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Convective Boundary Layer Flow of Williamson Hybrid Ferrofluid over a Moving Flat Plate with Viscous Dissipation

Wan Muhammad Hilmi Wan Rosli[1,2], Muhammad Khairul Anuar Mohamed[1,*], Norhafizah Md Sarif[1], Huei Ruey Ong[3]

1 Centre for Mathematical Sciences, Universiti Malaysia Pahang Al-Sultan Abdullah, Lebuh Persiaran Tun Khalil Yaakob, 26300 Kuantan, Pahang, Malaysia
2 UMW M&E Sdn. Bhd., Lot 29138, Mukim Bandar Serendah 48200 Serendah, Selangor Darul Ehsan, Malaysia
3 Faculty of Engineering Technology, DRB-HICOM University of Automotive Malaysia, Jaya Industrial Area, 26607 Pekan, Pahang, Malaysia

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ABSTRACT

Specifically, the oxide ferroparticles in ferrofluid has low thermal conductivity thus limiting its potential in the application of heat transfer. Increasing the volume fraction ferroparticles is one of the ways to increase the efficiency but this will cause clogging in the flow system. The present research investigated the convective boundary layer flow of a Williamson hybrid ferrofluid over a moving flat plate with viscous dissipation effects. Magnetite (Fe_3O_4) and copper (Cu), taken as hybrid ferroparticles, are suspended in Williamson fluid represented by human blood are believed to improve the heat transfer capabilities of the ferrofluid. The governing equations in the form of partial differential equations are reduced to ordinary differential equations by using the similarity transformation. The Runge-Kutta-Fehlberg (RKF45) method is used to numerically solve the transformed equations obtained. The effects of the magnetic parameter, the Williamson fluid parameter, the moving plate parameter, and the Eckert number on the velocity profiles, the temperature profiles as well as the reduced skin friction coefficient and the reduced Nusselt number are analyzed and discussed. It is revealed that Williamson hybrid ferrofluid has higher heat transfer capabilities and lower skin friction compared to the Williamson ferrofluid at the same volume fraction. In addition, the magnetic parameter increases the skin friction while the moving plate parameter increases the reduced Nusselt number.

1. Introduction

One of the ways to improve the conventional base fluid for thermal conductivity is by dissolving the non-identical nanoparticles into the based fluid. Called "hybrid nanofluid", the goal of dispersing the non-identical nanoparticles into the based fluid is to strengthen each component's ability to function alone or to compensate for any weaknesses to generate an optimum heat transfer rate. Before hybrid nanofluid subject attract the attention of researcher to study, Maxwell [1] pioneered

* *Corresponding author.*
E-mail address: mkhairulanuar@umpsa.edu.my

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the research of potential increasing heat transfer performance by dispersing large volume fraction solid particles into the base fluid but this results occurrence of sedimentation that inhibit heat transfer performance. To counter this problem, nanoparticles was developed to ensure suspension stability and thermal conductivity improvement. Choi and Eastman [2] instigated the research of improving fluid thermal conductivity by dispersing metallic nanoparticles. Due to this research, hybrid nanofluid has become popular topics among researchers to study and develop new types of fluid. Researchers explored this hybrid nanofluid with different types of aspects like thermal physical properties, viscosity, thermal conductivity, magnetic, moving plate, stretching, stagnation and others. Some examples which are from Mahesh *et al.,* [3], Kho *et al.,* [4], Usafzai and Aly [5], Merkin *et al.,* [6], Khashi'ie *et al.,* [7], Gumber *et al.,* [8] and Yaseen *et al.,* [9]. Researcher who studied blood as based fluid in their hybrid nanofluid research was done by Rosli *et al.,* [10,11], Waqas *et al.,* [12] and Saeed *et al.,* [13]. Rosli studied the behavior of blood based ferrofluid which is similar to the type of nanofluid used in this present research. They were using magnetite (Fe_3O_4) and copper (Cu) as nanoparticles.

The concept and formulation of boundary layer flow over a moving plate was first studied by Sakiadis [14] considering plate moving with uniform speed. Due to induction of ambient fluid this type of flow is distinct with Blasius flow [15]. The influenced of moving plate on a fractional Maxwell viscoelastic nanofluid was studied by Cao *et al.,* [16] using combination of finite-difference method with L1-algorithm. Sravanthi *et al.,* [15] studied the characteristics of magnetite-water nano liquid when applying variable pertinent parameters including permeable moving plate. Researchers who studied the influence of moving plate on the hybrid nanofluid was done by Usafzai and Aly [5], Khashi'ie *et al.,* [7], Yaseen *et al.,* [9] and Aladdin *et al.,* [17]. Bachok *et al.,* [18] studied the characteristics of three types of nanofluid which is, Copper (Cu), Alumina (Al_2O_3) and Titania (TiO_2), in extending Blasius and Sakiadis problems. Tiwari and Das as well as Buongiorno nanofluid model was formulated by Asshaari *et al.,* [19] to study the characteristics of heat and mass transfer of carbon nanotubes based-water nanofluids through moving plate. BVP4C method was deployed to solve numerical equations developed.

Viscous dissipation is the conversion of the kinetic energy of the fluid into thermal energy. The effect of viscous dissipation on the thermal boundary layer was first identified by Gebhart [20] and often ignored in unsteady conditions. From a practical point of view, this effect is important in several flow issues, and it is also the source of temperature rise and geodynamic heating. In the behavior of dynamic temperature which is equivalent to the attributed difference in heat transfer temperature, the impact of viscous dissipation cannot be ignored except for the lower velocity method due to the small temperature profile. Hasanuzzaman *et al.,* [21] study the effect of viscous dissipation as well as radiative on the transfer of unsteady magnetic-conductive heat mass across a vertically porous sheet. They were using Finite Difference Method (FDM) in their studies to solve the non-dimensional ODE's. The impact of viscous dissipation on hybrid nanofluid was studied by Mahesh *et al.,* [3], Kho *et al.,* [4], Mohamed *et al.,* [22] and Famakinwa *et al.,* [23]. Using Runge-Kutta fourth-order method, Loh *et al.,* [24] investigate the impact of viscous dissipation on the Alumina-water nano fluids transport in asymmetrical heated microchannel. Mohamed *et al.,* [25,26] studied viscous dissipation effects of boundary layer flow of nanofluid on a circular cylinder and solid sphere.

During the time of doing this research, latest studies on Williamson hybrid ferrofluid was done by Rosli *et al.,* [10,11]. They explored a few parameters that affect the characteristics of blood-based hybrid ferrofluid except for moving plate parameters and viscous dissipation. Other studies that explore the aspects of moving plate and viscous dissipation of Williamson hybrid ferrofluid were not available. Present research will extend the characteristics studies of blood-based hybrid ferrofluid or Williamson hybrid ferrofluid with aspects of moving plate parameter and viscous dissipation. The intension of doing this research is to provide cheap and fast theoretical knowledge of the fluid characteristics in terms of heat transfer performance and skin friction using mathematical modeling and numerical solution.

2. Mathematical Formulation

A two-dimensional moving flat plate immersed in a steady Williamson hybrid ferrofluid with ambient temperature (T_∞) is illustrated in Figure 1 below. It is assumed that the wall temperature, T_w, is equal to the boundary layer temperature (T), $T = T_w$. Velocity components along the x and y axes are defined as u and v. U_∞ defined as the free stream while $u_w = \varepsilon U_\infty$ is the moving plate velocity with ε and B_0 as the plate velocity parameter and magnetic field strength proportional to $y-$directional normal to the moving flat plate.

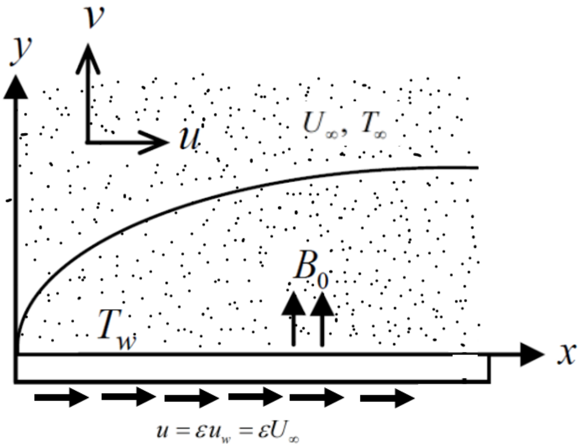

Fig. 1. Physical model and the coordinate system

From Figure 1 above, the boundary layer equation that can be formed are [10,11,22,27,28]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu_{hnf}\frac{\partial^2 u}{\partial y^2} + \sqrt{2}\,\nu_{hnf}\,\Gamma\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_o^2(x)}{\rho_{hnf}}u, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{hnf}}{\left(\rho C_p\right)_{hnf}}\frac{\partial^2 T}{\partial y^2} + \frac{\mu_{hnf}}{\left(\rho C_p\right)_{hnf}}\left(\frac{\partial u}{\partial y}\right)^2. \tag{3}$$

with boundary conditions:

$$\begin{gathered} u = \varepsilon u_w = \varepsilon U_\infty,\ v = 0,\ T = T_w \text{ at } y = 0 \\ u \to U_\infty,\quad T \to T_\infty, \text{ as } y \to \infty \end{gathered} \tag{4}$$

The hybrid ferrofluid kinematic viscosity, a dynamic viscosity, a density, and the electric conductivity are denoted as ν_{hnf}, μ_{hnf}, ρ_{hnf} and σ_{hnf} respectively. Furthermore, Γ, k_{hnf} and $(C_p)_{hnf}$ are the time constant, the thermal conductivity and the heat capacity of Williamson hybrid ferrofluid respectively. Other properties related to base fluid and the nanoparticles are denoted with subscript $_{bf}$ and $_{s1,s2}$ respectively. The hybrid ferrofluid properties are given as [29]:

$$\nu_{hnf}=\frac{\mu_{hnf}}{\rho_{hnf}},\quad \rho_{hnf}=(1-\phi_2)\left[(1-\phi_1)\rho_f+\phi_1\rho_{s1}\right]+\phi_2\rho_{s2},\quad \mu_{hnf}=\frac{\mu_f}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}},$$
$$\left(\rho C_p\right)_{hnf}=(1-\phi_2)\left[(1-\phi_1)\left(\rho C_p\right)_f+\phi_1\left(\rho C_p\right)_{s1}\right]+\phi_2\left(\rho C_p\right)_{s2}, \tag{5}$$
$$\frac{k_{hnf}}{k_{bf}}=\frac{k_{s2}+2k_{bf}-2\phi_2(k_{bf}-k_{s2})}{k_{s2}+2k_{bf}+\phi_2(k_{bf}-k_{s2})},\quad \frac{k_{bf}}{k_f}=\frac{k_{s1}+2k_f-2\phi_1(k_f-k_{s1})}{k_{s1}+2k_f+\phi_1(k_f-k_{s1})}$$

the similarity variable considered are as follows [22,27]

$$\eta=\left(\frac{U_\infty}{\nu x}\right)^{1/2} y,\ \psi=\left(a\nu x\right)^{1/2} f(\eta),\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \tag{6}$$

where η, ψ and θ is a non-dimensional variable, dimensional stream function and temperature, respectively. The similarity variables (6) satisfy the continuity Eq. (1) by definition

$$u=\frac{\partial\psi}{\partial y}\text{ and } v=-\frac{\partial\psi}{\partial x}. \tag{7}$$

Next, substitute the similarity variables Eq. (6) and Eq. (7) into governing Eq. (2) and Eq. (3) gives the following transformed ordinary differential equations:

$$\frac{v_{hnf}}{v_f}\left(f'''+\lambda f''f'''\right)+\frac{1}{2}ff''-M\left(f'-1\right)=0, \tag{8}$$

$$\frac{1}{\Pr}\frac{k_{hnf}}{k_f}\frac{\left(\rho C_p\right)_f}{\left(\rho C_p\right)_{hnf}}\theta''+\frac{1}{2}f\theta'+\frac{v_{hnf}}{v_f}\frac{\rho_{hnf}\left(C_p\right)_f}{\left(\rho C_p\right)_{hnf}}E_C f''^2=0. \tag{9}$$

Other quantities related to hybrid nanofluid are as follows:

$$\frac{v_{hnf}}{v_f}=\frac{1}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}\left[(1-\phi_2)+\left[(1-\phi_1)+\phi_1(\rho_{s1}/\rho_f)\right]+\phi_2(\rho_{s2}/\rho_f)\right]},$$

$$\frac{\left(\rho C_p\right)_f}{\left(\rho C_p\right)_{hnf}}=\frac{1}{(1-\phi_2)\left[(1-\phi_1)\rho_f+\phi_1(\rho C_p)_{s1}/(\rho C_p)_f\right]+\phi_2(\rho C_p)_{s2}/(\rho C_p)_f},$$

$$\frac{k_{hnf}}{k_f}\frac{\left(\rho C_p\right)_f}{\left(\rho C_p\right)_{hnf}}=\frac{k_{hnf}/k_f}{(1-\phi_2)\left[(1-\phi_1)\rho_f+\phi_1(\rho C_p)_{s1}/(\rho C_p)_f\right]+\phi_2(\rho C_p)_{s2}/(\rho C_p)_f},$$

$$\frac{v_{hnf}}{v_f}\frac{\rho_{hnf}\left(C_p\right)_f}{\left(\rho C_p\right)_{hnf}}=\frac{v_{hnf}}{v_f}\frac{(1-\phi_2)\left[(1-\phi_1)\rho_f+\phi_1\rho_{s1}\right]+\phi_2\rho_{s2}}{(1-\phi_2)\left[(1-\phi_1)\rho_f+\phi_1(\rho C_p)_{s1}/(\rho C_p)_f\right]+\phi_2(\rho C_p)_{s2}/(\rho C_p)_f}.$$

The boundary conditions (4) become

$$f(0)=0,\ f'(0)=\varepsilon,\ \theta(0)=1,$$
$$f'(\eta)\to 1,\ \theta(\eta)\to 0,\ \text{as } y\to\infty. \tag{10}$$

By definition, $\Pr=\dfrac{v_f\left(\rho C_p\right)_f}{k_f}$ is a Prandtl number, $\lambda=x\Gamma\sqrt{\dfrac{2a^3}{v_{hnf}}}$ is the Williamson fluid parameter, $M=\dfrac{\sigma B_o^{\,2}(x)}{a\rho_{nf}}$ is the magnetic parameter and $E_C=\dfrac{U_\infty^2}{\left(C_p\right)_f\left(T_w-T\infty\right)}$ is an Eckert number. The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x are given by:

$$C_f=\frac{\tau_w}{\rho_f u_w^2},\ Nu_x=\frac{xq_w}{k_f(T_w-T_\infty)}, \tag{11}$$

with the surface shear stress τ_w and the surface heat flux q_w are given by

$$\tau_w=\mu_{hnf}\left(\frac{\partial u}{\partial y}+\frac{\Gamma}{\sqrt{2}}\left(\frac{\partial u}{\partial y}\right)^2\right)_{\bar{y}=0},\ q_w=-k_{hnf}\left(\frac{\partial T}{\partial y}\right)_{\bar{y}=0} \tag{12}$$

Using variables in (6) and Eq. (11) give

$$C_f\,\mathrm{Re}_x^{\,1/2}=\frac{1}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}}\left(f''(0)+\frac{\lambda}{2}f''(0)^2\right)\text{ and } Nu_x\,\mathrm{Re}_x^{\,-1/2}=-\frac{k_{hnf}}{k_f}\theta'(0) \tag{13}$$

3. Results

Runge-Kutta-Fehlberg (RKF45) method programmed in Maples software is used to compute the non-linear ordinary differential Eq. (8) and Eq. (9) with boundary conditions (10). Developed by German mathematician, Erwin Fehlberg, it is a method of order 4 with an error estimator of order 5 on the large class of Runge-Kutta method. This method is used for its stability, accuracy and does not require initial value profile.

Table 1
Thermophysical properties of fluid and particles that are used in this research [22,30-34]

Physical Properties	Water	Blood	Magnetite (Fe_3O_4), ϕ_1	Copper (Cu), ϕ_2	Cobal ferrite ($CoFe_2O_4$)	Manganese-zinc ferrite ($Mn\text{-}ZnFe_2O_4$)	Silver (Ag)	Gold (Au)
ρ (kg/m^3)	997	1053	5180	8933	4907	4900	10500	19300
C_p (J/kg·K)	4179	3594	670	385	700	800	235	129
k(W/m·K)	0.613	0.492	9.7	400	3.7	5	429	318

Prandtl number, Pr is kept constants of, 21, throughout the results of this research due to considerations of blood as based fluid for all the parameter testing. Following Rosli *et al.,* [10,11] and Devi and Devi [29] the nanoparticle volume fractions (ϕ_1, ϕ_2) for magnetite and copper is taken as are added into blood to form the hybrid ferrofluid namely Fe_3O_4-Cu/blood hybrid ferrofluid. Blood hybrid ferrofluid is also compared with different types of hybrids ferrofluid and various nano particle volume fraction. Thermophysical properties of fluid and particles that are used in this research are provided in Table 1. The accuracy of numerical method is validated by comparing it with previous numerical result reported by Bachok *et al.,* [18] as shown in Table 2 using copper-water fluid, Pr = 6.2. Comparison with both previous results achieved good agreement.

Table 2
Comparison values of $f''(0)$ with previously published result when $\phi_1 = M = \lambda = \varepsilon = E_c = 0$

ϕ_2	$f''(0)$	
	Bachok *et al.,* [18]	Present Paper
0	0.3321	0.3321
0.1	0.3901	0.3901
0.2	0.4045	0.4045

Table 3 shows the result of $Nu_x \operatorname{Re}_x^{-1/2}$ and $C_f \operatorname{Re}_x^{1/2}$ for Williamson parameter tested. Both values of $Nu_x \operatorname{Re}_x^{-1/2}$ and $C_f \operatorname{Re}_x^{1/2}$ shows small changes when Williamson parameter increases. This results characteristic is parallel with previous results from Rosli *et al.,* [10,11].

Table 3
Values of $Nu_x \operatorname{Re}_x^{-1/2}$ and $C_f \operatorname{Re}_x^{1/2}$ various values of λ when $\Pr = 21,\ M = 0.5,\ \varepsilon = 0.1,\ E_c = 0.1$

λ	$Nu_x \operatorname{Re}_x^{-1/2}$	$C_f \operatorname{Re}_x^{1/2}$
0.1	2.2279	0.6951
0.2	2.2293	0.7005
0.25	2.2300	0.7031
0.3	2.2307	0.7056
0.4	2.2319	0.7107

Figure 2 and 3 illustrated the distributions of $Nu_x \mathrm{Re}_x^{-1/2}$ and $C_f \mathrm{Re}_x^{1/2}$ for four different ferroparticle volume fraction with the influence of viscous dissipation parameter known as Eckert number, E_c. Blood ($\phi_1 = \phi_2 = 0$), 0.1 vol. of Fe_3O_4/blood ferrofluid ($\phi_1 = 0.1, \phi_2 = 0$), 0.16 vol. of Fe_3O_4-Cu/blood hybrid ferrofluid ($\phi_1 = 0.1, \phi_2 = 0.06$), and 0.16 vol. of Fe_3O_4/blood ferrofluid ($\phi_1 = 0.16, \phi_2 = 0$) are the four ferroparticle volume fraction that is tested. Analyzing the results in Figure 2 and Figure 3, the increase of E_c parameter reduces the performance of convective heat transfer but does not affect skin friction of the fluid. This result is similar with previous research done Hasanuzzaman *et al.,* [21]. When comparing different ferroparticle volume fraction, Figure 3 illustrated that blood hybrid ferrofluid produces the highest skin friction compared to other ferroparticle volume fraction. The blood has the lowest skin friction due to no nanoparticle fraction in the fluid which resist the fluid flow. Considering the heat transfer performance, the 0.16 vol. of Fe_3O_4-Cu/blood hybrid ferrofluid has the best convective heat transfer performance as E_c = 0. The presence of E_c has eliminated the nanoparticle volume fraction effects thus high E_c producing similar performance in convective heat transfer capabilities. It is clearly seen in Figure 3 that the values of $Nu_x \mathrm{Re}_x^{-1/2}$ becomes similar with the other volume fractions as E_c increases.

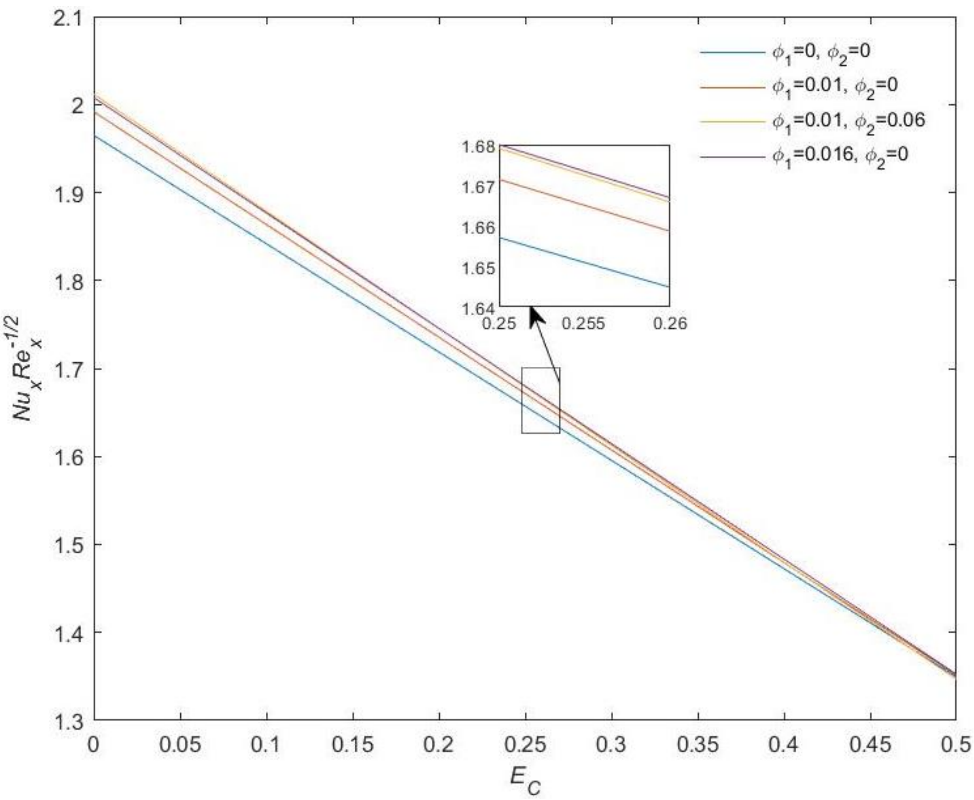

Fig. 2. Distribution of $Nu_x \mathrm{Re}_x^{-1/2}$ for various values of *E_c* when $\mathrm{Pr} = 21,\ \lambda = 0.1, M = 0.5,\ \varepsilon = 0.5$

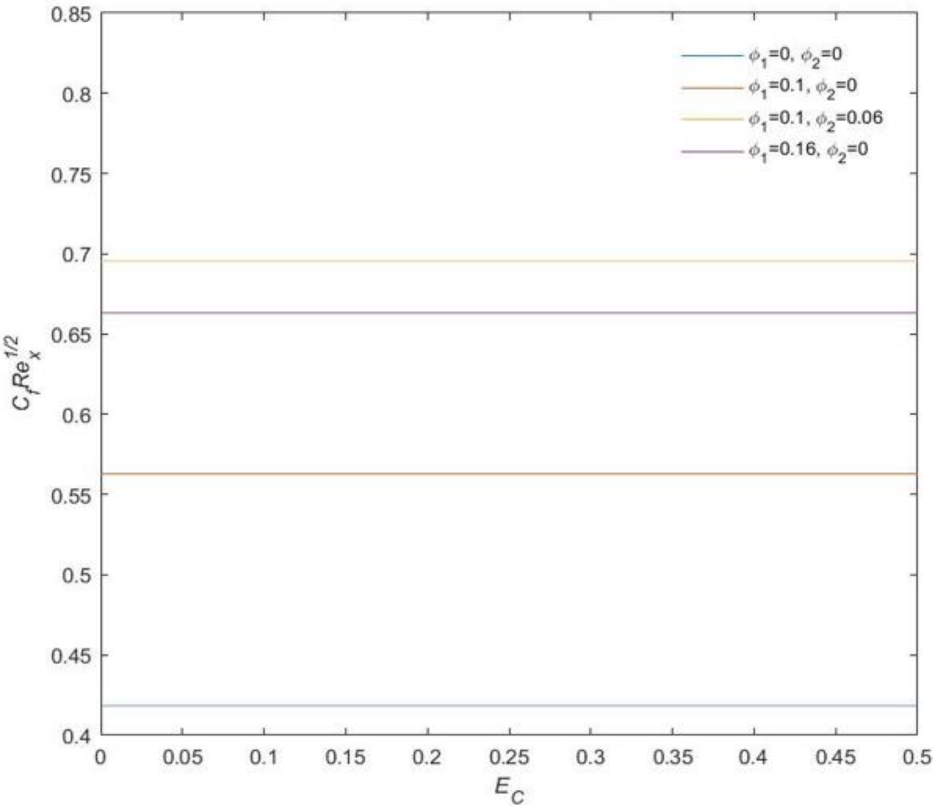

Fig. 3. Distribution of $C_f \mathrm{Re}_x^{1/2}$ for various values of E_c when $\mathrm{Pr}=21,\ \lambda=0.1, M=0.5,\ \varepsilon=0.5$

Figure 4 and 5 shows the distribution of $Nu_x \mathrm{Re}_x^{-1/2}$ and $C_f \mathrm{Re}_x^{1/2}$ for three types of blood-based hybrid ferrofluid with the influence of magnetic parameter, M. The three types of hybrids ferrofluid that is tested are blood with copper ferrite (Fe_3O_4-Cu/blood), cobalt ferrite with silver ($CoFe_2O_4$-Ag/blood) and manganese zinc ferrite with gold (Mn-$ZnFe_2O_4$-Au/blood). The ferroparticle volume is kept constant ($\phi_1 = 0.1, \phi_2 = 0.06$). Figure 4 illustrates that Fe_3O_4-Cu/blood has the highest convective heat transfer performance compared to $CoFe_2O_4$-Ag/blood and Mn-$ZnFe_2O_4$-Au/blood. It is noted that, the increase of magnetic parameter reduced the values of $Nu_x \mathrm{Re}_x^{-1/2}$. Physically, the increase in M, enhanced the magnetic force that attract the hybrid ferrofluid to a plate surface, thus promote the conductive heat transfer process which translate in reducing the convective heat transfer capabilities. From Figure 5, it is found that the increase in magnetic parameter M enhanced the skin friction coefficient. The Lorenz force is increases as magnetic parameter increases thus retard the fluid flow and resist the fluid flow for the hybrid ferrofluid tested. It is also found that the Mn-$ZnFe_2O_4$-Au/blood hybrid ferrofluid has the highest $C_f \mathrm{Re}_x^{1/2}$ values than $CoFe_2O_4$-Ag/blood and Fe_3O_4-Cu/blood hybrid ferrofluid. It is due to the high density of gold in the fluid thus produced high resistance in fluid flow.

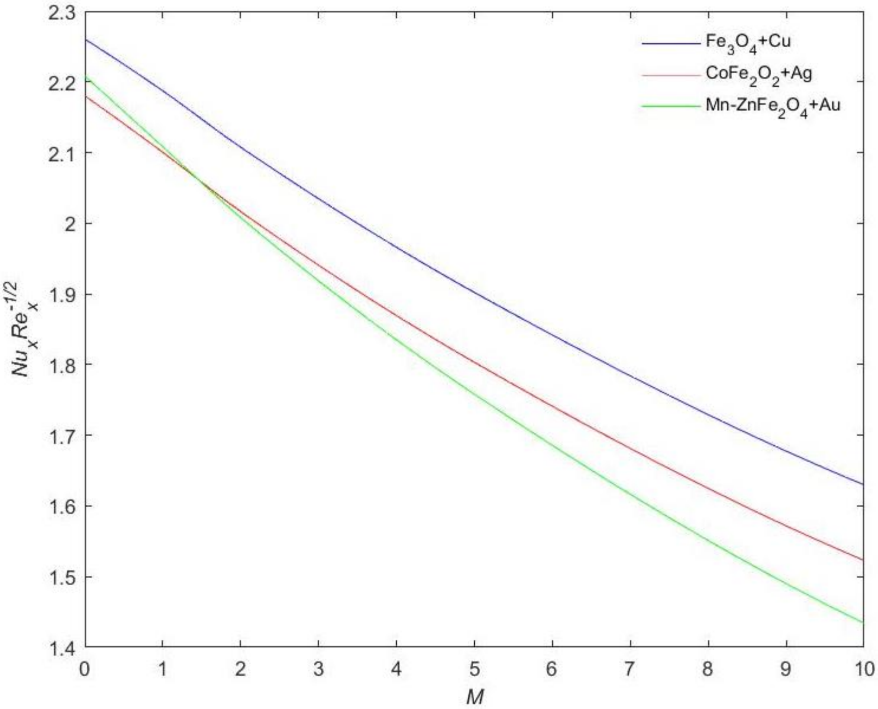

Fig. 4. Distribution of $Nu_x \mathrm{Re}_x^{-1/2}$ for various values of *M* when $\Pr = 21,\ \lambda = 0.1, E_c = 0.1,\ \varepsilon = 0.5$

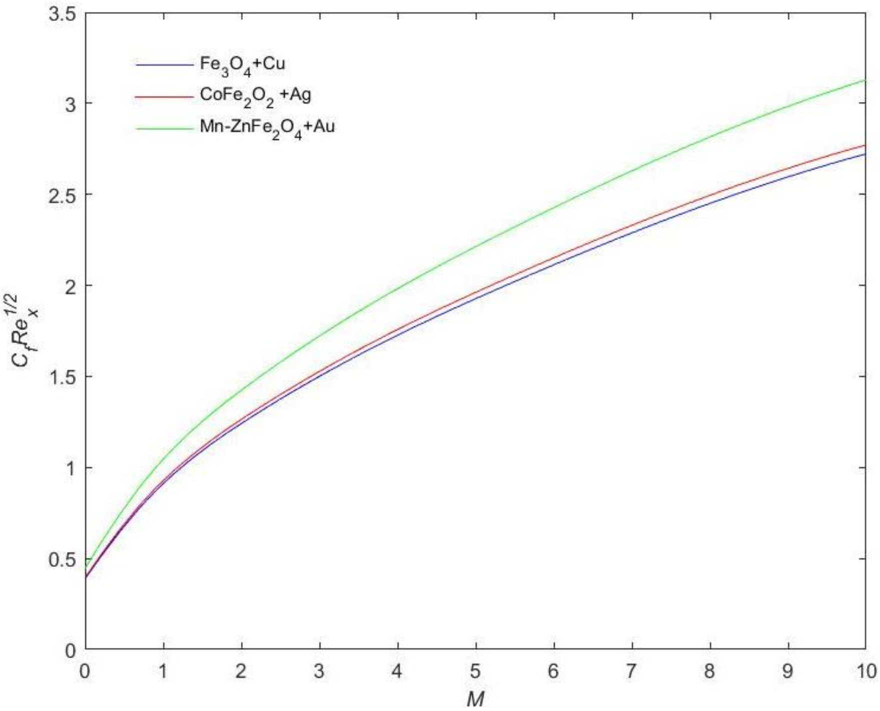

Fig. 5. Distribution of $C_f \mathrm{Re}_x^{1/2}$ for various values of *M* when $\Pr = 21,\ \lambda = 0.1, E_c = 0.1,\ \varepsilon = 0.5$

The effects of moving plate parameter on a for temperature and velocity profile are illustrated in Figure 6 and 7, respectively. In Figure 6, the boundary layer for temperature profile is decreasing as moving plate parameter increases. This situation promotes the increase in temperature gradient or physically increases the convective heat transfer capabilities in the fluid. This trend is similar to results from Mohamed *et al.,* [28]. In Figure 7, the velocity gradient is in inverted structured as $\varepsilon > 1$ and velocity boundary layer thickness decreases with moving plate parameter. As $\varepsilon < 1$, the flow has the boundary layer structured, which formed from the high plate velocity compared to the free stream velocity. Noticed that, the boundary layer thickness is increases with ε.

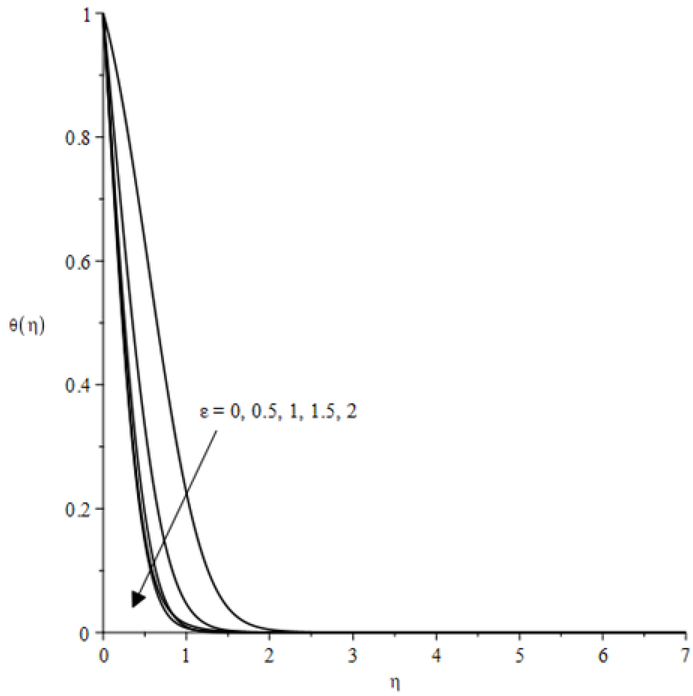

Fig. 6. Temperature profile for various values of ε when $\Pr = 21,\ M = 0.5,\ E_c = 0.1,\ \lambda = 0.1$

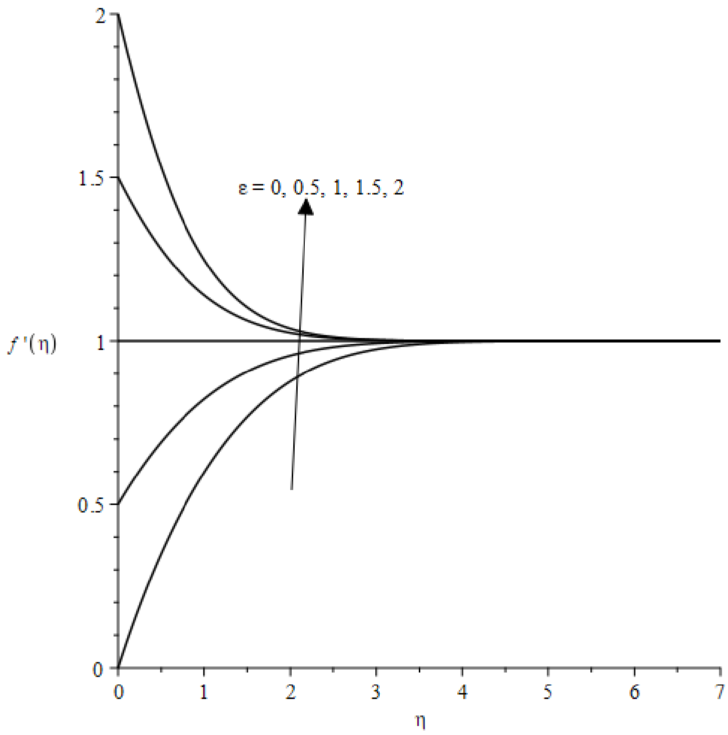

Fig. 7. Velocity profile for various values of ε when $\Pr = 21,\ M = 0.5,\ E_c = 0.1,\ \lambda = 0.1$

4. Conclusions

Convective boundary layer flow of Williamson hybrid ferrofluid over a moving flat with viscous dissipation were numerically studied. The present numerical method is validated with the sample result from Bachok *et al.,* [18] and the comparison achieved good agreement. Four parameters were tested in this research: Eckert number, magnetic, Williamson and moving plate parameter. Eckert number and magnetic parameter were tested with various ferroparticle volume fraction and different types of hybrids ferrofluid respectively. Summarization of results are as follows:

i. Williamson hybrid ferrofluid produces slightly better convective heat transfer performance as viscous dissipation is neglected.
ii. Increase in magnetic parameter results to a decrease in $Nu_x \mathrm{Re}_x^{-1/2}$ but increase in $C_f \mathrm{Re}_x^{1/2}$ value.
iii. Williamson parameter produces small effect on the boundary layer for temperature and velocity profiles.
iv. Moving plate parameter reduced the temperature profile. while increases the velocity profile.

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