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The relativistic effect of critical temperature reduction in the two dimensional Ising model

O efeito relativístico da redução crítica de temperatura no modelo bidimensional de Ising

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Rafael Santos da Costa
Graduando do Curso de Licenciatura em Matemática pela Universidade Federal do Pará
Instituição: Universidade Federal do Pará, Faculdade de Matemática - UFPA
Endereço: Rua Raimundo Santana Cruz, s/n, Bairro São Tomé, Salinópolis – PA, Brasil
E-mail: rafael_santos723@outlook.com

Jair Rodrigues Neyra
Graduando do Curso de Engenharia de Exploração e Produção de Petróleo pela Universidade Federal do Pará
Instituição: Universidade Federal do Pará, Faculdade de Engenharia - UFPA
Endereço: Rua Raimundo Santana Cruz, s/n, Bairro São Tomé, Salinópolis – PA, Brasil
E-mail: jairneyra30@gmail.com

Maria Liduína das Chagas
Doutora em Física Pela Universidade Federal do Rio Grande do Norte
Instituição: Universidade Federal do Sul e Sudeste do Pará, Faculdade de Física - Instituto de Ciências Exatas
Marabá – PA
Endereço: Cidade Universitária Campus Do Tauarizinho, Marabá-PA, Brasil
E-mail: liduina@unifesspa.edu.br

Maria das Graças Dias da Silva
Doutora em Física Pela Universidade Federal do Rio Grande do Norte
Instituição: Universidade do Estado do Rio Grande do Norte, Faculdade de Ciências Exatas e Naturais - UERN
Mossoró – RN
Endereço: Rua Almino, 478 – Centro – Mossoró/RN
E-mail: gracinha_dias@hotmail.com

Thiago Crisóstomo Carlos Nunes
Doutor em Física Pela Universidade Federal do Rio Grande do Norte
Instituição: Universidade Federal do Rio Grande do Norte
Endereço: Avenida Senador Salgado Filho, 3000, Bairro Lagoa Nova, Natal – RN, Brasil
E-mail: thiago.cris@yahoo.com.br

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Thiago Rafael da Silva Moura
Doutor em Física Pela Universidade Federal do Rio Grande do Norte
Instituição: Universidade Federal do Pará, Faculdade de Física - UFPA
Endereço: Rua Raimundo Santana Cruz, s/n, Bairro São Tomé, Salinópolis – PA, Brasil
E-mail: trsmoura@yahoo.com.br

ABSTRACT
We investigated the impact of Kaniadakis statistics on thermodynamic properties for a square magnetic grid. We used the Ising model. We reported numerical results for a two-dimensional magnetic network in a thermal bath. We calculated the probabilities of transition between states using κ-statistics, the Metropolis dynamics for stochastic processes of the finite-sized magnetic network, considering that the system interacts with a thermal reservoir. We investigated the behavior of various thermodynamic properties. We observed typical measurements of magnetization, energy and specific heat. Increasing the κ parameter reduces the critical temperature. We observed by the measurements of the fourth order Binder cumulative of magnetization, that for different network sizes and different values of the parameter κ, the system transition temperature magnetic decreases as κ increases.

Keywords: κ-exponential, Ising model, phase transitions.

RESUMO
Investigamos o impacto das estatísticas Kaniadakis nas propriedades termodinâmicas de uma grade magnética quadrada. Utilizamos o modelo de Ising. Relatamos resultados numéricos para uma rede magnética bidimensional em banho termal. Calculamos as probabilidades de transição entre estados usando a estatística k, a dinâmica Metropolis para processos estocásticos da rede magnética de tamanho finito, considerando que o sistema interage com um reservatório térmico. Investigamos o comportamento de várias propriedades termodinâmicas. Observamos medidas típicas de magnetização, energia e calor específico. Aumentar o parâmetro κ reduz a temperatura crítica. Observamos pelas medições do Fichário de quarta ordem cumulativo de magnetização, que para diferentes tamanhos de rede e diferentes valores do parâmetro κ, a temperatura de transição do sistema magnético diminui à medida que κ aumenta.

Palavras-chave: κ-exponencial, modelo de Ising, transições de fase.

1 INTRODUCTION

The area of phase transitions and critical phenomena is one of the central themes in Physics. This type of study is carried out in several physical systems: fluids, liquid crystals, metal alloys, spin glasses, magnetic materials, piezoelectric, ferroelectric materials, biological systems, etc. Magnetic systems are of particular interest for the potential of biomedical applications in magnetic hyperthermia and for the construction of new technological devices such as: magnetic memories, nano-antennas, magnetic tunneling junctions, etc. [1-8] The Ising model is a prototype model, proposed to provide an explanation of the phase transition of a magnetic system. The Ising model consists of N spins, subjected to short-range interactions, spread over the sites of a two-dimensional network.

Since Gibbs proposed the first generalization of Maxwell-Boltzmann's entropy, many other generalizations, whether classical or quantum, have emerged within the context of Statistical Mechanics, among these several we mentioned Tsalis [9] and Kaniadakis [10], [11]. A common point between these new entropies is the fact that these distributions, which form the basis of such generalizations, depend on some deforming parameter.

In 2001, G. Kaniadakis proposed a new statistic, the κ-statistic, which generalizes the Maxwell-Boltzmann-Gibbs statistic by varying the deformation parameter κ. The theory is guided by the principle of kinetic interaction (PKI). According to PKI, the temporal evolution of the function of distribution of identical particles subject to binary collisions leads us to a functional that is always increasing over time, satisfying the statement of the irreversibility of the second law of Thermodynamics. Kaniadakis concluded that such a function was related to a type of entropy defined as:

$$S_\kappa = -\langle ln_\kappa[f(x)]\rangle = -\int dx\, f(x) ln_\kappa[f(x)] \tag{1}$$

in which $f(x)$ is the velocity distribution of the particles and ln_κ is the logarithm deformed by the parameter κ. The ln_κ is a real and decreasing function $\forall x \in R$ given by:

$$ln_\kappa(x) = \frac{x^\kappa - x^{-\kappa}}{2\kappa} \tag{2}$$

its inverse function is called κ-exponential [12]. It is quantified by

$$exp_\kappa(\pm x) = \left(\sqrt{1+\kappa^2 x^2} \pm \kappa x\right)^{\frac{1}{\kappa}} \tag{3}$$

which is set for the interval $0 \leqslant x < 1$. When we take the limit of $\kappa \to 0$, the function that is κ-exponential becomes an ordinary exponential.

$$\lim_{\kappa \to 0} exp_\kappa\,(\pm x) = exp(\pm x) \tag{4}$$

2 THE THEORETICAL MODEL

The model deals with a network of short-range interactions between spins, described by the following Hamiltonian

$$E = -J\sum_i \sum_j \hat{\sigma}_i \cdot \hat{\sigma}_j + \vec{H}.\sum_j \hat{\sigma}_j \quad (5)$$

J is the exchange constant, $\vec{H}$ the applied magnetic field and $\{ \hat{\sigma} = \pm 1 \}$ is the set of spins in the network. The first term of equation (5) describes how the exchange interaction occurs between adjacent spins, the pair $\langle i, j\rangle$. The second term of equation (5) Zeeman energy, where $\vec{H} = H_x \hat{t}$ is the magnetic field applied along the axis $\hat{x}$.

Magnetization is defined as the sum of all spins in the network,

$$\langle M \rangle = \frac{1}{N}\sum_{i=1}^{N} \sigma_i \quad (6)$$

Deforming the Boltzmann distribution function in terms of κ-statistics, the probability that a given specific state will occur in our model is

$$W_\kappa(\alpha) = \frac{1}{Z} exp_\kappa(-\beta E_\alpha) \quad (7)$$

in which $\beta = \frac{1}{K_B T}$ and Z is the system partition function, which can be written as

$$Z = \sum_{i=1}^{N} exp_\kappa\,(\beta E_i) \quad (8)$$

in which K_B is the Boltzmann constant and T is the absolute temperature.

2.1 NUMERICAL EXPERIMENTATIONS

We proceeded with numerical experiments to obtain the measurements of the physical observables. We estimated the observables concerning the solutions of the master equation. The master equation is

$$\frac{dP(i,j)}{dt} = \sum_{i \neq j}\{W(i,j)P(j,t) - W(j,i)P(i,t)\} \quad (9)$$

in which $W(i,j)$ is the state's probability transition rate i for j and $P(j,t)$ is the state probability j in the instant of time: t. Given the transition rates, we found the stationary solution that satisfies the condition:

$$\sum_{i \neq j}\{W(i,j)P(j,t) - W(j,i)P(i,t)\} = 0 \quad (10)$$

The numerical solution of the master equation for the equilibrium state equation (10) served as the basis for the development of the Metropolis algorithm [13].

The Metropolis algorithm is summarized as follows:

1. Configure an initial j configuration for the system;
2. Generate tentative configuration of the j-th simulation cell;
3. If $\Delta E_{j,i} = E_j - E_i < 0$, the configuration i is accepted;
4. Otherwise, a random number is generated $\delta \in (0,1)$. If $\delta < P(\Delta E_{j,i}) \propto exp_\kappa(\Delta E_{i,j}/K_B T)$ the new configuration is accepted.
5. Otherwise, the j configuration remains.

To obtain our numerical results we worked with reduced constants $J = 1$ and $K_B = 1$. The exchange interaction occurs between the first neighbors of a variable on a network site. Naturally, the so-called edge effect arises, a problem that we overcome with the use of periodic boundary conditions, where a variable contained in one edge interacts, according to equation (5), with another located in an opposite edge. We defined the initial state of the magnetic grid with all spins aligned in the same direction, for instance, they are coupled ferromagnetically. This type of coupling is characteristic of materials in the ferromagnetic state. Starting from this initial network, we implemented the Metropolis algorithm, we will not describe the algorithm here, but the interested reader can consult the reference [14]. For the quantities $\langle E \rangle$ and $\langle m \rangle$ we have the following relationships:

$$\langle E \rangle = \frac{1}{N} \sum_{i=1}^{N} E_i \tag{11}$$

$$\langle m \rangle = \frac{1}{N} \sum_{i=1}^{N} M_i \tag{12}$$

Specific heat at a constant volume, C_v, is proportional to the energy variance. According to reference [15], we have:

$$C_v = \frac{1}{N K_B T^2} [\langle E^2 \rangle - \langle E \rangle^2] \tag{13}$$

Besides, we measured the fourth order Binder cumulative of magnetization [14].

$$U_4(M) = 1 - \frac{\langle M^4 \rangle}{3\langle M^2 \rangle^2} \quad (14)$$

3 RESULTS

We carried out numerical experiments on networks with lateral dimensions of the order of equivalent to $L = 4,8,14,16$ and 24, using 10^6 Monte Carlo steps. We discarded $2.5x10^4$, starting the accounting of physical observables after reaching statistical stability. In figure 1, we presented typical measurements of magnetization and Helmholtz free κenergy.

We observed that the average magnetization reaches the transition temperature more quickly with increasing κ. The expected value of energy reaches its saturation point, more quickly, the higher the quantitative values of κ.

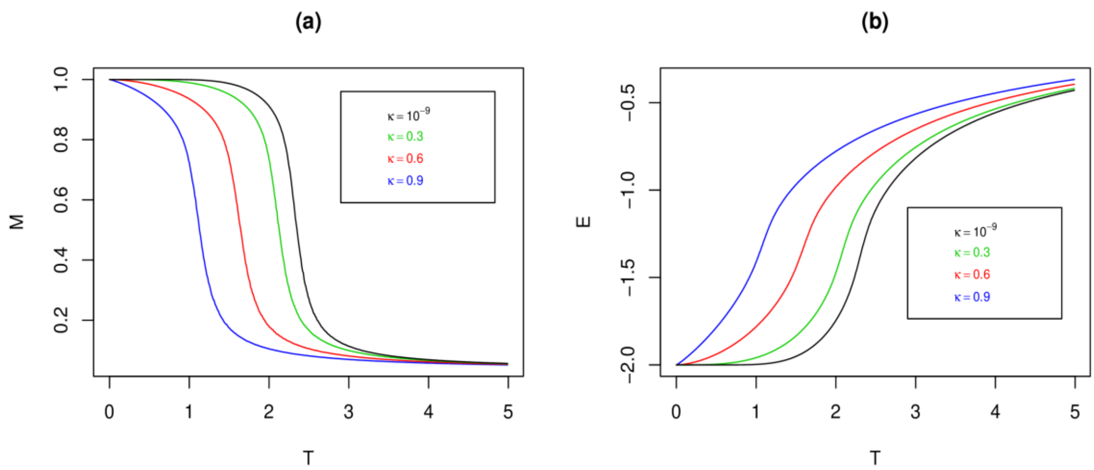

Figure 1 - In (a) we have the average magnetization per particle. In (b) the average energy per particle. In both, we observed that as κ increases, the phase transition occurs at an increasingly lower temperature. The curve for $\kappa = 10^{-9}$ in the two graphs, confirm that at the limit: $\kappa \rightarrow 0$ our model falls back to the classic Ising model.

In figure 2, we show the results of the specific heat for different values of κ. We noted that as κ grows, the second derivative of energy, has maximums closer to $T = 0$. The results shown in figures 1 and 2, show the shift in transition temperature T_C as κ increases.

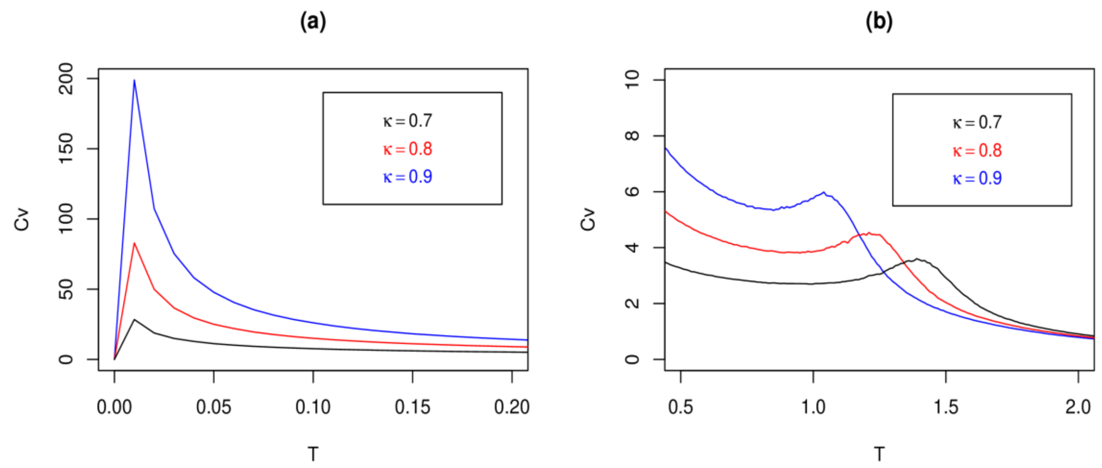

Figure 2 - In (a) we observed the presence of peaks occurring around a region close to $T = 0$. In (b) observed that in other region there are other minor peaks, which in turn are related to T_C described by the Binder Cumulative $U_4(M)$.

Figure 3 shows typical Binder Cumulative measurements for $\kappa = 10^{-9}$ and $\kappa = 0.9$, with different network sizes, depending on the temperature. Square side nets were used $L = 4,8,14,16$ and 24. Throughout the cumulant measurements, the region where the lines cross is the critical point measurement region, which is $T_C = 2.27$ for $\kappa = 10^{-9}$ and $T_C = 1.07$ for $\kappa = 0.9$.

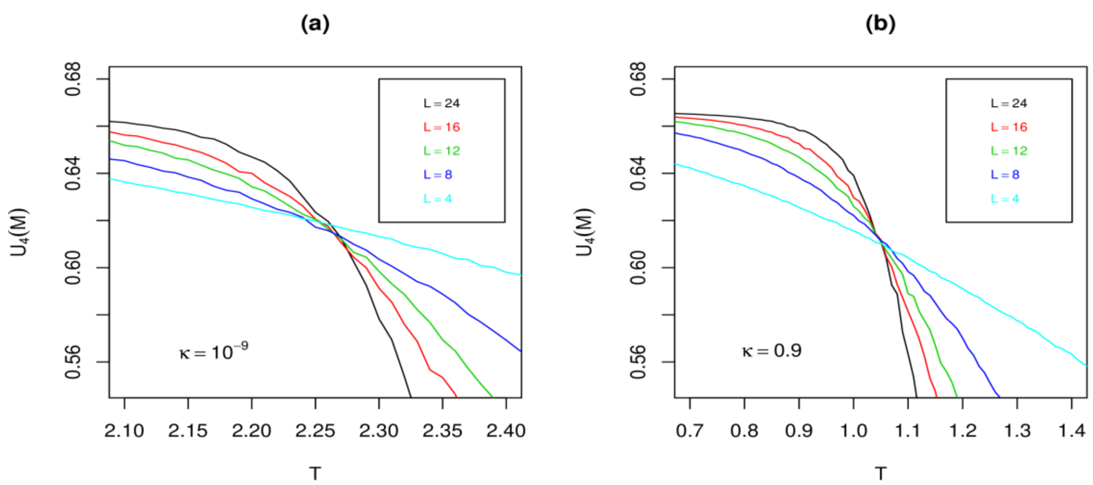

Figure 3 - Typical measurements of the fourth order Binder Cumulative for magnetization $U_4(M)$ as a function of temperature T for different network sizes, in (a) with $\kappa = 10^{-9}$, we observed that $T_C = 2.27$, in (b) we observed that with the increase of the parameter, it has a reduction in T_C , because taking $\kappa = 0.9$ we observed $T_C = 1.07$.

4 CONCLUSION

We studied a magnetic system, square network, in thermal bath, with the Hamiltonian of Ising. We used κ-exponential distribution, generalization of Boltzmann statistics, to measure typical physical observables. We analyzed the effect of the disturbance caused by the parameter κ on the system, we observed that with the reduction ofκ our model tries to fall back on the classic Ising model. Still, we obtained that with the increase of the parameterκ, T_C is reduced, leading to the transition between the ferromagnetic and paramagnetic phases occurring at lower temperatures. We estimated the critical point related to each value of the parameterκ, using the fourth order Binder Cumulative for magnetization $U_4(M)$.

For future contributions we intended to investigate, through the study of critical exponents, the class of universality of our model.

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