EXPLORING THE TRIGONOMETRIC CIRCLE AND TRIGONOMETRIC LAWS WITH A GEOGEBRA VISUALIZATION

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Abstract

Mathematical functions that establish a relationship between the angles of a right triangle and the lengths of its sides are known as trigonometric functions. In this project, we will explore two of the six trigonometric functions and their properties using the trigonometric circle. Initially, trigonometric functions were defined only for acute angles related to right triangles. To extend the sine and cosine functions to functions whose domain is the entire real line, geometric definitions using the unit circle with radius 1 are often used. Then, the domain of the other trigonometric functions is the real line with some isolated points removed. Modern definitions express trigonometric functions as infinite series or as solutions of differential equations. This allows the extension of the definition of sine and cosine functions to the entire complex plane and the definition of other trigonometric functions on the complex plane with some isolated points removed. This project discusses the use of GeoGebra in studying trigonometric functions, the Law of Sine, and Law of Cosine. GeoGebra is a powerful mathematical software that allows for the visualization and exploration of trigonometric functions.

Keywords: Trigonometry, functions, sine, cosine, GeoGebra, visualization.

1. Introduction

Trigonometry has a rich history, dating back to ancient civilizations such as the Babylonians, Egyptians, and Greeks. The Greeks are credited with developing the foundation of modern trigonometry, including the use of the six trigonometric functions: sine, cosine, tangent, cosecant, secant, and cotangent. The famous mathematician Hipparchus is often referred to as the "father of trigonometry" for his pioneering work in creating chord tables used to calculate the positions of celestial bodies.

The word "trigonometry" comes from the Greek words "trigonon" (triangle) and "metron" (measure), reflecting the fundamental concept of measuring triangles.

Trigonometry is a mathematical subject that focuses on analyzing the connections between the angles and sides of triangles. It has been used for centuries in various fields, including engineering, astronomy, physics, and even art.

Today, trigonometry is an essential tool in many fields, and its applications are endless. From calculating distances to determining the angles of a building, trigonometry plays a vital role in modern life.

Trigonometric functions, also known as circular functions, angle functions, or goniometric functions, are real functions that relate the angle of a right triangle to the ratios of two of its side lengths.

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circle with radius 1 are often used. Then, the domain of the other trigonometric functions is the real line with some isolated points removed. Modern definitions express trigonometric functions as infinite series or as solutions of differential equations. This allows the extension of the definition of sine and cosine functions to the entire complex plane and the definition of other trigonometric functions on the complex plane with some isolated points removed.

GeoGebra

GeoGebra is a dynamic mathematical software that allows users to create and explore mathematical models, including trigonometric functions. GeoGebra is a free, open-source software primarily used for geometry and graphing. It is particularly useful for visualizing mathematical concepts, as it allows users to create interactive diagrams and graphs that can be manipulated and explored in real time. GeoGebra also has built-in features for solving trigonometric equations and performing other trigonometric calculations, making it a valuable tool for students and educators.

2. Exploring the Trigonometric Circle: A GeoGebra Visualization

A unit circle, also known as a trigonometric circle, is a circle with a radius of one unit. It is commonly used to investigate and comprehend the characteristics of trigonometric functions like sine, and cosine.
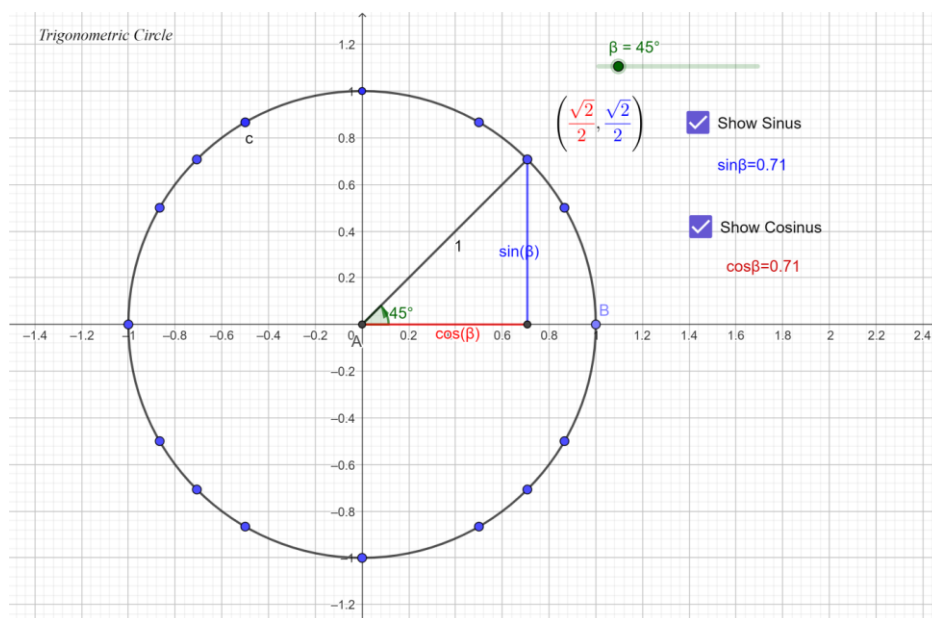

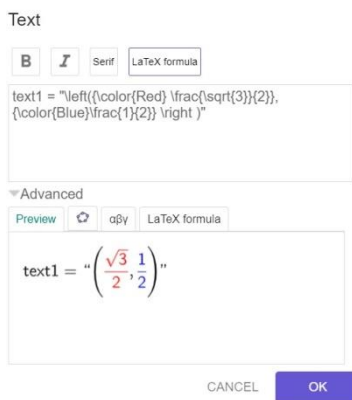**Figure 1.** Trigonometric Circle

Using the GeoGebra software, I created a unit circle with its center at the origin. By creating a circle with a center at the origin and a radius of 1 unit, we can easily represent angles and their corresponding trigonometric functions. This allows us to see the relationships between angles and their sine and cosine.

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Within the circle, I plotted points and connected them with line segments to form a right triangle. By manipulating a slider, the geometric data changes dynamically, showcasing the values of sine or cosine for various angles.

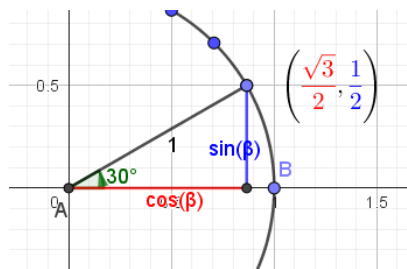Construction of the trigonometric circle:

We construct the trigonometric circle with a center at (0,0) and a radius of 1. We add a slider for an angle from 0 degrees to 360 degrees with a step of 15 degrees. Then, we construct a line segment that indicates the angle of the slider, with one end on the positive part of the x-axis and the other end on the constructed line segment $\overline{OD}$. From the Text Input window, we enter the following text in the editing field:

text1 = "\left ({\color {Red} \frac{\sqrt{3}} {2}},{\color{Blue}\frac{1}{2}} \right)"

Figure 2. Text Input

The points are displayed on the drawing surface $\left(\dfrac{\sqrt{3}}{2}, \dfrac{1}{2} \right)$.

Figure 3. Trigonometric Circle Points

We repeat this procedure for each of the characteristic points of the trigonometric circle, which are related to the values from the table. To make the image interactive, as the slider moves with the angle value, point D moves along the trigonometric circle with a step of 15 degrees. When it coincides with the corresponding value from the Text windows, only that Text window should be displayed. Therefore, for each Text window, a

condition is added to determine its visibility. In the properties of each Text in the Advanced tab, a condition is added to show the object, for example, $D\overset{?}{=}E$, if the Text window should only be displayed when point D coincides with point E, and so on.

Figure 4. Condition to show object

In this way, by moving the point on the angle slider, the angle, the values of the corresponding points, and the values of sine and cosine of the respective angle are visually displayed.

Note:

To determine the values of sine and cosine, users have two options. The first is to utilize the displayed values on the screen, while the second involves typing in commands for the desired trigonometric functions into the "Input" bar located at the bottom of the screen. For example, to find the sine value of an angle, input "sin (60°)" into the Input bar and press enter. GeoGebra will calculate and display the result accordingly.

3. Visualization of the Law of Sine using GeoGebra

The Law of Sines, also known as the Sine Rule, is a trigonometric law used to solve triangles when given certain information about their sides and angles.

The Law of Sines states that the ratio of the length of a side of a triangle to the sine of the angle opposite that side is constant for all three sides of the triangle. This law can be written as:

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} = 2R \qquad (1)$$
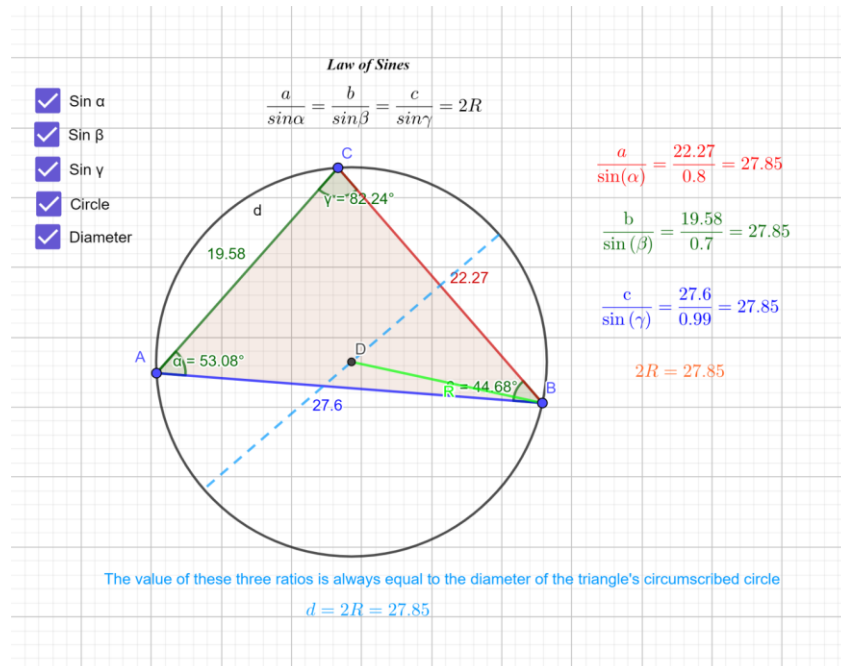
Figure 5. Law of Sines

When constructing a triangle using GeoGebra, you can move the points around and adjust the angles to see how the relationship between the sides and opposite angles remains consistent with the Law of Sines. As you move the points, GeoGebra will dynamically adjust the shape of the triangle and the corresponding values of the sides and angles. However, you will notice that the ratio of the sine of each angle to the length of its opposite side remains constant, consistent with the Law of Sines. This provides a visual representation of the Law of Sines and how it holds true for any triangle, regardless of its size or shape.

## 4. Visualization of the Law of Cosine using GeoGebra

The Law of Cosines, also known as the Cosine Formula or Cosine Rule, is a mathematical law that relates the lengths of the sides of a triangle to the cosine of one of its angles.
The Law of Cosines states that for any triangle with sides a, b, and c, and angles A, B, and C opposite those sides, the following equation holds true:

$$c^2 = a^2 + b^2 - 2ab \cdot \cos(C) \qquad (2)$$
$$a^2 = b^2 + c^2 - 2bc \cdot \cos(A) \qquad (3)$$
$$b^2 = a^2 + c^2 - 2ac \cdot \cos(B) \qquad (4)$$

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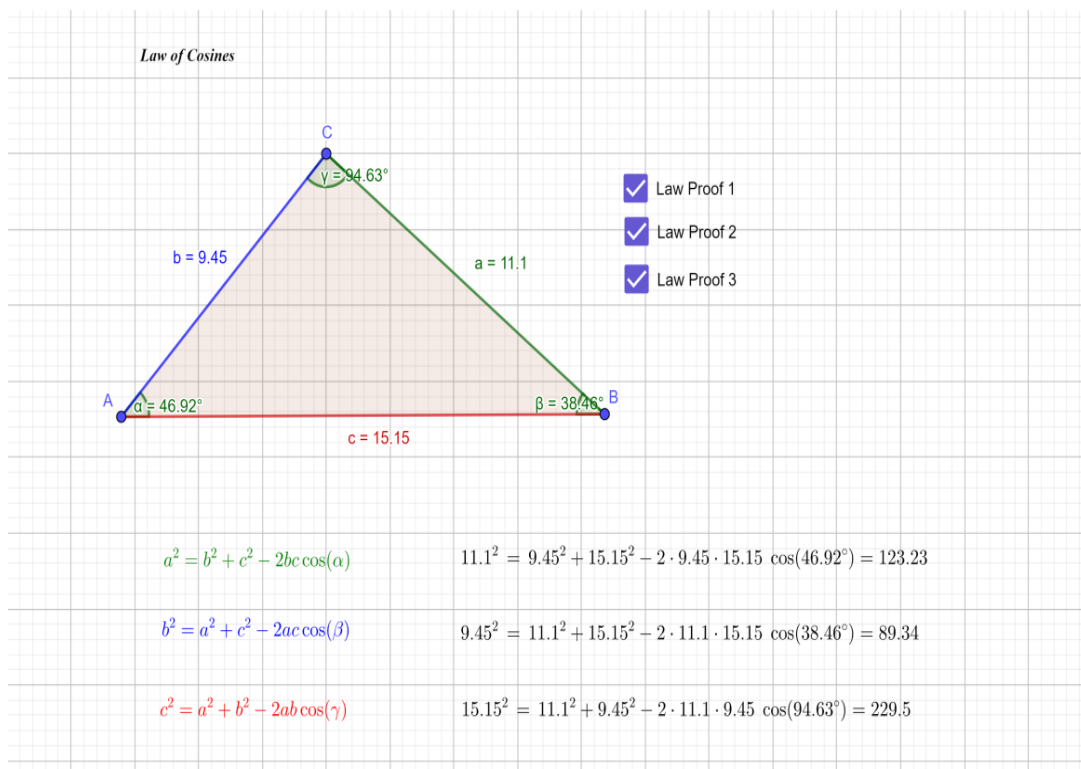
Figure 6. Law of Cosines

Using GeoGebra, I constructed a triangle and its angles, then used the Law of Cosines formula to prove a relationship between the lengths of its sides and angles. When constructing a triangle with GeoGebra, you can move the points and adjust the angles to see how the relationship between the sides and angles remains consistent with the Law of Cosines.

As you move the points around, GeoGebra will dynamically adjust the shape of the triangle and the corresponding values of the sides and angles. However, you will notice that the relationship between the sides and angles, as described by the Law of Cosines, remains the same.

By adjusting the angles and sides of the triangle in GeoGebra, you can see that this relationship between the sides and angles is always maintained, regardless of the size or shape of the triangle. This provides a visual representation of the Law of Cosines and how it holds true for any triangle.

5. Conclusion

In conclusion, the application of trigonometric functions in science is broad and decisive. These functions are utilized extensively in the study of natural phenomena, such as the behavior of light, wave motion, molecular structure, and celestial movement.

GeoGebra provides a user-friendly interface for visualizing and manipulating geometric objects, making it easy to solve problems involving angles, triangles, and trigonometric functions. With GeoGebra, you can easily

construct triangles, measure angles, and calculate trigonometric functions using the built-in commands and tools.

The use of trigonometric functions has revolutionized many fields of science and engineering, providing a powerful tool for modeling, analyzing, and predicting complex systems. With the continuous advancement of technology and research, the application of trigonometric functions will likely continue to expand and play an increasingly important role in shaping our understanding of the natural world.

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