Journal of Advanced Research in Fluid Mechanics and Thermal Sciences

Journal homepage:
https://semarakilmu.com.my/journals/index.php/fluid_mechanics_thermal_sciences/index
ISSN: 2289-7879

MHD Natural Convection Flow of Casson Ferrofluid over a Vertical Truncated Cone

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ARTICLE INFO

Article history:
Received 25 August 2023
Received in revised form 16 November 2023
Accepted 24 November 2023
Available online 15 December 2023

Keywords:
Boundary layer; Casson; ferrofluid; Keller-box; truncated cone

ABSTRACT

This study considers the mathematical model of MHD free or natural convection boundary layer flow of a Casson ferrofluid over a truncated cone. The set of non-linear partial differential equations that governed the model is first reduced to a simpler set of equations using the non-similar transformation. This set of equations then is solved numerically using the implicit finite difference method known as the Keller-box method. Blood and magnetite are taken as the based-fluid and the ferroparticles for the Casson ferrofluid, respectively. From the numerical study, it was found that the increase of ferroparticles volume fraction results in the increase in the skin friction coefficient and the Nusselt number while the magnetic parameter does the contrary. Further, both parameter shows diminishing effects, whereby if one of the parameter values is set to be increases, the friction between fluid and surface can be controlled by manipulating the other parameter values.

1. Introduction

NASA invented liquid rocket fuel to operate in the zero-gravity condition. Coined as a ferrofluid, it is the based fluid that contains magnetic nanoparticles called ferroparticles that can be directional into the combustion chamber using the magnetic field [1].

Today, the applications of ferrofluid extend not only to operate in aerospace but also employed in electronics devices such as the hi-fi speakers where ferrofluid replaces the functions of permanent magnet. In medicine, the ability to transport drug into a specific targeted body tissue or cell via magnetic field has promoted the widely used of ferrofluid specifically in cancer therapy, bleeding stopping agent, magnetic resonance imaging and other diagnostic tests [2]. The investigation on flow and heat transfer of ferrofluid has been done included from the works by Ramli *et al.,* [3] who

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https://doi.org/10.37934/arfmts.112.1.94105

investigated the ferrofluid flow on a Sakiadis plate. Study on MHD rotating flow towards exponential stretching sheet and stagnation point flow with thermal radiation effects has been carried out by Jusoh *et al.,* [4] and Yasin *et al.,* [5]. It is found that the increase in ferroparticles in fluid raised the velocity and thermal boundary layer thicknesses.

Considering the potential of ferrofluid as a transport agent in body, the study of this fluid flow and the characteristic has attracted researchers' attention. According to Rathod and Tanveer [6] and Venkatesan *et al.,* [7], the human blood shows the Newtonian fluid flow behavior in the wide arteries while in the narrow arteries, the blood flows mimic the non-Newtonian fluid flow behavior. In modelling the blood flows as the non-Newtonian fluid, Casson model has been introduced and has been identified as the preferred rheological model for characterizing the human blood flow Khalid *et al.,* [8]. This model characterized the fluid elastic solid behavior. Recent studies on Casson ferrofluid past a flat plate shows that by adding 10% of magnetite in Casson ferrofluid has enhanced 15% in Nusselt number compared to its based fluid Mohamed *et al.,* [9].

Study on the convective flow past a cone is important as these geometries are applied in many industrial, medicine and engineering devices such as the solder tip, the medicine bottle, the conical heater as well as the transmission pulley in continuous variable transmission (CVT) in a modern car.

Pioneer studies by Na and Chiou [10,11] who consider the laminar natural convection over a slender horizontal and vertical frustum of a cone shows that the surface temperature decreases as the Prandtl number increases. Further, the surface temperature at truncated cone is recorded higher than the full cone. The analysis on circular cone then have been extended with radiation effects by Yih [12]. The transformed governing equations are solved numerically using the Keller-box method. Further studies included the works by Chamkha [13] on magnetohydrodynamic flow, the porous medium Cheng [14], the gyrotactic microorganisms by Mahdy [15] and Khan *et al.,* [16] as well as pressure work effect Ajay and Srinivasa [17]. Recently, study on a truncated cone has been extended considering the advanced fluid like power-law nanofluid, micropolar nanofluid, pseudoplastic fluid and hybrid nanofluid, respectively [18-21].

Therefore, the objective of the present study is to investigate the free convection flow of blood based Casson ferrofluid over a vertical truncated cone. This study gives benefit of preliminary idea regarding the fluid parameters effects on the fluid flow and the heat transfer. Such studies have never been done before via numerical or experimentally analysis, thus the reported results in this study are new.

2. Mathematical Formulation

Figure 1 shows a steady two-dimensional MHD free convection flow and heat transfer over a vertical truncated cone in a Casson ferrofluid where blood is a based fluid. The *x* and *y* are the Cartesian coordinate with the x- axis measured along the surface of the cone from the origin, and *y*-axis is the coordinate measured normal to the surface of the cone. The radius of the truncated cone is represented by r. The origin of the coordinate system is placed at the vertex of the full cone, where $\bar{x} = x - x_o$, the magnetic field strength is denoted by B_o and the constant surface temperature is T_w while the temperature of the ambient fluid is T_∞. It is assumed that the boundary layer develops at the leading edge of the truncated cone $\left(x = x_o\right)$. Considering the boundary layer approximations, this fluid flow can be governed in 2-dimensional coordinate system as follows [10,22]:

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(1+\frac{1}{\beta}\right)v_{nf}\frac{\partial^2 u}{\partial y^2}+\frac{(\rho\beta)_{nf}}{\rho_{nf}}g(T-T_\infty)\cos A+\frac{\sigma_{nf}B_o{}^2u}{\rho_{nf}} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{nf}}{\left(\rho C_p\right)_{nf}}\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

with

$$\begin{aligned} &u=v=0,\ T=T_w \text{ at } y=0,\\ &u\to 0,\ T\to T_\infty \text{ as } y\to\infty. \end{aligned} \tag{4}$$

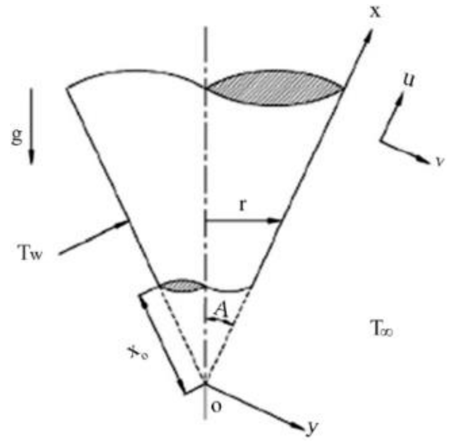

Fig. 1. Physical model of the vertical truncated cone

where u and v are the velocity components along x- and y- axes, respectively. T is the fluid temperature in the boundary layer, β is the Casson parameter, g is the gravitation acceleration. Further, A is the half angle of the full cone, σ_{nf}, v_{nf}, $\mu_{nf}, \rho_{nf}, \left(\rho C_p\right)_{nf}, \beta_{nf}$ and k_{nf} assume as the electrical conductivity, kinematic viscosity, dynamic viscosity, density, heat capacity, thermal expansion and thermal conductivity of the Casson ferrofluid. Detail equations that related the properties of the based fluid $_f$ and ferroparticles $_s$ in a ferrofluid $_{nf}$ is given in Eq. (5) where ϕ represent the ferroparticles volume fractions [23,24].

$$\begin{aligned} &v_{nf}=\frac{\mu_{nf}}{\rho_{nf}},\quad \rho_{nf}=(1-\phi)\rho_f+\phi\rho_s,\quad \mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}},\\ &\left(\rho C_p\right)_{nf}=(1-\phi)\left(\rho C_p\right)_f+\phi\left(\rho C_p\right)_s,\quad \frac{k_{nf}}{k_f}=\frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+\phi(k_f-k_s)}. \end{aligned} \tag{5}$$

Noticed that the non-linear governing Eq. (1) to Eq. (3) are in dimensional form and contain many dependent variables, thus it is uneasy to be solved. Thus, the non-dimensional variable η, ξ, the non-dimensional temperature θ and dimensional stream function ψ are consider as follows:

$$\xi = \frac{\bar{x}}{x_o} = \frac{x - x_o}{x_o}, \quad \eta = \frac{y}{\bar{x}} Gr^{1/4}, \quad f(\xi,\eta) = \frac{\psi}{r\nu Gr^{1/4}}, \quad \theta(\xi,\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{6}$$

with $Gr = \dfrac{g\beta(T_w - T_\infty)\bar{x}^3}{\nu^2}$ is a Grashof number. The Eq. (1) is satisfying by definition of $ru = \dfrac{\partial \psi}{\partial y}$ and $rv = -\dfrac{\partial \psi}{\partial x}$, such that

$$u = \frac{\nu Gr^{1/2}}{\bar{x}} \frac{\partial f}{\partial \eta}, \quad v = -\frac{\nu Gr^{1/4}}{\bar{x}} \left\{ \left(\frac{\xi}{1+\xi} + \frac{3}{4} \right) f + \xi \frac{\partial f}{\partial \xi} - \frac{1}{4} \eta f' \right\} \tag{7}$$

where $'$ denotes the differentiation with respect to η. Employing the variables in (6), the Eq. (2) and Eq. (3) are transformed to the following partial differential equations differential equations:

$$\left(1 + \frac{1}{\beta}\right) \frac{\nu_{nf}}{\nu_f} f''' + \left(\frac{3}{4} + \frac{\xi}{1+\xi} \right) f f'' - \frac{1}{2} f'^2 - M f' + \frac{(\rho\beta)_{nf}}{\rho_{nf}\beta_f} \theta = \xi \left[f' \frac{\partial f'}{\partial \xi} - f'' \frac{\partial f}{\partial \xi} \right], \tag{8}$$

$$\frac{k_{nf}/k_f}{(\rho C_p)_{nf}/(\rho C_p)_f} \frac{1}{\Pr} \theta'' + \left(\frac{3}{4} + \frac{\xi}{1+\xi} \right) f \theta' = \xi \left[f' \frac{\partial \theta}{\partial \xi} - \theta' \frac{\partial f}{\partial \xi} \right], \tag{9}$$

where $\Pr = \dfrac{\nu_f (\rho C_p)_f}{k_f}$ is a Prandtl number and $M = \dfrac{\sigma_{nf} B_o^{\,2} (x - x_o)^2}{Gr^{1/2} \nu_f \rho_{nf}}$ is a magnetic parameter. Detailed ferrofluid expression are as follows:

$$\frac{\nu_{nf}}{\nu_f} = \frac{1}{(1-\phi)^{2.5} \left[(1-\phi) + \phi(\rho_s/\rho_f) \right]}, \quad \frac{(\rho\beta)_{nf}}{\rho_{nf}\beta_f} = \frac{(1-\phi)\rho_f + \phi(\rho\beta)_s/\beta_f}{(1-\phi_1)\rho_f + \phi_1\rho_{s1}},$$

$$\frac{k_{nf}(\rho C_p)_f}{k_f(\rho C_p)_{nf}} = \frac{k_{nf}/k_f}{(1-\phi) + \phi(\rho C_p)_s/(\rho C_p)_f}, \quad \frac{\rho_{nf}(C_p)_f}{(\rho C_p)_{nf}} = \frac{(1-\phi_1)\rho_f + \phi_1\rho_{s1}}{(1-\phi_1)\rho_f + \phi_1(\rho C_p)_{s1}/(C_p)_f}.$$

The boundary conditions become:

$$f(0,\xi) = 0, \ \frac{\partial f}{\partial \eta}(0,\xi) = 0, \ \theta(0,\xi) = 1,$$
$$\frac{\partial f}{\partial \eta}(\eta,\xi) \to 0, \ \theta(\eta,\xi) \to 0, \text{ as } \eta \to \infty. \tag{10}$$

In the event of truncated cone ($\xi = 0$), the Eq. (8) and Eq. (9) reduce to the following ordinary differential equations

$$\left(1+\frac{1}{\beta}\right)\frac{\nu_{nf}}{\nu_f} f''' + \frac{3}{4} ff'' - \frac{1}{2} f'^2 - Mf' + \frac{\beta_{nf}}{\beta_f}\theta = 0, \tag{11}$$

$$\frac{k_{nf} / k_f}{(\rho C_p)_{nf} / (\rho C_p)_f} \frac{1}{\Pr}\theta'' + \frac{3}{4} f\theta' = 0, \tag{12}$$

while for full cone ($\xi \to \infty$), it reduces to

$$\left(1+\frac{1}{\beta}\right)\frac{\nu_{nf}}{\nu_f} f''' + \frac{7}{4} ff'' - \frac{1}{2} f'^2 - Mf' + \frac{\beta_{nf}}{\beta_f}\theta = 0, \tag{13}$$

$$\frac{k_{nf} / k_f}{(\rho C_p)_{nf} / (\rho C_p)_f} \frac{1}{\Pr}\theta'' + \frac{7}{4} f\theta' = 0. \tag{14}$$

Both the system of Eq. (11), Eq. (12) and Eq. (13), Eq. (14) are subjected to the boundary conditions

$$\begin{aligned} &f(0) = f'(0) = 0,\ \theta(0) = 1, \\ &f'(\eta) \to 0,\ \theta(\eta) \to 0,\ \text{as}\ \eta \to \infty. \end{aligned} \tag{15}$$

The skin friction coefficient C_f and the local Nusselt number Nu_x which given by

$$C_f = \frac{2\tau_w}{\rho_f u_r^2}, \qquad Nu_x = \frac{xq_w}{k_f (T_w - T_\infty)}. \tag{16}$$

The surface shear stress τ_w and the surface heat flux q_w are given by

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0}, \qquad q_w = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{17}$$

Using variables in Eq. (5) and Eq. (17) give

$$C_f Gr^{1/4} = \frac{2}{(1-\phi)^{2.5}}\left(1+\frac{1}{\beta}\right) f''(\xi,0) \quad \text{and} \quad Nu_x Gr^{-1/4} = -\left(\frac{k_{nf}}{k_f}\right)\theta'(\xi,0). \tag{18}$$

3. Numerical Method

The set of transformed partial differential Eq. (8) and Eq. (9) with Eq. (10) are solved numerically using the implicit finite difference method known as the Keller-box method. Pioneered by Keller [25], this numerical method is known for its unconditionally stable and suitable for solving non-linear equations at any order.

The Keller-box algorithm starts with reducing the Eq. (8) and Eq. (9) to a first-order system. Next, the midpoint of the net rectangle is written by using the central differences. Noticed that the non-linear equations need to be linearize before being solved, thus Newtons method is implemented. The resulting algebraic equations are then written in matrix-vector the form and lastly, being solved by the block tridiagonal elimination technique. Detail regarding the Keller-box method have been discussed included the works by Na [26], Cousteix and Cebeci [27] and recently by Mohamed *et al.,* [21] and Mohamed [28].

4. Results

A steady, two-dimensional MHD free convection flow and heat transfer over a vertical truncated cone in a Casson ferrofluid where blood as a based fluid. The transformed partial differential Eq. (7) and Eq. (8) with the boundary conditions (9) computed numerically considering the fluid flow parameters which are the ferroparticle volume fraction ϕ, the Casson parameter β and the magnetic parameter M. The results is obtained for the truncated cone $(\xi = 0)$ extending to the end of the cone $(\xi = \infty)$. For the numerical computation purpose, the Prandtl number $\Pr$ is set to 21 with respect to the blood as the based fluid while the ferroparticle properties is taken considering the magnetite ferroparticle Fe_3O_4. The thermophysical properties of blood and magnetite ferroparticles are shown in Table 1.

In order to validate the numerical scheme used, the comparison values of heat transfer coefficient $-\theta'(0)$ with previously published results are shown in Table 2. The results obtained in Table 2 are in good agreement, hence shows the precision for the whole results presented in this study.

Table 1
Thermophysical properties of blood and magnetite [5,29]

Physical Properties	Blood (f)	Magnetite Fe_3O_4 (ϕ)
ρ (kg/m^3)	1053	5180
C_p (J/kg·K)	3594	670
k(W/m·K)	0.492	9.7
β (1/K)	0.74x10-4	1.3x10-5

Table 2
Comparison values of $-\theta'(0)$ with previous published results for various values of $\Pr$ when $\phi_1 = \phi_2 = 0$

$\Pr$	$\xi = 0$ (truncated cone)			$\xi = \infty$ (full cone)		
	Na and Chiou [10]	Chamkha [13]	Present	Na and Chiou [10]	Chamkha [13]	Present
0.01	0.05742	0.0574	0.0595	0.07493	0.0751	0.0767
0.7	0.35320	-	0.3469	0.45101	-	0.4511
1	0.40110	0.4015	0.4009	0.51039	0.5111	0.5104
7	-	-	0.7455	-	-	0.9342
10	0.82690	0.8274	0.8269	1.03397	1.0342	1.0341
100	1.54930	1.5503	1.5496	1.92197	1.9230	1.9234

The graphs of the reduced skin friction coefficient $C_f Gr^{1/4}$ and the reduced Nusselt number $Nu_x Gr^{-1/4}$ along the non-dimensional streamwise coordinate ξ for various values of ϕ and M are shown in Figure 2 and Figure 3, respectively.

From Figure 2, it was observed that the values of $C_f Gr^{1/4}$ is unique, where $C_f Gr^{1/4} = 0$ at a truncated cone $(\xi = 0)$. The values of $C_f Gr^{1/4}$ increase linearly as ξ increases, sign that the skin friction increases with the length of the cone. Considering the ferroparticle volume fraction ϕ and the magnetic parameter M, it is found that the increase of ϕ raised the fluid friction. The increase of M does contradict with ϕ. From the numerical calculation, it is found that the fluid flow with $\phi = 0, M = 0.1$ gave the identical values of $C_f Gr^{1/4}$ compared to the fluid flow with $\phi = 0.05, M = 0.5$. Same goes to the fluid flow with $\phi = 0.05, M = 0.1$ and $\phi = 0.15, M = 0.5$. This behavior shows diminishing effects between ϕ and M, thus suggested that if one of the parameter values is set to be increases, the friction between fluid and surface can be controlled by manipulate the other one parameter values.

The increase $Nu_x Gr^{-1/4}$ is influence by the increase of ϕ and a decrease of M same as in the Figure 2. As shown in Figure 3, it is clearly gives us information that the presence of ϕ in blood-based fluid increase the fluid convective heat transfer capabilities. Next, as the fluid flow along the cone surface, it is suggested that for small values of ξ, the values of $Nu_x Gr^{-1/4}$ rapidly increasing from the truncated cone $(\xi = 0)$, then, as the cone reach to the end, the raising of the $Nu_x Gr^{-1/4}$ is slowing.

The temperature profiles $\theta(\eta)$ and the velocity profiles $f'(\eta)$ of a truncated cone $(\xi = 0)$ for various values of ϕ and M are illustrated in Figure 4 and 5, respectively. From Figure 4, it is found that the increase in ϕ and M results to the increase in the thermal boundary layer thickness. Physically, the increase of ϕ in fluid has raised the fluid thermal conductivity which led to the increase in energy spreading ability in the fluid, thus thickening the thermal boundary layer thickness.

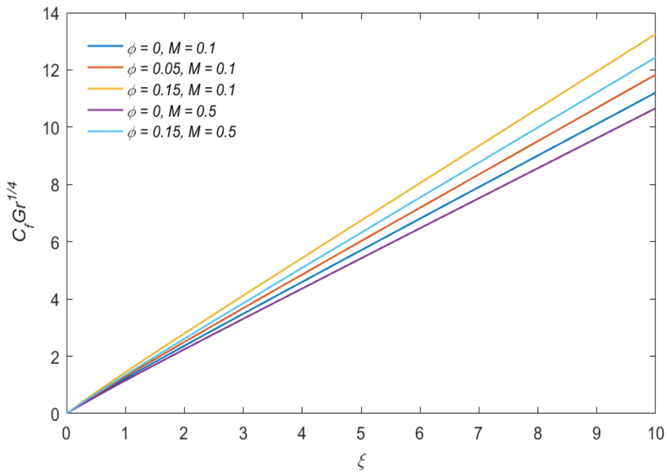

Fig. 2. Graph of $C_f Gr^{1/4}$ for various values of ϕ and M when $\Pr = 21, \beta = 0.5$

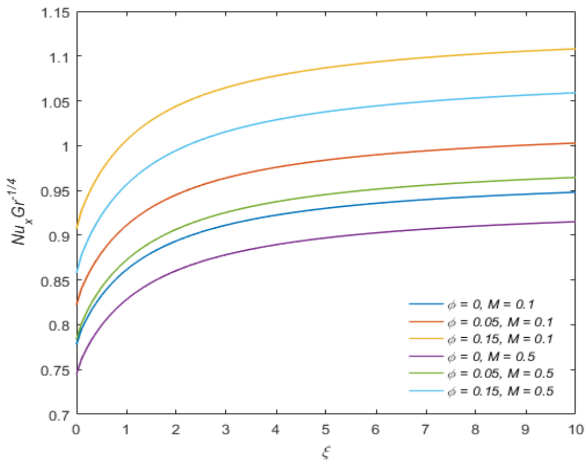

Fig. 3. Graph of $Nu_x Gr^{-1/4}$ for various values of ϕ and M when $\Pr = 21, \beta = 0.5$

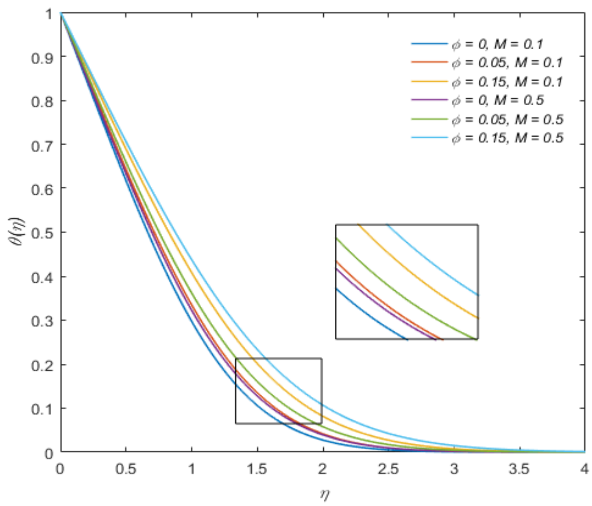

Fig. 4. Temperature profiles $\theta(\eta)$ for various values of ϕ and M when $\Pr = 21, \beta = 0.5$

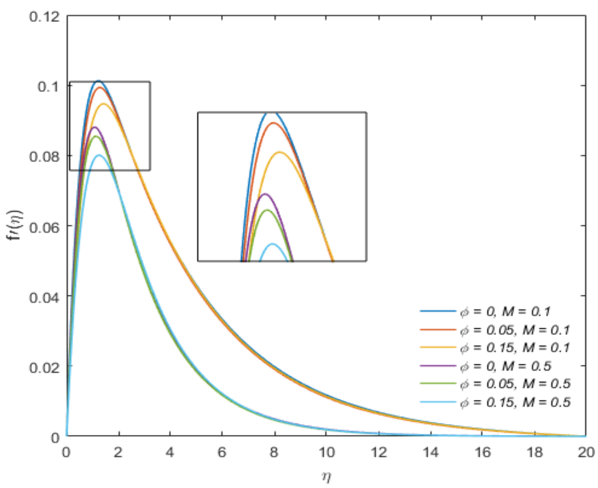

Fig. 5. Velocity profiles $f'(\eta)$ for various values of ϕ and M when $\Pr = 21, \beta = 0.5$

Meanwhile, ϕ gives no effects on the velocity boundary layer thickness in Figure 5. From figure, it is found that the fluid velocity is decreases as ϕ and M increases. This is due to the attraction between the magnetic effects at the cone surface with ferroparticle in the fluid has contributed to slowing down the fluid velocity. Further, the increase in magnetic effects has reduced the fluid ability to spread the fluid momentum thus thinning the velocity boundary layer thickness. Considering the effects of Casson parameter β in the fluid flow, Figure 6 to 9 are illustrated. Noticed that $\beta \to \infty$ refers to a Newtonian viscous fluid. The increase of β results to a decrease in $C_f Gr^{1/4}$ but increase in $Nu_x Gr^{-1/4}$. The variations of $C_f Gr^{1/4}$ and $Nu_x Gr^{-1/4}$ along the cone are depicted in Figure 6 and Figure 7, respectively. This indicate that the Casson ferrofluid generate higher friction but lower in heat transfer abilities compared to the Newtonian ferrofluid.

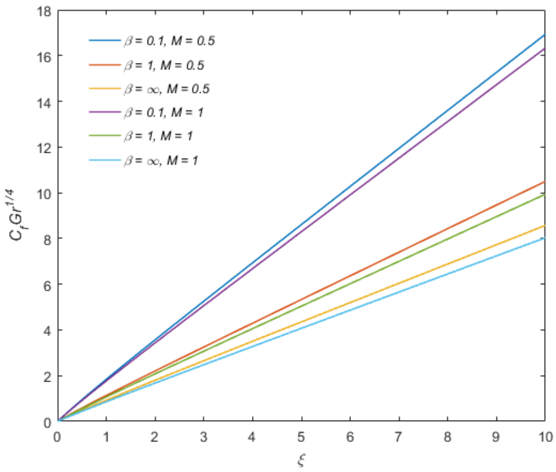

Fig. 6. Graph of $C_f Gr^{1/4}$ for various values of β and M when $\Pr = 21, \phi = 0.1$

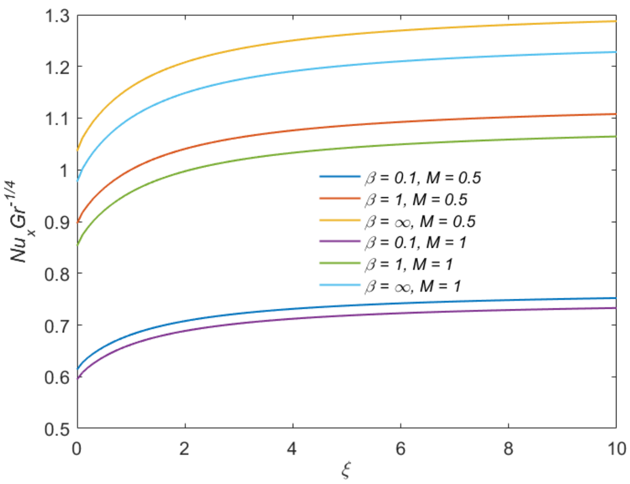

Fig. 7. Graph of $Nu_x Gr^{-1/4}$ for various values of β and M when $\Pr = 21, \phi = 0.1$

Lastly, the temperature profiles $\theta(\eta)$ and the velocity profiles $f'(\eta)$ for various values of β and M are shown in Figure 8 and Figure 9, respectively. From both figures, it is investigated that the increase in beta reduced both the thermal and velocity boundary layer thicknesses. This gave information that the Casson ferrofluid has more energy abilities to spread the energy and velocity in the fluid than the Newtonian ferrofluid. On the other hand, from Figure 9, it is found that the Newtonian ferrofluid has higher fluid velocity than Casson ferrofluid. The reduction in β indicates the increase of yield stress in a fluid thus reduced the effects of shear stress thus slowing down the fluid velocity [30].

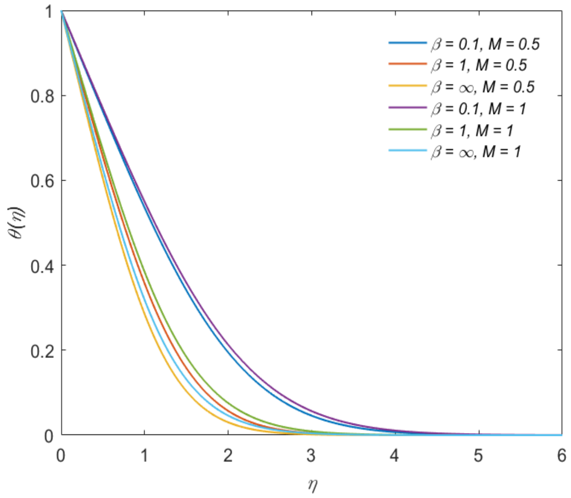

Fig. 8. Temperature profiles $\theta(\eta)$ for various values of β and M when $\Pr = 21, \phi = 0.1$

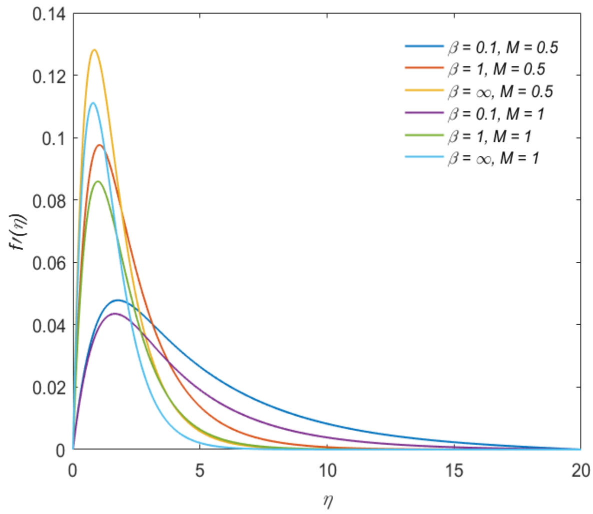

Fig. 9. Velocity profiles $f'(\eta)$ for various values of β and M when $\Pr = 21, \phi = 0.1$

5. Conclusion

In this paper, the magnetohydrodynamic (MHD) free convective boundary layer flow passes a truncated vertical cone in a blood based Casson ferrofluid is numerically studied. The effects of fluid flow parameter which are the ferroparticle volume fraction, the Casson parameter and the magnetic parameter on the skin friction coefficient, Nusselt number as well as the temperature and velocity profiles are analyzed and discussed.

As a conclusion, the increase in Nusselt number and the skin friction coefficient is affected by the increase of ferroparticle volume fraction and a reduction in magnetic parameter. Both parameters show a diminishing effect to each other thus, suggested that the friction can be controlled by manipulate the other one parameter values. Next, the Nusselt number and the skin friction coefficient increase with the length of the cone.

Considering the Casson characteristic over the Newtonian behavior, at the truncated cone, the thermal and velocity boundary layer thicknesses of Casson ferrofluid is greater than the Newtonian ferrofluid. At a full cone, it is found that the Casson ferrofluid is high in friction but low in Nusselt number compared to the Newtonian ferrofluid which physically shows the influence of conduction in the Casson ferrofluid.

Acknowledgement

Authors are grateful to acknowledge the Universiti Malaysia Pahang Al-Sultan Abdullah for providing the financial support under Tabung Persidangan Dalam Negara and the research grant no. RDU220388.

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