index obtained from call options and the future levels of the University of Michigan Consumer Sentiment index. However, in their study the other asymmetry indices were not able to accurately forecast the sentiment index. The different results could be motivated by the different computation methodology of the sentiment indicator being analysed, as well as the different periods considered (our study focused on 2007-2022, while the previous study analysed 1996-2017). Furthermore, the distinct institutional characteristics of the markets being analysed may also have contributed to the discrepancies.

To ensure the accuracy of the results, we followed the method used in Bevilaqua and Tunaru's (2021) study, but with an additional step. We also tested whether the asymmetry indices' ability to predict sentiment is still significant after controlling for past sentiment values, by running the following regression model:

$$SENT_t = \alpha + \beta_1\, asymmetry_{t-1} + \beta_2\, SENT_{t-1} + \varepsilon_t \tag{10}$$

where $SENT_t$ is the monthly level of the sentiment index at month t and is proxied alternatively by *ESI*, *SENTIX* and *ZEW*, $SENT_{t-1}$ represents the sentiment index level in the previous month, and $asymmetry_t$ is the value of option-implied asymmetry measured alternatively by the *CALL*, *PUT*, *SKEW*, and *RAX* indices, and recorded in the previous month (t-1).

In Table 4, Panel B, we report the result for the β_1 coefficient of the model. The results, presented in Panel B, indicate that the model's explanatory power significantly increases when past sentiment levels are added as a control variable. This also reduces the importance of option-implied asymmetry. However, for two out of the three sentiment indices used in our research, the *PUT* index remains a significant or marginally significant predictor of future levels.

4.4 Discussion of the results

The results presented in Section 4.3-4.4 suggest that asymmetry indices play a significant role in explaining both the contemporaneous and the future level of economic sentiment indicators. However, the economic interpretation of the results is not straightforward. High levels of sentiment are associated with a high asymmetry of the option-implied distribution, which means that investors buy more put than call options and attach a higher probability to events in the left tail of the distribution. There are two possible interpretations of this result. Firstly, investors might be willing to pay to hedge their gains in a high sentiment environment, which would make them more concerned about future market returns. This would increase the price of put options, shift the risk-neutral distribution to the left, and increase the asymmetry indices. This phenomenon is known as the "bubble theory", where high past returns indicate that a bubble is inflating, and a large drop can be expected when the bubble bursts. Harvey and Siddique (2000) found that when past returns were high in the US market, the investors' forecast of skewness became more negative (more skewed towards the left). Similarly, Xiong et al. (2016) found that high-priced stock markets are characterised by highly negative risk-neutral skewness, while stocks that have already fallen in price tend to be more positively skewed.

Another possible reason for the positive correlation could be the investment behaviour of institutional traders. If these large investors and institutions anticipate an improvement in the investment opportunities available, they may be encouraged to enter the underlying market by hedging in put options for extreme events. This kind of behaviour would contribute to the asymmetry of the distribution implied in option prices and consequently increase the asymmetry indices. However, since addressing this issue would involve obtaining information on the investment flows of those operators, we will leave this matter for further research.

5. Conclusion

Sentiment indicators play a crucial role in determining investor confidence. They provide valuable insights into market dynamics and empower both investors and regulators to assess market behaviour effectively. These indicators also help anticipate potential shifts in demand and asset valuations, which can prevent the formation of speculative bubbles. Previous studies conducted in the US market suggested the existence of a connection between measures obtained from option prices and investors sentiment (Stambaugh et al., 2012; Buraschi and Jiltsov, 2006; Han, 2008; Garleanu et al., 2009; Friesen et al., 2012; Lemmon and Ni, 2014; Bevilacqua and Tunaru, 2021).

As the Eurozone market lacks measures meant to capture asymmetry from option prices (Elyasiani et al. 2021), there are currently no studies on the role of option-implied asymmetry indices in explaining and forecasting sentiment levels. To address the gap, we have introduced four measures of asymmetry derived from option prices. First, we introduced a skewness index based on the CBOE procedure as a benchmark to measure risk-neutral skewness. Second, we followed the approach of Bevilacqua and Tunaru (2021) to decompose the *SKEW* index into its positive and negative components by considering call and put prices, respectively. This allowed us to obtain two indices, *CALL* and *PUT*, that account for asymmetry in specific parts of the distribution. Finally, we included the Risk-Asymmetry Index (RAX) developed by Elyasiani et al. (2018), which has proven to be a useful measure of risk in the European market (Gambarelli and Muzzioli, 2019).

Our findings suggest that asymmetry indices play a significant role in explaining the level of economic sentiment indicators. Among the four asymmetry indices, those based on the entire option-implied distribution (such as *RAX* and *SKEW*) are more appropriate for representing economic sentiment of the manufacturing industry, services, retail trade, construction and consumers. On the other hand, the index obtained from call (put) options is better suited to capture the sentiment of investors and analysts (financial intermediaries).

Additionally, the asymmetry index obtained from the left tail of the risk-neutral distribution (put prices) embeds useful information to forecast the level of sentiment in the following month. The *PUT* index has the highest forecasting power among the option-implied asymmetry indices, and it remains a significant or marginally significant predictor of future sentiment levels for both the ESI and the *SENTIX* indices even after controlling for their lagged levels.

Accurate measurement and prediction of investor sentiment hold paramount significance for managers, policymakers, and investors alike, shaping the landscape of financial decision-making. Managers can benefit from a nuanced understanding of investor sentiment as it provides invaluable insights into market dynamics and aids in the formulation of strategic investment plans. Policymakers, on the other hand, can leverage sentiment analysis to anticipate market trends and tailor regulatory frameworks accordingly, fostering stability and growth. For investors, a precise gauge of sentiment serves as a crucial tool for risk management and portfolio optimization, empowering them to make informed choices in an ever-changing financial environment. As financial markets become increasingly interconnected and influenced by psychological factors, the ability to correctly measure and predict investor sentiment emerges as a linchpin for informed decision-making across diverse stakeholders in the financial ecosystem.

Acknowledgements

The authors gratefully acknowledge financial support from University of Modena and Reggio Emilia for the FAR2021, FAR2022 and FAR2023 projects.

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Table 1. Descriptive statistics for the sentiment indices.

	CALL	PUT	SKEW	RAX	ESI	SENTIX	ZEW
Mean	79.35	137.56	117.74	102.52	99.24	100.96	112.70
Median	79.35	136.47	116.62	102.45	101.45	104.28	116.30
Maximum	77.19	126.05	108.93	101.60	59.70	57.13	36.30
Minimum	81.39	163.76	142.37	104.40	118.70	142.02	184.00
Std. Dev.	0.92	6.23	5.96	0.54	10.72	18.99	35.91
Skewness	-0.05	1.23	1.67	1.05	-0.98	-0.28	-0.29
Kurtosis	2.40	5.14	6.41	1.18	1.42	2.43	2.23
Jarque-Bera	2.92	85.19	181.22	4.12	2.23	5.07	7.42
p-value	0.23	0.00	0.00	0.00	0.02	0.08	0.02

Note: The table shows the descriptive statistics of the risk-neutral skewness indices and the economic sentiment indices adopted in our study. SKEW is the index we compute using the CBOE method, CALL and PUT are obtained by applying the CBOE SKEW formula only to call and put option prices, respectively (see Appendix A for further details about the computation of the indices). RAX is the risk-asymmetry index based on Elyasiani et al. (2018) (see Appendix B). The ESI is obtained from surveys addressed to representatives of the manufacturing industry, services, retail trade, construction, and consumers of the Euro area economies. SENTIX is the European Sentix Investor Confidence, based on information processed through a monthly survey conducted with around 5,500 investors and analysts. ZEW is the Eurozone ZEW Economic Sentiment, a monthly index based on the ZEW Financial Market Test, where 300 experts from banks, insurance companies, and financial departments of selected companies are interviewed every month about their assessments and forecasts on important international financial market data, including the economy, inflation rates, interest rates, stock markets, and exchange rates. Their expectations for the next six months are recorded and used to form the ZEW index. To facilitate the comparison with the ESI and the asymmetry indices (characterised by a baseline level of 100), we have added 100 to both the Sentix and the ZEW.

Table 2. Correlation matrix

	CALL	PUT	SKEW	RAX	ESI	SENTIX	ZEW
CALL	1.000						
PUT	0.147	1.000					
SKEW	0.214	0.905	1.000				
RAX	0.416	0.693	0.915	1.000			
ESI	0.374	0.359	0.409	0.417	1.000		
SENTIX	0.336	0.323	0.306	0.288	0.849	1.000	
ZEW	0.046	0.447	0.354	0.214	0.089	0.385	1.000

Note: the table reports the correlation between the monthly series of the indices. For a definition of the measures, see Table 1.

Table 3. Regression output for linear regression model in equation (8)

	ESI		*SENTIX*		*ZEW*	
CALL	4.355*** (3.263)	13.55%	6.922*** (3.068)	10.81%	1.798 (0.412)	0.00%
PUT	0.620*** (3.316)	12.44%	0.987*** (3.533)	9.94%	2.585*** (4.946)	19.56%
SKEW	0.737*** (3.859)	16.25%	0.979*** (3.437)	8.90%	2.139*** (3.753)	12.06%
RAX	8.293*** (3.586)	16.97%	10.132*** (2.863)	7.79%	14.244* (1.862)	4.07%

Note: the table presents the estimated output of the following regressions:

$$SENT_t = \alpha + \beta\, asymmetry_t + \varepsilon_t \tag{8}$$

where $SENT_t$ is the monthly level of the sentiment index and is proxied alternatively by *ESI*, *SENTIX* and *ZEW*, and $asymmetry_t$ is the value of option-implied asymmetry measured alternatively by the *CALL*, *PUT*, *SKEW*, and *RAX* indices. All the regressions are run by using the Ordinary Least Squares (OLS), with the Newey-West heteroskedasticity and autocorrelation consistent (HAC) covariance matrix (t-stats in parentheses). For the definition of the measures see Table 1. Significance at the 1% level is denoted by ***, at the 5% level by **, and at the 10% level by *.

Table 4. Regression output for linear regression model in equations (9-10)

	ESI		*SENTIX*		*ZEW*	
Panel A: regression output for model in equation (9)						
CALL	3.504*** (2.429)	8.61%	4.668*** (2.007)	4.67%	-2.742 (-0.699)	0.00%
PUT	0.730*** (3.935)	17.49%	1.082*** (3.974)	12.17%	2.390*** (4.286)	16.62%
SKEW	0.807*** (4.842)	19.65%	0.993*** (3.723)	9.27%	1.883*** (3.018)	9.23%
RAX	8.155*** (3.692)	16.48%	8.695*** (2.448)	5.67%	10.186 (1.251)	1.82%
Panel B: regression output for model in equation (10)						
CALL	-0.725* (-1.775)	90.47%	-2.012*** (-3.596)	90.01%	-4.270 (-3.767)	85.06%
PUT	0.161*** (2.618)	90.90%	0.155* (1.715)	89.38%	-0.010 (-0.053)	83.85%
SKEW	0.128** (2.574)	90.56%	0.068 (0.737)	89.19%	-0.110 (-0.637)	83.88%
RAX	0.361 (0.456)	90.16%	-0.933 (-0.805)	89.21%	-3.041 (-1.394)	84.05%

Note: the table presents the estimated output of the following regressions:

Panel A Model: $SENT_t = \alpha + \beta_1\, asymmetry_{t-1}$ (9)

Panel B Model: $SENT_t = \alpha + \beta_1\, asymmetry_{t-1} + \beta_2\, SENT_{t-1} + \varepsilon_t$ (10)

where $SENT_t$ is the monthly level of the sentiment index at month *t* and is proxied alternatively by *ESI*, *SENTIX* and *ZEW*, $SENT_t$ represents the sentiment index level in the previous month, and $asymmetry_t$ is the value of option-implied asymmetry measured alternatively by the *CALL*, *PUT*, *SKEW*, and *RAX* indices, and recorded in the previous month (t-1). All the regressions are run by using the Ordinary Least Squares (OLS), with the Newey-West heteroskedasticity and autocorrelation consistent (HAC) covariance matrix (t-stats in parentheses). For the definition of the measures see Table 1. Significance at the 1% level is denoted by ***, at the 5% level by **, and at the 10% level by *.

Appendix A. The Bakshi et al. (2003) model-free skewness formula, the SKEW index and its decomposition

In this section we provide further details about the model-free formula proposed in Bakshi et al. (2003) in order to compute the SKEW index and its components based on call and put options. According to Bakshi et al. (2003) model-free skewness is obtained from the following equation as:

$$SK(\mathrm{t},\tau) \equiv \frac{E_t^q\left\{\left(R(\mathrm{t},\ \tau)-E_t^q\left[R(\mathrm{t},\ \tau)\right]\right)^3\right\}}{\left\{\left(E_t^q(R(\mathrm{t},\ \tau)-E_t^q\left[R(\mathrm{t},\ \tau)\right]\right)^2\right\}^{3/2}} = \frac{e^{r\tau}W(\mathrm{t},\tau)-3e^{r\tau}\mu(\mathrm{t},\tau)V(\mathrm{t},\tau)+2\mu(\mathrm{t},\tau)^3}{\left[e^{r\tau}V(\mathrm{t},\tau)-\mu(\mathrm{t},\tau)^2\right]^{3/2}} \qquad \text{(A1)}$$

where $\mu(\mathrm{t},\tau)$, $V(\mathrm{t},\tau)$, $W(\mathrm{t},\tau)$ and $X(t,\tau)$ are the prices of the contracts, at time *t* with maturity *τ*, based on first, second, third and fourth moment of the distribution, respectively; their values are can be obtained from a cross-section of call and put option prices as:

$$\mu(t,\tau) \equiv E^q \ln\left[S(t+\tau)/S(t)\right] = e^{r\tau}-1-\frac{e^{r\tau}}{2}V(t,\tau)-\frac{e^{r\tau}}{6}W(t,\tau)-\frac{e^{r\tau}}{24}X(t,\tau) \qquad \text{(A2)}$$

$$V(t,\tau)=\int_{S(t)}^{\infty}\frac{2\left(1-\ln\left[K/S(t)\right]\right)}{K^2}C(t,\tau;K)dK+\int_{0}^{S(t)}\frac{2(1+ln\left[S(t)/K\right]}{K^2}P(t,\tau;K)dK \qquad \text{(A3)}$$

$$\begin{aligned}W(t,\tau)=&\int_{S(t)}^{\infty}\frac{6\ln\left[K/S(t)\right]-3\left(\ln\left[K/S(t)\right]\right)^2}{K^2}C(t,\tau;K)dK\\&-\int_{0}^{S(t)}\frac{6\ln[S(t)/K]+3\left(\ln\left[S(t)/K\right]\right)^2}{K^2}P(t,\tau;K)dK\end{aligned} \qquad \text{(A4)}$$

$$\begin{aligned}X(t,\tau)=&\int_{S(t)}^{\infty}\frac{12\left(\ln\left[K/S(t)\right]\right)^2-4\left(\ln\left[K/S(t)\right]\right)^3}{K^2}C(t,\tau;K)dK+\\&\int_{0}^{S(t)}\frac{12\left(\ln\left[S(t)/K\right]\right)^2+4\left(\ln\left[S(t)/K\right]\right)^3}{K^2}P(t,\tau;K)dK\end{aligned} \qquad \text{(A5)}$$

where $C(t,\tau;K)$ and $P(t,\tau;K)$ are the prices of a call and a put option at time *t* with maturity *τ* and strike *K*, respectively, $S(t)$ is the underlying asset price at time t.

To obtain the daily estimate of the SKEW index based on the CBOE (2010) methodology, SK is computed starting from a portfolio of options with payoff reflecting the skewness payoff:

$$SK=\frac{\mathbb{E}[R^3]-3\mathbb{E}[R]\mathbb{E}[R^2]+2\mathbb{E}[R]^3}{([R^2]-\mathbb{E}^2[R])^{3/2}} \qquad \text{(A6)}$$

For simplicity, *SK* can be written as $SK = \frac{P_3 - 3P_1P_2 + 2P_1^3}{(P_2 - P_1^2)^{3/2}}$, where P_1, P_2, and P_3, can be obtained as follows (CBOE, 2010):

$$P_1 = \mu = \mathbb{E}[R_\tau] = e^{r\tau}\left(-\sum_i \frac{1}{K_i^2} Q_{K_i}\,\Delta_{K_i}\right) + \varepsilon_1 \tag{A7}$$

$$P_2 = \mu = \mathbb{E}[R_\tau^2] = e^{r\tau}\left(\sum_i \frac{2}{K_i^2}\left(1 - \ln\left(\frac{K_i}{F_0}\right)\right) Q_{K_i}\,\Delta_{K_i}\right) + \varepsilon_2 \tag{A8}$$

$$P_3 = \mu = \mathbb{E}[R_\tau^3] = e^{r\tau}\left(\sum_i \frac{3}{K_i^2}\left\{2\ln\left(\frac{K_i}{F_0}\right) - \ln^2\left(\frac{K_i}{F_0}\right)\right\} Q_{K_i}\,\Delta_{K_i}\right) + \varepsilon_3 \tag{A9}$$

where F_0 denotes the forward price of the underlying index calculated from the put-call parity as $F_0 = e^{r\tau}\left[c(K,\tau) - p(K,\tau)\right] + K$, K_0 is the reference price, the first exercise price less or equal to the forward level $F_0(K \le F_0)$, K_i is the strike price of i-th out-of-the-money options used in the calculation, *r* is the risk-free rate with expiration τ , Δ_{K_i} is the sum divided by two of the two nearest prices to the exercise price *K*, Q_{K_i} is a generic price of a European call (resp. put) option with strike price above (resp. below) K_0. Finally, ε_1, ε_2, and ε_3 represent the adjustments for the difference between K_0 and F_0 and can be obtained as:

$$\varepsilon_1 = -\left(1 + \ln\left(\frac{F_0}{K_0}\right) - \frac{F_0}{K_0}\right) \tag{A10}$$

$$\varepsilon_2 = 2\ln\left(\frac{K_0}{F_0}\right)\left(\frac{F_0}{K_0} - 1\right) + \frac{1}{2}\ln^2\left(\frac{K_0}{F_0}\right) \tag{A11}$$

$$\varepsilon_3 = 3\ln^2\left(\frac{K_0}{F_0}\right)\left(\frac{1}{3}\ln\left(\frac{K_0}{F_0}\right) - 1 + \left(\frac{F_0}{K_0}\right)\right) \tag{A12}$$

Appendix B. Alternative measures of risk: the RAX index

Given the importance of disentangling positive and negative shocks to volatility, which are seen by investors, as good or bad news respectively, the information on upside and downside corridor implied volatilities is exploited in Elyasiani et al. (2018) in order to measure the asymmetry of the return distribution. Upside and downside corridor implied volatilities are aggregated into the risk-asymmetry index (RAX), which measures the difference between upside and downside corridor implied volatilities standardised by total volatility. The RAX index is meant to measure the investors' pricing asymmetry towards upside gains and downside losses.

Corridor implied volatility can be computed as the square root of corridor implied variance (*CIV*), that is obtained from model-free implied variance, due to Britten-Jones and Neuberger (2000) by truncating the integration domain between two barriers (see Carr and Madan, 1998; Andersen and Bondarenko, 2007):

$$\hat{E}\left[CIV\left(t,\tau\right)\right]=\hat{E}\left[\frac{1}{T}\int_{t}^{T}\sigma^{2}\left(t,\ldots\right)I_{t}\left(B_{1}B_{2}\right)dt\right] \tag{B1}$$

where $I_t\left(\ldots\right)$ is an indicator function that accumulates variance only if the underlying asset lies between the two barriers (B_1 and B_2). According to Demeterfi et al. (1999) and Britten-Jones and Neuberger (2000), it is possible to compute the expected value of corridor implied variance (CIV), under the risk-neutral probability measure, by using a portfolio of options with strikes ranging from B_1 to B_2, as:

$$\hat{E}\left[CIV\left(t,\tau\right)\right]=\frac{2e^{r\tau}}{\tau}\int_{B_1}^{B_2}\frac{M\left(K,\tau\right)}{K^{2}}dK \tag{B2}$$

where $M\left(K,\tau\right)$ is the minimum between a call or put option price with strike price K and maturity τ, r is the risk-free rate, and B_1 and B_2 are the barrier levels within which the variance is accumulated. Downside corridor implied variance is obtained by setting B_1 equal to zero and B_2 equal to the forward price, F_t, on the other hand, upside corridor implied variance is computed by setting B_1 equal to the forward price, F_t, and B_2 equal to infinity (∞). Downside (σ_{DW}) and upside (σ_{UP}) corridor implied volatility are the square root of downside and upside corridor implied variance, respectively:

$$\sigma_{DW}\left(t,\tau\right)=\sqrt{\frac{2e^{r\tau}}{\tau}\int_{t}^{F_t}\frac{M\left(K,\tau\right)}{K^{2}}dK} \tag{B3}$$

$$\sigma_{UP}\left(t,\tau\right)=\sqrt{\frac{2e^{r\tau}}{\tau}\int_{F_t}^{\infty}\frac{M\left(K,\tau\right)}{K^{2}}dK} \tag{B4}$$

and $F_t = K^* e^{r\tau} * difference$, where K^* is the reference strike price (i.e. the strike at which the $difference$ in absolute value between the at-the money call and put prices is the smallest).

Following Elyasiani et al. (2018), we aggregate upside and downside corridor implied volatilities into the risk-asymmetry index (RAX), which measures the difference between upside and downside corridor implied volatilities standardised by total volatility. The numerator is standardised by total volatility so that the RAX index is not influenced by the level of volatility in bullish or bearish market periods:

$$RAX\left(t,\tau\right) = \frac{\sigma_{UP}\left(t,\tau\right) - \sigma_{DW}\left(t,\tau\right)}{\sigma_{TOT}\left(t,\tau\right)} \tag{B5}$$

where $\sigma_{TOT}\left(t,\tau\right)$ is the sum of the upside and downside corridor implied volatilities and coincides with model-free implied volatility.