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Water Distribution Networks Optimization Considering Uncertainties in the Demand Nodes

Gustavo H. B. Cassiolato[1] · Jose Ruben Ruiz-Femenia[2] · Raquel Salcedo-Diaz[2] · Mauro A. S. S. Ravagnani[1]

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Abstract

The fluctuation in the consumption of treated water is a situation that distribution networks gradually face. In times of greater demand, this consumption tends to suffer unnecessary impacts due to the lack of water. The uncertainty that occurs in water consumption can be mathematically modeled by a finite set of scenarios generated by a normal distribution and attributed to the network design. This study presents an optimization model to minimize network installation and operation costs under uncertainties in water demands. A Mixed Integer Nonlinear Programming model is proposed, considering the water flow directions in the pipes as unknown. A deterministic approach is used to solve the problem in three steps: First, the problem is solved with a nominal value for each uncertain parameter. In the second stage, the problem is solved for all scenarios, with the independent variables of the scenario being fixed and obtained from the solution reached in the first stage, known as the deterministic solution. Finally, all scenarios are solved without fixing any variable values, in a stochastic approach. Two case studies were used to test the applicability of the model and global optimization techniques were used to solve the problem. The results show that the stochastic solution can lead to optimal solutions for robust and flexible water distribution networks, capable of working under different conditions, considering the uncertainties of node demand and variable pipe directions.

Keywords Water distribution networks · Disjunctive programming · Uncertainties in the nodes demand · Unknown flow directions

✉ Mauro A. S. S. Ravagnani
massravagnani@uem.br

1 Chemical Engineering Graduate Program, State University of Maringá, Av. Colombo, 5790 – CEP 87020900, Maringá, Brazil

2 Chemical Processes Engineering Institute, University of Alicante, Carretera de San Vicente s/n – 03004, Alicante, Spain

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1 Introduction

Water supply systems are fundamental in industrial processes and urban centers. These systems involve the water catchment, the distribution network, and the pumping station. Water Distribution Networks (WDN) are made up of treated water reservoirs, pipes that connect the reservoirs to demand nodes, hydraulic pumps and, in most cases, loops between demand nodes. These components are responsible for the most important part of the network cost.

The design of WDN is an important research field and the number of papers published in this subject is increasing. The synthesis of WDN can be formulated as an optimization problem, involving, mainly, the minimization of the network installation cost, which depends on the pipeline diameters. Normally, flow directions are considered known in the demand nodes loop formed in the network. The optimization problem can have a Nonlinear Programming (NLP), or a Mixed Integer Nonlinear Programming (MINLP) formulation and stochastic and deterministic approaches have been used to solve the problem.

Generally, drinking water consumption is variable, causing fluctuations in the demand nodes in different periods of operation and these types of uncertainties need to be considered in the network design. Water demand variability can be modeled as a set of finite scenarios. A scenario can be understood as the situation of generating water demands for each node in the network, with the hydraulic conditions of the problem. Thus, each set of generated demands is treated as a scenario for the network. This situation is consistent with what occurs in WDN, considering the variability of water consumption in the network.

In the present paper an optimization model for WDN synthesis considering uncertainties in node demand is proposed. The problem has a MINLP formulation, and the flow directions are considered unknown and treated as optimization variables. The objective function is the total cost of WDN, considering the installation costs, which depend on the pipe diameters, and the annualized pumping cost, due to the use of hydraulic pumps in the network. Disjunctive programming is used to reformulate the problem and linearization techniques are used to avoid problems with the nonlinear Hazen-Williams equation, used in hydraulic calculations. No additional software or hydraulic simulators are used, as all velocity and pressure drop calculations are included in the model. The variation in water demand for a set of finite scenarios is generated by a normal distribution in Excel.

GAMS (General Algebraic Modeling System) environment is used to implement the developed model, in three steps. First, only a unique nominal value for each uncertain parameter is used. In the sequence, for all scenarios, the independent variables are fixed to the solution achieved in the first step. It corresponds to a deterministic approach. Finally, the problem is solved for all scenarios, without fixing variables (stochastic solution). Two case studies are used to test the developed model and global optimization techniques are used to achieve the problem solution.

Fluctuation in the consumption of treated water supplied by WDN is an everyday situation, due, for example, to natural variation and population growth. In times of greater demand, this consumption tends to suffer unpleasant impacts due to the lack of water. The uncertainty that occurs in water consumption being addressed in the WDN synthesis provides designers with an adequate solution to the problem and contributes to the network design. Furthermore, working with a project where the water flow directions are not fixed makes the network more realistic, providing different situations and structures.

2 Literature Review

The WDN design formulated as an optimization problem normally seeks to minimize the installation cost, related to the pipe diameter, subject to a set of constraints involving mass balances at demand nodes, energy balances if there are network loops, and pressure and velocity limits. For this, a set of commercial tubes with appropriate costs and roughness coefficients to be chosen is considered, aiming to minimize the total cost. It is common to solve hydraulic equations using additional software. Different formulations have been used, but MINLP is more representative of real problems, and, recently, the use of this type of optimization models by research groups has increased. In general, global optimality cannot be guaranteed due to the non-linear and non-convex behavior of the model.

In a recent paper, Mala-Jetmarova et al. (2018) presented a detailed review of the types of WDN optimization models and methods used to solve the problem. Designs of new WDN, expansion, and rehabilitation of existing water distribution systems, strengthening, design timing, parameters uncertainty, water quality, and operational considerations were reviewed. As pointed out by the authors, different deterministic and stochastic approaches have been used to solve the optimization problem. Stochastic approaches are used in large-scale problems, where deterministic approaches normally fail. Some important methods used are Particle Swarm Optimization, in Ezzeldin et al. (2014), Surco et al. (2017) and Surco et al. (2021), Harmony Search (HS) in Geem (2009), Simple Benchmarking Algorithm, in Shende and Chau (2019), Whale Optimization Algorithm, in Ezzeldin and Djebedjian (2020) and Genetic Algorithm, in Reca et al. (2017), Sangroula et al. (2022), Egito et al. (2023), Shekofteh et al. (2023) and Parvaze et al. (2023).

There are less papers focusing on deterministic approaches to solve the WDN optimization problem. This is due to the intrinsic limitations of deterministic solvers in getting trapped in local optimal solutions in nonlinear problems and in the difficulties in using global optimality methods in large scale problems. However, important advances have been published in this research field. Bragalli et al. (2012) proposed, for the optimization of WDN with fixed topologies, a nonconvex continuous NLP relaxation and an MINLP search approach. D'Ambrosio et al. (2015) presented a complete review of Mathematical Programming approaches in the optimization of WDN considering the notion of the network design and the network operation. Caballero and Ravagnani (2019) proposed an MINLP model considering unknown flow directions in the network loops and used global optimization techniques to solve the problem. In Cassiolato et al. (2023) an MINLP model was developed for the synthesis of WDN considering the minimization of the WDN total cost, given by the sum of installation and operational costs. Cassiolato et al. (2022a) considered in the model unknown flow directions and SBB and BARON solvers were used to achieve the problem solution. Cassiolato et al. (2022b) considered installation and energy costs, with unknown flow directions.

As mentioned before, the majority of the published papers use non-deterministic approaches and consider fixed and known flow directions and a hydraulic simulator to solve the velocities and pressure calculation. In real WDN, variations in nominal values can occur and these variations can influence the optimum network operation conditions, causing an unappropriated unexpected behavior. Thus, the assessment of uncertainties in different periods of operation is a recent and important field of research that needs to be considered in the final stage of the WDN project.

Branisavljevic et al. (2009) used a Genetic Algorithm to find optimal solutions considering uncertainties in the water nodal demand by a Monte Carlo simulation. Sivakumar et al.

(2016) studied the uncertainties in the tube rugosity and evaluated the tube flowrate and the different pressures between two adjacent nodes. Dongre and Gupta (2017) considered uncertainties in the water demand and in the tube rugosity using fuzzy logic. Calvo et al. (2018) considered non-correlated functions of log-normal probability distributions in the management of valves. Geranmehr et al. (2019) also used a fuzzy model to evaluate uncertainties in the nodes demand in the reservoir and in the rugosity coefficient. Salcedo-Díaz et al. (2020) modeled the uncertainties in the nodes demand by a set of correlated scenarios generated by a Monte Carlo simulation, assuming a log-normal probability distribution.

Pankaj et al. (2022) used a cuckoo search optimization algorithm to solve a WDN design problem with water quality uncertainties with Monte Carlo simulations. In the same sense, Wang and Zhu (2022) developed work that considers, in addition to uncertainty in water quality, the hydraulic reliability of the network. Cunha et al. (2023) proposed a statistical methodology for generating uncertain scenarios in WDN. The authors applied it to multi-objective problems. Ucler and Kocken (2023) developed a multi-objective model considering several criteria, based on scenarios generated for the network, providing alternatives for designers. A more complete review was presented by Dandy et al. (2023), in which the authors presented a comprehensive review of the relative importance of the various uncertainties apparent in WDN projects.

In this paper, the existence of uncertainties in the demand of nodes in WDN synthesis is considered. The optimization model has an MINLP formulation and disjunctive programming is used to deal with integer variables related to tube diameter, cost, roughness coefficient, and flow direction in the tubes. The objective function is the total cost of installing the WDN and the constraints are the mass balances at the demand nodes, the energy balances in the network loops, and the velocity and pressure limits, with no additional software required for the hydraulic calculations. The model is coded in GAMS and a deterministic approach is used to solve the problem with global optimization techniques. Uncertainties in node demand are thought of as a set of correlated scenarios generated by a Monte Carlo simulation, assuming a normal probability distribution.

3 Optimization Model

The system is modeled by a set of reservoirs and demand nodes described by their expected demands, their values, and by a set of tubes with initial and final nodes, described by their lengths whose diameters are chosen from a set of commercial available diameters. Each diameter is associated with a cost per length and a specific roughness coefficient. Among the demand nodes may exist closed loops. For each demand node, there is a minimum pressure limit and the velocity in the tubes is between upper and lower limits.

WDN design is considered an optimization problem with MINLP formulation, in which the objective function to be minimized is the total cost of the network, given by the summation of installation and operation costs, subject to a set of algebraic constraints. These constraints are composed of a mass balance in each node, the pressure difference between two adjacent nodes considering the existence of loops, the equation for the volumetric flowrate in each tube, and the equation of Hazen-Williams for pressure loss calculation. These constraints form a nonlinear equations system. To further complicate the problem, there are inequalities in the velocity within the pipes and pressure limits in the demand nodes. Disjunctive programming is used to determine optimal WDN topology, with the assignment of binary variables and linear equations.

Uncertainties in water demand nodes are modeled using a finite set of scenario sampling from a probability distribution. The problem is solved in three stages. In the first stage, the problem is solved without considering the existence of uncertainties. Then, decisions made in the first stage, before considering uncertainties, are given to the project variables. In the second stage, decisions made after considering uncertainties allow to calculate operational variables. Finally, in the third stage, decisions are given by operational and project variables.

The indexes, sets, variables, and parameters are described as:

Indexes

i, j	Demand node
k	Available diameter
s	Scenario

Sets

$\mathcal{D}$	Available commercial diameters (*k*)
$\mathcal{E}_{i,j}$	Presence of a pipe between node *i* and node *j* (*i-j*)
$\mathcal{N}$	Demand nodes (*i, j*)
$\mathcal{S}$	Scenarios (*s*)

Parameters

$CostD(D_k)$	Cost per length of pipe with diameter D_k [$/m]
$d_{i,s}$	Water demand for node *i* in the scenario *s* [volume/time]
D_k	Available commercial diameter *k* [m]
e_1	Annual interest ratio [%]
$Ep_{i,j}^{min}$ and $Ep_{i,j}^{max}$	Minimum and maximum values for the pump energy in pipe *i-j* [m]
FAI	Annualization factor for the installation cost [year^{-1}]
h_i	Node *i* elevation [m]
$L_{i,j}$	Pipe *i-j* length [m]
n_a	Design lifetime [year]
P_i^{min}	Minimum pressure in node *i* [m]
$prob_s$	Probability of occurrence of scenario *s* [%]
$q_{i,j}^{min}$ and $q_{i,j}^{max}$	Minimum and maximum values for the volumetric flowrate in pipe *i-j* [m^3/s]
R_k	Rugosity coefficient in pipe with diameter D_k [non-dimensional]
$v_{i,j}^{min}$ and $v_{i,j}^{max}$	Minimum and maximum values for the velocity in pipe *i-j* [m/s]
α	Hazen-Williams numerical conversion factor [depends on the system being used]
β e γ	Hazen-Williams equation coefficients [non-dimensional]
$\Delta P_{i,j}^{min}$ and $\Delta P_{i,j}^{max}$	Minimum and maximum values for the pressure loss in the pipe *i-j* [m]

Boolean variables

$W_{i,j}^1$	True, if water flows from node *i* to node *j* or false, on the contrary
$W_{i,j}^2$	True, if water flows from node *j* to node *i* or false, on the contrary

$Y_{i,j,k}$ True, if in the pipe *i-j* diameter D_k is selected or false, on the contrary

Binary variables

$w^1_{i,j}$ 1, If water flows from node *i* to node *j* or 0, on the contrary
$w^2_{i,j}$ 1, If water flows from node *j* to node *i* or 0, on the contrary
$y_{i,j,k}$ 1, If in the pipe *i-j* diameter D_k is selected or false, on the contrary

Variables

$Cp_{i,j,s}$	Pump in the pipe *i-j* annualized operational cost in scenario *s* [$/year]
$Cost_{i,j}$	Pipe *i-j* cost [$]
$Diam_{i,j}$	Pipe *i-j* diameter [m]
$E^{pow}_{i,j,s}$	Pump energy in pipe *i-j* in scenario *s* [kW]
$Ep_{i,j,s}$	Pipe *i-j* pump in scenario *s* [m]
$Ep^1_{i,j,s}\,(Ep^2_{i,j,s})$	Equal to $Ep_{i,j,s}$ if water flows from node *i* (*j*) to node *j* (*i*) in scenario *s*
$expTAC$	Expected total annual cost [$/year]
$P_{i,s}$	Pressure in node *i* in scenario *s* [m]
$q_{i,j,s}$	Volumetric flowrate in pipe *i-j* in scenario *s* [m^3/s]
$\underline{q}^1_{i,j,s}\,(q^2_{i,j,s})$	Equal to $q_{i,j,s}$ if water flows from node *i* (*j*) to node *j* (*i*) in scenario *s*
$q_{i,j,s}$	Logarithm of $q_{i,j,s}$ in pipe *i-j* in scenario *s*
$Rug_{i,j}$	Rugosity coefficient in pipe *i-j* [nondimensional]
TAC_s	Total annual cost in scenario *s* [$/year]
$\mathrm{v}_{\mathrm{i,j,s}}$	Water velocity in pipe *i-j* [m/s]
$\underline{v}^1_{i,j,s}\,(v^2_{i,j,s})$	Equal to $v_{i,j,s}$ if water flows from node *i* (*j*) to node *j* (*i*) in scenario *s*
$\mathrm{v}_{\mathrm{i,j,s}}$	Logarithm o $v_{i,j,s}$ f in pipe *i-j* in scenario *s*
$\Delta P_{i,j,s}$	Pressure loss in pipe *i-j* in scenario *s* [m]
$\Delta \underline{P}^1_{i,j,s}\,(\Delta P^2_{i,j,s})$	Equal to $\Delta P_{i,j,s}$ if water flows from node *i* (*j*) to node *j* (*i*) in scenario *s*
$\Delta P_{i,j,s}$	Logarithm of $\Delta P_{i,j,s}$ in pipe *i-j* in scenario *s*

The WDN is evaluated by its total annual cost (TAC), given by the annual installation cost plus the annual pump energy cost. For each scenario s $\in \mathcal{S}$, a value for TAC_s is calculated and, to evaluate the performance of the WDN under uncertainties in a unique metric, the expected value for TAC is minimized, given by:

$$expTAC = \sum_{s\in\mathcal{S}} prob_s TAC_s \tag{1}$$

in this equation $prob_s$ is the inverse of the number of generated scenarios, being used the same probability of occurrence for all scenarios.

The model constraints are the algebraic equations and inequalities that must be solved in each scenario, which constitute a nonlinear system.

Mass balance in each demand node:

$$\sum_{j\in\mathcal{E}_{j,i}} \left(q^1_{j,i,s} - q^2_{j,i,s}\right) - \sum_{j\in\mathcal{E}_{i,j}} \left(q^1_{i,j,s} - q^2_{i,j,s}\right) = d_{i,s}, \forall i \in \mathcal{N} and\, s \in \mathcal{S} \tag{2}$$

Energy balance in the network loops:

$$P_{i,s} + h_i + Ep^1_{i,j,s} - Ep^2_{i,j,s} = P_{j,s} + h_j + \Delta P^1_{i,j,s} - \Delta P^2_{i,j,s}, \forall i,j \in \mathcal{E}_{i,j}\, and\, s \in \mathcal{S} \tag{3}$$

being $P^{min}_i \leq P_{i,s}$, for all $i \in \mathcal{N}$ and $s \in \mathcal{S}$.

The choice of the pipe diameter depends on its cost, rugosity coefficient, volumetric flowrate, and pressure loss, given by the Hazen-Williams equation. Logarithms can be used to linearize the last two equations. The exclusive disjunction can be thought:

$$\underset{k \in \mathcal{D}}{\veebar} \begin{bmatrix} Y_{i,j,k} \\ Diam_{i,j} = D_k \\ Cost_{i,j} = L_{i,j}\, CostD\left(D_k\right) \\ Rug_{i,j} = R_k \\ \bar{v}_{i,j,s} = \bar{q}_{i,j,s} - \ln\left(\frac{\pi}{4} D_k^2\right) \\ \Delta\bar{P}_{i,j,s} = \ln\left(\alpha\, L_{i,j}\right) + \beta\, \bar{q}_{i,j,s} - \ln\left(R_k^\beta\, D_k^\gamma\right) \end{bmatrix}, \quad \forall i,j \in \mathcal{E}_{i,j} \quad and \quad s \in \mathcal{S} \tag{4}$$

This disjunction can be written using a convex hull reformulation (Grossmann and Lee 2003). The binary variable $y_{i,j,k}$ associated to the pipe *i-j* with diameter D_k, for all $k \in \mathcal{D}$, is equal to 1 if in the pipe *i-j* the diameter D_k is selected and 0, on the contrary. In this way:

$$Diam_{i,j} = \sum_{k \in \mathcal{D}} D_k\, y_{i,j,k}, \forall i,j \in \mathcal{E}_{i,j} \tag{5}$$

$$Cost_{i,j} = \sum_{k \in \mathcal{D}} L_{i,j}\, CostD\left(D_k\right) y_{i,j,k}, \forall i,j \in \mathcal{E}_{i,j} \tag{6}$$

$$Rug_{i,j} = \sum_{k \in \mathcal{D}} R_k\, y_{i,j,k}, \forall i,j \in \mathcal{E}_{i,j} \tag{7}$$

$$\bar{v}_{i,j,s} = \bar{q}_{i,j,s} - \sum_{k \in \mathcal{D}} ln\left(\frac{\pi}{4} D_k^2\right) y_{i,j,k}, \forall i,j \in \mathcal{E}_{i,j} \text{and} s \in \mathcal{S} \tag{8}$$

$$\Delta\bar{P}_{i,j,s} = ln\left(\alpha\, L_{i,j}\right) + \beta\bar{q}_{i,j,s} - \sum_{k \in \mathcal{D}} ln\left(R_k^\beta\, D_k^\gamma\right) y_{i,j,k}, \forall i,j \in \mathcal{E}_{i,j} \text{ and } s \in \mathcal{S} \tag{9}$$

$$\sum_{k \in \mathcal{D}} y_{i,j,k} = 1, \forall i,j \in \mathcal{E}_{i,j} \tag{10}$$

The original variables can be found by exponentiation:

$$e^{\bar{v}_{i,j,s}} = v_{i,j,s}, \forall i,j \in \mathcal{E}_{i,j}\, e\, s \in \mathcal{S} \tag{11}$$

$$e^{\bar{q}_{i,j,s}} = q_{i,j,s}, \forall i,j \in \mathcal{E}_{i,j} \text{ e } s \in \mathcal{S} \tag{12}$$

$$e^{\Delta\bar{P}_{i,j,s}} = \Delta P_{i,j,s}, \forall i,j \in \mathcal{E}_{i,j} \text{ e } s \in \mathcal{S} \tag{13}$$

The flow direction in each pipe is given by the exclusive disjunction:

$$\begin{bmatrix} W^1_{i,j} \\ v_{i,j,s} = v^1_{i,j,s} \\ q_{i,j,s} = q^1_{i,j,s} \\ \Delta P_{i,j,s} = \Delta P^1_{i,j,s} \\ Ep_{i,j,s} = Ep^1_{i,j,s} \\ v^{min}_{i,j} \le v_{i,j,s} \le v^{max}_{i,j} \\ \mathrm{q}^{\mathrm{min}}_{\mathrm{i,j}} \le \mathrm{q}_{\mathrm{i,j,s}} \le \mathrm{q}^{\mathrm{max}}_{\mathrm{i,j}} \\ \Delta P^{min}_{i,j} \le \Delta P_{i,j,s} \le \Delta P^{max}_{i,j} \\ Ep^{min}_{i,j} \le Ep_{i,j,s} \le Ep^{max}_{i,j} \end{bmatrix} \underline{\vee} \begin{bmatrix} W^2_{i,j} \\ v_{i,j,s} = v^2_{i,j,s} \\ q_{i,j,s} = q^2_{i,j,s} \\ \Delta P_{i,j,s} = \Delta P^2_{i,j,s} \\ Ep_{i,j,s} = Ep^2_{i,j,s} \\ v^{min}_{i,j} \le v_{i,j,s} \le v^{max}_{i,j} \\ q^{min}_{i,j} \le q_{i,j,s} \le q^{max}_{i,j} \\ \Delta P^{min}_{i,j} \le \Delta P_{i,j,s} \le \Delta P^{max}_{i,j} \\ Ep^{min}_{i,j} \le Ep_{i,j,s} \le Ep^{max}_{i,j} \end{bmatrix}, \forall i,j \in \mathcal{E}_{i,j} \text{ and } s \in \mathcal{S} \tag{14}$$

This disjunction can be written using a convex hull reformulation. The binary variable $w^1_{i,j}$ is equal to 1 if water flows from node i to node j and 0, on the contrary, and the binary variable $w^2_{i,j}$ is equal to 1 if water flows from node j to node i e 0, on the contrary. In this way:

$$v_{i,j,s} = v^1_{i,j,s} + v^2_{i,j,s}, \forall i,j \in \mathcal{E}_{i,j} \text{ and } s \in \mathcal{S} \tag{15}$$

$$v^{min}_{i,j} w^1_{i,j} \le v^1_{i,j,s} \le v^{max}_{i,j} w^1_{i,j}, \forall i,j \in \mathcal{E}_{i,j} \text{ and } s \in \mathcal{S} \tag{16}$$

$$v^{min}_{i,j} w^2_{i,j} \le v^2_{i,j,s} \le v^{max}_{i,j} w^2_{i,j}, \forall i,j \in \mathcal{E}_{i,j} \text{ and } s \in \mathcal{S} \tag{17}$$

$$q_{i,j,s} = q^1_{i,j,s} + q^2_{i,j,s}, \forall i,j \in \mathcal{E}_{i,j} \text{ and } s \in \mathcal{S} \tag{18}$$

$$q^{min}_{i,j} w^1_{i,j} \le q^1_{i,j,s} \le q^{max}_{i,j} w^1_{i,j}, \forall i,j \in \mathcal{E}_{i,j} \text{ and } s \in \mathcal{S} \tag{19}$$

$$q^{min}_{i,j} w^2_{i,j} \le q^2_{i,j,s} \le q^{max}_{i,j} w^2_{i,j}, \forall i,j \in \mathcal{E}_{i,j} \text{ and } s \in \mathcal{S} \tag{20}$$

$$\Delta P_{i,j,s} = \Delta P^1_{i,j,s} + \Delta P^2_{i,j,s}, \forall i,j \in \mathcal{E}_{i,j} \text{ and } s \in \mathcal{S} \tag{21}$$

$$\Delta P^{min}_{i,j} w^1_{i,j} \le \Delta P^1_{i,j,s} \le \Delta P^{max}_{i,j} w^1_{i,j}, \forall i,j \in \mathcal{E}_{i,j} \text{ and } s \in \mathcal{S} \tag{22}$$

$$\Delta P^{min}_{i,j} w^2_{i,j} \le \Delta P^2_{i,j,s} \le \Delta P^{max}_{i,j} w^2_{i,j}, \forall i,j \in \mathcal{E}_{i,j} \text{ and } s \in \mathcal{S} \tag{23}$$

$$Ep_{i,j,s} = Ep^1_{i,j,s} + Ep^2_{i,j,s}, \forall i,j \in \mathcal{E}_{i,j} \text{ and } s \in \mathcal{S} \tag{24}$$

$$Ep^{min}_{i,j} w^1_{i,j} \le Ep^1_{i,j,s} \le Ep^{max}_{i,j} w^1_{i,j}, \forall i,j \in \mathcal{E}_{i,j} \text{ and } s \in \mathcal{S} \tag{25}$$

$$Ep^{min}_{i,j} w^2_{i,j} \le Ep^2_{i,j,s} \le Ep^{max}_{i,j} w^2_{i,j}, \forall i,j \in \mathcal{E}_{i,j} \text{ and } s \in \mathcal{S} \tag{26}$$

$$w^1_{i,j} + w^2_{i,j} = 1, \forall i,j \in \mathcal{E}_{i,j} \tag{27}$$

The uncertainties in the demand nodes imply in the use of pumps which could be necessary to satisfy the water demands in the network, depending on the scenario being evaluated. The pump energy located in the pipe *i-j* in scenario *s* and the pump annualized operation cost are given by:

$$E^{pow}_{i,j,s} = \frac{9.81}{0.82} q_{i,j,s}\, Ep_{i,j,s},\ \forall i,j \in \mathcal{E}_{i,j} \text{ and } s \in \mathcal{S} \tag{28}$$

$$Cp_{i,j,s} = 0.24 \cdot 8{,}000\, E^{pow}_{i,j,s},\ \forall i,j \in \mathcal{E}_{i,j} \text{ and } s \in \mathcal{S} \tag{29}$$

The pump efficiency is 0.82, the energy cost per kWh is 0.24 and the number of total hours of pumping is 8,000.

The total annual cost in each scenario is composed by the pipe installation annual cost and by the pumps annual operation cost, given by:

$$TAC_s = \sum_{i,j \in \mathcal{E}_{i,j}} \left(FAI \cdot Cost_{i,j} + Cp_{i,j,s} \right),\ \forall s \in \mathcal{S} \tag{30}$$

The annualization factor of the installation cost for the extension of the WDN design in n_a years, subject the annual interest rate e_1, is:

$$FAI = \frac{e_1 \left(1 + e_1 \right)^{n_a}}{\left(1 + e_1 \right)^{n_a} - 1} \tag{31}$$

As a result, the complete optimization model for the WDN synthesis considering uncertainties in the demand nodes and unknown flow directions is:

$$\min \sum_{s \in \mathcal{S}} prob_s \cdot TAC_s$$

$$s.a \sum_{j \in \mathcal{E}_{j,i}} \left(q^1_{j,i,s} - q^2_{j,i,s} \right) - \sum_{j \in \mathcal{E}_{i,j}} \left(q^1_{i,j,s} - q^2_{i,j,s} \right) = d_{i,s},\ \forall \mathrm{i} \in \mathcal{N} \text{ and s} \in \mathcal{S}$$

$$P_{i,s} + h_i + Ep^1_{i,j,s} - Ep^2_{i,j,s} = P_{j,s} + h_j + \Delta P^1_{i,j,s} - \Delta P^2_{i,j,s},\ \forall \mathrm{i,j} \in \mathcal{E}_{i,j} \text{ and s} \in \mathcal{S}$$

$$Diam_{i,j} = \sum_{k \in \mathcal{D}} D_k\, y_{i,j,k},\ \forall i,j \in \mathcal{E}_{i,j}$$

$$Cost_{i,j} = \sum_{k \in \mathcal{D}} L_{i,j}\, CostD\left(D_k\right) y_{i,j,k},\ \forall i,j \in \mathcal{E}_{i,j}$$

$$Rug_{i,j} = \sum_{k \in \mathcal{D}} R_k\, y_{i,j,k},\ \forall i,j \in \mathcal{E}_{i,j}$$

$$\bar{v}_{i,j,s} = \bar{q}_{\mathrm{i,j,s}} - \sum_{k \in \mathcal{D}} ln\left(\frac{\pi}{4} D^2_k \right) y_{i,j,k},\ \forall i,j \in \mathcal{E}_{i,j} \text{ and } s \in \mathcal{S}$$

$$\Delta\bar{P}_{i,j,s} = ln\left(\alpha L_{i,j}\right) + \beta\bar{q}_{i,j,s} - \sum_{k\in\mathcal{D}} ln\left(R_k^{\beta} D_k^{\gamma}\right) y_{i,j,k},\ \forall i,j \in\ \mathcal{E}_{i,j} \text{ and } s \in\ \mathcal{S}$$

$$\sum_{k\in\mathcal{D}} y_{i,j,k} = 1,\ \forall i,j \in\ \mathcal{E}_{i,j}$$

$$Diam_{i,j} = \sum_{k\in\mathcal{D}} D_k\, y_{i,j,k}, \forall i,j \in \mathcal{E}_{i,j}$$

$$e^{\bar{v}_{i,j,s}} = v_{i,j,s},\ \forall\, i,j \in\ \mathcal{E}_{i,j} \text{ e } s \in\ \mathcal{S}$$

$$e^{\bar{q}_{i,j,s}} = q_{i,j,s},\ \forall\, i,j \in\ \mathcal{E}_{i,j} \text{ e } s \in\ \mathcal{S}$$

$$e^{\Delta\bar{\mathrm{P}}_{\mathrm{i,j,s}}} = \Delta\mathrm{P}_{\mathrm{i,j,s}},\ \forall\, i,j \in\ \mathcal{E}_{i,j} \text{ e } s \in\ \mathcal{S}$$

$$P_i^{min} \leq\ P_{i,s},\ \forall\, i \in\ \mathcal{N} \text{ and } s \in\ \mathcal{S}$$

$$E_{i,j,s}^{pow} = \frac{9.81}{0.82} q_{i,j,s}\, Ep_{i,j,s},\ \forall\, i,j \in\ \mathcal{E}_{i,j} \text{ and } s \in\ \mathcal{S}$$

$$Cp_{i,j,s} = 0.24 \cdot 8{,}000\, E_{i,j,s}^{pow},\ \forall\, i,j \in\ \mathcal{E}_{i,j} \text{ and } s \in\ \mathcal{S}$$

$$TAC_s = \sum_{i,j\in\mathcal{E}_{i,j}} \left(FAI \cdot Cost_{i,j} + Cp_{i,j,s}\right), \forall s \in \mathcal{S}$$

$$v_{\mathrm{i,j,s}} = v_{i,j,s}^{1} + v_{i,j,s}^{2},\ \forall\, i,j \in\ \mathcal{E}_{i,j} \text{ and } s \in\ \mathcal{S}$$

$$v_{i,j}^{min}\, w_{i,j}^{1} \leq\ v_{i,j,s}^{1} \leq\ v_{i,j}^{max}\, w_{i,j}^{1},\ \forall\, i,j \in\ \mathcal{E}_{i,j} \text{ and } s \in\ \mathcal{S}$$

$$v_{i,j}^{min}\, w_{i,j}^{2} \leq\ v_{i,j,s}^{2} \leq\ v_{i,j}^{max}\, w_{i,j}^{2},\ \forall\, i,j \in\ \mathcal{E}_{i,j} \text{ and } s \in\ \mathcal{S}$$

$$q_{i,j,s} = q_{i,j,s}^{1} + q_{i,j,s}^{2},\ \forall\, i,j \in\ \mathcal{E}_{i,j} \text{ and } s \in\ \mathcal{S}$$

$$q_{i,j}^{min}\, w_{i,j}^{1} \leq\ q_{i,j,s}^{1} \leq\ q_{i,j}^{max}\, w_{i,j}^{1},\ \forall\, i,j \in\ \mathcal{E}_{i,j} \text{ and } s \in\ \mathcal{S}$$

$$q_{i,j}^{min}\, w_{i,j}^{2} \leq\ q_{i,j,s}^{2} \leq\ q_{i,j}^{max}\, w_{i,j}^{2},\ \forall\, i,j \in\ \mathcal{E}_{i,j} \text{ and } s \in\ \mathcal{S}$$

$$\Delta P_{i,j,s} = \Delta P_{i,j,s}^{1} + \Delta P_{i,j,s}^{2},\ \forall\, i,j \in\ \mathcal{E}_{i,j} \text{ and } s \in\ \mathcal{S}$$

$$\Delta P_{i,j}^{min}\, w_{i,j}^{1} \leq \Delta P_{i,j,s}^{1} \leq \Delta P_{i,j}^{max}\, w_{i,j}^{1}, \;\forall\, i,j \in \mathcal{E}_{i,j} \text{ and } s \in \mathcal{S}$$

$$\Delta P_{i,j}^{min}\, w_{i,j}^{2} \leq \Delta P_{i,j,s}^{2} \leq \Delta P_{i,j}^{max}\, w_{i,j}^{2}, \;\forall\, i,j \in \mathcal{E}_{i,j} \text{ and } s \in \mathcal{S}$$

$$Ep_{i,j,s} = Ep_{i,j,s}^{1} + Ep_{i,j,s}^{2}, \;\forall\, i,j \in \mathcal{E}_{i,j} \text{ and } s \in \mathcal{S}$$

$$Ep_{i,j}^{min}\, w_{i,j}^{1} \leq Ep_{i,j,s}^{1} \leq Ep_{i,j}^{max}\, w_{i,j}^{1}, \;\forall\, i,j \in \mathcal{E}_{i,j} \text{ and } s \in \mathcal{S}$$

$$Ep_{i,j}^{min}\, w_{i,j}^{2} \leq Ep_{i,j,s}^{2} \leq Ep_{i,j}^{max}\, w_{i,j}^{2}, \;\forall\, i,j \in \mathcal{E}_{i,j} \text{ and } s \in \mathcal{S}$$

$$w_{i,j}^{1} + w_{i,j}^{2} = 1, \;\forall\, i,j \in \mathcal{E}_{i,j}$$

The three-stages procedure for considering uncertainties in WDN nodes demand can be described as:

(i) In the first step, the expected TAC is minimized with a probability of occurrence of 100% for the original scenario. In this step, the optimum design is achieved by considering a unique nominal value for each uncertain parameter. In this case, the annual pumping cost is zero.
(ii) In the second step the expected TAC with probability of occurrence in all scenarios is minimized but with fixed values for the discrete variables, given by the solution obtained in the first step. In this case, the annual installation cost is fixed. The solution found in this stage is called deterministic solution.
(iii) In the third stage the TAC is minimized with probability of occurrence in all scenarios with no fixed values for the variables. In this way, the optimal design considering uncertainties in the nodes demand is achieved. The solution found in this stage is called stochastic solution.

4 Case Studies

To test the applicability of the developed optimization model for the synthesis of WDN considering uncertainties in the nodes demand, two case studies from the literature were used. In both cases, the parameters considered for the Hazen-Williams equation are $\alpha = 10.667, \beta = 1.852$ and$\gamma = 4.871$. The scenarios were generated by using a normal distribution in Excel. The model was coded in GAMS and the problem was solved using the BARON global optimization solver. This solver was used because it is suitable for solving problems with nonlinear programming formulation and mixed nonlinear programming with integer variables, as it implements deterministic global optimization techniques with Branch and Bound algorithms. The lifetime of the WDN design is 20 years and the interest rate is 5% per year. Hydraulic pumps are considered if it is necessary to attend the demand in some node in each one of the generated scenarios.

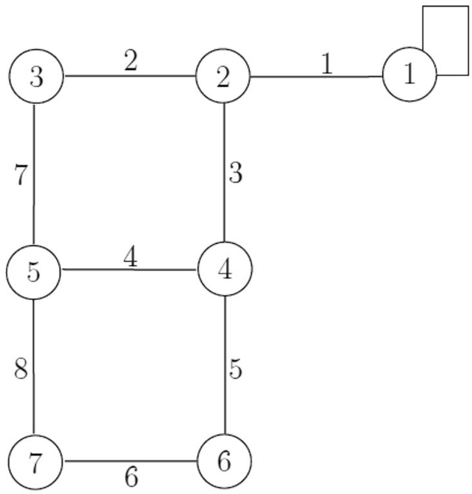

Fig. 1 Two loop WDN, the lines are the tubes, and the circles are the nodes

5 Case Study 1

The first benchmark problem is known as Two loop WDN and is presented in Fig. 1. The network has 8 pipes, all 1000 m long, linking 7 nodes demand, whose elevations are 210, 150, 160, 155, 150, 165 and 160 m, and 2 loops involving the nodes. The reservoir is considered the first node, and its demand is the summation of all other nodes. The water velocity must be bounded by 0.3 and 3 m/s and the minimum acceptable pressure for all nodes is 30 m. The pipe diameter is selected from a set of available commercial diameters, given in Table 1. The rugosity coefficient is 130 for all pipes.

For this network, 30 scenarios were generated, with expected values and standard deviation for the water demand in each one of the nodes given in Table 2. The problem has 4435 variables, being 128 discrete variables. The deterministic solutions found by fixing the design variables, according to the problem solution without assuming variations in the demand nodes are compared with the stochastic solution obtained without fixing the variables, following the solution of the problem assuming the variation of water demand in the nodes.

Table 3 presents results achieved by the diameters and pipe flow directions and the expected total annual cost, according to each solution. Results show that the WDN design by the stochastic approach reduces the expected TAC by more than 2.6% in comparison to the deterministic solution. As the annual installation cost is greater in the stochastic solution, the reduction in the expected TAC is due to the necessity of using pumps to satisfy

Table 1 Available diameters for the Two loop WDN

Diameter (m)	Cost ($/m)	Diameter (m)	Cost ($/m)
0.0254	2	0.3048	50
0.0508	5	0.3556	60
0.0762	8	0.4064	90
0.1016	11	0.4572	130
0.1524	16	0.5080	170
0.2032	23	0.5588	300
0.2540	32	0.6096	550

Table 2 Water demand in the nodes for the Two loop WDN

Water demand (m^3/h)	Node 2	Node 3	Node 4	Node 5	Node 6	Node 7
Expected value	100	100	120	270	330	200
Standard deviation	4.88	3.92	8.13	18.29	31.52	15.42

the nodes demand in some scenarios in the deterministic design. Table 3 presents the differences between the stochastic and deterministic solutions in some pipe diameters. It is related to the water velocity in the pipes and to the use of energy in piping. To satisfy the necessary water demand, the greater the selected diameter the greater the water velocity, implying in more energy use.

6 Case Study 2

The second case study considered a real WDN located in Brazil, named Grande Setor WDN. Figure 2 presents the network topology, with 8 pipes, whose lengths are 2540, 1230, 1430, 1300, 1490, 1210, 1460, 1190 m, linking 7 demand nodes, whose elevations are 6, 5.5, 5.5, 6, 4.5 and 4 m, with 2 loops and a water reservoir, represented by the seventh node. Its demand is given as the summation of all other nodes. The water velocity must be between 0.2 and 3 m/s and the acceptable minimum pressure is 25 m for all nodes. The pipe diameters are selected from a set of available commercial diameters, given in Table 4.

In this case study, 20 scenarios were generated with the expected values and the standard deviation for the water demand in each node presented in Table 5. The problem has 2981 variables, 96 being discrete variables. As in case study 1, the deterministic solution obtained fixing the design variables, according to the problem solution without assuming variability in the water nodes demand is compared with the stochastic solution obtained without fixing the variables, following the problem solution assuming the variation in the nodes demand.

Table 6 presents the results obtained for the diameters and the pipe flow directions and the expected total annual cost, according to each solution. Results shown that the stochastic solution reduce the TAC in 1.2% when compared to the deterministic solution. It is due to the need of using pumping to satisfy the water nodes demand in some scenarios in the deterministic solution. As the installation cost is greater in the stochastic solution, the

Table 3 Results for the Two loop WDN

Pipe	Deterministic solution	Stochastic solution
1	0.4572 (1–2)	0.4572 (1–2)
2	0.2540 (2–3)	0.3556 (2–3)
3	0.4064 (2–4)	0.3556 (2–4)
4	0.1016 (4–5)	0.0254 (4–5)
5	0.4064 (4–6)	0.3556 (4–6)
6	0.2540 (6–7)	0.0254 (6–7)
7	0.2540 (3–5)	0.3556 (3–5)
8	0.0254 (7–5)	0.3048 (5–7)
expTAC($/year)	35,757.05	34,798.27

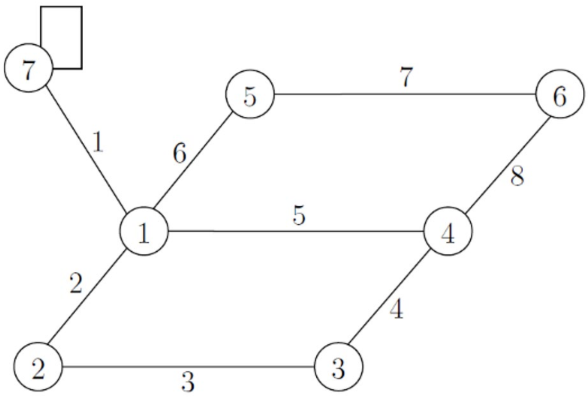

Fig. 2 Grande Setor WDN

Table 4 Available diameters for the Modified Grande Setor WDN

Diameter (m)	Cost (US$/m)	Rugosity coefficient	Diameter (m)	Cost (US$/m)	Rugosity coefficient
0.1084	23.55	145	0.3662	158.93	130
0.1564	31.90	145	0.4164	187.50	130
0.2042	43.81	145	0.4666	218.12	130
0.2520	59.30	145	0.5180	257.80	130
0.2998	76.12	145	0.6196	320.15	130

Table 5 Water demand for the Modified Grande Setor WDN

Water demand (L/s)	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6
Expected value	84.29	47.78	80.32	208.60	43.44	40.29
Standard deviation	8.05	6.85	11.28	19.93	3.35	7.41

Table 6 Results for the Modified Grande Setor WDN

Pipe	Deterministic solution	Stochastic solution
1	0.6196 (7–1)	0.6196 (7–1)
2	0.2520 (1–2)	0.2998 (1–2)
3	0.1084 (2–3)	0.2042 (2–3)
4	0.2998 (4–3)	0.2998 (4–3)
5	0.6196 (1–4)	0.6196 (1–4)
6	0.2520 (1–5)	0.2520 (1–5)
7	0.1084 (5–6)	0.1084 (5–6)
8	0.2520 (4–6)	0.2520 (4–6)
expTAC (US$/year)	150,877.70	148,942.16

reduction in the expected TAC is due to the pumping system to satisfy the nodes demand in some scenarios of the deterministic solution.

Some existing works in the literature also covered uncertainties in nodal demands in the WDN project. The objective presented in Branisavljevic et al. (2009) was to reduce nodal uncertainties using flow data in modeling the problem. Dongre and Gupta (2017) considered uncertainties in water demand and pipe roughness to consider various levels of pressure acceptance. Salcedo-Díaz et al. (2020) obtained a result similar to that provided here, but the authors did not use the flow direction as a variable in the problem. Differently, this work proposes a deterministic method to highlight that not considering such uncertainties can lead to suboptimal results. In general, everyone agrees that the WDN project must consider uncertainties, even if each one approached them in a specific way.

The proposed model can be used for solving large scale problems. However, deterministic approaches to solve the model, like the one used in the paper, cannot be effective in all cases. The calculation time spent in solving the model can also be high.

7 Conclusions

In this article, a MINLP formulation optimization model was developed using disjunctive mathematical programming for the synthesis of WDN, considering uncertainties in the demand of nodes and unknown flow directions. The main objective was to minimize the total annual cost, consisting of installation cost and the annual cost of pumping energy. The variation of water demand was modeled as a set of finite scenarios generated from a normal distribution in Excel. The model was implemented in GAMS and the global BARON optimization solver was used.

The problem was solved in three stages. First, only a unique nominal value for each uncertain parameter was used. Second, for all scenarios, in which the independent scenarios variables are fixed to the solution obtained in the first step (the deterministic solution). Third, for all scenarios, no variable is fixed (the stochastic solution). Two case studies were used to test the applicability of the developed model and results showed that under uncertainties, the stochastic solution improve the deterministic one.

The WDN optimization problem combined with uncertainties in node demands and unknown flow directions is a new but non-trivial problem due to the inherent complexity of the system's nonlinearity. A finite set of applications of the developed model is necessary according to the number of scenarios generated for WDN evaluation. For large-scale problems, the model developed can be effective, but the fact that it is deterministic contributes to delays in solving the problem and even to non-convergence to an appropriate optimum.

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Declarations

Competing Interests The authors have no relevant financial or non-financial conflicts of interest to disclose.

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