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Article

Decision-Maker's Preference-Driven Dynamic Multi-Objective Optimization

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Abstract: Dynamic multi-objective optimization problems (DMOPs) are optimization problems where elements of the problems, such as the objective functions and/or constraints, change with time. These problems are characterized by two or more objective functions, where at least two objective functions are in conflict with one another. When solving real-world problems, the incorporation of human decision-makers (DMs)' preferences or expert knowledge into the optimization process and thereby restricting the search to a specific region of the Pareto-optimal Front (POF) may result in more preferred or suitable solutions. This study proposes approaches that enable DMs to influence the search process with their preferences by reformulating the optimization problems as constrained problems. The subsequent constrained problems are solved using various constraint handling approaches, such as the penalization of infeasible solutions and the restriction of the search to the feasible region of the search space. The proposed constraint handling approaches are compared by incorporating the approaches into a differential evolution (DE) algorithm and measuring the algorithm's performance using both standard performance measures for dynamic multi-objective optimization (DMOO), as well as newly proposed measures for constrained DMOPs. The new measures indicate how well an algorithm was able to find solutions in the objective space that best reflect the DM's preferences and the Pareto-optimality goal of dynamic multi-objective optimization algorithms (DMOAs). The results indicate that the constraint handling approaches are effective in finding Pareto-optimal solutions that satisfy the preference constraints of a DM.

Keywords: dynamic multi-objective optimization; constrained optimization; decision-maker preference; differential evolution; performance measures

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1. Introduction

Dynamic multi-objective optimization problems (DMOPs) have multiple goals or objectives, and the objectives and/or constraints change over time [1–4]. However, the goals are usually in conflict with one another, thereby making the process of finding a single optimal solution a very difficult task [5]. Finding a set of optimal trade-off solutions is therefore the norm, with the Pareto-dominance relation [6] being used to compare the quality of the trade-off solutions. The set of optimal trade-off solutions in the decision space is called the Pareto-optimal Set (POS), while in the objective space, the set is referred to as the Pareto-optimal Front (POF) or Pareto Frontier [7].

DMOPs occur frequently in the real-world in a diverse range of domains, such as structural engineering [8]; plant control and scheduling [9–13]; and process optimization in manufacturing, for example material carbonization [14], copper removal in hydrometallurgy [15], and balancing disassembly lines [16].

However, the set of trade-off solutions may be overwhelming in number; a subset that better reflects the decision-maker (DM)'s preferences, may be required [17,18]. Some research has been conducted on incorporating a DM's preferences for static multi-objective optimization problems (MOPs) [9,19–27]. Most of these studies used a priori, interactive, and a posteriori approaches. It is noteworthy to state that to the best of the authors' knowledge, a priori and interactive preference incorporation methods have not been applied to DMOPs. Posteriori could have been applied when real-world problems were solved and a set of solutions were provided to the real-world DM.

Introducing DM preferences, however, leads to a reformulation of DMOPs as constrained problems, which are then solved by dynamic multi-objective optimization algorithms (DMOAs) using a variation of a penalty function [28–32]. The constraints imparted on DMOPs as a result of DM preferences are defined in the objective space; thereby, the constraints partition the objective space into feasible and infeasible regions.

The contributions of this study are

- A preference incorporation method adapted for DMOPs that is partly a priori and partly interactive and enables a DM to specify their preferences. The a priori incorporation of DM preferences occurs through a procedure, named bootstrap. The interactive incorporation of preferences is employed whenever a change occurs in the dynamic environment such that the DM preference set may be significantly affected.
- A bounding box approach (refer to Equation (2) in Section 2) to specify a DM's preferences in the dynamic multi-objective optimization (DMOO) search process. The proposed bounding box, unlike the proposal in [33], is employed in the context of DMOPs, thus making it the first of its kind.
- New approaches that can drive a DMOA's search constrained by the DM's preferences, as well as a comparative analysis of the constraint managing approaches incorporated into a DMOA. The proposed constraint managing approaches are fundamentally different from one another in terms of how they penalize solutions that violate a DM's preferences.
- New performance measures that measure how well a found solution adheres to the preferences of a DM. In this article, a solution will henceforth be referred to as a decision.

The base DMOA used in this study is a hybrid form of differential evolution (DE) [34], combining non-dominated sorting [35] with vector-evaluation schemes for selecting target vectors and the vectors that survive to the next generation during the optimization process, since it has been shown to perform well in solving DMOPs [36]. The proposed constraint managing approaches are incorporated in the same DMOA (the hybrid DE) to ensure a fair comparison of their performance. Their performance is measured using current (traditional) DMOO measures [1,37,38] and the new measures proposed in this article. It should be noted, however, that the constraint managing approaches and the preferences incorporation approaches can be incorporated into any DMOA.

The rest of the article is organized as follows: Section 2 presents background concepts and theories required to understand the rest of the article. The experimental setup, including the algorithmic setup, benchmark functions, performance measures, and statistical analysis employed in the study are discussed in Section 3. Section 4 presents and discusses the results of the experiments. Finally, conclusions are presented in Section 5 based on the results obtained from the experiments.

2. Background

This section discusses the key concepts which underlie the proposals in this study. Section 2.1 discusses the mathematical formulation of DMOPs that are addressed in this study. It also discusses the mathematics of the proposed bounding box approach and the limiting behaviors of the penalty function employed in this study. Section 2.2 discusses the mathematics required for new performance measures proposed in this article. Lastly, Section 2.3 discusses the core DMOA on which the based DMOA used in this study is based.

2.1. Bounding Box Mathematics

Let a composite function F be defined as follows:

$$F : \Omega_x \times \Omega_t \longrightarrow O \tag{1}$$

where $\Omega_x = \Re^n$, with $n \geq 2$, refers to the decision space, $\Omega_t \subseteq R$ refers to the time space, $t \in \Omega_t$ is a real-valued time instance and $t = \frac{1}{n_t}\left\lfloor \frac{\tau}{\tau_t} \right\rfloor$, with n_t referring to the severity of change, τ referring to the iteration counter, and τ_t referring to the frequency of change.

Let the objective space, O, be defined as [39]

$$O = \begin{cases} \Re^2 & \text{(e.g., FDA1 [10], dMOP2 [39])} \\ \Re^3 & \text{(e.g., FDA5 [10])} \end{cases}$$

Then, a decomposition of F follows:

$$F(\mathbf{x}, t) = \begin{cases} (f1, f2) & \text{(e.g., FDA1 [10], dMOP2 [39])} \\ (f1, f2, f3) & \text{(e.g., FDA5 [10])} \end{cases}$$

Each objective function f_i is defined as

$$f_i : \{\ \Omega_x \times \Omega_t \longrightarrow R\}, \quad i = 1, 2, 3$$

Let a DM's preference set be defined as

$$\texttt{Box}(\mathbf{z}, \mathbf{p}) = \{\mathbf{z} \in O | d(\mathbf{z}, \mathbf{p}) \leq r, \mathbf{p} \in O, r \in \Re\} \tag{2}$$

where d is the Euclidean distance measure, $\mathbf{p}$ is the center of the box formed by the points in this set, $\text{Box}(\mathbf{z}, \mathbf{p})$, r is the radius of the box, O is the objective space as defined in Equation (1), and the values of $\mathbf{p}$ and r are interactively selected by the DM.

Let a penalty function and its limiting behaviours be defined as

$$\text{penalty}(\mathbf{z}_k \in O, \lambda) = \begin{cases} 0, & if \quad d(\mathbf{z}_k, \mathbf{p}) \leq r \\ \lambda(d(\mathbf{z}_k, \mathbf{p}) - r), & if \quad d(\mathbf{z}_k, \mathbf{p}) > r \end{cases} \tag{3}$$

$$\lim_{\lambda \to c} \text{penalty}(\mathbf{z}_k \in O, \lambda) = \begin{cases} 0, & if \quad d(\mathbf{z}_k, \mathbf{p}) \leq r \\ c(d(\mathbf{z}_k, \mathbf{p}) - r), & if \quad d(\mathbf{z}_k, \mathbf{p}) > r \end{cases} \tag{4}$$

$$\lim_{\lambda \to \text{realmax}} \text{penalty}(\mathbf{z}_k \in O, \lambda) = \begin{cases} 0, & if \quad d(\mathbf{z}_k, \mathbf{p}) \leq r \\ \text{realmax}, & if \quad d(\mathbf{z}_k, \mathbf{p}) > r \end{cases} \tag{5}$$

$$\lim_{\lambda \to \text{realmax}} \mathbf{z}_k + \text{penalty}(\mathbf{z}_k \in O, \lambda) = \begin{cases} \mathbf{z}_k, & if \quad d(\mathbf{z}_k, \mathbf{p}) \leq r \\ I_1 \cdot \text{realmax}, & if \quad d(\mathbf{z}_k, \mathbf{p}) > r \end{cases} \tag{6}$$

$$\lim_{\lambda \to c} \mathbf{z}_k + \text{penalty}(\mathbf{z}_k \in O, \lambda) = \begin{cases} \mathbf{z}_k, & if \quad d(\mathbf{z}_k, \mathbf{p}) \leq r \\ \mathbf{z}_k + I_1 \cdot c(d(\mathbf{z}_k, \mathbf{p}) - r), & if \quad d(\mathbf{z}_k, \mathbf{p}) > r \end{cases} \tag{7}$$

where λ ($\geq$0) is a penalty control parameter whose value is determined by each algorithm, and $\mathbf{p}$, r, and d are defined as in Equation (2).

Then, a penalized outcome, $\mathbf{z}_k^* \in O$, is defined as $\mathbf{z}_k^* = \mathbf{z}_k + I_1 \cdot \text{penalty}(\mathbf{z}_k, \lambda)$ where $\mathbf{z}_k$ is a non-penalized outcome in the objective space, $\mathbf{z}_k = F(\mathbf{x}_k, t), \mathbf{x}_k \in \Omega_x$, F is as defined in Equation (1), and I_1 is an all-ones vector in the objective space (e.g., $(1, 1) \in \Re^2$).

2.2. Mathematics for Newly Proposed Performance Measures

This section discusses the mathematics required for two newly proposed performance measures. Section 2.2.1 discusses a measure that calculates the deviation of the violating decisions. The calculation of the spread of non-violating decisions that are found in the bounding box is discussed in Section 2.2.2.

2.2.1. Deviation of Violating Decisions

Solution space vectors whose objective values are outside the preference set are referred to as violating decisions, since they violate DM preferences. Depending on the control parameters used in the implementation of the penalty function of the proposed algorithms, the violating decisions may occasionally find their way into the archive, especially in situations where all the non-dominated solutions violate DM preferences and non-violating decisions are not found. However, it is a rare scenario: the non-violating decisions, if they are found in the archive, are very likely to dominate the violating and penalized decisions in the Pareto-dominance sense. However, when violating decisions find their way into the archive, a measure of the proximity of these violating decisions to the preferred bounding box is required. The smaller the total proximity, the better the violating decisions are. This section presents the mathematics underlying the calculation of the total proximity/deviation of the violating decisions.

Let $\mathbf{p}$, r, and the distance measure $d(\mathbf{z}, \mathbf{p})$ be as defined in Section 2.1, and let a set of violating decisions, Z, be defined as follows:

$$Z = \{\mathbf{z}_k \in O | d(\mathbf{z}, \mathbf{p}) > r\}, k = 1, \dots, |Z| \tag{8}$$

Let the cardinality, N, of Z be defined as

$$N = |Z| \tag{9}$$

Let the deviation of $\mathbf{z}_k \in Z$ be defined as

$$d_k = d(\mathbf{z}_k, \mathbf{p}) - r \; (d_k > 0) \tag{10}$$

Then the total deviation of all elements in Z is

$$\text{dVD} = \sqrt{\frac{\sum(1 + d_k)^2}{N}} \tag{11}$$

2.2.2. Spread of Non-Violating Decisions

The spread of non-violating decisions is one of the four measures proposed in this article. This measure estimates how well spread out the preferred decisions are within the bounding box located in the objective space. The greater the value of this measure, the better the performance of an algorithm. The calculation of this measure is presented in Algorithm 1.

Algorithm 1 Spread of Non-violating decisions

procedure SPREADESTIMATOR(outcomes)
▷ outcomes: objective vectors preferred by DM
  Get count of outcomes, N ← count(outcomes)
  if $N \leq 1$ **then**
    return 0 ▷ 1 or zero outcomes, spread is zero
  if $N == 2$ **then** ▷ 2 outcomes
    return $\text{norm}_2(\text{outcomes}(2) - \text{outcomes}(1))$ ▷ return spread between 2 values
  dtot ← 0 ▷ more than 2 outcomes, calculation required - initialize total spread, *dtot*
  firstNode ← outcomes(1) ▷ get a node
  currentNode ← firstNode ▷ set current node
  while unProcessedNodes(outcomes) > 1 **do** ▷ process each outcome
    MarkNodeAsProcessed(currentNode) ▷ mark outcome as processed
    nearestNode=getNearestNode(currentNode,outcomes) ▷ find nearest node to outcome being processed
    dist = $norm_2$(currentNode − nearestNode) ▷ calcuated distance between these two solutions
    dtot ← dtot + dist ▷ add their distance to total distance
    currentNode ← nearestNode
  dist = $norm_2$(currentNode − firstNode) ▷ finally calculate distance to first solution
  dtot ← dtot + dist ▷ add last distance to total distance
  return dtot ▷ return total distance

2.3. Core Dynamic Multi-Objective Optimization Algorithm

The core DMOA used in this study is presented in Algorithm 2. The algorithm starts with a set of randomly generated solutions, after initializing its run-time parameters, such as the population size, maximum archive size, maximum number of iterations, etc. The non-dominated solutions are added to the archive. A loop is performed until the number of iterations exceeds the maximum number of iterations. The non-dominated solutions, which are found at the end of the loop's execution, constitute the final solutions to the associated optimization problem. The algorithm uses sentry solutions to check whether a change in the environment has occurred.

Algorithm 2 Dynamic Multi-objective Optimization Algorithm

procedure DMOA(freq, severity, maxiteration, dMOP)
  Set population size, N
  Set archive max size, SizeArchive
  Initialize the iteration counter, iteration ← 0
  Initialize time, t ← 0
  Initialize(P_t, freq, severity, dMOP, t) ▷ initialize population of solutions, P_t
  AssignNonDominatedToArchive(P_t, dMOP, t) ▷ initialize archive
  while iteration ≤ maxiteration **do** ▷ check if stopping condition has been reached
    $t \leftarrow 1/\text{severity} \cdot \lfloor \text{iteration}/\text{freq} \rfloor$ ▷ calculate the current time
    Optimizer(P_t, dMOP, t) ▷ perform the search optimization
    Pick sentry solutions ▷ select sentry solutions to check for change
    if ENV changes(P_t, dMOP, t) **then** ▷ check for change in environment
      ProcessChange(P_t, freq, severity, dMOP, t) ▷ respond to change
    iteration ← iteration+1 ▷ increase iteration count

3. Experimental Setup

This section discusses the experimental setup used for this study. Section 3.1 discusses the algorithm setup. The DM preferences are discussed in Section 3.2. Section 3.3 discusses the benchmark functions, and the performance measures are highlighted in Section 3.4. The statistical analysis approach is discussed in Section 3.5.

3.1. Algorithm Setup

The approach that was followed to incorporate the DM's preference into the search process of the DMOA is discussed in Section 3.1.1.

3.1.1. Decision-Maker's Preference Incorporation

The different procedures and how they are used with a DMOA for the preference-driven search process are presented in Figure 1. Before the normal run of the DMOA starts, the a priori preference incorporation procedure is used to define the DM's preference. During the run of the DMOA, any of the constraint managing approaches can be used. If a small environment change occurs, the DMOA's change response approach is executed during the normal DMOA run. However, if the change is large, requiring a change in the boundary box placement, the interactive preference incorporation procedure is first completed before the DMOA's change response approach is executed during the normal DMOA run.

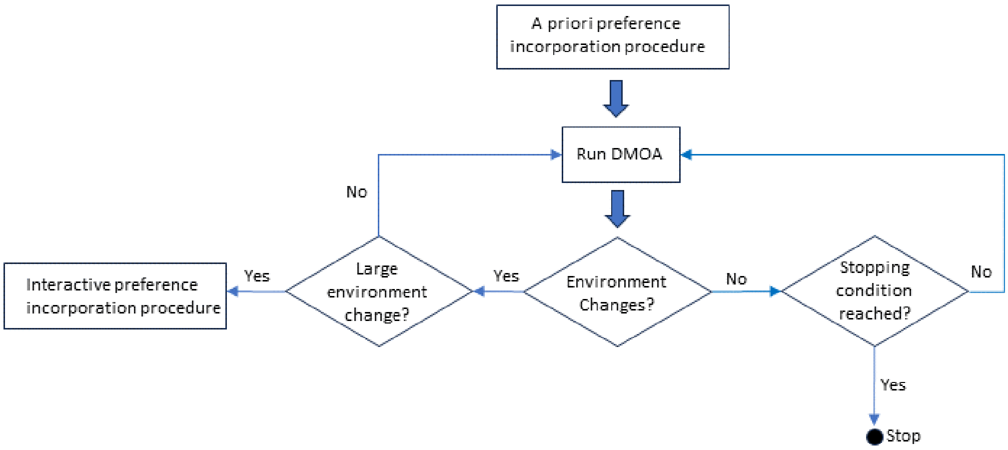

Figure 1. Preference-driven search process of a DMOA.

A Priori Preference Incorporation: A single run of the algorithm is executed. A series of POFs is presented to the DM. The DM selects one of these POFs, and then selects one of the points on the POF, which will become his $\mathbf{x}_p$ and $\mathbf{p}$, i.e., the preferred decision/solution vector and the preferred outcome, respectively. This preference, together with the radius of the bounding box specified by the DM during the bootstrap procedure (refer to Algorithm 3), is used to drive the DMOA's search to optimize the DMOPs under the constraints of the DM's preferences. The time complexity of the bootstrap procedure is similar to the time complexity of the DMOA that is used to produce the POSs.

Algorithm 3 Bootstrap Procedure

```
1: procedure BOOTSTRAP(freq, severity, iteration, F)
2:     Call DMOA(freq, severity, iteration, F)
   ▷ DMOA returns {POS_t^k}, ∀k = 1,...,n
   ▷ where k is the kth environment change
3:     i ← DMChooseIn(1,...,n)                       ▷ DM indicates their preferred POS
4:     x_p ← DMChooseIn(POS_t^i)                     ▷ DM indicates their preferred solution
5:     p ← F(x_p, t)                                 ▷ DM's preference is formulated
6:     DM Choose box radius, r ← random()            ▷ DM indicates their preferred boundary box size
7:     return (x_p : p : r)
```

Interactive Preference Incorporation: A significant change in the environment may occur where the resulting POF may shift in such a way that the DM preference, $\mathbf{p}$, is no longer part of the new POF. In this scenario, the DM interactively redefines the position of the bounding box ensuring that its preference lies on the new POF. A few scenarios may emerge when this shift of the POF occurs. The initial preferred outcome, p, may no longer lie on the new POF, but the functional value of the corresponding decision variable, x_p, may still lie on the new POF. The second possibility is that both p and the functional value of x_p do not lie on the new POF. In both cases, a new bounding box position needs to be defined. The interactive redefinition of the bounding box position is presented in Algorithm 4. The time complexity of the interactive preference incorporation procedure is a constant value, i.e., low time complexity.

Algorithm 4 Interactive Incorporation of Preferences

procedure REPOSITIONBOUNDINGBOX(F, x_p, p, r, t)
▷ F: multi-objective function to be evaluated
▷ x_p: DM preferred decision vector
▷ p: DM preferred outcome as defined in Equation (2), page 3
▷ r: DM preferred box radius
▷ Archive: POS_t
    $POF_t \leftarrow F(POS_t, t)$ ▷ POF is corresponding objective values of POS
    if p $\in POF_t$ **then** ▷ DM preferred outcome still lies on POF
        return (x_p:p:r)
    if F(x_p,t) $\in POF_t$ **then** ▷ DM's preferred decision lies on POF, preferred outcome does not
        Reposition center of box, p $\leftarrow$ F(x_p, t) ▷ Automatically reposition center of box
        return (x_p:p:r)
    $x_p \leftarrow$ DMChooseIn(POS_t) ▷ DM selects a new position for x_p and p
    Reposition center of box, p $\leftarrow F(x_p, t)$ ▷ Reposition center of box based on DM's input
    return (x_p : p : r)

3.1.2. Algorithms

The following three approaches employ a penalty function (refer to Equation (3), Section 2) to penalize violating decisions which do not satisfy the DM's preferences. Each of these approaches were incorporated into the hybrid DE and are the three constraint managing approaches evaluated in this study:

1. Proportionate Penalty: With this approach, the penalty is proportional to the violation, and violating decisions are penalized during function evaluation. Algorithm 5 presents this approach, referred to as PPA for the rest of the article.

Algorithm 5 Proportionate Penalty Algorithm

procedure FUNCEVALUATE(F, x, t)
▷ F: multi-objective function to be evaluated
▷ x: decision vector
▷ p: as defined in Equation (2), page 3
▷ r: as defined in Equation (2), page 3
▷ λ: as defined in Equation (2), page 3
▷ λ: a random number between 100 and 1000
▷ I_1: as defined in Equation (2), page 3
    z $\leftarrow$ F(x,t) ▷ calculate objective value of x
    d $\leftarrow$ $norm_2$(z-p) $-$ r ▷ calculate violation of z
    if d ≤ 0 **then**
        return z ▷ x is a non-violating decision, no penalty applied
▷ x is a preference violating decision, proceed to penalize it for violation
    penalty $\leftarrow \lambda \cdot$ d ▷ calculate penalty
    penalty $\leftarrow I_1 \cdot$ penalty ▷ vectorize penalty
    z $\leftarrow$ z + penalty ▷ impose proportionate penalty to objective value of x
    return z ▷ return new penalized objective value of x

2. Death Penalty: Maximum/death penalty is imposed on violating decisions during function evaluation. Some penalty, which is death, is administered on a decision irrespective of the magnitude of the violation of that decision. With maximum penalty,

it becomes very unlikely that violating decisions will find their way into the archive, because they will be dominated by non-violating decisions. Violating decisions are computationally eliminated during the search process, and the optimization process is driven towards a region of the search space dominated by non-violating decisions. The Death Penalty Algorithm, referred to as DPA in the rest of the article, is presented in Algorithm 6.

Algorithm 6 Death Penalty

```
procedure FUNCEVALUATE(F, x, t)
  ▷ F: multi-objective function to be evaluated
  ▷ x: decision vector
  ▷ p: as defined in Equation (2), page 3
  ▷ r: as defined in Equation (2), page 3
  ▷ I_1: as defined in Equation (2), page 3
  ▷ realmax: maximum real value on a machine
    z ← F(x,t)                                   ▷ calculate objective value of x
    d ← norm_2(z-p) − r                          ▷ calculate violation of z
    if d ≤ 0 then
        return z                                 ▷ x is a non-violating decision, no penalty applied
  ▷ x is a preference violating decision, proceed to penalize it for violation
    penalty ← realmax                            ▷ calculate penalty
    penalty ← I_1 · penalty                      ▷ vectorize penalty
    z ← penalty                                  ▷ impose death/max penalty
    return z                                     ▷ return new penalized objective value of x
```

3. Restrict Search To Feasible Region: Feasibility is preserved by starting the search within the preferred bounding box and employing the death penalty to prevent preference violating decisions from entering the archive. This approach restricts the search to the feasible region, unlike [40], and it improves the exploring capability of this algorithm. Preferred decisions start the search during initialization of the population of decisions. A pool of preferred decisions is aggregated with the DM preference and the current decisions in the archive. Then, a loop is performed where nearly identical clones of the pool of preferred decisions are created using polynomial mutation [41]. These new clones constitute a new population from where the search will start. Some of the non-dominated decisions in the new population are added to the archive. Algorithm 7 presents the Restrict Search To Feasible Region Algorithm, referred to as RSTFRA in the rest of the article.

Algorithm 7 Restrict Search to Feasible Region

```
procedure INITIALIZE(P_t, freq, severity, F, t)
  ▷ x_p: DM preferred decision vector)
  ▷ archive: POS
  ▷ F: multi-objective function to be evaluated
  ▷  N: population size, fixed for this study
    pool ← [x_p ; archive]                       ▷ pooled preferences
    i ← 1                                        ▷ initialize counter
    while  i ≤  N  do
        iNumberAttempts ← 1
        while  (iNumberAttempts ≤ 100) && (!isPreferredDecision(solution, F, t) )  do    ▷
  searching for prefered decision
            solution ← randomlyChooseIn(pool)            ▷ randomly select solution from pool
            solution ← polynomial_mutate(solution)       ▷ apply mutation to solution
            iNumberAttempts ← iNumberAttempts + 1        ▷ increment number of attempts
        addSolutionToPopulation(P_t, solution)           ▷ add mutated solution to the population
        i ← i + 1
    AssignNonDominatedToArchive(P_t, F, t )              ▷ add non-dominated decisions to archive
```

The time complexity of the constraint managing approaches PPA and DP is a constant value. The time complexity of RSTFRA is $O(m)$ due to adding the non-dominated solution to the archive with size m.

3.1.3. Differential Evolution Algorithm Control Parameters

The following settings were used for the DE algorithm in this study:

1. The base algorithm (refer to Algorithm 8) is characterized as DE/best/1/bin.
2. To generate a trial vector from a parent vector during the mutation phase of the algorithm, the best vector in the adjacent hypercube or sub-population of the parent vector is selected as the target vector. The number of hypercubes employed by the algorithm is the same as the number of objective functions in the underlying DMOP.
3. Two randomly selected vectors from the parent vector's hypercube are used to form a difference vector.
4. Binary crossover [42] is used due to its viability as a crossover method in DE algorithms.
5. The scaling factor, β, amplifies the effects of the difference vector. It has been shown that a larger β increases the probably of escaping local minima, but can lead to premature convergence. On the other hand, a smaller value results in smaller mutation step sizes slowing down convergence, but facilitating better exploitation of the search space [43,44]. This leads to a typical choice for β in the range $(0.4, 0.95)$ [43,44]. Therefore, in this study, the algorithm randomly chooses $\beta \in (0.4, 1)$. The recombination probability is $p_r = 0.8$, since DE convergence is insensitive to the control parameters [42,45] and a large value of p_r speeds up convergence [43,45,46].

Algorithm 8 Differential Evolution Algorithm

```
procedure OPTIMIZER(P, F, t)
    ▷ β: scaling factor set per algorithm
    ▷ p_r: recombination prob set per algorithm
    ▷ maxgen(≥1): number of function evaluations set per algorithm
    ▷ P: current population of vectors
    ▷ F: multi-objective function to be optimized
    ▷ t: current time
    gen = 1                                          ▷ set the generation counter
    P_gen ← P                                        ▷ initialize current population
    V ← Ø                                            ▷ initialize set of vectors
    while gen ≤ maxgen do                            ▷ check if stopping condition has been reached
        while moreUnprocessed(v ∈ P_gen) do          ▷ process all individuals of population
            v' ← getTrialVector(β, v, P_gen, F, t)   ▷ calculate trial vector
            v'' ← getChildVector(p_r, v', v, F, t)   ▷ produce child vector
            V ← V ∪ {v, v''}                         ▷ add trial and child vectors to set of vectors
            markAsProcessed(v ∈ P_gen)
        P_gen ← getNextGenerationVectors(V)          ▷ produce next generation
        gen ← gen+1                                  ▷ increment counter
        V ← Ø                                        ▷ reset set of vectors
    AssignNonDominatedToArchive(P_maxgen, F, t )     ▷ add non-dominated solutions to archive
```

The time complexity of the static non-dominated sorting genetic algorithm II (NSGA-II) is $O(iN^2)$, where i is the number of objective functions and N is the population size [35]. The non-dominated sorting has a time complexity of $O(iN^2)$, the crowding distance calculation has a time complexity of $O(iN \log N)$, and elitist sorting has a time complexity of $O(iN^2)$ [35].

The DE algorithm used in this study uses the same non-dominated sorting and elitist sorting as NSGA-II. In addition, the time complexity of adding a solution to the archive is $O(im)$, where m is the size of the archive. When a change in the environment occurs, the re-evaluation of the archive has a time complexity of $O(im^2)$. However, it should be noted that all DMOAs that incorporate a change response would typically re-evaluate the solutions. Therefore, the time complexity of the DE is $O(im^2)$. Furthermore, since the base algorithm used in this study is only for demonstration purposes, if these approaches are incorporated into another DMOA, the time complexity will depend on that of the chosen DMOA.

3.2. Decision-Maker's Preferences

The DM preferences are correspondingly associated, serially, with each of the eighteen experimental configurations in Section 3.3. For instance, the first experimental preference in Table 1 is associated with the first experimental preference in Table 2, while both are associated with the first experimental configuration in Table 3.

Table 1. Experimental preferences for decision variables.

S/N	x_1	x_2	x_3	x_4	x_5	x_6
1	0.47600	0.53000	0.5877	0.59104	0.5324	0.4989
2	0.47700	0.33000	0.1630	0.42817	0.3250	0.2654
3	0.47600	0.19000	0.0912	0.17928	0.1616	0.2331
4	0.81600	0.95000	0.8768	0.86623	0.4892	0.7924
5	0.76100	−0.10000	0.1513	−0.17267	0.0387	0.1229
6	0.16700	0.18000	0.0032	−0.00311	0.1462	−0.0284
7	0.96400	3.5×10^{-6}	0.5378	0.51880	0.3188	0.5433
8	0.00000	2.8×10^{-5}	0.3660	0.28965	0.4592	0.3918
9	1.00000	0.00030	0.3992	0.39925	0.5292	0.4621
10	0.35700	1.00000	0.5083	0.74667	0.8383	0.7472
11	0.00000	0.00091	0.7330	0.49810	0.8207	0.6616
12	0.00000	0.08000	0.6419	0.87802	0.8485	0.8020
13	0.14600	0.31000	0.2970	0.30878	0.3052	0.3065
14	0.73400	0.16000	0.1657	0.13508	0.1101	0.1729
15	0.31700	0.31000	0.3230	0.32197	0.3290	0.2881
16	0.06100	0.00460	0.0027	0.00225	0.0032	0.0079
17	1.00000	0.04400	0.0391	0.10593	0.0204	0.0658
18	0.00000	0.00280	0.0015	0.00042	0.0016	0.0045

Table 2. Experimental preferences for objective values.

S/N	f_1	f_2	f_3
1	0.4800	0.3200	N/A
2	0.4800	0.6200	N/A
3	0.4800	0.9300	N/A
4	0.8200	2.4000	N/A
5	0.7600	0.1700	N/A
6	0.1700	0.6300	N/A
7	0.9300	4.5×10^{-71}	1.3843
8	1.8000	2.9×10^{-59}	0
9	1.0×10^{-16}	4.5×10^{-62}	1.6781
10	2.8×10^{-8}	2.8×10^{-8}	1.6354
11	2.9000	0.0041	0
12	3.5000	0.4400	0
13	0.1500	4.6000	N/A
14	0.7300	9.5000	N/A
15	0.3200	4.3000	N/A
16	0.0610	0.9700	N/A
17	1.0000	0.2100	N/A
18	0.0000	1.0000	N/A

3.3. Benchmark Functions

Three DMOPs with various τ_t-n_t combinations were used in this study. The experimental configurations used for these benchmarks are presented in Table 3.

Table 3. Benchmark function configurations.

S/N	DMOP	τ_t	n_t	Iterations	c(f(x))	σ(runs)
1	FDA1	4	10	16	20	30
2	FDA1	5	10	20	20	30
3	FDA1	2	10	8	20	30
4	FDA1	4	1	16	20	30
5	FDA1	5	1	20	20	30
6	FDA1	2	1	8	20	30
7	FDA5	4	10	16	20	30
8	FDA5	5	10	20	20	30
9	FDA5	2	10	8	20	30
10	FDA5	4	1	16	20	30
11	FDA5	5	1	20	20	30
12	FDA5	2	1	8	20	30
13	dMOP2	4	10	16	20	30
14	dMOP2	5	10	20	20	30
15	dMOP2	2	10	8	20	30
16	dMOP2	4	1	16	20	30
17	dMOP2	5	1	20	20	30
18	dMOP2	2	1	8	20	30

The following symbols were used in Table 3: τ_t: frequency of change; n_t: severity of change; $c(f(x))$: count of function evaluations per iteration; $\sigma(runs)$: number of runs per configuration; $FDA1$: type I DMOP (POS is dynamic, POF is static), POF = $1 - \sqrt{f_1}$ and is convex, POS is $x_i = G(t)$ [10,38]; $FDA5$: type II DMOP (POS and POF are dynamic), for 3 objectives, POF = $f_1^2 + f_2^2 + f_3^2 = (1 + G(t))^2$ and is non-convex, POS is $x_i = G(t)$ [10,38]; $dMOP2$: POF changes from convex to concave, type II DMOPs, POF = $1 - f_1^{H(t)}$, POS is $x_i = G(t)$ [38,39].

3.4. Performance Measures

Each of the performance measures were calculated immediately before a change in the environment occurred. This was performed for thirty runs. An average of the values of the thirty runs was then calculated for each measure in each environment.

The following traditional DMOO performance measures were used in this study:

- Accuracy (acc) measures how accurately a DMOA is able to approximate the true POF of a DMOP [1,37,38]. The lower the value of acc, the better the performance of the DMOA.
- Stability (stab) quantifies the effect of environment changes on the accuracy measure value [1,37,38,47]. The lower the value of this measure, the better the DMOA's performance.
- Hypervolume Ratio (hvr) [48] measures the proportion of the objective space that is covered by a non-dominated set without suffering from the bias of a convex region as seen with the hypervolume measure [49]. The higher the value of this measure, the better the DMOA's performance.
- Reactivity (react) [50] measures how long it takes a DMOA to recover after a change in environment occurred, i.e., the length of time it takes to reach a specified accuracy threshold after the change occurred [38]. The lower the value of this measure, the better the DMOA's performance.

The following newly proposed measures were used in this study:

- Number of Non-Violating Decisions (nNVD) measures the number of decisions that fall within the DM's preference set. The higher the value of this measure, the better a DMOA's performance.
- Spread of Non-Violating Decisions (sNVD) measures the spread of decisions within the preference set. A high value indicates a good DMOA performance.

- Number of Violating Decisions (nVD) measures the number of violating decisions in the archive. These are decisions that do not lie within the preference set. The lower the value of this measure, the better the performance of the DMOA.
- Total Deviation of Violating Decisions (dVD) measures the total deviation from the preference set for all violating decisions in the archive. It is calculated based on the steps that are highlighted in Section 2.2.1. The lower the value of this measure, the better the DMOA's performance.

The four new performance measures proposed in this article specifically measure the performance of a DMOA with regards to DM preference constraints, and thus facilitate the comparative analysis of DMOAs in the context of a DM's preferences.

3.5. Statistical Analysis

A statistical analysis of the performance measure values was conducted in accordance with the wins-losses$_B$ algorithm proposed in [1]. The wins-losses$_B$ algorithm was implemented in R [51], and the Kruskal–Wallis and Mann–Whitney U statistical functions in R were used as stipulated in [1]. The calculation of wins and losses by the wins-losses$_B$ algorithm is presented in Algorithm 9 [52].

A win or loss is only recorded if there was a statistical significant difference in performance of the two algorithms that are being compared with the pair-wise Mann–Whitney U test. Therefore, $Diff > 0$ indicates a good performance, since the DMOA obtained more wins than losses. On the other hand, $Diff < 0$ indicates a poor performance, since the DMOA was awarded more losses than wins.

Algorithm 9 wins-losses$_B$ algorithm for wins and losses calculation [52]

for each benchmark **do**
    for each n_t-τ_t combination **do**
        for each performance measure, p_m **do**
            for each algorithm alg **do**
                Calculate the average value pm_{avg} for each of the 30 runs
                Perform Kruskal–Wallis test on the average values, pm_{avg}
                if statistically significant difference **then**
                    for each pair of algorithms **do**
                        Perform Mann–Whitney U test
                        if statistically significant difference **then**
                            for each environment env **do**
                                Assign a win to algorithm with best average over all pm_{avg} for env
                                Assign a loss to algorithm with worst average over all pm_{avg} for env
Calculate $Diff = \#wins - \#losses$
▷ calculate $Diff$ for each parameter (benchmark, n_t-τ_t combination, performance measure, algorithm) as required for analysis

4. Results

This section presents a summary of the results obtained from this study. Detailed results are, however, presented in Appendix A.

The summarized results are presented in Tables 4–6. For all of these tables, any column with a bold entry signifies the winning algorithm for the particular measure of performance, or for the experimental configuration, in the corresponding row.

Figures 2 and 3 present the objective space for a selected DMOP which is constrained by a bounding box representing a DM's preferences for two selected experimental configurations. The bounding box in these specific instances is a sphere. The two figures present the results for a randomly chosen run and a randomly chosen environment among many environments (changes) that are typical of a single run of a DMOA solving a DMOP.

Table 4 presents the results for six experimental configurations, for various n_t–τ_t combinations, i.e., different types of environment changes. It highlights the total number of wins and losses obtained by each constraint managing approach (algorithm) over all benchmarks and measures for each of the environment types. The death-penalty algorithm (DPA) performed the best for four types of environments (n_t–τ_t combinations), while the proportionate-penalty algorithm (PPA) outperformed the other DMOAs in the other two environment types ($n_t = 10, \tau = 2$ and $n_t = 10, \tau = 4$). In those six $n_t - \tau_t$ combinations, the RSTFRA never outperformed DPA but it performed better than PPA in two types of environments ($n_t = 1, \tau = 4$ and $n_t = 1, \tau = 2$). DPA was the only DMOA that obtained more wins than losses for all environment types. The Restrict-search-to-feasible-region algorithm (RSTFRA) obtained more losses than wins for all environment types, except for $n_t = 10, \tau = 2$. On the other hand, PPA obtained more wins than losses for all environments, except $n_t = 1, \tau = 4$ and $n_t = 1, \tau = 2$.

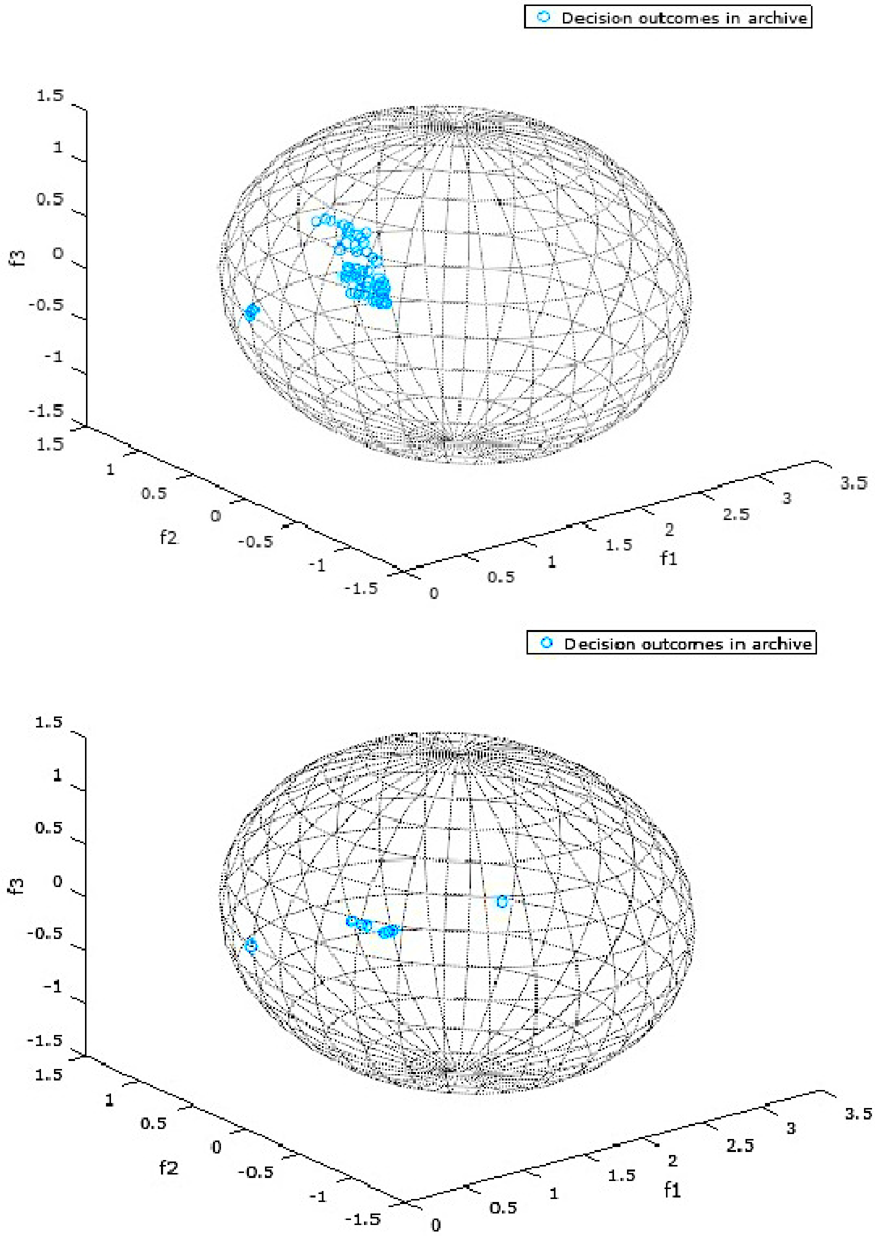

Figure 2. Decisions found by DPA (above) and RSTFRA (below) for FDA5 with Sphere Spec = $(1.7808, 2.9185 \times 10^{-59}, 0.0, 1.5)$, $n_t = 10$ and $\tau_t = 5$.

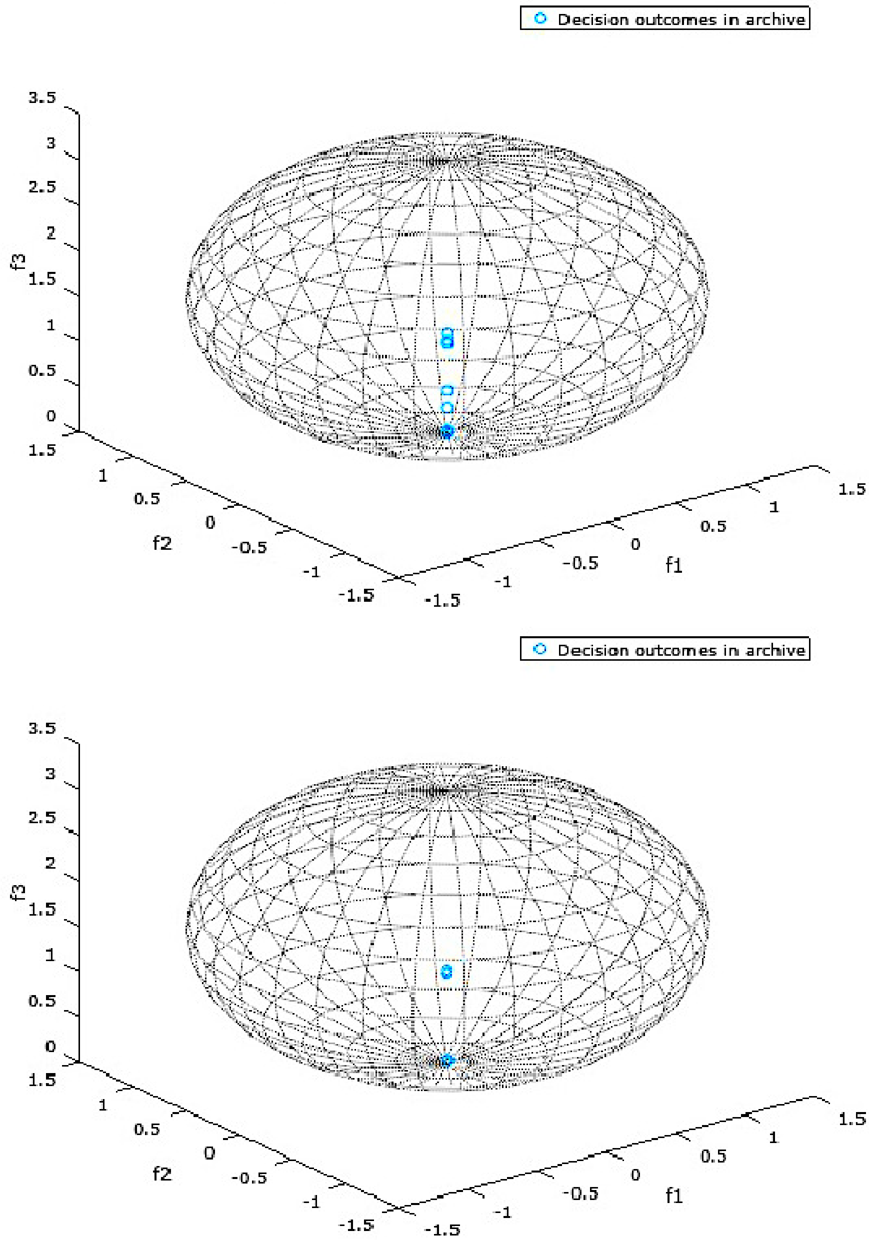

Figure 3. Decisions found by DPA (above) and PPA (below) for FDA5 with Sphere Spec = $(1.0276 \times 10^{-16}, 4.4955 \times 10^{-62}, 1.68781, 1.5)$, $n_t = 10$ and $\tau_t = 2$.

Table 4. Overall wins and losses for various frequency and severity of change combinations using wins-losses$_B$ [1].

n_t	τ_t	RESULTS	PPA	DPA	RSTFRA
10	4	Wins	64	59	53
10	4	Losses	54	58	64
10	4	Diff	10	1	−11
10	4	Rank	**1**	2	3
10	5	Wins	61	67	32
10	5	Losses	45	40	75
10	5	Diff	16	27	−43
10	5	Rank	2	**1**	3
10	2	Wins	73	60	35
10	2	Losses	40	54	74
10	2	Diff	33	6	−39
10	2	Rank	**1**	2	3
1	4	Wins	41	67	56
1	4	Losses	66	44	54
1	4	Diff	−25	23	2
1	4	Rank	3	**1**	2
1	5	Wins	56	73	43
1	5	Losses	53	47	72
1	5	Diff	3	26	−29
1	5	Rank	2	**1**	3
1	2	Wins	51	77	55
1	2	Losses	64	49	70
1	2	Diff	−13	28	−15
1	2	Rank	3	**1**	2

Table 5 presents the performance of the proposed algorithms with respect to the performance measures discussed in Section 3.4. It highlights the total number of wins and losses obtained by each algorithm for all the benchmarks and environment types for each of the performance measures. DPA performed the best for five of the eight measures and second-best for the rest. Two (react and dVD) of its five wins were ties with RSTFRA. Results for the first four measures in Section 3.4 indicated that DPA performed the best for three (acc, hvr, react) out of the four measures. It won with the least number of losses for the accuracy measure, acc, making it the most accurate of the proposed algorithms. For those four measures, RSTFRA won once (stab), but obtained the same number of losses as PPA for the win. RSTFRA also obtained the highest number of worst rankings. None of the algorithms obtained more wins than losses for all of the measures, with all algorithms obtaining more losses than wins for at least three measures.

DPA had the highest number of wins for the measures proposed in this study, i.e., it performed the best for two out of four measures, making DPA the best performing algorithm for all the performance measures discussed in Section 3.4. PPA recorded the highest number of wins for the nNVD measure, while DPA ranked first for the sNVD measure. Thus, PPA and DPA performed better than RSTFRA in finding non-violating decisions of a DM's preferences. Although RSTFRA ranked best for nVD and dVD, the magnitude of wins recorded by RSTFRA for those two measures were negligible. Despite RSTFRA ranking first for nVD and dVD, PPA never lost to any of the other algorithms on those measures. DPA tied with RSTFRA on the dVD measure.

Table 6 presents the overall results, presenting the total number of wins and losses obtained by each algorithm over all performance measures and all environment types for all benchmarks. PPA ranked first with 403 wins, DPA recorded 346 wins, while RSTFRA ranked last with 274 wins. In addition, RSTFRA recorded the highest number of overall losses (409), resulting in the most negative $Diff$ value. DPA recorded the least number of overall losses and the most positive $Diff$ value. These overall results are consistent with the earlier results, which indicate that DPA performed the best on most of the performance measures and $n_t - \tau_t$ combinations, while RSTFRA consistently lagged behind the other two proposed algorithms.

Table 5. Overall Wins and Losses for various performance measures, and frequency and severity of change combinations, using wins-losses$_B$ [1].

PM	RESULTS	PPA	DPA	RSTFRA
acc	Wins	69	83	63
acc	Losses	75	60	80
acc	Diff	−6	23	−17
acc	Rank	2	**1**	3
stab	Wins	23	23	32
stab	Losses	22	34	22
stab	Diff	1	−11	10
stab	Rank	2	2	**1**
hvr	Wins	82	94	40
hvr	Losses	62	50	104
hvr	Diff	20	44	−64
hvr	Rank	2	**1**	3
react	Wins	14	45	45
react	Losses	57	25	22
react	Diff	−43	20	23
react	Rank	3	**1**	**1**
nNVD	Wins	91	65	39
nNVD	Losses	38	67	90
nNVD	Diff	53	−2	−51
nNVD	Rank	**1**	2	3
sNVD	Wins	67	87	47
sNVD	Losses	68	48	85
sNVD	Diff	−1	39	−38
sNVD	Rank	2	**1**	3
nVD	Wins	0	2	4
nVD	Losses	0	4	2
nVD	Diff	0	−2	2
nVD	Rank	3	2	**1**
dVD	Wins	0	4	4
dVD	Losses	0	4	4
dVD	Diff	0	0	0
dVD	Rank	3	**1**	**1**

Table 6. Overall wins and losses for various DMOA using wins-losses$_B$ [1].

RESULTS	PPA	DPA	RSTFRA
Wins	346	403	274
Losses	322	292	409
Diff	24	111	−135
Rank	2	**1**	3

Figures 2 and 3 present the objective space where the preferred objective vectors, or preferred outcomes, are contained in a DM's preference set. The preference set, or the bounding box, in these instances is a sphere whose defining properties are specified by a DM in the bootstrap procedure described in Algorithm 3. For the sphere specifications in Figures 2 and 3, the first three numbers represent the location of the center of the sphere, while the last number represents the radius of the sphere.

Figures 2 and 3 are simply snapshots and are thus incapable of showing the dynamics of the preference set. They are, however, presented in this section to provide a one-time view into the state of the objective space during the optimization process.

In all the snapshots presented by Figures 2 and 3, all the decisions in the archive are preferred by the DM, since all the objective vectors lie within the spheres representing the DM's preferences. This is a testament to the fact that the proposed algorithms are effective in finding optimal trade-off solutions/decisions that reflect a DM's preferences within the search space.

DPA in Figure 2 had the highest number of preferred vectors/outcomes within its spheres, which is consistent with earlier results in this section, indicating its overall superiority over the other algorithms proposed in this study. As a matter of fact, it is ranked best for the experimental configuration represented by Figure 2, and RSTFRA is ranked the worst performing algorithm.

In the experimental configuration represented by Figure 3, PPA ranked best, though only marginally better than DPA. Both algorithms effectively found the DM's preferred decisions, as none of the algorithms produced violating decisions.

5. Conclusions

This article investigated the incorporation of a DM's preferences when solving DMOPs. The following contributions were made: an approach that is partly a priori and partly interactive that enables a decision-maker to indicate its preferences for dynamic problems; a bounding box approach to incorporate the preferences in the DMOA's search; constraint managing approaches to drive the search of a DMOA constrained by the preferences; and new performance measures measuring how well a DMOA's found solutions adhere to the preferences of a DM.

The results show that a DM's preferences can effectively be specified using the proposed approach which is partly a priori and interactive. The results further indicate that the proposed bounding box specification is an effective mathematical abstraction of a DM's preferences. The three proposed constraint managing approaches showed varying degree of performance. The DPA performed the best, while RSTFRA lagged behind the other proposed approaches. Furthermore, the four new performance measures proposed in this article that specifically evaluate the performances of DMOAs in the context of a DM's preferences proved to effectively evaluate the performance of the DMOAs.

Future work will include experimenting with some of the geometric properties of the bounding box and the impact that these properties have on being able to specify the preferences of the DM in various ways.

It will not be trivial to compare the performance of different approaches that define a decision-maker's preferences in the traditional way that DMOAs' performance is evaluated. The way in which a specific approach defines the decision-maker's preferences will directly influence the solutions that a DMOA finds. This article took the first step towards this, by proposing new performance measures for measuring the performance of DMOAs based on how well their found solutions adhere to the DM's preferences. However, the question remains: if you compare two approaches that incorporate decision-maker preferences, how will you be able to determine whether one will be better than the other? As long as both approaches find solutions that do adhere to the DM preferences, in the end, the best (or most preferred) approach will be dependent on the application and the preference of the real decision-maker. Future work will investigate this further, i.e., in which ways can the performance of DMOAs incorporating DM preferences be efficiently compared.

In the future, the proposed bounding box approach and constraint managing approaches will be incorporated into various state-of-the-art DMOAs, evaluating the DMOAs' performance on a range of DMOPs with varying characteristics [38] and measuring their performance with the newly proposed measures. Lastly, approaches to incorporate uncertainty in a DM's preferences and the performance of the proposed approaches in this article in the presence of uncertainty will also be investigated.

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Appendix A. Detailed Results

Table A1. acc and stab for each DMOA with various frequency and severity of change in different environments.

DMOOP	n_t	τ_t	PM	PPA	DPA	RSTFRA	PM	PPA	DPA	RSTFRA
fda1	10	4	acc	0.7100	0.7360	**0.6633**	stab	**0.0000**	**0.0000**	**0.0000**
fda1	10	4	acc	**0.5959**	0.6533	0.7089	stab	**0.1913**	0.2382	0.2055
fda1	10	4	acc	**0.6236**	0.6581	0.6701	stab	**0.1583**	0.1845	0.1725
fda1	10	4	acc	0.6355	**0.5756**	0.6087	stab	0.2044	**0.1675**	0.1932
fda1	10	5	acc	**0.6645**	0.6721	0.7355	stab	**0.0000**	**0.0000**	**0.0000**
fda1	10	5	acc	0.6443	0.6303	**0.6217**	stab	0.1826	0.1801	**0.1188**
fda1	10	5	acc	**0.6204**	0.6630	0.6485	stab	0.2073	0.2242	**0.1906**
fda1	10	5	acc	0.5634	**0.5295**	0.5401	stab	0.2073	0.2242	**0.1906**
fda1	10	2	acc	0.6993	**0.5643**	0.6647	stab	**0.0000**	**0.0000**	**0.0000**
fda1	10	2	acc	**0.5952**	0.6773	0.6496	stab	**0.1347**	0.1697	0.1500
fda1	10	2	acc	**0.6311**	0.6796	0.6332	stab	**0.1091**	0.1621	0.1531
fda1	10	2	acc	**0.5669**	0.5971	0.5843	stab	0.2105	**0.1919**	0.1923
fda1	1	4	acc	0.9975	0.9936	**0.9823**	stab	**0.0000**	**0.0000**	**0.0000**
fda1	1	4	acc	0.7261	**0.6684**	0.7465	stab	**0.0000**	0.0002	**0.0000**
fda1	1	4	acc	**0.9671**	0.9961	0.9836	stab	0.0329	**0.0039**	0.0164
fda1	1	4	acc	**0.6606**	0.6645	0.7412	stab	0.0004	**0.0000**	**0.0000**
fda1	1	5	acc	0.9644	**0.7128**	0.7965	stab	**0.0000**	**0.0000**	**0.0000**
fda1	1	5	acc	**0.7928**	0.8884	0.8250	stab	**0.0000**	**0.0000**	**0.0000**
fda1	1	5	acc	0.9139	**0.6744**	0.8290	stab	**0.0845**	0.278	0.1030
fda1	1	5	acc	0.7700	0.8612	**0.7521**	stab	**0.0000**	**0.0000**	0.0012
fda1	1	2	acc	0.9580	0.9370	**0.8967**	stab	**0.0000**	**0.0000**	**0.0000**
fda1	1	2	acc	**0.8832**	0.9017	0.8840	stab	**0.0000**	**0.0000**	**0.0000**
fda1	1	2	acc	0.9587	0.9519	**0.8972**	stab	0.0410	**0.0252**	0.0882
fda1	1	2	acc	0.8834	**0.7915**	0.8299	stab	**0.0000**	**0.0000**	**0.0000**

Table A2. hvr and react for each DMOA for FDA1 with various frequency and severity of change.

DMOOP	n_t	τ_t	PM	PPA	DPA	RSTFRA	PM	PPA	DPA	RSTFRA
fda1	10	4	hvr	1.7400	1.7865	**1.8379**	react	**13.0000**	**13.0000**	**13.0000**
fda1	10	4	hvr	1.5609	1.7001	**1.9762**	react	**9.0000**	**9.0000**	**9.0000**
fda1	10	4	hvr	**1.7016**	1.6661	1.5562	react	**5.0000**	**5.0000**	**5.0000**
fda1	10	4	hvr	1.7977	**2.0063**	1.5598	react	**1.0000**	**1.0000**	**1.0000**
fda1	10	5	hvr	1.6588	1.592	**1.9064**	react	**16.0000**	**16.0000**	**16.0000**
fda1	10	5	hvr	**1.7341**	1.7048	1.5894	react	**11.0000**	**11.0000**	**11.0000**
fda1	10	5	hvr	1.6116	**1.7846**	1.7651	react	**6.0000**	**6.0000**	**6.0000**
fda1	10	5	hvr	1.4383	**1.733**	1.624	react	**1.0000**	**1.0000**	**1.0000**
fda1	10	2	hvr	**1.7493**	1.5363	1.7156	react	**7.0000**	**7.0000**	**7.0000**
fda1	10	2	hvr	**1.6243**	1.5522	1.5465	react	**5.0000**	**5.0000**	**5.0000**
fda1	10	2	hvr	**1.5960**	1.5437	1.4597	react	**3.0000**	**3.0000**	**3.0000**
fda1	10	2	hvr	**1.6373**	1.5162	1.4294	react	**1.0000**	**1.0000**	**1.0000**
fda1	1	4	hvr	1.4955	1.5851	**1.7424**	react	**13.0000**	**13.0000**	**13.0000**
fda1	1	4	hvr	2.9606	2.7286	**2.9838**	react	8.3000	**8.0667**	8.4000
fda1	1	4	hvr	1.4713	1.6797	**1.7775**	react	**5.0000**	**5.0000**	**5.0000**
fda1	1	4	hvr	2.7961	2.7168	**2.8980**	react	**1.0000**	**1.0000**	**1.0000**
fda1	1	5	hvr	**2.0055**	1.4014	1.4275	react	12.5000	**7.5000**	9.5000
fda1	1	5	hvr	3.7040	**4.3578**	3.6041	react	10.4000	10.6333	**10.3667**
fda1	1	5	hvr	**1.8095**	1.2873	1.2637	react	4.8333	**3.0000**	3.1667
fda1	1	5	hvr	3.6045	**4.2051**	3.5012	react	**1.0000**	**1.0000**	**1.0000**
fda1	1	2	hvr	2.0333	1.7847	**2.0553**	react	4.2000	3.5000	**2.8000**
fda1	1	2	hvr	**4.2680**	4.2409	4.0464	react	4.6333	**4.5667**	**4.5667**
fda1	1	2	hvr	1.9415	1.7069	**2.4759**	react	2.8000	2.6000	**2.3333**
fda1	1	2	hvr	**3.9993**	3.5754	3.7061	react	**1.0000**	**1.0000**	**1.0000**

Table A3. nNVD and sNVD for each DMOA for FDA1 with various frequency and severity of change.

DMOOP	n_t	τ_t	PM	PPA	DPA	RSTFRA	PM	PPA	DPA	RSTFRA
fda1	10	4	nNVD	99.4333	98.1333	**99.8000**	sNVD	0.0297	**0.0301**	0.0296
fda1	10	4	nNVD	**99.5000**	96.1667	96.0667	sNVD	0.0298	**0.0308**	**0.0308**
fda1	10	4	nNVD	**99.5000**	94.3667	95.6000	sNVD	0.0298	**0.0314**	0.0310
fda1	10	4	nNVD	**99.0333**	96.3000	96.8333	sNVD	0.0299	**0.0308**	0.0306
fda1	10	5	nNVD	**100.0000**	**100.0000**	**100.0000**	sNVD	**0.0295**	**0.0295**	**0.0295**
fda1	10	5	nNVD	**100.0000**	**100.0000**	**100.0000**	sNVD	**0.0295**	**0.0295**	**0.0295**
fda1	10	5	nNVD	**100.0000**	99.9667	**100.0000**	sNVD	0.0295	0.0295	**0.0296**
fda1	10	5	nNVD	**100.0000**	**100.0000**	**100.0000**	sNVD	**0.0295**	**0.0295**	**0.0295**
fda1	10	2	nNVD	64.9667	64.9000	**67.1333**	sNVD	0.0457	**0.0458**	0.0441
fda1	10	2	nNVD	**67.4333**	58.6667	57.6667	sNVD	0.0442	0.0506	**0.0514**
fda1	10	2	nNVD	**67.5667**	58.7000	56.0000	sNVD	0.044	0.0505	**0.0532**
fda1	10	2	nNVD	**67.7000**	57.4667	57.7333	sNVD	0.0438	**0.0518**	0.0512
fda1	1	4	nNVD	26.2667	**27.6333**	25.6333	sNVD	0.0671	0.0648	**0.0686**
fda1	1	4	nNVD	26.2333	**26.5667**	21.6333	sNVD	0.0719	0.0777	**0.0853**
fda1	1	4	nNVD	**26.1667**	23.1333	22.7000	sNVD	0.0675	0.0773	**0.0784**
fda1	1	4	nNVD	25.0000	25.3667	**25.6000**	sNVD	0.0735	0.0756	**0.0769**
fda1	1	5	nNVD	**100.0000**	**100.0000**	**100.0000**	sNVD	**0.0296**	0.0295	0.0295
fda1	1	5	nNVD	71.4333	99.0333	**100.0000**	sNVD	**0.0422**	0.0301	0.0296
fda1	1	5	nNVD	**100.0000**	98.9333	99.3000	sNVD	0.0296	**0.0299**	0.0298
fda1	1	5	nNVD	76.2667	**98.1667**	95.7667	sNVD	**0.0394**	0.0303	0.0291
fda1	1	2	nNVD	66.0333	65.7000	**66.2000**	sNVD	0.0448	**0.0453**	0.0449
fda1	1	2	nNVD	31.7000	30.2333	**40.3000**	sNVD	0.0973	**0.1029**	0.0726
fda1	1	2	nNVD	**62.6000**	43.2333	43.4333	sNVD	0.04708	**0.0690**	0.0670
fda1	1	2	nNVD	**31.4667**	31.3667	28.8667	sNVD	0.0981	0.1001	**0.1089**

Table A4. nVD and dVD for each DMOA for FDA1 with various frequency and severity of change.

DMOOP	n_t	τ_t	PM	PPA	DPA	RSTFRA	PM	PPA	DPA	RSTFRA
fda1	10	4	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda1	10	4	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda1	10	4	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda1	10	4	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda1	10	5	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda1	10	5	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda1	10	5	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda1	10	5	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda1	10	2	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda1	10	2	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda1	10	2	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda1	10	2	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda1	1	4	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda1	1	4	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda1	1	4	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda1	1	4	nVD	0.0667	**0.0000**	**0.0000**	dVD	0.0667	**0.0000**	**0.0000**
fda1	1	5	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda1	1	5	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda1	1	5	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda1	1	5	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda1	1	2	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda1	1	2	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda1	1	2	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda1	1	2	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**

Table A5. acc and stab for each DMOA for FDA5 with various frequency and severity of change.

DMOOP	n_t	τ_t	PM	PPA	DPA	RSTFRA	PM	PPA	DPA	RSTFRA
fda5	10	4	acc	0.6404	0.5962	**0.5380**	stab	**0.0000**	**0.0000**	**0.0000**
fda5	10	4	acc	**0.6751**	0.7731	0.7557	stab	0.1793	0.1457	**0.0884**
fda5	10	4	acc	**0.4445**	0.4619	0.6266	stab	0.4251	0.4261	**0.2747**
fda5	10	4	acc	0.2282	0.2270	**0.1946**	stab	0.4743	0.5300	**0.4098**
fda5	10	5	acc	**0.2683**	0.4080	0.8316	stab	**0.0000**	**0.0000**	**0.0000**
fda5	10	5	acc	**0.4298**	0.6803	0.8570	stab	0.1992	0.1543	**0.0168**
fda5	10	5	acc	**0.5140**	0.6998	0.8183	stab	0.2266	0.1100	**0.0883**
fda5	10	5	acc	**0.5382**	0.6913	0.7904	stab	0.2461	0.1599	**0.1331**
fda5	10	2	acc	**0.6239**	0.6727	0.9835	stab	**0.0000**	**0.0000**	**0.0000**
fda5	10	2	acc	**0.8799**	0.8925	0.9526	stab	0.0985	0.0860	**0.0000**
fda5	10	2	acc	**0.6285**	0.6549	0.9234	stab	0.3367	0.2923	**0.0277**
fda5	10	2	acc	**0.4285**	0.4320	0.9186	stab	0.4402	0.4291	**0.0455**
fda5	1	4	acc	**0.9984**	1.0000	1.0000	stab	**0.0000**	**0.0000**	**0.0000**
fda5	1	4	acc	0.9986	**0.9955**	0.9970	stab	**0.0014**	0.0045	0.0030
fda5	1	4	acc	1.0000	**0.9991**	**0.9991**	stab	**0.0000**	0.0009	0.0009
fda5	1	4	acc	0.9996	**0.9945**	0.9956	stab	**0.0004**	0.0055	0.0044
fda5	1	5	acc	1.0000	0.9993	**0.9958**	stab	**0.0000**	**0.0000**	**0.0000**
fda5	1	5	acc	0.9953	0.9915	**0.9746**	stab	**0.0047**	0.0085	0.0254
fda5	1	5	acc	1.0000	**0.9971**	0.9976	stab	**0.0000**	0.0029	0.0024
fda5	1	5	acc	0.9963	0.9962	**0.9793**	stab	**0.0037**	0.0038	0.0207
fda5	1	2	acc	1.0000	0.9993	**0.9948**	stab	**0.0000**	**0.0000**	**0.0000**
fda5	1	2	acc	0.9963	0.9990	**0.9781**	stab	0.0037	**0.0010**	0.0219
fda5	1	2	acc	1.0000	0.9993	**0.9980**	stab	**0.0000**	0.0007	0.0020
fda5	1	2	acc	1.0000	0.9987	**0.9853**	stab	**0.0000**	0.0013	0.0147

Table A6. hvr and react for each DMOA for FDA5 with various frequency and severity of change.

DMOOP	n_t	τ_t	PM	PPA	DPA	RSTFRA	PM	PPA	DPA	RSTFRA
fda5	10	4	hvr	**1.6654**	1.3814	1.2592	react	12.9667	12.9667	**12.5000**
fda5	10	4	hvr	**2.4693**	1.7110	1.5734	react	8.9333	**8.8333**	**8.8333**
fda5	10	4	hvr	**3.2302**	1.8911	1.6400	react	5.0000	**4.9667**	5.0000
fda5	10	4	hvr	**2.0206**	2.0017	1.7762	react	**1.0000**	**1.0000**	**1.0000**
fda5	10	5	hvr	2.1537	**2.7315**	2.4955	react	16.0000	16.0000	**15.9333**
fda5	10	5	hvr	2.2018	**2.6023**	1.9044	react	11.0000	**10.8000**	10.9333
fda5	10	5	hvr	2.267	**2.3623**	2.0773	react	**6.0000**	**6.0000**	**6.0000**
fda5	10	5	hvr	2.1636	**2.4252**	1.8647	react	**1.0000**	**1.0000**	**1.0000**
fda5	10	2	hvr	1.5414	**1.6826**	1.2248	react	**6.9000**	7.0000	7.0000
fda5	10	2	hvr	**3.3034**	3.2674	1.4156	react	5.0000	**4.9667**	5.0000
fda5	10	2	hvr	4.9233	**5.1433**	1.1358	react	**3.0000**	**3.0000**	**3.0000**
fda5	10	2	hvr	3.5145	**4.0314**	1.2700	react	**1.0000**	**1.0000**	**1.0000**
fda5	1	4	hvr	**4.4565**	4.0032	3.8261	react	**12.6000**	13.0000	13.0000
fda5	1	4	hvr	2.4313	**2.5719**	2.2178	react	7.7667	**7.3000**	7.5333
fda5	1	4	hvr	**4.7446**	3.4347	3.5272	react	5.0000	**4.8667**	**4.8667**
fda5	1	4	hvr	2.1979	**2.7897**	2.2656	react	**1.0000**	**1.0000**	**1.0000**
fda5	1	5	hvr	**2.9987**	2.6455	1.9198	react	16.0000	15.0000	**11.5000**
fda5	1	5	hvr	**2.6475**	2.1308	1.0979	react	8.2000	7.3000	**2.8000**
fda5	1	5	hvr	**3.088**	2.8665	2.5523	react	6.0000	**5.3333**	5.5000
fda5	1	5	hvr	**2.5349**	2.3717	1.3460	react	**1.0000**	**1.0000**	**1.0000**
fda5	1	2	hvr	3.0785	**3.5184**	1.8674	react	4.0000	3.9000	**3.0000**
fda5	1	2	hvr	2.2679	**3.1541**	1.0209	react	3.3000	3.7000	**1.2000**
fda5	1	2	hvr	2.9413	**3.3106**	3.2396	react	3.0000	2.9333	**2.6667**
fda5	1	2	hvr	3.2166	**3.2704**	2.2137	react	**1.0000**	**1.0000**	**1.0000**

Table A7. nNVD and sNVD for each DMOA for FDA5 with various frequency and severity of change.

DMOOP	n_t	τ_t	PM	PPA	DPA	RSTFRA	PM	PPA	DPA	RSTFRA
fda5	10	4	nNVD	39.5000	**50.0333**	45.3000	sNVD	**0.1175**	0.0966	0.1008
fda5	10	4	nNVD	57.1667	67.3333	**68.4333**	sNVD	**0.0906**	0.0791	0.0726
fda5	10	4	nNVD	63.5000	**74.6333**	63.1333	sNVD	**0.1100**	0.0915	0.0962
fda5	10	4	nNVD	71.9000	**72.9333**	58.8667	sNVD	0.1237	0.1122	**0.1245**
fda5	10	5	nNVD	42.0000	**49.9667**	27.4000	sNVD	**0.1652**	0.1327	0.1010
fda5	10	5	nNVD	**72.0667**	70.2000	35.0667	sNVD	**0.1105**	0.1015	0.0915
fda5	10	5	nNVD	**74.4333**	65.4667	36.4000	sNVD	**0.1089**	0.1005	0.0940
fda5	10	5	nNVD	**79.0667**	60.1667	34.0333	sNVD	0.1017	**0.1079**	0.1059
fda5	10	2	nNVD	**12.2667**	11.8333	2.9333	sNVD	**0.2712**	0.2546	0.0040
fda5	10	2	nNVD	**13.1667**	9.1000	1.5667	sNVD	0.1928	**0.2910**	0.0032
fda5	10	2	nNVD	**19.3000**	9.8000	2.6333	sNVD	0.2200	**0.3729**	0.0005
fda5	10	2	nNVD	**41.5667**	33.8000	2.6000	sNVD	0.1413	**0.1716**	0.0062
fda5	1	4	nNVD	**12.9667**	12.7000	9.6333	sNVD	0.1967	**0.2117**	0.0279
fda5	1	4	nNVD	**32.4667**	18.6667	12.3667	sNVD	**0.2258**	0.1087	0.0253
fda5	1	4	nNVD	**13.6667**	12.9667	9.3667	sNVD	**0.2683**	0.2654	0.0122
fda5	1	4	nNVD	**29.2333**	28.9000	16.5333	sNVD	**0.2238**	0.0853	0.0124
fda5	1	5	nNVD	19.4000	**23.2000**	21.7333	sNVD	**0.0897**	0.0592	0.0568
fda5	1	5	nNVD	35.8333	34.2000	**67.6333**	sNVD	**0.0946**	0.0687	0.0368
fda5	1	5	nNVD	21.0333	**23.4333**	21.0333	sNVD	**0.1374**	0.1011	0.0651
fda5	1	5	nNVD	39.1	33.6333	**62.8333**	sNVD	**0.0757**	0.0693	0.0407
fda5	1	2	nNVD	22.1667	23.6333	**26.4333**	sNVD	**0.1600**	0.1365	0.0837
fda5	1	2	nNVD	**20.8667**	17.8667	6.6000	sNVD	0.0726	**0.0964**	0.0577
fda5	1	2	nNVD	**18.8000**	17.5000	17.4333	sNVD	0.2013	**0.2357**	0.1891
fda5	1	2	nNVD	**20.2667**	18.0333	11.4667	sNVD	0.0617	**0.0747**	0.0391

Table A8. nVD and dVD for each DMOA for FDA5 with various frequency and severity of change.

DMOOP	n_t	τ_t	PM	PPA	DPA	RSTFRA	PM	PPA	DPA	RSTFRA
fda5	10	4	nVD	0.0333	**0.0000**	**0.0000**	dVD	0.0333	**0.0000**	**0.0000**
fda5	10	4	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda5	10	4	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda5	10	4	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda5	10	5	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda5	10	5	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda5	10	5	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda5	10	5	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda5	10	2	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda5	10	2	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda5	10	2	nVD	0.0333	**0.0000**	**0.0000**	dVD	0.0333	**0.0000**	**0.0000**
fda5	10	2	nVD	0.0333	**0.0000**	**0.0000**	dVD	0.0333	**0.0000**	**0.0000**
fda5	1	4	nVD	0.0667	**0.0000**	**0.0000**	dVD	0.0667	**0.0000**	**0.0000**
fda5	1	4	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda5	1	4	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda5	1	4	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda5	1	5	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda5	1	5	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda5	1	5	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda5	1	5	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda5	1	2	nVD	0.0333	**0.0000**	**0.0000**	dVD	0.0333	**0.0000**	**0.0000**
fda5	1	2	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda5	1	2	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
fda5	1	2	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**

Table A9. acc and stab for each DMOA for dMOP2 with various frequency and severity of change.

DMOOP	n_t	τ_t	PM	PPA	DPA	RSTFRA	PM	PPA	DPA	RSTFRA
dmop2	10	4	acc	0.8700	**0.7811**	0.8707	stab	**0.0000**	**0.0000**	**0.0000**
dmop2	10	4	acc	**0.8284**	0.8287	0.8715	stab	**0.0000**	0.0164	**0.0000**
dmop2	10	4	acc	**0.8124**	0.8805	0.8840	stab	**0.0000**	0.0110	0.0002
dmop2	10	4	acc	0.9980	**0.9717**	0.9995	stab	0.0006	0.0165	**0.0001**
dmop2	10	5	acc	0.7858	**0.7680**	0.9103	stab	**0.0000**	**0.0000**	**0.0000**
dmop2	10	5	acc	**0.6857**	0.7187	0.9217	stab	0.0099	0.0261	**0.0000**
dmop2	10	5	acc	0.8374	**0.7731**	0.9080	stab	**0.0000**	0.0159	**0.0000**
dmop2	10	5	acc	0.9773	**0.9755**	0.9951	stab	0.0209	0.0226	**0.0036**
dmop2	10	2	acc	**0.8127**	0.8490	0.8298	stab	**0.0000**	**0.0000**	**0.0000**
dmop2	10	2	acc	0.8232	0.8581	**0.7979**	stab	**0.0000**	**0.0000**	**0.0000**
dmop2	10	2	acc	0.9443	**0.8277**	0.8558	stab	**0.0000**	**0.0000**	**0.0000**
dmop2	10	2	acc	0.9980	**0.9975**	0.9988	stab	0.0009	0.0003	**0.0002**
dmop2	1	4	acc	0.9266	0.3449	**0.3097**	stab	**0.0000**	**0.0000**	**0.0000**
dmop2	1	4	acc	0.9309	0.8833	**0.5509**	stab	**0.0224**	0.1167	0.4491
dmop2	1	4	acc	1.0000	**0.3802**	0.6802	stab	**0.0000**	0.6198	0.3198
dmop2	1	4	acc	0.5236	**0.4389**	0.4764	stab	0.0073	**0.0045**	0.0396
dmop2	1	5	acc	0.9478	**0.2882**	0.9615	stab	**0.0000**	**0.0000**	**0.0000**
dmop2	1	5	acc	**0.9393**	0.9508	0.9574	stab	**0.0177**	0.0492	0.0426
dmop2	1	5	acc	0.9991	**0.3737**	0.9778	stab	**0.0009**	0.6263	0.0222
dmop2	1	5	acc	0.4861	**0.3884**	0.7584	stab	**0.0103**	0.0710	0.2271
dmop2	1	2	acc	0.9548	**0.7587**	0.9164	stab	**0.0000**	**0.0000**	**0.0000**
dmop2	1	2	acc	0.9833	**0.8949**	0.9059	stab	**0.0059**	0.1051	0.0941
dmop2	1	2	acc	0.9942	**0.8278**	0.9069	stab	**0.0058**	0.1722	0.0931
dmop2	1	2	acc	**0.4735**	0.5391	0.7011	stab	**0.0169**	0.0625	0.2519

Table A10. hvr and react for each DMOA for dMOP2 with various frequency and severity of change.

DMOOP	n_t	τ_t	PM	PPA	DPA	RSTFRA	PM	PPA	DPA	RSTFRA
dmop2	10	4	hvr	**1.2872**	1.2594	1.2577	react	12.8667	12.9667	**12.3667**
dmop2	10	4	hvr	1.1982	1.2284	**1.3295**	react	**8.0333**	8.6333	8.6000
dmop2	10	4	hvr	**1.5950**	1.5624	1.4643	react	4.9667	**4.8333**	**4.8333**
dmop2	10	4	hvr	1.3038	**1.4336**	1.4014	react	**1.0000**	**1.0000**	**1.0000**
dmop2	10	5	hvr	**1.9143**	1.8882	1.1107	react	**16.0000**	**16.0000**	**16.0000**
dmop2	10	5	hvr	1.6730	**1.7919**	1.1055	react	**11.0000**	**11.0000**	**11.0000**
dmop2	10	5	hvr	**1.7423**	1.6500	1.0242	react	6.0000	**5.9333**	5.9667
dmop2	10	5	hvr	1.8847	**2.0522**	1.1175	react	**1.0000**	**1.0000**	**1.0000**
dmop2	10	2	hvr	1.3575	**1.5524**	1.4858	react	**6.8000**	6.9000	**6.8000**
dmop2	10	2	hvr	1.3303	1.4589	**1.4937**	react	**4.8000**	4.9333	4.8333
dmop2	10	2	hvr	**1.7886**	1.3657	1.3977	react	2.9333	**2.8000**	2.9000
dmop2	10	2	hvr	**1.5989**	1.5930	1.4986	react	**1.0000**	**1.0000**	**1.0000**
dmop2	1	4	hvr	1.5361	**2.0163**	1.6181	react	8.6000	2.6000	**1.4000**
dmop2	1	4	hvr	2.4648	**9.2045**	4.3700	react	7.8000	5.2667	**2.3333**
dmop2	1	4	hvr	0.9826	6.7302	**10.8249**	react	5.0000	**2.4667**	4.6000
dmop2	1	4	hvr	2.0165	**5.4407**	2.7435	react	**1.0000**	**1.0000**	**1.0000**
dmop2	1	5	hvr	1.651	**4.8469**	0.9488	react	12.0000	**2.5000**	12.5000
dmop2	1	5	hvr	2.1529	**14.7819**	0.9485	react	9.6333	**8.3333**	8.6667
dmop2	1	5	hvr	0.9861	**6.5701**	0.9669	react	6.0000	**2.8333**	6.0000
dmop2	1	5	hvr	1.7927	**6.4797**	0.7462	react	**1.0000**	**1.0000**	**1.0000**
dmop2	1	2	hvr	1.9387	**7.6131**	1.4606	react	3.3000	**1.5000**	2.9000
dmop2	1	2	hvr	2.3369	**5.3866**	1.5312	react	4.7667	3.1333	**2.8667**
dmop2	1	2	hvr	0.9523	**10.4355**	2.5348	react	3.0000	**2.9333**	3.0000
dmop2	1	2	hvr	**1.7974**	1.3771	0.7159	react	**1.0000**	**1.0000**	**1.0000**

Table A11. nNVD and sNVD for each DMOA for dMOP2 with various frequency and severity of change.

DMOOP	n_t	τ_t	PM	PPA	DPA	RSTFRA	PM	PPA	DPA	RSTFRA
dmop2	10	4	nNVD	5.3667	5.8333	**6.4333**	sNVD	**0.0597**	0.0511	0.0466
dmop2	10	4	nNVD	**4.0667**	3.9333	3.6000	sNVD	0.0698	**0.0835**	0.0716
dmop2	10	4	nNVD	**3.1333**	3.0000	3.1000	sNVD	0.0775	**0.1051**	0.0987
dmop2	10	4	nNVD	**4.3000**	3.3333	3.0667	sNVD	0.0721	0.0806	**0.0951**
dmop2	10	5	nNVD	12.6000	**13.3333**	3.8333	sNVD	**0.1300**	0.1264	0.0291
dmop2	10	5	nNVD	**10.4000**	9.8667	3.0000	sNVD	0.1548	**0.1585**	0.0395
dmop2	10	5	nNVD	**11.0000**	9.3333	2.9667	sNVD	0.1465	**0.1710**	0.0382
dmop2	10	5	nNVD	**9.6667**	7.9333	2.5667	sNVD	0.1740	**0.2079**	0.0364
dmop2	10	2	nNVD	5.8333	5.8667	**6.5667**	sNVD	0.1150	**0.1180**	0.1052
dmop2	10	2	nNVD	**3.9000**	3.1333	2.8667	sNVD	0.1543	0.1751	**0.2310**
dmop2	10	2	nNVD	**3.7333**	2.9333	2.8333	sNVD	0.1752	0.1745	**0.2025**
dmop2	10	2	nNVD	**4.6333**	3.2000	3.0333	sNVD	0.1419	0.1825	**0.2077**
dmop2	1	4	nNVD	68.1667	91.5667	**94.4667**	sNVD	**0.0441**	0.0326	0.0319
dmop2	1	4	nNVD	49.3000	24.8000	**60.7000**	sNVD	0.0633	0.1534	**0.1541**
dmop2	1	4	nNVD	50.8000	**63.1667**	30.2333	sNVD	0.0585	0.0660	**0.1982**
dmop2	1	4	nNVD	0	**0.0333**	0	sNVD	**0.0000**	**0.0000**	**0.0000**
dmop2	1	5	nNVD	**81.0667**	75.4333	0.9333	sNVD	0.0366	**0.0596**	0
dmop2	1	5	nNVD	**59.5333**	3.4333	1.0000	sNVD	0.0519	**0.0929**	0
dmop2	1	5	nNVD	60.6333	**66.3333**	0.9	sNVD	0.0492	**0.0532**	0
dmop2	1	5	nNVD	**0.0000**	**0.0000**	**0.0000**	sNVD	**0.0000**	**0.0000**	**0.0000**
dmop2	1	2	nNVD	31.4333	**32.0333**	5.8000	sNVD	0.1038	**0.1133**	0.0060
dmop2	1	2	nNVD	**13.4333**	1.7000	2.1000	sNVD	0.2290	**0.3570**	0.0454
dmop2	1	2	nNVD	**19.9333**	12.5333	0.6333	sNVD	0.1599	**0.5381**	0.0247
dmop2	1	2	nNVD	**0.0000**	**0.0000**	**0.0000**	sNVD	**0.0000**	**0.0000**	**0.0000**

Table A12. nVD and dVD for each DMOA for dMOP2 with various frequency and severity of change.

DMOOP	n_t	τ_t	PM	PPA	DPA	RSTFRA	PM	PPA	DPA	RSTFRA
dmop2	10	4	nVD	0.0667	**0.0000**	**0.0000**	dVD	0.0667	**0.0000**	**0.0000**
dmop2	10	4	nVD	0.0333	**0.0000**	**0.0000**	dVD	0.0333	**0.0000**	**0.0000**
dmop2	10	4	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
dmop2	10	4	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
dmop2	10	5	nVD	0.0333	**0.0000**	**0.0000**	dVD	0.0333	**0.0000**	**0.0000**
dmop2	10	5	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
dmop2	10	5	nVD	0.1333	**0.0000**	**0.0000**	dVD	0.1333	**0.0000**	**0.0000**
dmop2	10	5	nVD	0.0667	**0.0000**	**0.0000**	dVD	0.0667	**0.0000**	**0.0000**
dmop2	10	2	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
dmop2	10	2	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
dmop2	10	2	nVD	0.0333	**0.0000**	**0.0000**	dVD	0.0333	**0.0000**	**0.0000**
dmop2	10	2	nVD	0.0333	**0.0000**	**0.0000**	dVD	0.0333	**0.0000**	**0.0000**
dmop2	1	4	nVD	**0.0000**	**0.0000**	**0.0000**	dVD	**0.0000**	**0.0000**	**0.0000**
dmop2	1	4	nVD	**0.0000**	23.3333	**0.0000**	dVD	**0.0000**	8.0838	**0.0000**
dmop2	1	4	nVD	**0.0000**	**0.0000**	3.3333	dVD	**0.0000**	**0.0000**	1.1525
dmop2	1	4	nVD	**1.0000**	96.6667	100.0000	dVD	78.3002	**64.215**	85.5742
dmop2	1	5	nVD	**0.0000**	6.6667	6.6667	dVD	**0.0000**	1.0863	1.0156
dmop2	1	5	nVD	**0.0000**	66.6667	**0.0000**	dVD	**0.0000**	25.4565	**0.0000**
dmop2	1	5	nVD	**0.0000**	13.3333	10.0000	dVD	**0.0000**	2.5970	1.7496
dmop2	1	5	nVD	**1.0000**	100.0000	100.0000	dVD	83.0633	**65.1873**	93.9007
dmop2	1	2	nVD	**0.0000**	**0.0000**	6.6667	dVD	**0.0000**	**0.0000**	1.1100
dmop2	1	2	nVD	**0.0000**	26.7333	1.7000	dVD	**0.0000**	9.0173	0.4143
dmop2	1	2	nVD	**0.0000**	**0.0000**	24.1667	dVD	**0.0000**	**0.0000**	7.6244
dmop2	1	2	nVD	**1.0000**	62.4667	57.6333	dVD	**85.1725**	90.4603	93.0091

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