

# A visual approximation of a Fourier transforms operation using GeoGebra

*Djoko Untoro Suwarno*\*

Electrical Engineering, Faculty of Science and Technology, Sanata Dharma Univesity, Paingan, Maguwoharjo, Sleman 55282, Indonesia

**Abstract.** Many engineering fields use the Fourier Transform. The purpose of the Fourier Transform is to decompose periodic signals into fundamental frequencies and harmonic frequencies. The Fourier transform operation in ordinary textbooks is mathematically analytical, this often makes it difficult for students who study it. This paper describes the mathematical operations that occur in the Fourier transform. Discrete Fourier transform operations consist of conversion operations of continuous-time signals into data sequences, exponential complex number multiplication operations, and integration operations of number data sequences. Visualization of Fourier transform operations using the GeoGebra application. This research obtains visualization results of Fourier transform operations. The resulting visualization includes the effect of increasing signal frequency and the integral limit. The influence of the greater integral limit causes the peak of the frequency spectrum to become larger. The integral result of the pulse signal is in the form of a sinc function and corresponds to the illustration in the textbook. The limitation of the results of this research is that the Fourier transform visualization was carried out at low frequencies

**Keywords.** Visualization of transforms operations, GeoGebra, frequency domain, time domain.

## 1 Introduction

Fourier transform is a mathematical operation to convert signals in the time domain into the frequency domain. The Fourier transform operation consists of an infinite integral operation, the operation of multiplying the time domain function  $f(t)$  with a complex exponential. The Fourier transform is a tool used in the field of signal processing and is explained in [1]. Computing the discrete Fourier transform using the Winograd method [2] is carried out by summarizing the calculation of complex number multiplication. The application of the Fourier transform to sound and vibration applications was carried out by Shin [3] and Smith [4]. In Sundararajan's paper [5] the DFT (Discrete Fourier Transform) operation is explained along with an illustration of the operation. Students often have difficulty learning the mathematical operations that occur in the Fourier transform, both for continuous Fourier

---

\* Corresponding author: [Joko\\_unt@usd.ac.id](mailto:Joko_unt@usd.ac.id)

transform operations and discrete Fourier transform operations. This article will discuss the Fourier transform operation graphically using GeoGebra. GeoGebra is a web-based tool for Mathematics Teaching and Learning, including classroom activities, graphing, geometry, collaborative whiteboards, and more. In the GeoGebra application, there is no facility to perform Fourier transform operations directly. Fourier transform operations are carried out with basic operations in the form of multiplication of complex numbers, and integral operations with limits. This paper will investigate the Fourier transform with input single-frequency sine signals, multiple-frequency sine signals, and the effect of adding offset to the sine signal. Fourier Transform Computing which was developed by Cooley in 1969 continues to develop, but the Fourier Transform command has not been implemented in GeoGebra. The author tries to use GeoGebra to perform Fourier transformations using the basic commands and operations in GeoGebra.

## 2 The Fourier transform

The Continuous-time Fourier transform is shown in Equation (1).

$$X(f) = \int_{t_1}^{t_2} x(t) e^{-i 2\pi f t} dt, \tag{1}$$

where,  $x(t)$  is a signal in the time domain,  
 $X(f)$  is a signal resulting from the Fourier transform in the frequency domain,  
 $e^{-i 2\pi f t}$  is a complex exponential function,  
 $t_1$  and  $t_2$  are the integral limit time for Fourier Transform operation.

Fourier transform computations are performed on discrete signals and are called Discrete Fourier Transformations. The Fourier transform computational operation is carried out for discrete data, so the Fourier transform required is in the form of a discrete Fourier transform. The Discrete-time Fourier transform shown in Equation (2).

$$X_k = \sum_{n=0}^{N-1} x_n e^{-i \frac{2\pi}{N} nk}, \tag{2}$$

where,  $x_n$  is a discrete sequence obtained from the signal sampling of the time domain,  
 $X_k$  is the result of the discrete-time Fourier transform,  
 $N$  is the number of samples in the  $2\pi$  period,  
 $n, k$  are data index, integer values  
 $e^{-i \frac{2\pi}{N} nk}$  is a complex exponential sequence number.

The continuous-time signal is converted into time series data by multiplying the continuous-time signal with the Dirac function with a delta  $t$  time interval. The equation for converting a continuous signal into discrete data is shown in the following Equation (3).

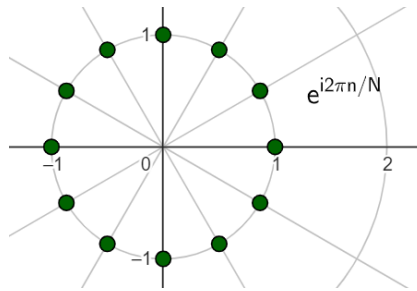
$$X_{[n]} = \sum_{n=0}^N x(t) \delta(t - n \Delta T), \tag{3}$$

where  $\delta(t - n \Delta T)$  is a Dirac function that is a series of unit impulse signals with time interval  $\Delta T$ . Euler's identity states that  $e^{i\theta}$  can be decomposed into a sine function and a cosine function as shown in Equation (4).

$$e^{i\theta} = \cos(\theta) + i \sin(\theta). \tag{4}$$

Based on Euler's identity equation, the exponential function of complex number  $i$  forms the cosine function and the sine function. The real part of the imaginary power exponential

function is a cosine function, while the sine part is the imaginary part. A series of complex exponential numbers  $e^{\frac{i2\pi}{N}n}$  for n from 0 to N-1 forms a circular pattern shown in Fig. 1.



**Fig. 1.** Location of a series of complex exponential numbers  $e^{\frac{i2\pi}{N}n}$ , for N=12, n=0 to N-1.

### 3 Methods

The Fourier transform operation in command form is not available in GeoGebra. This research will describe the operations that occur in the Fourier transform using GeoGebra. The Fourier transform operation consists of multiplying the signal  $f(t)$  by the complex exponential series  $e^{i\theta}$ , and the finite integration operation. After the Fourier transform operation was formed using GeoGebra, it was tested with input in the form of a sine signal. The input signal is a sine function with a single frequency and an input sine function with multiple frequencies.

Discrete Fourier transform operations can be broken down into sub-operations:

1. converting time-domain signals  $x(t)$  into discrete data  $x(n)$  using Equation (3),
2. mapping operations from time-domain data  $x(t)$  into a polar form using multiplication  $x(n)e^{-i2\pi t/T}$ ,
3. finite integral operations to locate mass center points  $\int_{t_1}^{t_2} x(n)e^{-i2\pi t/T} dt$ ,
4. display integral results.

The Fourier transform operation in GeoGebra is carried out using multiplication operations and integral operations. GeoGebra Construction protocol for Fourier Transform is shown in Table 1.

**Table 1.** GeoGebra construction protocol for Fourier transformation.

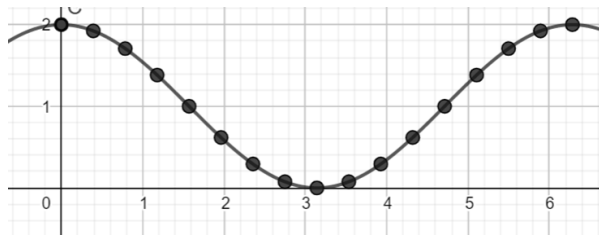
No.	GeoGebra command	Description
1	$f(t)=if(-1<t<1,1,0)$	Dirac impulse function as signal input
2	$W=slider(-2\pi, 2\pi)$	window limits from $\pi$ to $\pi$
3	$a=f(t) e^{i w/N t}$	Mapping time domain $f(t)$ into the frequency domain in polar form
4	$Integral(a,-2\pi,2\pi)$	Integral data in the frequency domain with a lower and upper limit
5	$(W, a)$	Plotting of Freq. Response

Fourier Transform operation testing is carried out by providing input  $f(t)$  for a cosine signal with a single frequency and a signal consisting of two frequencies. The next Fourier transformation test is in the form of changing the integral limits.

1.  $f(t) = \cos(t)$ ,
2.  $f(t) = A_1 \cos(f_1 t) + A_2 \cos(f_2 t) + \text{offset}$ ,
3.  $\text{integral}(a, t1, t2)$ .

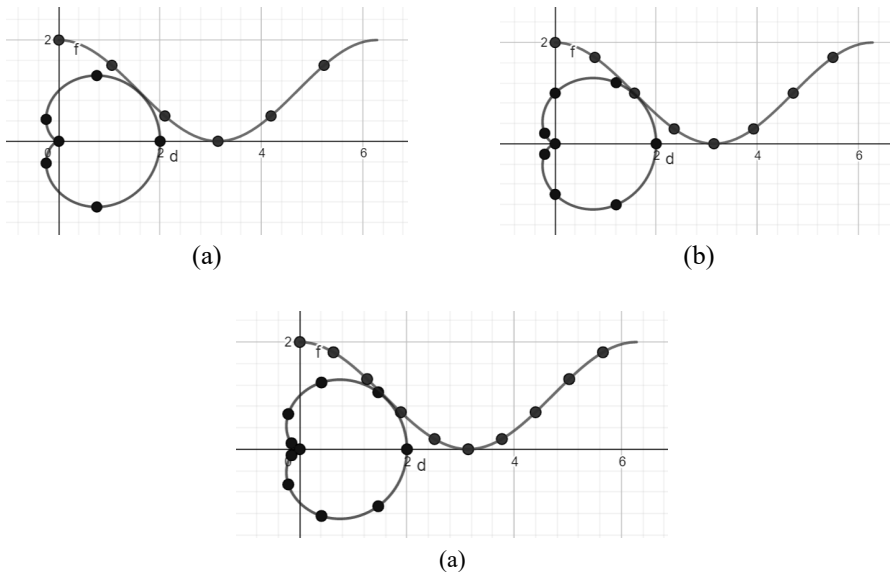
## 4 Results and discussion

Continuous signal conversion into discrete data is shown in Fig. 2. A signal with a period of  $2\pi$  is sampled with the number  $N = 16$  with a sampling time interval of  $2\pi/N$ .



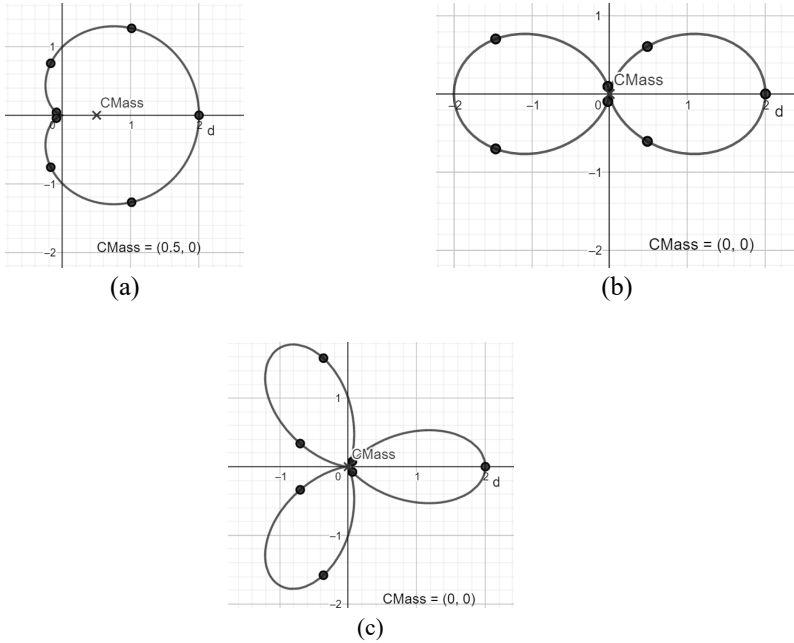
**Fig. 2.** The cosine signal with period  $2\pi$  sampled  $N=16$  data.

Mapping time-domain signals or discrete sequences into polar form is done through complex exponential multiplication as shown in Fig. 3.



**Fig. 3.** Mapping sampled signals with varying numbers of  $N$  samples into polar form, (a) for  $N=6$  points, (b) for  $N=8$  points, and (c)  $N=10$ .

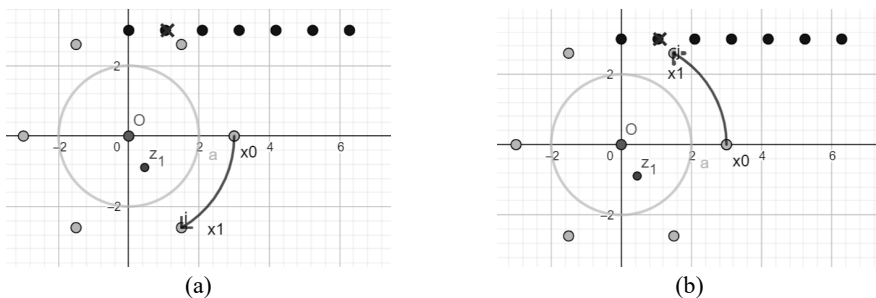
Signals in the time domain are converted to series with  $n$  indexes indicating the sequence order from 0 to  $N-1$ . The index  $n$  is the angle while the amplitude becomes magnitude in polar form. Polar patterns for various cosine signals with integer frequencies are shown in Fig. 4.



**Fig. 4.** Polar pattern for signals with various frequency harmonics and center of mass points location (a) Signal with fundamental frequency, (b) Signal with 2nd harmonics frequency, (c) Signal with 3rd harmonics frequency.

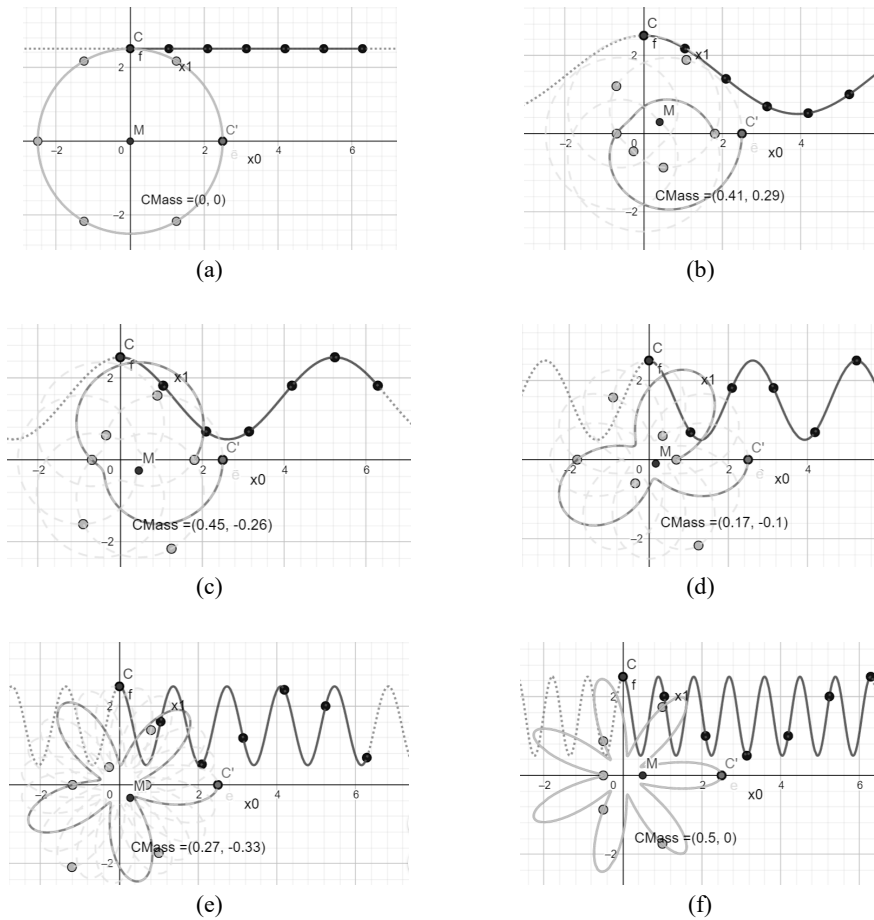
The point mass calculation is carried out by performing an averaging operation for the points after mapping them to polar coordinates. The center of mass is used as the frequency response position for a specific frequency.

The direction of the curve in multiplication with the positive exponential complex is CW (ClockWise) and multiplication with the negative exponential complex produces the direction of CCW (counterclockwise) as seen in Fig. 5.



**Fig. 5.** The direction of mapping on polar shapes. (a) the negative exponential complex has CW direction, (b) positive exponential complex has CCW direction.

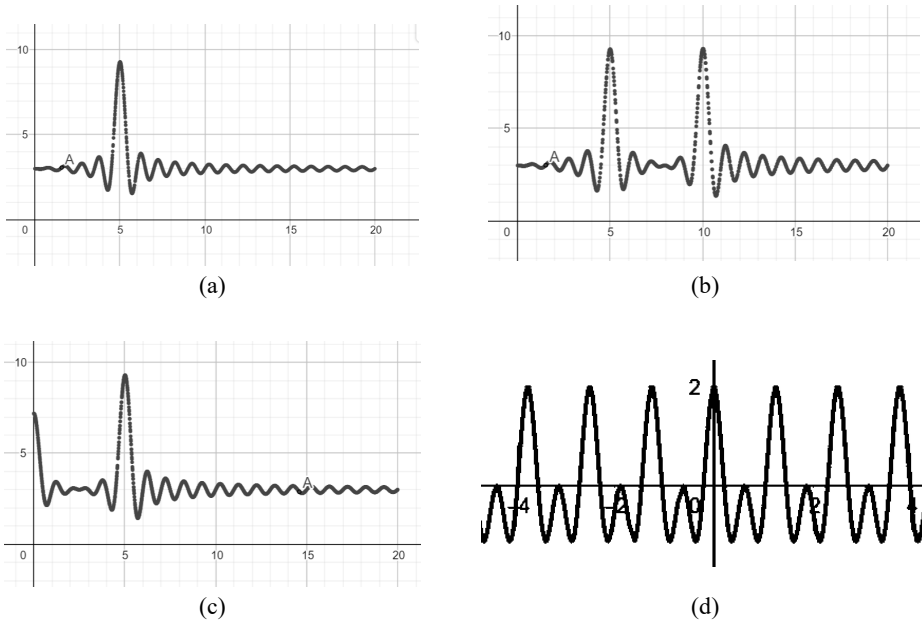
Mapping for signals with fractional frequencies results in complicated curves, curves require several cycles to return to their original positions. A fractional frequency of 1.2 requires 5 cycles to return to its original position. The graph for various signals with fractional frequencies is shown in Fig. 6.



**Fig. 6.** Mapping sequence data input into polar form with various fractional frequency. (a)  $\cos(0t)$ , (b)  $\cos(0.4t)$ , (c)  $\cos(0.6t)$ , (d)  $\cos(1.2t)$ , (e)  $\cos(2.2t)$ , (f)  $\cos(3.6t)$ .

Figure 7a shows the Fourier transform for an input signal in the form of  $\cos(5t)$  producing a scale number of 5 on the horizontal axis. The transformation results correspond exactly to the input signal. Likewise, for Fig. 7b there is a signal with two frequencies  $\cos(5t) + \cos(10t)$ . The transformation results produce a spectrum that matches the input. Adding an offset to the input signal and carrying out a transformation will shift the transformation curve vertically. Figure 7c and 7d show adding an offset to the signal means increasing the DC value and not shifting the spectrum in the horizontal direction but in the vertical direction.

The effect of integral borders on responsiveness is shown in Fig. 8. Figure 8a with an integral border of  $-\pi$  to  $\pi$  produces an amplitude of 1, while Fig. 8b with an integral border of  $-3\pi$  to  $3\pi$  produces 3 times the amplitude in Fig. 8a.

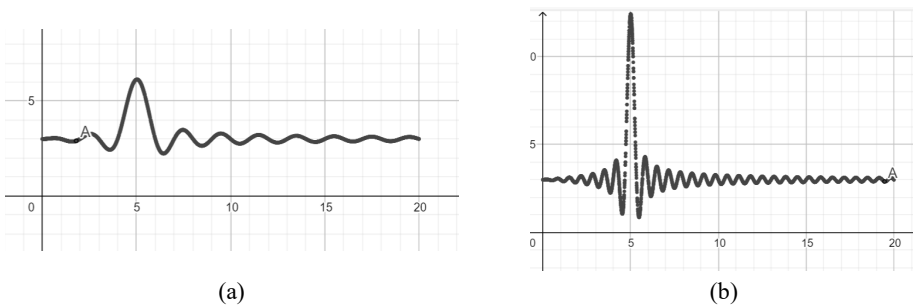


**Fig. 7.** Frequency spectrum in  $\cos(5t)$ , and  $\cos(10t)$  signals with offset increments. (a) spectrum freq  $\cos(5t)$ , (b) spectrum freq  $\cos(5t) + \cos(10t)$ , (c) spectrum freq  $\cos(5t) + \text{offset}(0.3)$ , (d) signal  $\cos(5t) + 1/5 \cos(10t)$ .

Fourier transform for pulse signal with a certain pulse width  $T$  is shown in Fig. 9. Pulse signals are similar to impulse signals of a certain width. Fourier transforms result for  $rect(t)$  pulse signal in the form of  $sinc(f)$  function. The exploratory solution to Fourier's transformation of pulse signals is shown in the Equation (5) and Equation (6).

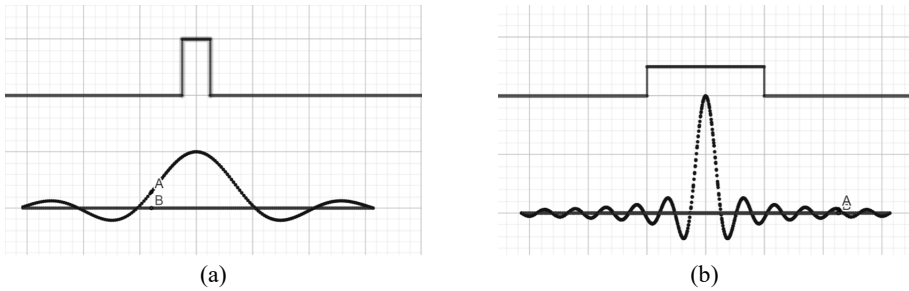
$$rect(t) = \begin{cases} 1 & -\tau/2 < t < \tau/2, \\ 0 & |t| > \tau/2 \end{cases} \quad (5)$$

$$X(\omega) = \int_{-\tau/2}^{\tau/2} rect(t) e^{-i\omega t} dt = \tau \frac{\sin\left(\frac{\omega\tau}{2}\right)}{\left(\frac{\omega\tau}{2}\right)} = \tau sinc\left(\frac{\omega\tau}{2}\right), \quad (6)$$



**Fig. 8.** Integral boundary effect (a) integral boundary from  $-\pi$  to  $\pi$  results from a small response amplitude, (b) integral boundary from  $-3\pi$  to  $3\pi$  n amplitude response is getting higher.

The Fourier transformation for input in the form of a pulse function produces a sinc function and is shown in Equation 6. This result follows the Fourier transform in [4] and can be seen from the resulting graph.



**Fig. 9.** (a) Frequency Response for narrow pulse signal, (b) wide pulse signal.

## 5 Conclusion

From the investigation, the following results were obtained. Fourier transformation can be implemented in GeoGebra by using a multiplication stage between  $f(t)$  and complex exponential, then integrating to a certain limit and displaying the results of the integral calculation. For  $f(t)=\cos(5t)+\cos(10t)$  it produces a spectrum consisting of two signals  $\text{sinc}(5t)+\text{sinc}(10t)$  which is a spectrum with frequencies corresponding to the input signal.

## Acknowledgements

The author expressed his gratitude for the support from Martanto, S.T, M.T. head of the Electrical Department FST USD for the financing to attend this conference.

## References

1. W. T. Cochran et al., *IEEE* **55**, 1664-1674 (1967).
2. S. Winograd, *Math. Of Comput.* **32**, 175-199 (1978).
3. K. Shin and J. Hammond, in *Fundamentals of signal processing for sound and vibration engineers* (John Wiley & Sons, 2008).
4. J. O. Smith, in *Mathematics of the discrete Fourier transform (DFT)* (Center for Computer Research in Music and Acoustics (CCRMA), Department of Music, Stanford University, Stanford, California, 2002).
5. D. Sundararajan, in *The discrete Fourier transform theory, algorithms, and applications* (World Scientific, 2001).