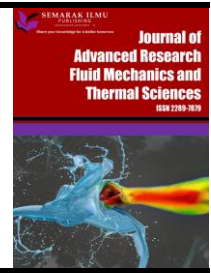




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Stagnation Bioconvection Flow of Titanium and Aluminium Alloy Nanofluid Containing Gyrotactic Microorganisms over an Exponentially Vertical Sheet

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ABSTRACT

The pivotal aim of this research is to address a natural stagnation bioconvection flow of a hybrid nanofluid containing gyrotactic microorganisms over an exponentially stretching and shrinking vertical sheet. The mathematical formulation of simplified Navier-Stokes equations is made in the presence of a few parameters such as Prandtl number, concentration to thermal buoyancy ratio, microorganism to thermal buoyancy ratio, Lewis number, bioconvection Peclet number, bioconvection Lewis number, microorganisms concentration difference and buoyancy parameter. The two types of nanofluid containing titanium alloy (Ti6Al4V) and aluminium alloy (AA7075) immersed in water are considered for the investigation. In the analysis, the governing partial differential equations (PDEs) are transformed into a set of ordinary differential equations (ODEs) by a similarity transformation. The resulting equations are rewritten in MATLAB software through the Bvp4c method to obtain the solutions. The effects of hybrid nanofluid of titanium alloy (Ti6Al4V) and aluminium alloy (AA7075), microorganisms' concentration difference parameter, and bioconvection Lewis Number are observed in this mathematical model in the presence of stretching and shrinking sheets. The numerical values are obtained for the skin friction coefficient, local Nusselt number, local Sherwood number, and local density of motile microorganisms for the reporting purpose. In addition, the profiles of the velocity, temperature, concentration, and microorganism are visualized as the main findings of this article.

1. Introduction

Hybrid nanofluid is a very novel type of nanofluids that contains two or more various nanoparticles. Ali [1] state that hybrid nanofluid is prepared by mixing two various kinds of nanoparticles in the equivalent base fluid to have greater thermophysical, optical, rheological, and

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morphological properties. Hybrid nanofluids have been tested for different applications like solar collectors and automotive cooling [1]. The flow and heat transfer of a thin film containing aluminium (AA7075) and titanium (Ti6Al4V) alloy nanoparticles enhance the surface coating for corrosion protection in chemical processing equipment's, wire, fiber, transpiration cooling and heat exchangers design [2]. Xian *et al.*, [3] found that hybrid nanofluids show better thermal conductivity and heat transfer performance compared to mono nanofluids and base fluids at most of the time. Similar to mono nanofluids, hybrid nanofluids deteriorate heat transfer performance when concentration of nanoparticles is higher than its upper limit.

Khashi'ie *et al.*, [4] focused on the MHD hybrid nanofluid flow with heat transfer on a moving plate with Joule heating. They combine metal (Cu) and metal oxide (Al₂O₃) nanoparticles with water (H₂O) as the base fluid and concluded that two solutions are obtained when the plate is moved oppositely from the free stream flow. Bilal *et al.*, [5] explore the effects of electro-magneto-hydrodynamics electroosmotic flow of hybrid nanofluid through circular cylindrical microchannels. The conversion of simple fluid to hybrid nanofluid has greatly alteration in the present model where it enhances the thermal properties of fluid and become useful for the designing of effectual electromagnetic appliances and exquisite. Recently, the published reports regarding to the mathematical model of the hybrid nanofluid has been described in details [6-15]. This increasing fascination with hybrid nanofluids in numerical studies has compelled us to delve into their characteristics, particularly their fluid flow and heat transfer properties.

Microorganisms is an arbitrary grouping, defined as organisms that are too small to be seen solely and they include bacteria, fungi, viruses, protozoa and green algae. It plays an important role in photosynthetic productivity, in decay and as parasites of all groups of living things [16]. Meanwhile, bioconvection happens when microorganisms are swimming in upward direction of fluid and yield instability and amorphous pattern. Due to upward swimming, the gyrotactic microorganisms such as algae tends to concentrate on the upper portion of fluid and produces dense stratification that usually develops wobbly. Nanofluid bioconvection phenomenon describes the density stratification and nebulous pattern formation formed by the synchronized edge of nanoparticles, buoyancy forces and denser self-propelled microorganisms. Such microorganisms might include gyrotaxis, gravitaxis, or oxytaxis organisms [17].

Azam [18] studied a mathematical modelling and numerical simulation for the unsteady Bioconvection flow of chemically reactive Sutterby nanofluid under the influences of gyrotactic microorganisms and nonlinear radiation. He found that microorganism field is upgraded for higher estimation of microorganism difference parameter and Peclet number. Muhammad *et al.*, [19] observed the characteristics of the Jeffrey nanofluid flow with the influence of activation energy and motile microorganisms over a sheet is considered. The dynamic physical declaration of attained results reveals that buoyancy ratio parameter and bioconvection Rayleigh number plays a vibrant role in the declining flow of Jeffery nanofluid while contrasting nature is analyzed for mixed convection parameter [19]. Besides, the additional references on these topics can be found in the review papers by [20-23]. Hence, it has been emphasized in these mentioned literatures that the inclusion of bioconvection in fluid flow models to effectively simulate and capture certain real phenomena is important and could give a significant impact.

The combination of nanofluid and bioconvection has produced some excellent results in microfluidic devices including micro-reactors and micro-channel. Kuznetsov and Avramenko [24] have firstly used nanoparticles to investigate the bioconvection of gyrotactic microorganism. It is established that the critical Rayleigh number increases with the increase of the average number density of small particles, which means that the particles make the suspension more stable. Abdelsalam and Bhatti [25] researched about the peristaltic flow of non-Newtonian nanofluid with

swimming oxytactic microorganisms through a space between two infinite coaxial conduits. The output obtained from the study shows that the Peclet number initially increases after nondimensionalizing the governing equations of microorganisms. It is evident that Peclet number reduces the motile microorganisms substantially. There are also other studies on bioconvection that can be found in [26-29]. However, the study regarding the bioconvection within a hybrid nanofluid flow is still limited in certain cases and a continuous investigation is needed.

Stagnation point flow is significant in forced convective heat transfer and fluid mechanics as it has tremendous applications like cooling of electronic devices using fans. Stagnation-point flows toward an exponentially shrinking sheet by considering alumina (Al₂O₃), copper (Cu) and titania (TiO₂) nanoparticles were considered by Bachok *et al.*, [30]. In these studies, they found that the heat transfer rate enhanced with the increase of nanoparticle volume fractions and intensified for Cu compared to Al₂O₃ and TiO₂ nanoparticles. Yousefi *et al.*, [31] compared the influences of a Titania-copper/water hybrid nanofluid, Titania/water nanofluid, and regular fluid (viscous) on the stagnation point flow past a wavy cylinder. After that, the problem of stagnation point flow of hybrid nanofluid flow was extended to the different aspects discussed in [32-35].

According to the abovementioned literature, it is seen that most of the studies considered alumina and copper as hybrid nanoparticles. Hence it is crucial to widen the scope of hybrid nanofluid that considered different kinds of nanoparticles and investigate their flow and heat transfer properties. In this study, we refer to the model by Waini *et al.*, [36] where they examined the hybrid nanofluid flow towards a stagnation point on an exponentially stretching/shrinking vertical sheet with buoyancy effects. We extended their paper by adding the nanoparticle concentration equation and density of gyrotactic microorganism (bioconvection flow) equation. These additional equations are required in order to implement the bioconvection parameter and the microorganism concentration parameter towards the model. We have also used different types of nanoparticles compared to those by Waini *et al.*, [36], which are titanium alloy (Ti6Al4V) and aluminium alloy (AA7075), while Waini *et al.*, [36] considered the common nanoparticles which are alumina (Al₂O₃) and copper (Cu). Titanium alloy (Ti6Al4V) and aluminium alloy (AA7075) are immersed in water to form the hybrid nanofluid. Hence, the findings of this study are still new and have not yet been discovered by any other researchers. It is worth to be mentioned that the reasons behind the usage of titanium alloy (Ti6Al4V) and aluminium alloy (AA7075) are due to the fact that the amalgamation of these two nanoparticles within a nanofluid yields synergistic effects, where the resultant properties surpass those of the individual components. For instance, the hybrid nanofluid exhibits enhanced thermal conductivity and mechanical strength, which are the combined qualities of the separate alloys. The synergistic combination enhances the performance and efficiency of the hybrid nanofluid in applications involving heat transfer and structural integrity.

Therefore, the pivotal aim of this research is to address a natural stagnation bioconvection flow of a hybrid nanofluid containing gyrotactic microorganisms over an exponentially stretching and shrinking vertical sheet using a numerical method, namely as Matlab *bvp4c*. We aim to provide the mathematical model for the problem and observe the impact of the considered parameters on the physical quantities of interest to simulate the fluid flow dynamics via the numerical perspective. These numerical findings (simulation of the fluid behaviour) could be served as guidance to those working with the fluid in experimental and practical activities.

2. Methodology

This study considers a hybrid nanofluid flow towards a stagnation point on a vertical sheet. The Cartesian coordinates of x - and y - axes with the origin O , where x is assigned vertically along to the

surface and y is orthogonal to it. The free stream velocity is taken as $u_e(x) = ae^{x/L}$ where ($a > 0$) and L is the reference length. Meanwhile, the surface is stretched ($b > 0$) or shrunk ($b < 0$) in an exponential velocity, $u_w(x) = be^{x/L}$ where $b = 0$ is for the static surface. The graphical model is depicted in Figure 1.

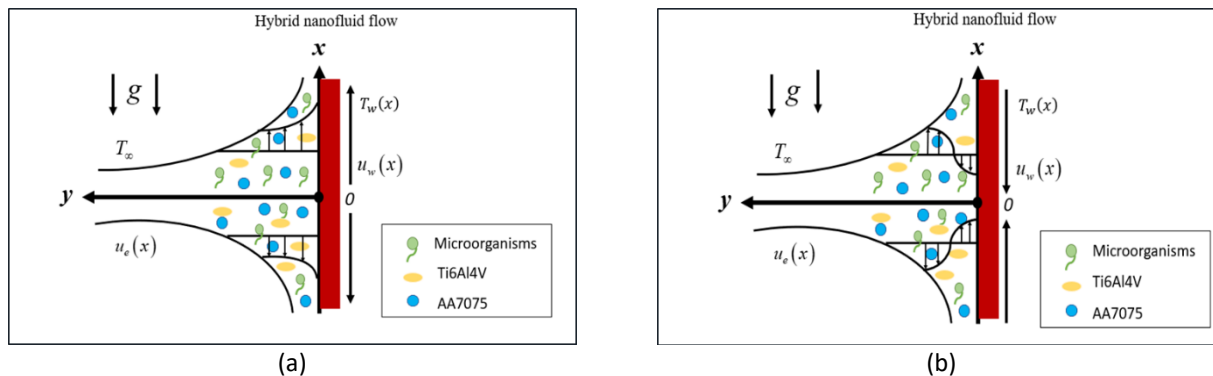


Fig. 1. Geometry for the flow problem

The fluid is considered to be a homogeneous mixture of nanoparticles, and gyrotactic microorganisms are also swimming in it. Microorganisms stabilize the suspension of nanoparticles by bio-convective flow which is generated by the combined simultaneous effects of nanoparticles and buoyancy. The nanoparticles are titanium alloy (Ti6Al4v) and aluminium alloy (AA7075) termed as φ_1 and φ_2 respectively. The total volume concentration of both nanoparticles is denoted by φ given as $\varphi = \varphi_1 + \varphi_2$. The thermophysical properties of the hybrid nanofluid and nanoparticles are presented in Table 1 and 2, respectively.

Table 1

Thermophysical properties of hybrid nanofluid [36]

Thermophysical properties	Hybrid nanofluid
Density (ρ)	$\rho_{hnf} = (1 - \varphi_2) [(1 - \varphi_1) \rho_f + \varphi_1 \rho_{n1}] + \varphi_2 \rho_{n2}$
Heat capacity (ρC_p)	$(\rho C_p)_{hnf} = (1 - \varphi_2) [(1 - \varphi_1) (\rho C_p)_f + \varphi_1 (\rho C_p)_{n1}] + \varphi_2 (\rho C_p)_{n2}$
Dynamic viscosity (μ)	$\mu_{hnf} = \frac{\mu_f}{(1 - \varphi_1)^{2.5} (1 - \varphi_2)^{2.5}}$
Thermal conductivity (k)	$\frac{k_{hnf}}{k_{nf}} = \frac{k_{n2} + 2k_{nf} - 2\varphi_2 (k_{nf} - k_{n2})}{k_{n2} + 2k_{nf} + \varphi_2 (k_{nf} - k_{n2})}$ and $\frac{k_{nf}}{k_f} = \frac{k_{n1} + 2k_f - 2\varphi_1 (k_f - k_{n1})}{k_{n1} + 2k_f + \varphi_1 (k_f - k_{n1})}$
Thermal expansion coefficient of temperature ($\rho\beta_T$)	$(\rho\beta)_{hnf} = (1 - \varphi_2) [(1 - \varphi_1) (\rho\beta)_f + \varphi_1 (\rho\beta)_{n1}] + \varphi_2 (\rho\beta)_{n2}$

Table 2
 Thermophysical properties of nanoparticles and water [37]

Thermophysical properties	Ti6Al4v	AA7075	Water
$\rho(\text{kg}/\text{m}^3)$	4420	2810	997.1
$C_p(\text{J}/\text{kgK})$	0.56	960	4,179
$k(\text{W}/\text{mK})$	7.2	173	0.613
$\beta \times 10^{-5} (1/\text{K})$	0.86	2.34	21

The governing equations are (see Waini *et al.*, [36])

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_e \frac{du_e}{dx} + \nu_{hnf} \frac{\partial^2 u}{\partial y^2} + (\beta_T)_{hnf} (T - T_\infty) g + (\beta_C)_{hnf} (C - C_\infty) g + (\beta_N)_{hnf} (N - N_\infty) g \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{hnf}}{(\rho C_p)_{hnf}} \frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_n \frac{\partial^2 C}{\partial y^2} \tag{4}$$

$$u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} + \frac{bW_c}{\Delta C} \frac{\partial}{\partial y} \left(N \frac{\partial C}{\partial y} \right) = D_m \frac{\partial^2 N}{\partial y^2} \tag{5}$$

where u is the component velocity along x -axis, v is the component velocity along the y -axis, u_e is the stagnation or external flow, ν_{hnf} is the kinematic viscosity of the hybrid nanofluid, $(\beta_T)_{hnf}$, $(\beta_C)_{hnf}$, $(\beta_N)_{hnf}$ are the thermal expansion coefficient of temperature, concentration and density of microorganism of hybrid nanofluid, g is the buoyancy parameter, T is the fluid temperature, C is the concentration of nanoparticles, N is the density motile of microorganism, k_{hnf} is the thermal conductivity of the hybrid nanofluid, $(\rho C_p)_{hnf}$ is the heat capacity of the hybrid nanofluid, D_n is the diffusivity of nanoparticles, D_m is the diffusivity of microorganism and W_c is the maximum cell swimming speed.

The governing Eq. (1) to Eq. (5) are subjected to the boundary conditions given as follows (see Waini *et al.*, [36]),

$$\begin{aligned} v = 0, \quad u = u_w, \quad T = T_w, \quad C = C_w, \quad N = N_w & \quad \text{at} \quad y = 0 \\ u \rightarrow u_e, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty, \quad N \rightarrow N_\infty & \quad \text{as} \quad y \rightarrow \infty \end{aligned} \tag{6}$$

where $T_w(x) = T_\infty + T_0 e^{2x/L}$ is the surface of temperature, $C_w(x) = C_\infty + C e^{x/L}$ is the surface concentration of nanoparticles, $N_w(x) = N_\infty + N e^{x/L}$ is the surface density of microorganism, where T_∞ , C_∞ and N_∞ are the constant ambient temperature, concentration of nanoparticles and density of microorganism, respectively.

The governing equations which are in the form of partial differential equations (PDEs) are transformed into ordinary differential equations (ODEs) by using similarity transformation method. The similarity variables used are as shown below,

$$\eta = \sqrt{\frac{a}{2\nu_f L}} e^{\left(\frac{x}{2L}\right)} y, \quad \psi = \sqrt{2av_f Le^{\left(\frac{x}{2L}\right)}} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \quad (7)$$

$$X(\eta) = \frac{N - N_\infty}{N_w - N_\infty}$$

where η is the similarity variable and ψ denotes the stream function with $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$.

By using the similarity variables (7) into Eq. (1) to Eq. (5), the PDEs are transformed into ODEs as follows,

$$\frac{P_1}{P_2} f'''(\eta) + f(\eta)f''(\eta) - 2f'(\eta)^2 + 2 + 2\lambda \left(\frac{P_5}{P_2} \theta(\eta) + N_c \phi(\eta) + N_N X(\eta) \right) = 0 \quad (8)$$

$$\frac{1}{Pr} \frac{P_4}{P_3} \theta''(\eta) + f(\eta)\theta'(\eta) - 4f'(\eta)\theta(\eta) = 0 \quad (9)$$

$$4f'(\eta)\phi(\eta) - f(\eta)\phi'(\eta) - \frac{1}{Pr Le} \phi''(\eta) = 0 \quad (10)$$

$$4f'(\eta)X(\eta) - f(\eta)X'(\eta) + \frac{Pe}{Pr Lb} \left(\phi''(\eta)X(\eta) + \Omega\phi'(\eta) + \phi'(\eta)X'(\eta) \right) - \frac{1}{Pr Lb} X''(\eta) = 0 \quad (11)$$

The usage of Eq. (7) into Eq. (6) produces the following new form of boundary conditions,

$$\begin{aligned} f(\eta) = 0, \quad f'(\eta) = \varepsilon, \quad \theta(\eta) = 1, \quad \phi(\eta) = 1, \quad X(\eta) = 1 & \quad \text{at } \eta = 0 \\ f'(\eta) \rightarrow 1, \quad \theta(\eta) \rightarrow 0, \quad \phi(\eta) \rightarrow 0, \quad X(\eta) \rightarrow 0 & \quad \text{as } \eta \rightarrow \infty \end{aligned} \quad (12)$$

where P_1, P_2, P_3, P_4 and P_5 are the thermophysical properties of hybrid nanofluids, Gr_x is the Grashof number, Re_x is the Reynolds number, λ is the mixed convection parameter, Pr is the Prandtl number, Le is the Lewis number, Pe is the bioconvection Peclet number, Lb is the bioconvection Lewis number, Ω is the microorganisms concentration difference parameter, N_c is the concentration-to-thermal-buoyancy ratio parameter, N_N is the microorganism-to-thermal-buoyancy ratio parameter and ε is the stretching/shrinking parameter, define as follows

$$Gr_x = \frac{g\beta_f T_0 L^3 e^{\left(\frac{2x}{L}\right)}}{\nu f^2}, \quad Re_x = \frac{aLe^{\left(\frac{x}{L}\right)}}{\nu f}, \quad \lambda = \frac{g(\beta_T)_f T_0 L}{a^2} = \frac{Gr_x}{Re_x^2}, \quad Le = \frac{a}{D_n}, \quad Pe = \frac{bW_c}{D_m}, \quad (13)$$

$$Lb = \frac{a}{D_m}, \quad \Omega = \frac{N_\infty}{N_w - N_0}, \quad N_c = \frac{(\beta_c)_{hmf} C_0}{(\beta_T)_f T_0}, \quad N_N = \frac{(\beta_N)_{hmf} N_0}{(\beta_T)_f T_0}$$

In this study, physical parameters are skin friction coefficient (C_f), and local Nusselt number (Nu_x) by Waini *et al.*, [36] together with new physical parameters local Sherwood number (Sh_x) and local density of motile microorganisms (Nn_x) by Zohra *et al.*, [38] are as follows,

$$C_f = \frac{\mu_{hmf}}{\rho_f u_e^2} \left(\frac{\partial u}{\partial y} \right)_{y=0}, \quad Nu_x = \frac{-Lk_{hmf}}{k_f (T_w - T_\infty)} \left(\frac{\partial T}{\partial y} \right)_{y=0}, \quad Sh_x = \frac{-L}{(C - C_\infty)} \left(\frac{\partial C}{\partial y} \right)_{y=0}, \quad (14)$$

$$Nn_x = \frac{-L}{(N_w - N_\infty)} \left(\frac{\partial N}{\partial y} \right)_{y=0}$$

Using the similarity variables Eq. (7) then,

$$\left(2Re_x \right)^{\frac{1}{2}} C_f = \frac{\mu_{hmf}}{\mu_f} f''(0), \quad \left(\frac{Re_x}{2} \right)^{\frac{1}{2}} Nu_x = -\frac{k_{hmf}}{k_f} \theta'(0), \quad \left(\frac{Re_x}{2} \right)^{\frac{1}{2}} Sh_x = -\phi'(0), \quad (15)$$

$$\left(\frac{Re_x}{2} \right)^{\frac{1}{2}} Nn_x = -X'(0)$$

3. Results

The transformed governing boundary layer equations are solved numerically by using *bvp4c* in MATLAB. The effects of titanium alloy ϕ_1 (Ti6Al4V) and aluminium alloy ϕ_2 (AA7075), stretching/shrinking parameter ε on velocity, temperature, concentration and microorganisms' profiles are illustrated graphically. The effects on the skin friction coefficient $(2Re_x)^{1/2}C_f$, local Nusselt number $(Re_x/2)^{-1/2}Nu_x$ (refers to the heat transfer rate), local Sherwood number $(Re_x/2)^{-1/2}Sh_x$ (refers to the mass transfer rate) and local Density of Motile Microorganism $(Re_x/2)^{-1/2}Nn_x$ were computed for various values of ϕ_1 and ϕ_2 with other parameters were set to be constant at $Pr = 6.2$ and $\lambda = N_c = N_N = Le = Pe = Lb = \Omega = 1$.

Table 3 shows the comparison of the skin friction coefficient $(2Re_x)^{1/2}C_f$ and the local Nusselt number $(Re_x/2)^{-1/2}Nu_x$ between the results obtained by Bachok *et al.*, [30], Rehman *et al.*, [39] and Waini *et al.*, [36] with present study.

Table 3

Comparison for Numerical Values of $(2Re_x)^{1/2}C_f$ and $(Re_x/2)^{-1/2}Nu_x$ for different ϕ_1 when Copper ($\phi_2 = 0$), $Pr = 6.2$ and $\lambda = 0$ (non-buoyant case)

ε	ϕ_1	$(2Re_x)^{1/2}C_f$				$(Re_x/2)^{-1/2}Nu_x$	
		Bachok <i>et al.</i> , [30]	Rehman <i>et al.</i> , [39]	Waini <i>et al.</i> , [36]	Present study	Waini <i>et al.</i> , [36]	Present study
-	0.0	2.1182		2.1182	2.1182	0.0588	0.0588
0.5	0.1	2.7531		2.7531	2.7531	0.4439	0.4439
	0.2	3.5372		3.5372	3.5372	0.7636	0.7636
0.0	0.0	1.6872	1.68720	1.6872	1.6872	2.5066	2.5066
	0.1	2.1929	2.19293	2.1930	2.1930	2.9655	2.9655
	0.2	2.8174	2.81750	2.8175	2.8175	3.4292	3.4292
0.5	0.0	0.9604	0.96040	0.9604	0.9604	4.0816	4.0816
	0.1	1.2483	1.24829	1.2483	1.2483	4.6637	4.6637
	0.2	1.6039	1.6039	1.6038	1.6038	5.2726	5.2726

The comparisons of the values in the Table 3, observed that the present results show an excellent agreement with the previous study. Therefore, this verifies the reliability and accuracy of the method used for this study. It is observed that the values of $(2Re_x)^{1/2}C_f$ decreases but the values of $(Re_x/2)^{-1/2}Nu_x$ increases when $\varepsilon = -0.5$. Meanwhile, these physical quantities enhance as ϕ_1 increases.

The effects of ϕ_1 on the $(2Re_x)^{1/2}C_f$, $(Re_x/2)^{-1/2}Nu_x$, $(Re_x/2)^{-1/2}Sh_x$ and $(Re_x/2)^{-1/2}Nn_x$ are presented in Figures 2-5, respectively for varies values of ϕ_1 ($\phi_1 = 0.0, 0.1, 0.2$) with fixed $\phi_2 = 0.1$. From Figure 2, it can be seen that $(2Re_x)^{1/2}C_f$ decreases as the sheet experiences reduced shrinkage and as ϕ_1 increases for upper branch, while the lower branch appears unsteady result. In logical interpretation, in the state of the shrinking sheet, which is opposed to the direction of the fluid flow can causes the wall shear stress at the surface to intensify due to the resistance force, which then causes the skin friction to improve. If the sheet is shrinkage extensively, then the skin friction can be improved extensively as well.

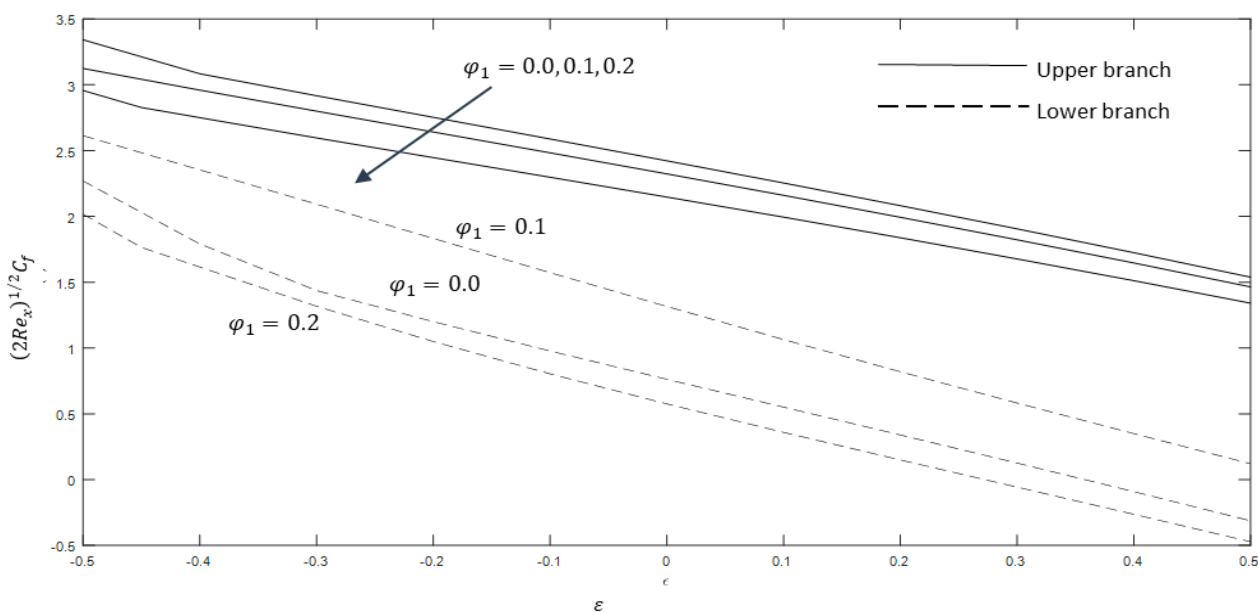


Fig. 2. Effects of various values of ϕ_1 on $(2Re_x)^{1/2}C_f$

Next, in Figure 3 shows that $(Re_x/2)^{-1/2}Nu_x$ decreases as the sheet is shrinking for upper branch. This physically implies that when the strength of the shrinking sheet decreases, the thermal conductivity of the sheet decreases due to the nanoparticles for upper branch, which invade the sheet and more surface area exposed to the nanofluid flow. Consequently, it will increase the heat flux at the surface of the sheet and affects the rate of heat transfer to increase. However, the lower branch views a different mode where $(Re_x/2)^{-1/2}Nu_x$ increases as the sheet are shrinking. In view of the fact to the increment in ϕ_1 . The permeability of the sheet traps the slow-moving molecules and reduces the thermal conductivity of the sheet.

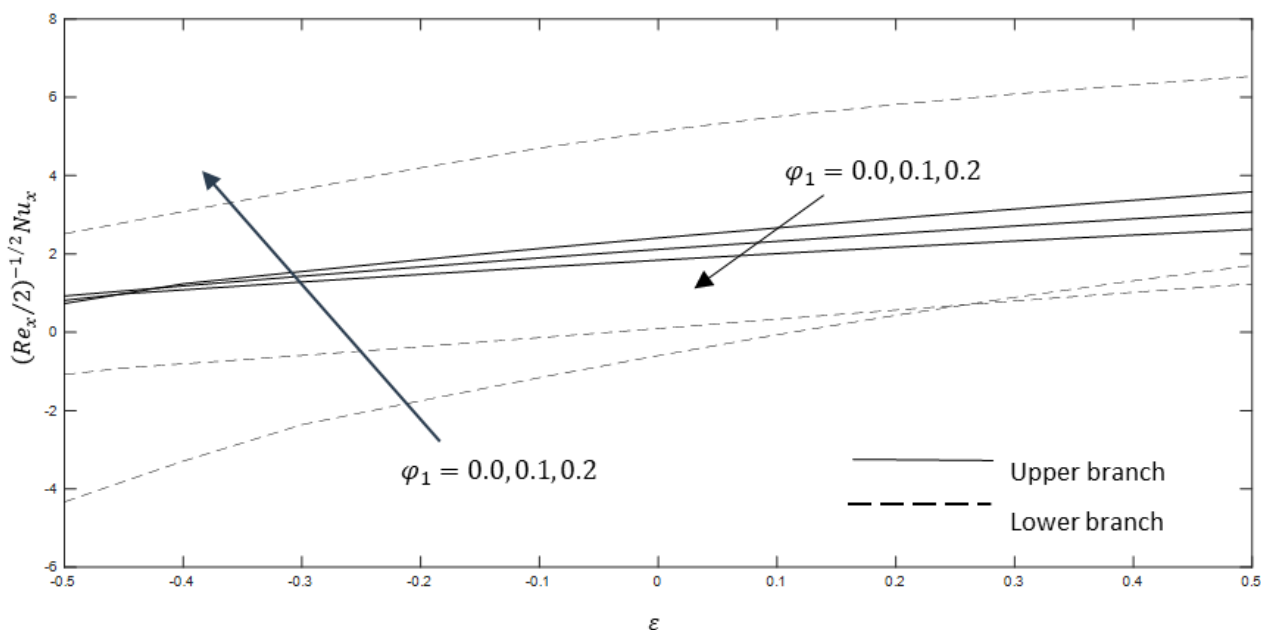


Fig. 3. Effects of various values of ϕ_1 on $(Re_x/2)^{-1/2}Nu_x$

Local Sherwood number, $(Re_x/2)^{-1/2}Sh_x$ is presented in Figure 4 where it shows that $(Re_x/2)^{-1/2}Sh_x$ decreases in the presence of shrinking sheet as ϕ_1 increases for the upper branch while lower branch appears unstable as ϕ_1 increases. In physical perspective, when the strength of the shrinking sheet increases, it would have less nanoparticles to fill the sheet and these reduce the mass diffusivity. Thus, the rate of mass transfer decreases as the sheet is shrunk.

And local density of motile microorganism, $(Re_x/2)^{-1/2}Nn_x$ is presented in Figure 5 where it state that $(Re_x/2)^{-1/2}Nn_x$ decreases as ϕ_1 increases for the upper branch while lower branch appears unstable as ϕ_1 increases. The upper branch solutions bring the decrement in the value of $(Re_x/2)^{-1/2}Nn_x$ as the strength of the shrinking sheet increases. Physically, the state of the shrinking sheet restraint the microorganism movement into the sheet and contributes the microorganism from the sheet to move out from the sheet. It concludes that the motile wall microorganism flux decreases as $(Re_x/2)^{-1/2}Nn_x$ decreases.

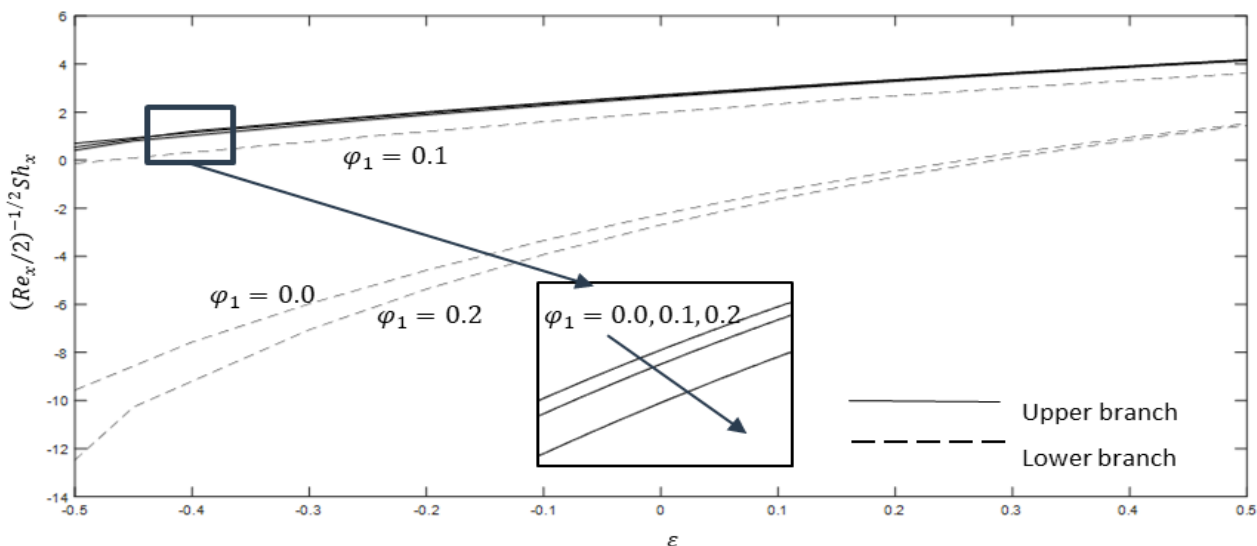


Fig. 4. Effects of various values of φ_1 on $(Re_x/2)^{-1/2}Sh_x$

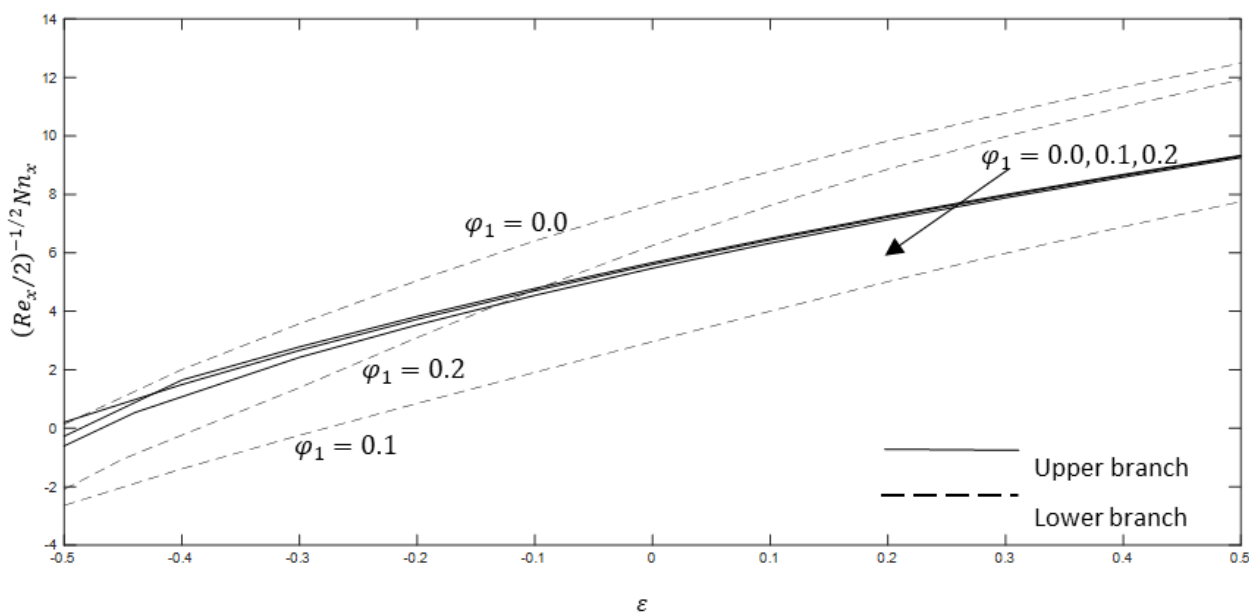


Fig. 5. Effects of various values of φ_1 on $(Re_x/2)^{-1/2}Nn_x$

The velocity, temperature, concentration and microorganism profiles for various values of φ_1 are illustrated graphically in Figures 6-9, for $\varepsilon=0.3$ with fixed values of $\varphi_2=0.1$. These profiles asymptotically satisfy the free stream Eq. (12), thus giving us confidence in the accuracy of the current solutions. Figure 6 shows that $f'(\eta)$ for both upper branch and lower branch are decreases as φ_1 increases. Therefore, the thickness of the boundary layer also decreases for both branches.

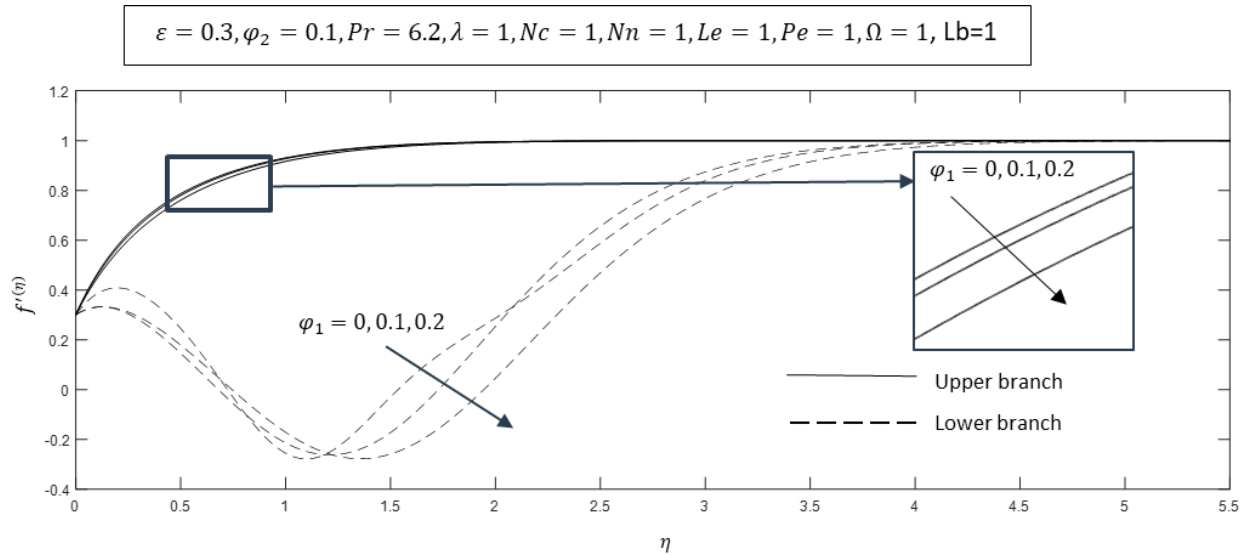


Fig. 6. Velocity profiles for varies φ_1

Figure 7 shows that the temperature $\theta(\eta)$ increases as φ_1 increases for upper branch while unstable for lower branch. This shows that the heat transfer increases as φ_1 increases for upper branch.

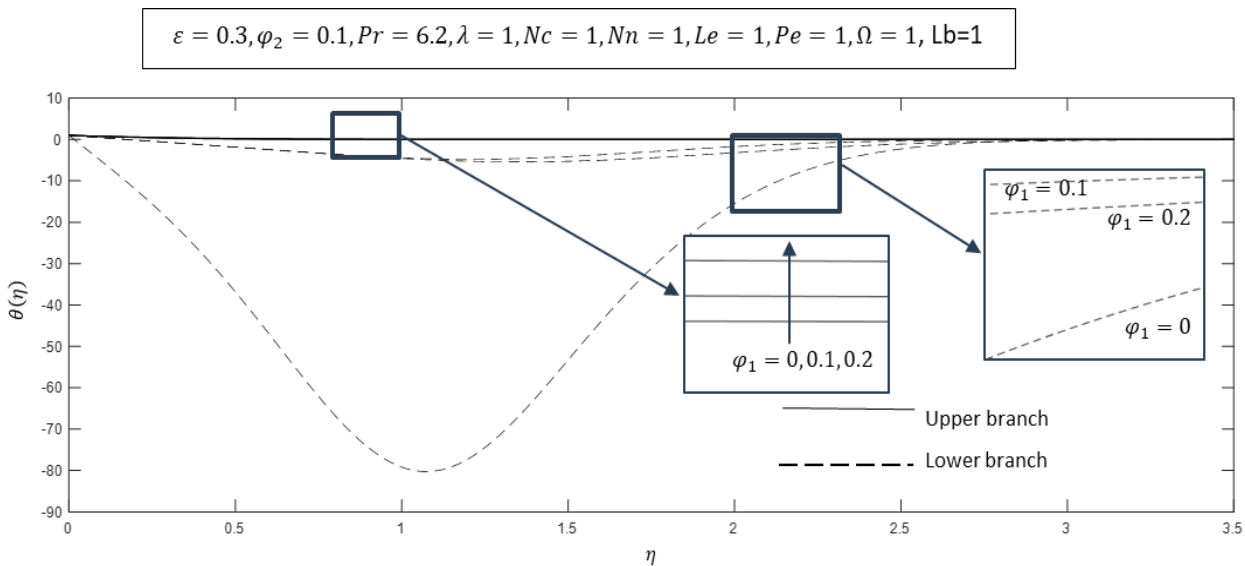


Fig. 7. Temperature profiles for varies φ_1

Concentration profile $\phi(\eta)$ is presented in Figure 8 shows that the $\phi(\eta)$ increases as φ_1 increases for both upper branch and lower branch. As consequent to the increment concentration, the mass transfer increases.

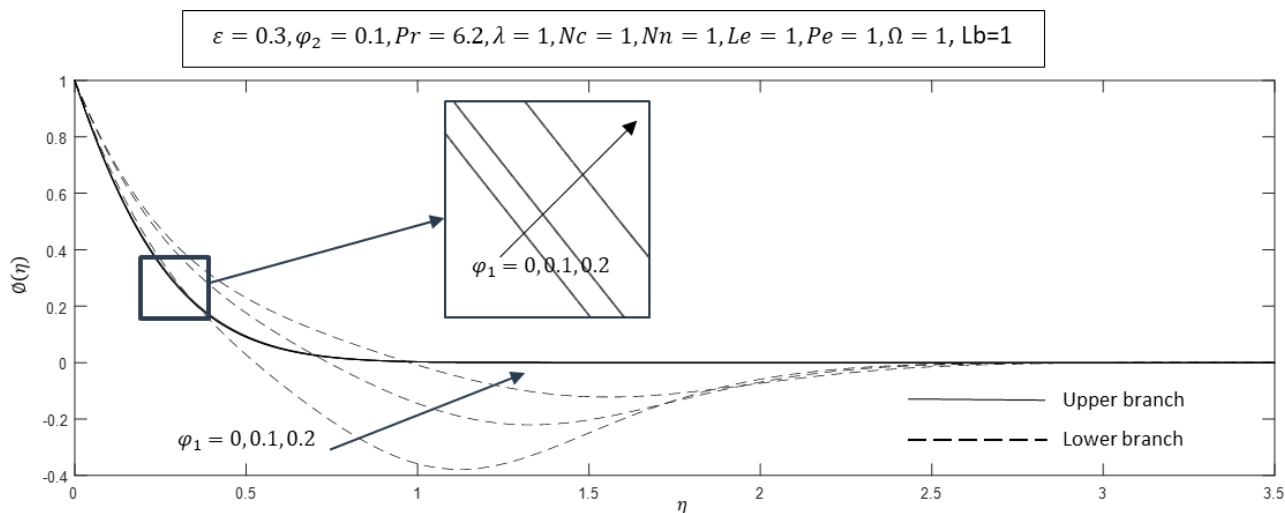


Fig. 8. Concentration profiles for varies φ_1

And microorganism profile $\chi(\eta)$ in Figure 9 shows that the microorganism decreases as φ_1 increases for both branches where it reduces the mass diffusivity.

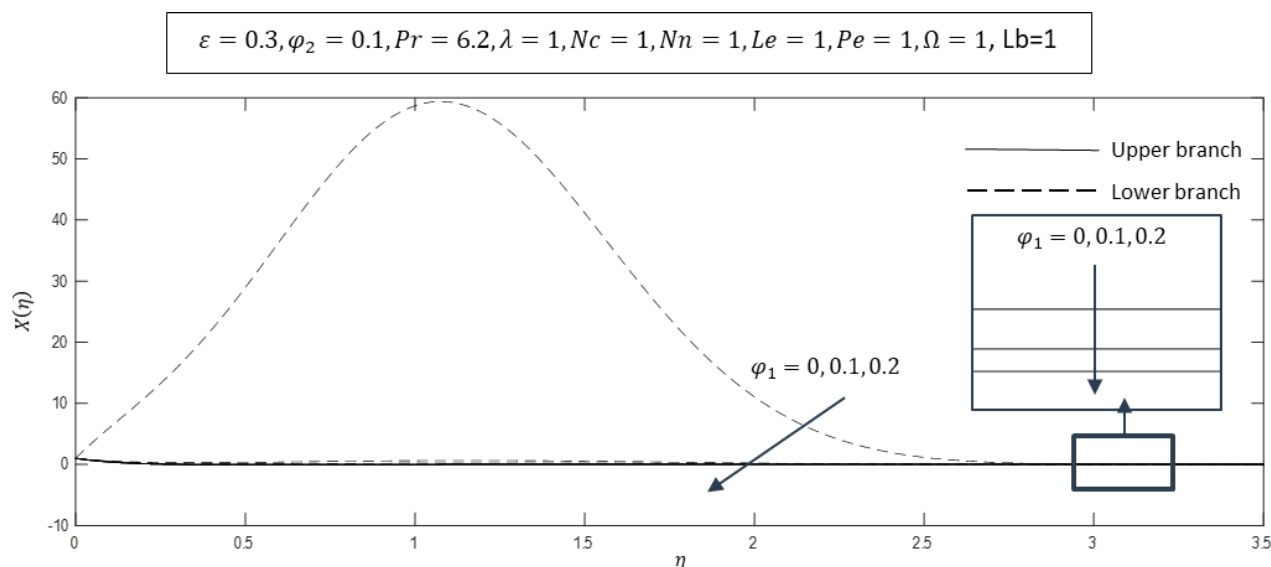


Fig. 9. Microorganism profiles for varies φ_1

The $(2Re_x)^{1/2}C_f$, $(Re_x/2)^{-1/2}Nu_x$, $(Re_x/2)^{-1/2}Sh_x$ and $(Re_x/2)^{-1/2}Nn_x$ for various values of φ_2 are tabulated in Table 4, for both $\varepsilon > 0$ and $\varepsilon < 0$ with fixed $\varphi_1 = 0.1$. From this table, it shows that the values of $(2Re_x)^{1/2}C_f$ decreases for both branches as φ_2 increases in the presence of shrinking. Conversely, the values of $(2Re_x)^{1/2}C_f$ decreases for upper and lower branch as φ_2 increases in the presence of stretching. Meanwhile, the values of $(Re_x/2)^{-1/2}Nu_x$ decreases for both upper and lower branch in the presence of shrinking and stretching sheet.

Next, the values of $(Re_x/2)^{-1/2}Sh_x$ decreases for upper branch as φ_2 increases while the values of $(Re_x/2)^{-1/2}Sh_x$ decreases when $\varphi_2 = 0.0$ to $\varphi_2 = 0.1$ and increases at $\varphi_2 = 0.2$ for lower branch in the presence of shrinking and stretching sheet. Lastly, the values of $(Re_x/2)^{-1/2}Nn_x$ decreases for upper branch and increases for lower branch as φ_2 increases in the presence of shrinking and

stretching. The sequence results are displayed in table form as it provides a clear coefficient value that is easy to be analysed.

Table 4

Values of $(2Re_x)^{1/2}C_f$, $(Re_x/2)^{-1/2}Nu_x$, $(Re_x/2)^{-1/2}Sh_x$ and $(Re_x/2)^{-1/2}Nn_x$ for φ_2 when $\varphi_1 = 0.1$ and $\lambda = N_C = N_N = Le = Pe = Lb = \Omega = 1$

Upper branch					
ε	φ_2	$(2Re_x)^{1/2}C_f$	$(Re_x/2)^{-1/2}Nu_x$	$(Re_x/2)^{-1/2}Sh_x$	$(Re_x/2)^{-1/2}Nn_x$
-0.5	0.0	3.47516	1.01087	0.89901	0.73350
	0.1	3.12375	0.92515	0.69699	0.20622
	0.2	2.74740	0.82780	0.45477	0.20384
0.0	0.0	2.60596	2.45430	2.77107	5.79384
	0.1	2.32245	2.11525	2.68325	5.60249
	0.2	2.05222	1.82303	2.59738	4.42828
0.5	0.0	1.65951	3.60025	4.20212	9.37824
	0.1	1.46411	3.07429	4.16592	9.30502
	0.2	1.23949	2.61533	4.12043	9.21446
Lower branch					
ε	φ_2	$(2Re_x)^{1/2}C_f$	$(Re_x/2)^{-1/2}Nu_x$	$(Re_x/2)^{-1/2}Sh_x$	$(Re_x/2)^{-1/2}Nn_x$
-0.5	0.0	3.25186	24.32979	0.56188	-22.30364
	0.1	2.61299	2.51404	-0.13779	-2.63788
	0.2	2.74740	0.82780	0.45477	0.20384
0.0	0.0	2.15233	59.86825	2.56340	-47.34707
	0.1	1.31550	5.13555	1.97858	2.96492
	0.2	0.55695	-1.75178	2.09830	9.06437
0.5	0.0	0.58224	56.47435	3.97237	-38.09695
	0.1	0.12085	6.53665	3.61750	7.75168
	0.2	-0.29604	1.27822	4.29381	12.05353

The profiles of velocity, temperature, concentration and microorganism for φ_2 are illustrated graphically in Figures 10-13, respectively, for $\varepsilon = 0.3$ with fixed values of $\varphi_1 = 0.1$. Figure 10 shows that $f'(\eta)$ are decreases for upper branch while lower branch appear unstable as the φ_2 increases.

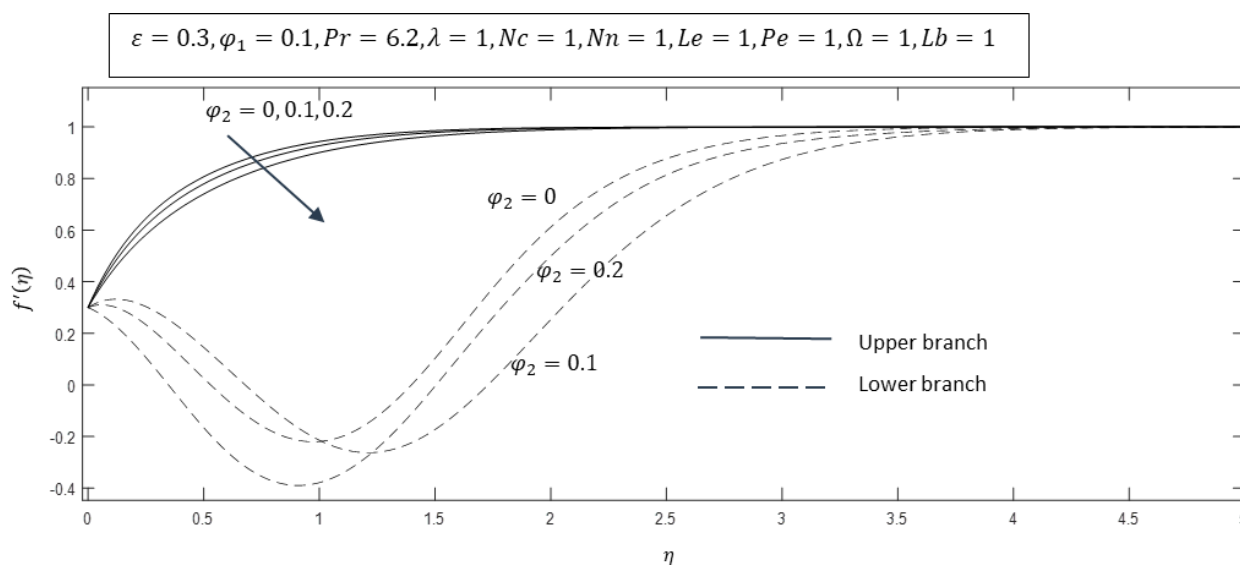


Fig. 10. Velocity profiles for varies φ_2

Figure 11 shows that the temperature $\theta(\eta)$ increases as φ_2 increases for upper branch while unstable for lower branch. This shows that the heat transfer increases as φ_2 increases for upper branch.

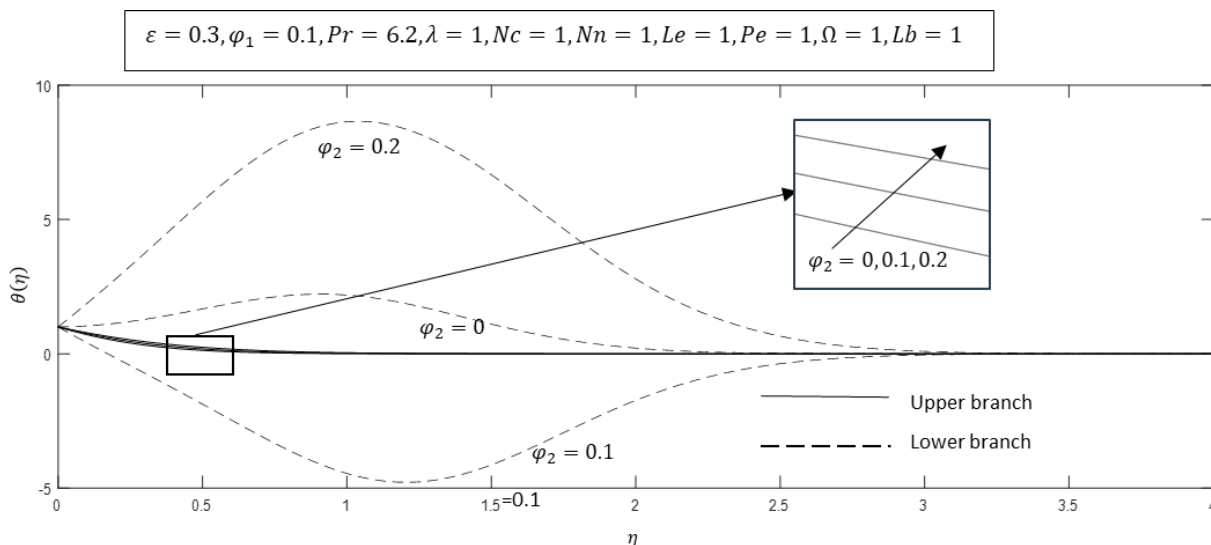


Fig. 11. Temperature profiles for varies φ_2

The concentration $\phi(\eta)$ increases as φ_2 increases for both branches as shown in Figure 12 which conclude that the mass transfer increases.

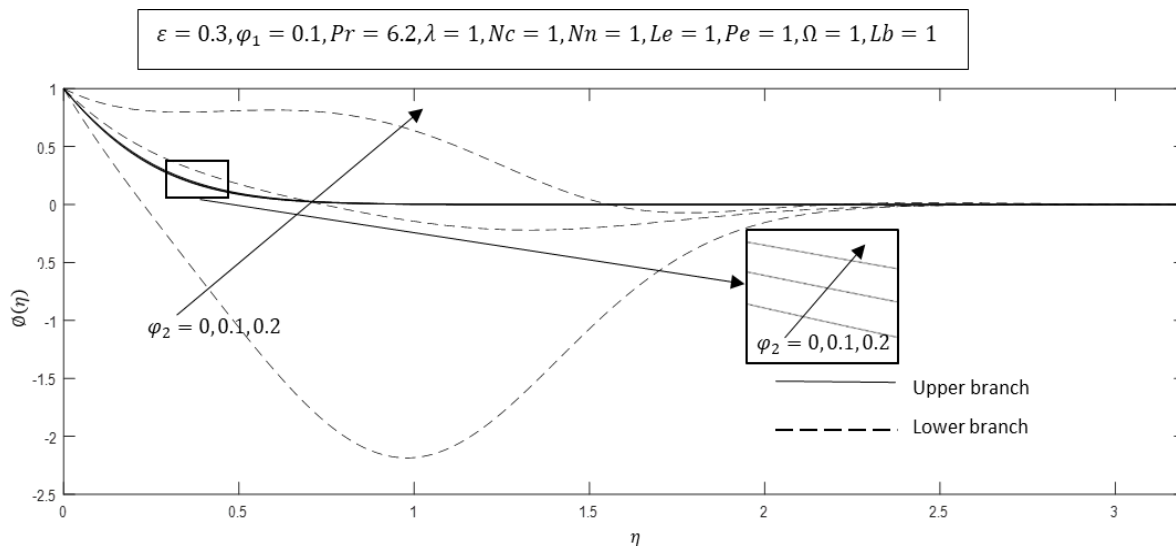


Fig. 12. Concentration profiles for varies φ_2

Lastly, in Figure 13, microorganism $\chi(\eta)$ decreases as φ_2 increases for upper branch while the lower branch appears unstable.

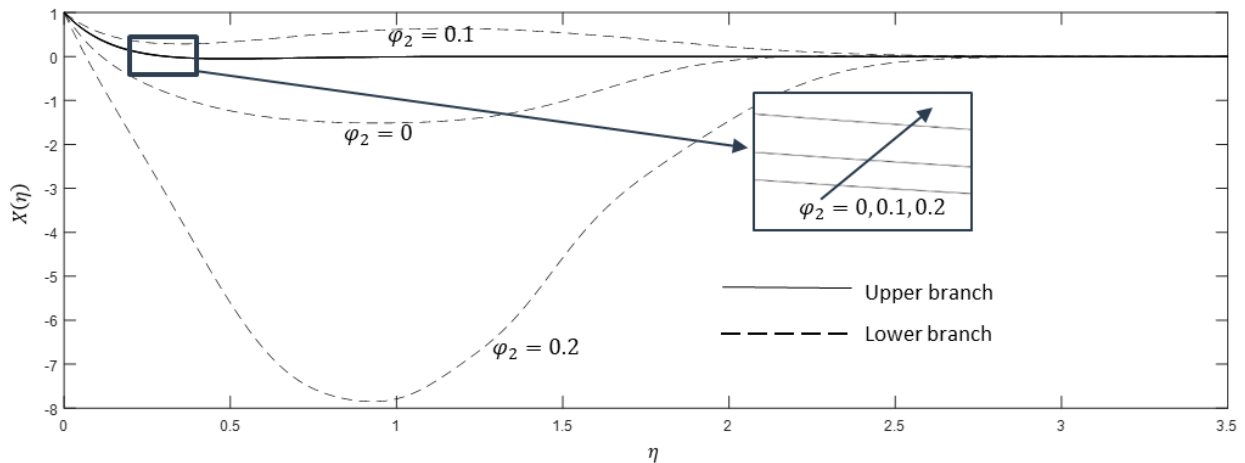


Fig. 13. Microorganism profiles for varies φ_2

4. Conclusions

In the present study, the hybrid nanofluid flow containing gyrotactic microorganisms towards a stagnation point on an exponentially stretching/shrinking vertical sheet with mixed convection effects has been accomplished. The governing equations has been transformed using similarity transformation which then solved by bvp4c solver in Matlab. Some parameters are being considered in this study which are volume fraction of titanium alloy (φ_1) and aluminium alloy (φ_2), stretching ($\varepsilon > 0$), and shrinking ($\varepsilon < 0$) to determine the behaviour of the $(2Re_x)^{1/2}C_f$, $(Re_x/2)^{-1/2}Nu_x$, $(Re_x/2)^{-1/2}Sh_x$ and $(Re_x/2)^{-1/2}Nn_x$, also the velocity, temperature, concentration and microorganism profiles respectively. The results obtained are as follow

- i. $(2Re_x)^{1/2}C_f$, $(Re_x/2)^{-1/2}Nu_x$ and $(Re_x/2)^{-1/2}Sh_x$ are decreases when both φ_1 and φ_2 increases,
- ii. $(Re_x/2)^{-1/2}Nn_x$ increases when φ_2 increases, but decreases when φ_1 increases,
- iii. Velocity profile decreases when φ_1 increases for both branches, but only for upper branch when φ_2 increases,
- iv. The temperature profile increases when φ_1 and φ_2 increases for upper branch,
- v. The concentration profiles increases when φ_1 and φ_2 increases for both branches,
- vi. The microorganism profiles increases when φ_2 for upper branch increases, but decreases when φ_1 for upper and lower branch increases.

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