Received: 19 April 2022 | Revised: 21 September 2022 | Accepted: 6 October 2022

DOI: 10.1002/sta4.512

ORIGINAL ARTICLE

WILEY

Nonparametric precedence chart with repetitive sampling

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Abstract

In most real-world applications, such as production and manufacturing processes, the underlying process distribution does not always follow a normal distribution. In such cases, statistical process control literature recommends the use of nonparametric (or distribution-free) control charts. This paper introduces a new distribution-free precedence chart using repetitive sampling. The performance of the proposed chart is investigated in terms of the average run-length (*ARL*) profile. The expressions of the in-control process probability and *ARL* of the proposed chart are introduced using integral formulas. The out-of-control performance of the new chart is compared to that of the existing precedence charts with and without runs-rules. A numerical example is provided using real-life data to demonstrate the application and implementation of the new chart.

KEYWORDS

distribution-free, order statistic, phase I, phase II, precedence chart, repetitive sampling

1 | INTRODUCTION

Most statistical techniques are based on one or more assumptions. One of the most popular assumptions is that continuous variables to be used in the analysis are normally distributed. When this assumption is violated, the use of parametric tests will lead to unreliable results and invalid inference; see, for example, Chakraborti et al. (2004). To remedy this problem, the literature recommends either the use of data transformation such as the log or square transformation (see Feng et al., 2012) or nonparametric tests such as the Wilcoxon rank-sum, sign, signed-rank, precedence and exceedance (see Human et al., 2010; Malela-Majika et al., 2021, 2022). The former technique does not allow the interpretation of the results based on the original data. Therefore, when the data transformation technique does not correct the violated assumption or not provide accurate information about the original data, nonparametric tests are recommended. Note though that parametric tests perform better than their nonparametric counterparts when the underlying distribution is known or normal (Chakraborti & Graham, 2019). In statistical process control (SPC), when the underlying process distribution is unknown or departures from normality, nonparametric (or distribution-free) control charts are more reliable than parametric control charts. The former charts include the Mann–Whitney, precedence, sign and signed-rank control charts just to cite a few (Malela-Majika et al., 2016a, 2016b, 2021). These control charts are designed either based on the assumption of known process parameters also known as Case K (e.g., the sign and signed rank charts) or the assumption of unknown process parameters also known as Case U (e.g., the precedence and Mann–Whitney charts). In Case U, these charts are implemented using two regimes, namely, Phases I and II. The process parameters and control limits are determined in Phase I using historical (or reference) data when the process is considered to be in-control (IC). However, in Phase II, the parameters and control limits found in Phase I are used to continuously monitor the process; see, for example, Montgomery (2013). The focus of this paper is on the precedence chart because of its advantage over other nonparametric control charts, which is its simplicity in the design, implementation and derivation of closed-form expressions of the characteristics of its run-length distribution.

The basic precedence chart is one of the most popular nonparametric charts where the quality process is monitored by plotting the j^{th} ($1 \le j \le n$) order statistic of the Phase II (or test) sample on the chart. The process is considered out-of-control (OOC) if the plotting statistic plots on or beyond two thresholds representing the lower and upper control limits defined by the a^{th} and b^{th} order statistics of the reference sample where $1 \le a < b \le m$ with $a+b=m+1$ (see Chakraborti et al., 2004). Chakraborti et al. (2004) investigated the run-length properties of a class of distribution-free charts, named median precedence (or simply precedence) chart. This chart is reported to be less sensitive in monitoring large shifts in the location parameter. To improve the performance of the precedence chart, Chakraborti et al. (2009) and Malela-Majika et al. (2016a) added supplementary run-rules to the basic precedence chart. Khoo and Ariffin (2006) introduced improved runs-rules to the Shewhart $\overline{X}$ control chart under the assumption of normality. The precedence chart with improved run-rules was introduced to enhance the ability of the basic precedence chart toward small and large shifts when the assumption normality is violated; see, for example, Malela-Majika et al. (2016a, 2022). More recently, Malela-Majika et al. (2021) introduced the double precedence chart and reported its superiority over the basic precedence chart especially in monitoring small to large shifts. For more details on control charts based on order statistics, readers are referred to Triantafyllou (2018, 2019).

The aforementioned charts are based on single samples. Sampling techniques can have a considerable impact on the performance of control charts. Besides the use of single samples (or simple random samples), other sampling techniques such as the repetitive and group repetitive sampling can be used in order to enhance shifts detection abilities of control charts. Aslam et al. (2015) introduced the S^2 chart with repetitive sampling. They showed that the S^2 chart with repetitive sampling outperforms the classical S^2 chart regardless of the size of the shift. Azam et al. (2016) proposed the Shewhart $\overline{X}$ control chart using repetitive sampling under the Burr distribution and reported that their chart performs better than the Shewhart $\overline{X}$ control chart using single sampling. Huang et al. (2021) introduced the generally weighted moving average (GWMA) using repetitive sampling and reported its superiority over the existing hybrid exponentially weighted moving average (HEWMA) counterpart when large design parameters and small adjustment parameters are used. The GWMA sign chart was recently proposed by Chen et al. (2022) to improve the sensitivity of the existing GWMA scheme in monitoring small shifts. For more details on control charts with repetitive sampling, readers are referred to Azam et al. (2015) and Shafqat et al. (2020). To improve the performance of the precedence chart toward the detection of small to moderate shifts, this paper introduces a new precedence control chart using repetitive sampling, named repetitive sampling (RS) precedence chart (hereafter, RS precedence chart) in this paper.

The remainder of this paper is organized as follows: The mathematical background of the new chart is presented in Section 2. In addition, the operation and main considerations of the nonparametric RS precedence chart as well as the IC and OOC average run-length (denoted as ARL_0 and ARL_1, respectively) and average sample number (ASN) expressions are provided. Section 3 investigates the performance of the proposed chart in terms of the average run-length (ARL). In addition, the RS precedence chart is compared to the existing precedence charts. Section 4 provides an illustrative example based on real-life data, and Section 5 presents some concluding remarks.

2 | MATHEMATICAL BACKGROUND OF THE RS PRECEDENCE CHART

Assume that a Phase I random sample of size m, $X=\{X_i, i=1, 2, \ldots, m\}$, is available from the IC process with an unknown c.d.f. (cumulative distribution function) $F_X(x)$. Let $Y=\{Y_{tk}, t=1, 2, \ldots; k=1, 2, \ldots, n\}$ be a t^{th} Phase II sample of size n with c.d.f. $G_Y(y)$, which is of the same nature as the one of the Phase I sample with a difference in the location parameter, i.e., $F_X(x)=G_Y(x+\delta)$, where δ is the change (or shift) in the location parameter (Chakraborti et al., 2004). When $\delta=0$, the process is said to be IC; in this case, $F_X(x)=G_Y(x)$. Otherwise, the process is said to be OOC. For simplicity in the notations, $F_X(x)$ and $G_Y(y)$ are simply denoted by F and G henceforth. Note that the Phase II samples are assumed to be i.i.d. (independent and identically distributed) of one another and of the Phase I sample.

The proposed RS precedence control chart is divided into three charting regions as shown in Figure 1: $A=\left(-\infty, X_{(a_2:m)}\right] \cup \left[X_{(b_2:m)}, +\infty\right)$, $B=\left(X_{(a_2:m)}, X_{(a_1:m)}\right] \cup \left[X_{(b_1:m)}, X_{(b_2:m)}\right)$ and $C=\left(X_{(a_1:m)}, X_{(b_1:m)}\right)$, where $X_{(\ell:m)}$ represents the ℓ^{th} ($\ell \in \{a_2, a_1, b_1, b_2\}$) order statistic of the Phase I sample of size m and ℓ represents the position of the order statistic on the Phase I sample, which is also referred to as the charting constant. The choices of the charting constants a_2, a_1, b_1 and b_2 of the RS precedence chart are such that $a_2=m-b_2+1$ and $a_1=m-b_1+1$. Thus, the outer lower and upper control limits (denoted as $OLCL$ and $OUCL$) and inner lower and upper control limits (denoted as $ILCL$ and $IUCL$) of the RS precedence chart are estimated from the IC Phase I sample of size m as follows: $\widehat{OLCL}=X_{(a_2:m)}$, $\widehat{ILCL}=X_{(a_1:m)}$, $\widehat{IUCL}=X_{(b_1:m)}$ and $\widehat{OUCL}=X_{(b_2:m)}$.

In Phase II, the operation procedure of the RS precedence control chart is as follows:

Step 1. Select a new set of test samples each of size n.

Step 2. At the t^{th} sampling time, take a test sample (Y) of size n and find the j^{th} order statistic, $Y_{(j:n)}$, where $j=r+1$ ($r=1,2,\ldots$) with $n=2r+1$.

Step 3. Declare the process as OOC if $Y_{(j:n)}$ falls in region A. Declare the process as IC if $Y_{(j:n)}$ falls in region C and return to Step 1; otherwise, we have an inconclusive state then return to Step 2 and use the next test sample.

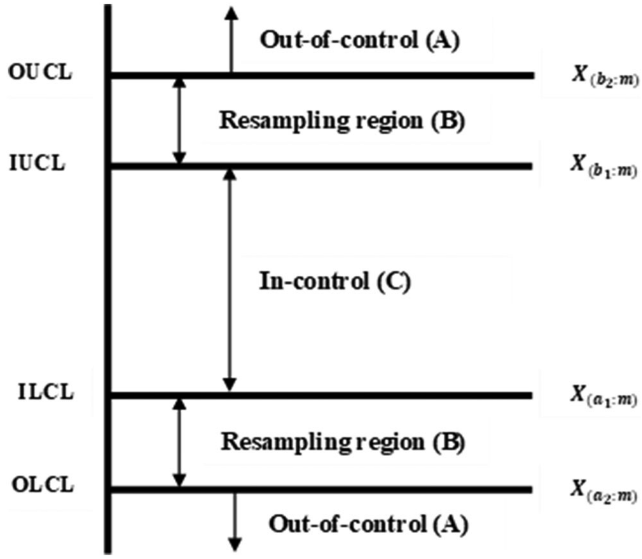


FIGURE 1 Regions of the nonparametric RS precedence chart

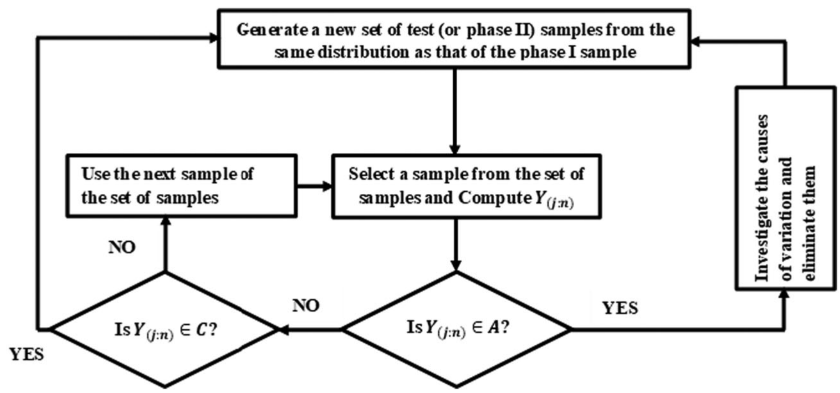


FIGURE 2 Flow chart of the proposed RS precedence chart in Phase II

Note that the OOC signal triggers an investigation that is conducted to find the causes of the OOC situation in order to eliminate them. Figure 2 summarizes the Phase II operation of the proposed nonparametric RS precedence control chart using the regions provided in Figure 1.

2.1 | IC *ARL* and *ASN* expressions of the nonparametric RS precedence chart

Let p_A, p_B and p_C denote the unconditional probabilities that the charting statistics $Y_{(j:n)}$ for any test sample $Y = \{y_1, y_2, \ldots, y_n\}$ plots in regions A, B and C, respectively. When the process is IC, these probabilities are defined by

$$p_A = \frac{2n!m!}{(j-1)!(n-j)!(b_2-1)!(m-b_2)!}\int_0^1 \sum_{h=0}^{n-j}\left\{\frac{(-1)^h}{j+h}\binom{n-j}{h}t^{j+h}\right\}\left[t^{b_2-1}(1-t)^{m-b_2}\right]dt, \tag{1}$$

$$p_B = \frac{2n!}{(j-1)!(n-j)!}\int_0^1 \sum_{h=0}^{n-j}\left\{\frac{(-1)^h}{j+h}\binom{n-j}{h}\left(t^{j+h}-s^{j+h}\right)\right\}f_{b_1 b_2}(s,t)\,ds\,dt, \tag{2}$$

and

$$p_C = \frac{2n!}{(j-1)!(n-j)!}\int_0^1 \sum_{h=0}^{n-j}\left\{\frac{(-1)^h}{j+h}\binom{n-j}{h}\left(t^{j+h}-s^{j+h}\right)\right\} f_{a_1 b_1}(s,t)\,ds\,dt, \tag{3}$$

where $f_{ab}(s,t) = \frac{m!}{(a-1)!(b-a-1)!(m-b)!} t^{a-1}(t-s)^{b-a-1}(1-t)^{m-b}$ (with $a < b$) represents the joint density function of $U_{(a:m)}$ and $U_{(b:m)}$ of a random sample of size m from the Uniform (0, 1) distribution with $a \in \{a_2, a_1\}$ and $b \in \{b_1, b_2\}$. Since $\psi^{-1}(.) = GF^{-1}(.)$, when the process is IC, $\psi^{-1}(t) = GF^{-1}(t) = t$ and $\psi^{-1}(s) = GF^{-1}(s) = s$ (see Table 1).

Thus, from Equations (1)–(3), the probability that the process is declared to be IC is defined by

$$p_{in} = \frac{p_C}{1-p_{rep}}, \tag{4}$$

where the probability of repetition, $p_{rep} = p_B$.

The IC ARL (ARL_0) is then defined by

$$ARL_0 = \frac{1}{1-p_{in}}. \tag{5}$$

Note that the charting constants are selected such that the attained ARL_0 is much closer or equal to the nominal ARL_0, denoted as $\overline{\alpha}$, which is chosen to be equal to some high desired values such as 370, 500 and 1,000.

The IC average sample number (ASN), denoted as ASN_0, which is the IC average number of samples needed to reach a decision, is given by

$$ASN_0 = \frac{n}{1-p_{rep}}. \tag{6}$$

2.2 | OOC *ARL* and *ASN* expressions of the RS precedence chart

When the process is OOC, the OOC ARL (ARL_1) and OOC ASN (ASN_1) are given by

TABLE 1 IC and OOC transformation function under the $N(0, 1)$, $G(1, 1)$ and $t(5)$ distributions

					$\psi(u) = GF^{-1}(u)$ (OOC case)		
Distribution	$f(x)$	**Parameters**	$F(x)(\delta = 0)$	$G(x)(\delta \geq 0)$	**Upper one-sided scheme**	**Two-sided scheme**	$\psi(u)$ **(IC case)**
Standard normal: $X \sim N(0,1)$ $x \in (-\infty, \infty)$	$\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right)$	$\mu = 0$ and $\sigma^2 = 1$	$\Phi(x)$	$\Phi(x-\delta)$	$\Phi\left(-\delta + \Phi^{-1}(u)\right)$	$\Phi\left(\delta + \Phi^{-1}(u)\right)$	u
Gamma[a]: $X \sim G(\alpha, \beta)$ $x \in [0, \infty)$ $\alpha > 0$ and $\beta > 0$	$x^{\alpha-1}\frac{e^{-x/\beta}}{\Gamma(\alpha)\beta^{\alpha}}$	$\alpha = 1$ and $\beta = 1$	$1-\exp(-x)$	$1-\exp\left(\frac{-x}{\delta+1}\right)$ $\alpha = 1$ and $\beta = 1$	$1-\exp\left(\frac{1}{\delta+1}\ln(1-u)\right)$	$1-\exp\left(\frac{1}{\delta+1}\ln(1-u)\right)$	u
Student's t: $X \sim t(v)$ $x \in (-\infty, \infty)$ $v > 0$	$\frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{v\pi}\,\Gamma\left(\frac{v}{2}\right)}\left(1+\frac{x^2}{v}\right)^{-\left(\frac{v+1}{2}\right)}$	$v = 5$	$F_5(x)$	$F_5\left(x-\sqrt{2}\delta\right)$	$F_5\left(-\sqrt{2}\delta + F_5^{-1}(u)\right)$	$F_5\left(\sqrt{2}\delta + F_5^{-1}(u)\right)$	u

[a]Note that the Gamma distribution is positively skewed and the skewness of the Gamma distribution increases as α decreases. Also note that the G(1, 1) distribution is the Exponential distribution with mean 1, EXP(1).

$$ARL_1 = \frac{1}{1 - p_{out}} \tag{7}$$

and

$$ASN_1 = \frac{n}{1 - p^1_{rep}}, \tag{8}$$

respectively, with

$$p_{out} = \frac{p_{1C}}{1 - p^1_{rep}} \tag{9}$$

and

$$p^1_{rep} = p_{1B}, \tag{10}$$

where p_{1A}, p_{1B} and p_{1C} are defined by

$$p_{1A} = \frac{2\,n!m!}{(j-1)!(n-j)!(b_2-1)!(m-b_2)!}\int_0^1 \sum_{h=0}^{n-j}\left\{\frac{(-1)^h}{j+h}\binom{n-j}{h}\left[\psi^{-1}(t)\right]^{j+h}\right\}\left[t^{b_2-1}(1-t)^{m-b_2}\right]dt, \tag{11}$$

$$p_{1B} = \frac{2\,n!}{(j-1)!(n-j)!}\int_0^1 \sum_{h=0}^{n-j}\left\{\frac{(-1)^h}{j+h}\binom{n-j}{h}\left[\left[\psi^{-1}(t)\right]^{j+h} - \left[\psi^{-1}(s)\right]^{j+h}\right]\right\} f_{b_1 b_2}(s,t)\,ds\,dt, \tag{12}$$

and

$$p_{1C} = \frac{2\,n!}{(j-1)!(n-j)!}\int_0^1 \sum_{h=0}^{n-j}\left\{\frac{(-1)^h}{j+h}\binom{n-j}{h}\left[\left[\psi^{-1}(t)\right]^{j+h} - \left[\psi^{-1}(s)\right]^{j+h}\right]\right\} f_{a_1 b_1}(s,t)\,ds\,dt, \tag{13}$$

respectively. For more details on the conversion function $\psi^{-1}(.) = GF^{-1}(.)$ for the OOC case, see Table 1. Note that these expressions are used in Mathcad Prime 8 to investigate the performance of the proposed nonparametric RS precedence chart.

3 | IC AND OOC PERFORMANCES OF THE RS PRECEDENCE CHART

In this section, the IC robustness and OOC performance of the proposed nonparametric RS precedence chart are investigated under the standard normal distribution (i.e., N[0, 1]), Student's t distribution with 5 degrees of freedom (i.e., t[5]) and Gamma distribution with shape and scale parameters both equal to 1 (i.e., G[1,1]). For a fair comparison, these distributions were transformed such that their means and variances are equal to 0 and 1, respectively. Table 1 presents the upper one-sided and two-sided IC and OOC transformation (or conversion) functions. For instance, the probability that the plotting $Y_{(j:n)}$ falls in region A is given by

$$p_A = \frac{2\,n!m!}{(j-1)!(n-j)!(b_2-1)!(m-b_2)!}\int_0^1 \sum_{h=0}^{n-j}\left\{\frac{(-1)^h}{j+h}\binom{n-j}{h}\left[\psi^{-1}(t)\right]^{j+h}\right\}\left[t^{b_2-1}(1-t)^{m-b_2}\right]dt.$$

Since $\psi^{-1}(t) = t$ when the process is IC, then regardless of the nature of the underlying distribution, for the IC case, p_A becomes

$$p_A = \frac{2\,n!m!}{(j-1)!(n-j)!(b_2-1)!(m-b_2)!}\int_0^1 \sum_{h=0}^{n-j}\left\{\frac{(-1)^h}{j+h}\binom{n-j}{h}t^{j+h}\right\}\left[t^{b_2-1}(1-t)^{m-b_2}\right]dt.$$

However, when the process is OOC, under the normal distribution, for a one-sided precedence chart, $\psi^{-1}(t)=\Phi\left(-\delta+\Phi^{-1}(t)\right)$, then p_A, which is denoted as p_{1A}, becomes:

$$p_{1A} = \frac{2n!m!}{(j-1)!(n-j)!(b_2-1)!(m-b_2)!}\int_0^1 \sum_{h=0}^{n-j}\left\{\frac{(-1)^h}{j+h}\binom{n-j}{h}\left[\Phi\left(-\delta+\Phi^{-1}(t)\right)\right]^{j+h}\right\}\left[t^{b_2-1}(1-t)^{m-b_2}\right]dt.$$

Under the gamma distribution, p_{1A} becomes

$$p_{1A} = \frac{2\,n!m!}{(j-1)!(n-j)!(b_2-1)!(m-b_2)!}\int_0^1 \sum_{h=0}^{n-j}\left\{\frac{(-1)^h}{j+h}\binom{n-j}{h}\left[1-\exp\left(\frac{1}{\delta+1}\ln(1-t)\right)\right]^{j+h}\right\}\left[t^{b_2-1}(1-t)^{m-b_2}\right]dt.$$

In this paper, we focus on the one-sided precedence chart for simplicity.

3.1 | IC robustness

Table 2 investigates the IC properties of the proposed nonparametric RS precedence chart under the distributions considered in this paper for different nominal ARL_0 values ($\overline{\alpha}$) when the Phase I sample size $m \in \{50, 100, 500\}$. For instance, for a Phase I sample size of 100 and a nominal $ARL_0 = 370$ ($\overline{\alpha} = 370$), the combination $(a_2, a_1, b_1, b_2) = (4, 26, 75, 97)$ yields attained ARL_0 and ASN_0 values of 376.8 and 6.5, respectively, across all the underlying process distributions considered in this paper. The results in Table 2 show that the IC characteristics (i.e., ARL_0 and ASN_0) of the RS precedence chart are the same across all the distributions considered in this paper. This reveals that the proposed RS precedence chart is IC robust. Since it is found to be IC robust, in the next subsection, its OOC performance is also investigated under different distributions. Note that the width of the inner and outer control limit dependent on the sizes of Phases I and II sample.

3.2 | OOC performance of the proposed chart

Since the proposed chart is IC robust, we can then proceed to the investigation of its OOC properties. Table 3 presents the OOC performance of the nonparametric RS precedence chart in terms of the ARL_1 profile along with the ASN_1 when $n \in \{5, 7, 11\}$ and $m \in \{100,500\}$. The findings in Table 3 reveal the following:

- The larger the Phase I (or Phase II) sample size, the more sensitive the proposed chart is.
- The proposed chart performs better under heavy-tailed distributions followed by symmetric distributions; see, for instance, the results under the $t(5)$ and $N(0, 1)$ distributions, respectively.
- Under skewed distributions, the proposed chart is slower in detecting shifts in the process location; see, for instance, the results under the $G(1, 1)$ distribution.

Since the proposed chart is faster in detecting shifts for large Phase I sample sizes, it is noticed that regardless of the nature of the underlying process distribution, the larger (smaller) the Phase I sample size, the smaller (larger) the ASN_1 values.

- It is also noticed that the ASN_1 value is maximum between the shift (δ) of sizes 1 and 1.25 standard deviation.

Figures 3 and 4 confirm that as the Phase I (or Phase II) sample size increases, the ability of the proposed chart to detect shifts in the location process increases as well.

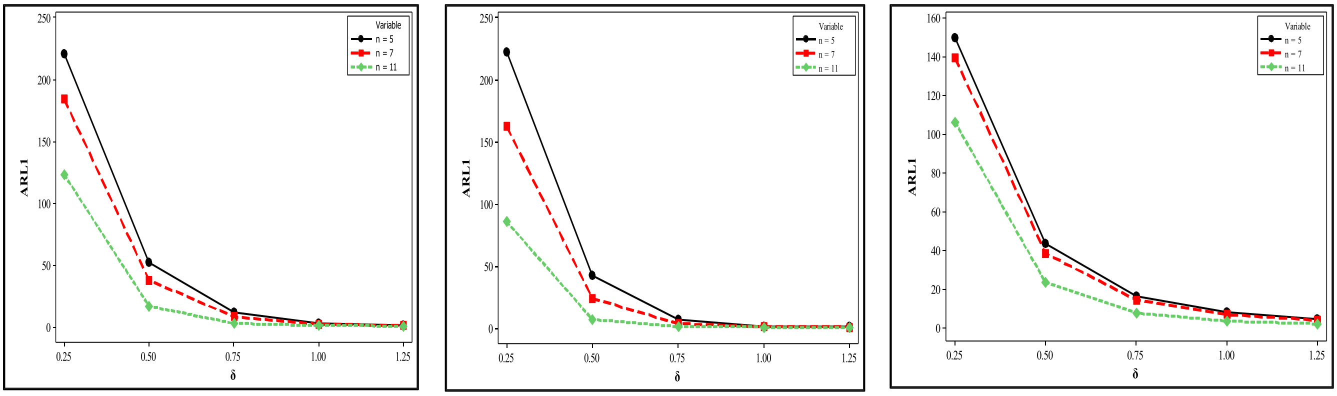
TABLE 2 Charting constants of the precedence chart with repetitive sampling along with the attained ARL_0 and ASN_0 when $n \in \{5, 7, 11\}$, $m \in \{50, 100, 500\}$ and $\overline{\alpha} \in \{200, 370, 500, 1{,}000\}$ under the $N(0, 1)$, $t(5)$ and $G(1,1)$ distributions

		$(n, j) = (5, 3)$						$(n, j) = (7, 4)$						$(n, j) = (11, 6)$					
m	$\bar{\alpha}$	a_2	a_1	b_1	b_2	ARL_0	ASN_0	a_2	a_1	b_1	b_2	ARL_0	ASN_0	a_2	a_1	b_1	b_2	ARL_0	ASN_0
50	200	2	16	35	49	209.5	8.1	3	19	32	48	206.8	14.0	6	8	43	45	195.4	11.1
	370	2	4	47	49	334.0	5.1	3	11	40	48	369.4	7.8	5	12	39	46	368.4	11.9
	500	None	None	None	None	None	None	2	20	31	49	507.6	16.3	5	7	44	46	396.1	11.1
	1,000	None	None	None	None	None	None	2	12	39	49	1017.9	8.1	4	6	45	47	912.2	11.0
100	200	5	29	72	96	199.1	7.1	8	29	72	93	203.4	9.1	13	27	74	88	198.6	12.3
	370	4	26	75	97	376.8	6.5	7	21	80	94	371.2	7.6	11	32	69	90	368.3	14.1
	500	3	35	66	98	514.1	9.4	6	29	72	95	505.8	9.1	10	35	66	91	498.8	15.9
	1,000	2	38	63	99	1041.9	11.4	4	39	62	97	1043.9	15.2	9	27	74	92	1022.1	12.4
500	200	28	145	356	473	200.5	7.1	41	179	322	460	200.8	12.2	65	192	309	436	202.8	19.1
	370	22	153	348	479	370.0	7.6	35	176	325	466	369.9	11.8	58	191	310	443	370.8	18.9
	500	19	165	336	482	500.8	8.5	32	178	323	469	501.8	12.0	55	189	312	446	502.4	18.4
	1,000	16	138	363	485	1004.6	6.8	26	183	318	475	1009.6	12.2	48	191	310	453	998.0	18.9

Note: **None** means no combination will yield an attained ARL_0 closer to the nominal ARL_0 value.

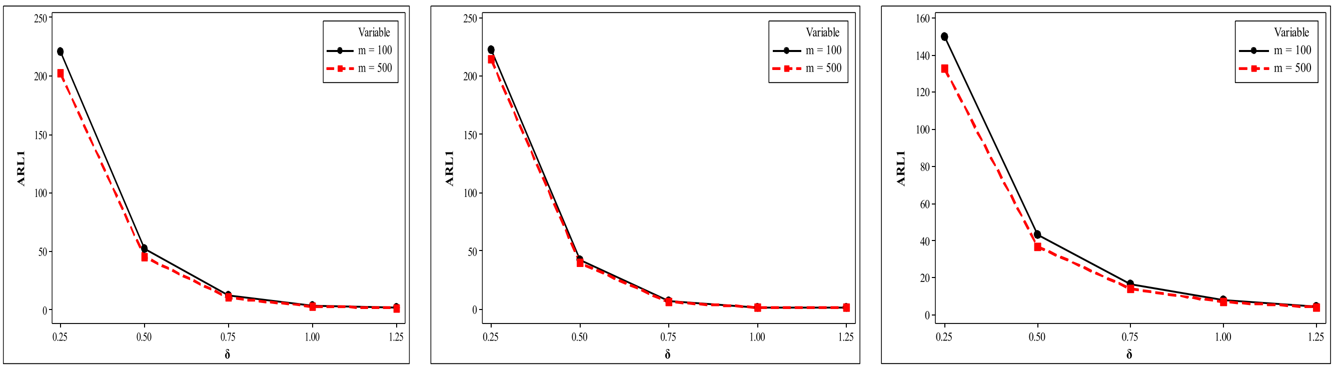
TABLE 3 ARL_1 and ASN_1 profiles of the RS precedence chart when $n \in \{5,7,11\}$ for a nominal $ARL_0 = 500$

		$(n, j) = (5, 3)$			$(n, j) = (7, 4)$			$(n, j) = (11, 6)$		
m	δ	$N(0, 1)$	$t(5)$	$G(1, 1)$	$N(0, 1)$	$t(5)$	$G(1, 1)$	$N(0, 1)$	$t(5)$	$G(1, 1)$
100	0.25	220.6 (10.2)	222.5 (11.0)	149.8 (10.4)	184.1 (9.9)	162.2 (10.5)	139.3 (9.9)	123.2 (18.2)	85.8 (20.4)	106.3 (18.0)
	0.50	52.0 (13.1)	42.6 (17.4)	43.4 (11.9)	38.0 (12.6)	24.1 (16.2)	38.5 (11.4)	17.1 (26.8)	7.2 (39.1)	23.5 (21.9)
	0.75	12.1 (19.0)	7.0 (32.9)	16.6 (13.7)	8.5 (17.8)	3.8 (28.0)	14.5 (13.0)	3.1 (40.9)	1.4 (49.1)	7.8 (26.3)
	1.00	3.4 (26.8)	1.8 (46.2)	8.0 (15.2)	2.5 (23.3)	1.3 (28.9)	7.0 (14.6)	1.3 (38.1)	1.0 (26.0)	3.6 (29.5)
	1.25	1.5 (27.8)	1.1 (31.6)	4.6 (16.3)	1.3 (21.6)	1.0 (17.5)	4.1 (15.7)	1.0 (23.9)	1.0 (15.5)	2.2 (30.1)
	1.50	1.1 (20.0)	1.0 (17.6)	3.1 (16.7)	1.1 (15.4)	1.0 (11.2)	2.7 (16.1)	1.0 (16.0)	1.0 (12.2)	1.6 (28.5)
	1.75	1.0 (13.2)	1.0 (10.9)	2.3 (16.6)	1.0 (11.1)	1.0 (8.6)	2.1 (16.0)	1.0 (12.8)	1.0 (11.2)	1.3 (25.9)
	2.00	1.0 (9.3)	1.0 (7.8)	1.9 (15.9)	1.0 (8.8)	1.0 (7.5)	1.7 (15.5)	1.0 (11.5)	1.0 (11.0)	1.2 (23.3)
500	0.25	202.2 (9.2)	214.2 (9.9)	132.6 (9.3)	157.7 (13.5)	142.1 (14.9)	113.2 (13.5)	104.3 (21.8)	72.0 (25.1)	87.1 (21.3)
	0.50	45.1 (11.7)	39.6 (15.6)	36.8 (10.7)	26.8 (18.7)	16.3 (26.9)	27.4 (16.0)	12.1 (34.4)	4.8 (52.7)	17.3 (26.5)
	0.75	10.3 (16.7)	6.2 (29.0)	14.0 (12.1)	5.2 (28.5)	2.3 (46.3)	9.7 (18.6)	2.2 (49.9)	1.2 (48.1)	5.6 (31.8)
	1.00	2.9 (22.6)	1.6 (36.8)	6.9 (13.4)	1.7 (32.8)	1.1 (31.3)	4.6 (20.6)	1.1 (36.3)	1.0 (21.6)	2.7 (34.1)
	1.25	1.4 (21.9)	1.1 (22.4)	4.1 (14.1)	1.1 (23.1)	1.0 (15.5)	2.8 (21.2)	1.0 (20.9)	1.0 (13.5)	1.7 (32.5)
	1.50	1.1 (15.4)	1.0 (12.0)	2.8 (14.3)	1.0 (14.5)	1.0 (9.8)	2.0 (20.6)	1.0 (14.4)	1.0 (11.5)	1.4 (29.0)
	1.75	1.0 (10.4)	1.0 (7.7)	2.1 (14.0)	1.0 (10.2)	1.0 (7.8)	1.6 (19.3)	1.0 (12.0)	1.0 (11.1)	1.2 (25.3)
	2.00	1.0 (7.6)	1.0 (6.0)	1.8 (13.4)	1.0 (8.3)	1.0 (7.2)	1.4 (17.7)	1.0 (11.2)	1.0 (11.0)	1.1 (22.2)



(a) $N(0,1)$ distribution **(b)** $t(5)$ distribution **(c)** $G(1,1)$ distribution

FIGURE 3 Effect of the test sample size of the Phase II performance of the proposed scheme when $m = 100$



(a) $N(0,1)$ distribution **(b)** $t(5)$ distribution **(c)** $G(1,1)$ distribution

FIGURE 4 Effect of the reference sample size of the Phase II performance of the proposed scheme when $n = 5$

3.3 | Performance comparison

Table 4 compares the performance of the nonparametric RS proposed precedence chart with those of the existing precedence-type control charts when $m = 500$ and $n = 5$ under the $N(0, 1)$, $t(5)$ and $G(1, 1)$ distributions when $\overline{\alpha} = 500$ and for the existing double sampling (DS) precedence chart we use the IC average sample size $ASS_0 = n = 5$ with $(n_1, n_2) = (3, 6)$ for a fair comparison. From Table 4, it can be observed that for symmetric and heavy-tailed distributions, the nonparametric precedence chart incorporated with improved runs-rules outperforms the proposed RS precedence chart for small shifts in the location parameter, whereas, for large shifts, the proposed chart outperforms all competing charts. Under skewed distributions, the proposed chart outperforms the precedence charts with supplementary runs-rules as well as the basic one regardless of the size (or magnitude) of the shifts. Compared to the DS precedence chart proposed by Malela-Majika et al. (2021), the proposed RS precedence chart performs better for moderate and large shifts under symmetric and heavy-tailed distributions, while the DS precedence chart performs better for small shifts in the location parameter. However, under skewed distributions, the RS precedence chart outperforms the DS precedence chart for small and moderate shifts, and the converse is true for large shifts.

4 | ILLUSTRATIVE EXAMPLE

The data from Castagliola et al. (2016) on a production process of 500 ml milk bottles where the quality of interest is the volume (in *ml*) of milk within each bottle are used to demonstrate the application and implementation of the proposed nonparametric RS precedence chart. The data consist of two datasets for which the goodness of fit test for normality is not rejected. The first dataset has 20 subgroups of size five each (i.e., $m = 100$) collected when the process was considered to be IC. Displayed in Table 5, the second dataset is composed of 20 test samples of size five each (i.e., $n = 5$) collected at different time points.

TABLE 4 Comparison of the proposed RS precedence chart with the existing precedence control charts for nominal $ARL_0 = 500$

	N(0, 1)					*t*(5)					G (1, 1)				
Shift	**1-*of*-1**	**2-*of*-2**	**1-*of*-1 or 2-*of*-2**	**Double sampling**	**Repetitive sampling**	**1-*of*-1**	**2-*of*-2**	**1-*of*-1 or 2-*of*-2**	**Double sampling**	**Repetitive sampling**	**1-*of*-1**	**2-*of*-2**	**1-*of*-1 or 2-*of*-2**	**Double sampling**	**Repetitive sampling**
0.00	520.27	490.21	505.24	502.18	500.80	520.27	490.21	505.24	504.54	500.80	520.27	490.21	505.24	502.18	500.80
0.25	261.60	170.07	**142.50**	171.75	202.20	293.52	179.43	151.87	**151.21**	214.20	172.76	310.12	186.98	380.15	**132.60**
0.50	77.73	39.37	**33.14**	37.14	45.10	90.30	**33.13**	35.20	**23.25**	39.60	58.49	88.52	91.54	120.15	**36.80**
0.75	25.79	12.99	15.18	10.46	**10.30**	26.57	9.05	12.93	**5.01**	6.20	26.16	28.03	29.53	38.15	**14.00**
1.00	10.26	5.99	6.04	4.05	**2.90**	8.87	4.05	3.89	2.02	**1.60**	14.48	10.26	9.20	13.40	**6.90**
1.25	4.88	4.94	4.98	2.09	**1.40**	3.65	2.66	2.39	1.59	**1.10**	9.27	6.81	6.71	5.04	**4.10**
1.50	2.80	2.67	2.10	1.42	**1.10**	1.95	2.21	1.36	1.05	**1.00**	6.6	2.61	2.51	**2.43**	2.80
1.75	1.80	2.34	1.86	1.22	**1.00**	1.34	2.06	1.13	1.08	**1.00**	5	2.33	2.24	**2.02**	2.10
2.00	1.40	2.1	1.50	1.04	**1.00**	1.10	2.02	1.05	**1.01**	**1.00**	4.02	2	1.86	**1.06**	1.80

Note: The ARL values of the best chart are boldfaced.

TABLE 5 Phase II data and plotting statistics of the nonparametric RS precedence chart using Castagliola et al. (2016) data when $\overline{\alpha} = 500$ and $(m, n) = (100, 5)$

	k						Control limits				
Time (t)	1	2	3	4	5	$Y_{(3:5)}$	OLCL	ILCL	IUCL	OUCL	Signal
1	499.92	500.74	501.76	497.76	499.10	499.92	498.89	500.06	500.88	502.78	NO
2	499.88	499.76	500.94	499.63	499.43	499.76	498.89	500.06	500.88	502.78	NO
3	499.87	501.11	499.15	501.12	500.03	500.03	498.89	500.06	500.88	502.78	NO
4	499.36	500.39	501.06	500.08	499.85	500.08	498.89	500.06	500.88	502.78	NO
5	500.41	499.18	501.97	500.27	501.41	500.41	498.89	500.06	500.88	502.78	NO
6	501.89	500.47	498.09	500.16	501.53	500.47	498.89	500.06	500.88	502.78	NO
7	498.72	499.78	499.11	499.77	501.20	499.77	498.89	500.06	500.88	502.78	NO
8	**500.20**	**498.55**	**498.84**	**498.52**	**500.10**	**498.84**	**498.89**	**500.06**	**500.88**	**502.78**	**YES**
9	498.96	499.79	497.38	500.09	498.95	498.96	498.89	500.06	500.88	502.78	NO
10	498.66	500.32	500.31	500.66	499.83	500.31	498.89	500.06	500.88	502.78	NO
11	500.69	498.65	498.37	500.33	500.52	500.33	498.89	500.06	500.88	502.78	NO
12	499.30	500.09	498.64	499.24	499.16	499.24	498.89	500.06	500.88	502.78	NO
13	500.77	499.72	497.88	501.04	500.00	500.00	498.89	500.06	500.88	502.78	NO
14	498.82	500.21	500.73	499.63	499.29	499.63	498.89	500.06	500.88	502.78	NO
15	**498.25**	**498.46**	**499.56**	**498.68**	**499.57**	**498.68**	**498.89**	**500.06**	**500.88**	**502.78**	**YES**
16	**500.08**	**498.44**	**498.06**	**500.29**	**498.21**	**498.44**	**498.89**	**500.06**	**500.88**	**502.78**	**YES**
17	**498.21**	**498.86**	**500.19**	**498.63**	**500.06**	**498.86**	**498.89**	**500.06**	**500.88**	**502.78**	**YES**
18	500.21	498.24	497.38	499.03	500.55	499.03	498.89	500.06	500.88	502.78	NO
19	499.96	499.56	499.23	498.42	498.63	499.23	498.89	500.06	500.88	502.78	NO
20	500.88	502.57	499.01	498.48	498.60	499.01	498.89	500.06	500.88	502.78	NO

Note: The ARL values of the best chart are boldfaced.

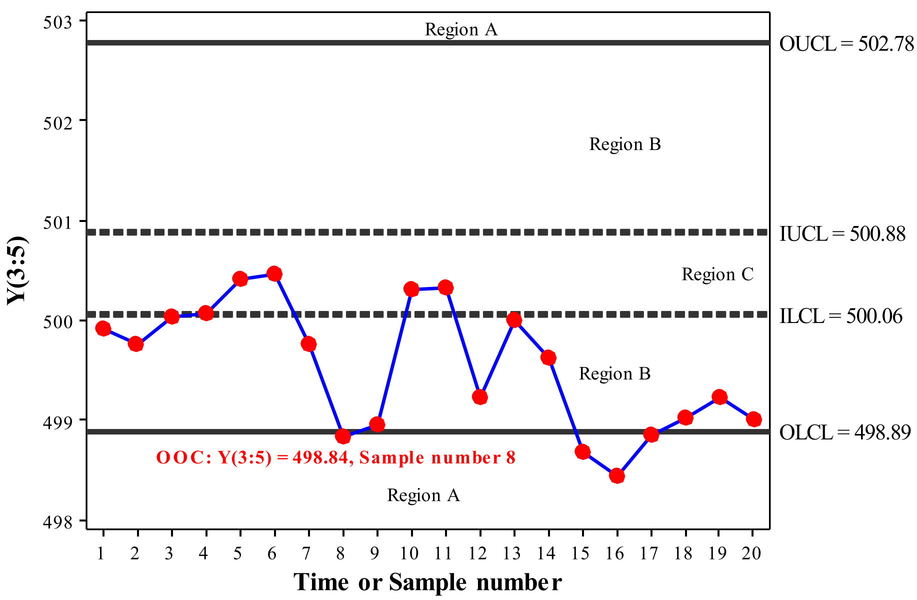


FIGURE 5 Illustrative example of the proposed nonparametric RS precedence chart using Castagliola et al. (2016) data on the production process of milk bottles for $\overline{\alpha} = 500$ and $(m, n) = (100, 5)$

The values of the charting constants for $\overline{\alpha} = 500$ when $n = 5$ and $m = 100$ are found in Table 2 where a_2, a_1, b_1 and b_2 are equal to 3, 35, 66 and 98, respectively, so that the RS precedence chart yields an attained $ARL_0 = 514.1$ with $ASN_0 = 9.4$. The plot of the charting statistic is shown in Figure 5, and the results are also displayed in Table 5. From Figure 5 and Table 5, it can be observed that the proposed nonparametric RS precedence chart gives a signal for the first time on the *eighth* sample in the prospective phase.

5 | CONCLUSION

This paper proposed a new nonparametric RS precedence chart for monitoring shifts in the process location regardless of the nature of the underlying process distribution. The exact expressions of the run-length properties of the proposed chart are provided and computed using Mathcad Prime 8. The proposed chart is found to be IC robust, and the results reveal that it outperforms the precedence chart with and without supplementary runs-rules for different shifts sizes except for small shifts under symmetric and heavy-tailed distributions. In addition, the proposed RS precedence chart outperforms the DS precedence chart in many cases.

In future, researchers who are interested to develop robust and efficient nonparametric control charts can look at the design of nonparametric RS precedence charts with variable sample sizes and sampling interval.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are openly available in Taylor & Francis at https://doi.org/10.1080/0740817X.2015.1056861, reference number 10.1080/0740817X.2015.1056861.

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How to cite this article: Malela-Majika, J.-C. (2023). Nonparametric precedence chart with repetitive sampling. *Stat*, *12*(1), e512. https://doi.org/10.1002/sta4.512