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Bootstrap resampling as a tool for calculating uncertainty measurement

Reamostragem bootstrap como uma ferramenta para calcular a incerteza de medição

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ABSTRACT
The objective of this paper is to provide an applied research comparing the traditional and bootstrap methods to calculate the measure uncertainty. For this purpose, were performed a dimension analysis for internal and external diameter on one lot with around one hundred parts from a Brazilian company. Following, were performed resamples with replacement – bootstrap samples – for each dimension obtained and then the uncertainty calculation. After that, it was concluded that the proposed method is the most appropriate, because decreases the bias of estimation when it works with small sample size, which is common on metrology works. The companies researched assert that they do not perform the uncertainty calculation on dimension

analysis in measuring; they only perform the involved instruments calibration. It justifies calculation using the bootstrap method proposed on this work.

Keywords: Metrology, Measuring Uncertainty, Resampling, Bootstrap.

RESUMO
O objetivo deste artigo é fornecer uma pesquisa aplicada comparando os métodos tradicionais e de bootstrap para calcular a incerteza de medição no contexto de metrologia. Para tanto, foi realizada uma análise de dimensões de diâmetro interno e externo em um lote com cerca de cem peças de uma empresa brasileira. A seguir, foram realizadas reamostragens com reposição - amostras bootstrap - para cada dimensão obtida e, posteriormente, o cálculo da incerteza. Em seguida, concluiu-se que o método proposto é o mais adequado, pois diminui o viés de estimação quando se trabalha com um pequeno tamanho de amostra, o que é comum em trabalhos de metrologia. As empresas pesquisadas afirmam que não realizam o cálculo da incerteza na análise de dimensões em medição; elas executam apenas a calibração dos instrumentos envolvidos. Isto justifica o cálculo usando o método de bootstrap proposto neste trabalho.

Palavras chave: Metrologia, Medição de Incerteza, Reamostragem, Bootstrap.

1 INTRODUCTION

The Metrology is the science that covers all theoretical and practical aspects related to measurements, whichever uncertainty in any field of science or technology. In this area to the Scientific and Industrial Metrology is an essential tool in technological innovation and growth, promoting competitiveness and creating a favorable environment for scientific and industrial development in any country (INMETRO, 2012) In this sense, accurate methods are needed to estimate how much variation - or dispersion – exists in Metrology experiments. Generally, it is performed by the estimation of a statistical parameter, known as standard deviation, which is used to measure the standard error of the estimated mean.

The following work consists of an applied research about measurement uncertainty in dimensional metrology, focused in the standard deviation estimative. The study aims to apply the statistical method called Bootstrap for improved accuracy of estimate of the measurement uncertainty of a real batch of industrial pieces. The main objective of the use of this procedure is based on the proceeding to calculate the measurement uncertainty used to characterize the quality of a result of measurement, as well as, obtain a quite precise estimation of standard error. The methodology used for this calculation is to gather at least three repetitions of each dimension.

Therefore, this paper proposes an improvement in data’s collection in order to increase it by calculating the estimated Bootstrap and ensure the quality and accuracy of the measurements.

2 ESTIMATION METHODS: TRADITIONAL versus BOOTSTRAP

Uncertainty measurement is the non-negative parameter which characterizes the dispersion of the values attributed to a measurand, based on the used information (INMETRO, 2009).

The uncertainty of a measurement result reflects the lack of accurate knowledge of the measurand. (Noronha, 2008).

In practice there are many possible sources of uncertainty in measurement, including:

- incomplete definition of the measurand;
- imperfect realization of the definition of the measurand;
- unrepresentative sampling of the sample measured may not represent the measurement;
- inadequate knowledge of the effects of environmental conditions on the measurement or imperfect measurement of environmental conditions;
- incorrect personal bias in reading analogue instruments;
- finite instrument resolution or discrimination threshold;
- inexact values of measurement standards and reference materials;
- inexact values of constants and other parameters obtained from external sources and used in data reduction algorithm;
- approach and assumptions incorporated on the measurement method and procedure;
- variations in repeated observations of the measurand under apparently identical conditions.

A. Standard Uncertainty

The standard uncertainty or model of a source of error is the dispersion range around the central value equal to one standard deviation.

The evaluation of the standard uncertainty can be classified into Type A and Type B. The classification's purpose of Type A and Type B is to indicate the two different ways of evaluating uncertainty components, and serves only to discussion, the classification is not intended to indicate that there is any difference in the nature of the components resulting two types of evaluation. (Incerpi, 2008).

B. Standard Uncertainty of type “A”

The uncertainty of the type "A" is the uncertainty calculated based on a probability distribution.

By adopting statistical factors for the determination of this type of uncertainty is necessary perform a number of repetitions in equal conditions, in other words, the dimensional aspect and type of industrial pieces must be the same, as well as, same measurement tool and same at room temperature, etc. For good results, the number of repetitions must be at least 10, and still have a measurand good quality. If this uncertainty is calculated in terms of calibration standards used should be of good quality.

It is noteworthy that the sample standard deviation (S) is an estimate of the population standard deviation (σ), which quantifies the dispersion of a population in study. The bias estimation is the difference between the sample standard deviation and the population standard deviation. In practice it's a difficult measure to obtain because the σ usually not known in practice.

The procedure to be adopted is shown below:

a) Execute the number "n" of measurements;
b) Calculate the sample standard deviation of the measurements using Eq. (1):

$$S = \sqrt{\frac{\sum_{k=1}^{n}(X_k - \bar{X})^2}{n-1}} \tag{1}$$

Being:

S = Sample standard deviation;
X_k = Results of the current measurement;
$\bar{X}$ = Average results;
n = Number of measurements;
k = Index of current measurement.

c) Calculate the uncertainty (depending on use):

Adopting individual values (worst situation):

$$u = s \tag{2}$$

Being:

u = Uncertainty;

s =Standard deviation.

Adopting mean values (when we consider the average as the result of measurement):

$$u = \frac{s}{\sqrt{n}} \tag{3}$$

Being:

s = Standard deviation;

n = Number of measurements.

D. Standard Uncertainty of type “b”

The uncertainty of type "B" is the uncertainty evaluation method performed by other ways that not the statistical analysis of a series of observations. There are some examples of uncertainty of type "B" below:

- Previous measurements data;
- Manufacturers specifications;
- Experience in the use and verification of the
- performance of the instrument with time;
- Data provided of calibration certificates;
- Environmental influences.

E. Combined Uncertainty (u_c)

The combined uncertainty consists in the quadratic sum of the different uncertainties of measurements performed by any measurement tool, in other words:

$$u_c = \sqrt{u_1^2 + u_2^2 + u_3^2 + \ldots + u_n^2} \quad (4)$$

This value is not adopted as real because it represents a statistical probability of approximately 68% to find the measurement's error, and thus does not constitute a good approximation. To determine the uncertainty with greater confidence level should be calculated expanded uncertainty, the value of which is within a 95% confidence (Incerpi, 2008).

The value of the combined uncertainty also includes the uncertainties inherited patterns corrected if necessary (according to the guidelines of certificates corresponding calibration).

F. Expanded Uncertainty (***U***)

It is defined as the quantity that expresses an interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measured (INMETRO, 2009).

This fraction can be seen as the coverage probability or level of confidence interval. To associate a specific level of confidence to the interval defined by the expanded uncertainty are necessary assumptions expressed or implied with respect to the probability distribution characterized by the measurement result and its combined uncertainty. The confidence level can be assigned to this interval can be known only to the extent that such assumptions may be justified. The expanded uncertainty is given by Eq. (5).

$$U = k * u_c \quad (5)$$

where k is the coverage factor for the desired confidence level.

It is very common to be expanded uncertainty represented by the symbol "U" and the "k" is the coverage factor for specific statistical confidence level, to 95% confidence level k is approximately 2.

G. Degrees of Freedom

The degree of freedom corresponds to the number n of independent observations of a given measurement. However, a degree of freedom is lost for each constraint that exists on the

n observations. According to INMETRO (2009), the effective degrees of freedom is the estimated value that the combination of degrees of freedom (v_i) associated with each of the standard uncertainties, are weighted by their standard uncertainties. The effective degrees of freedom (v_{eff}) is obtained by Welch–Satterthwaite equation (Eq. 6).

$$v_{eff} = \frac{u_c{}^4}{\frac{u_1^4}{v_1} + \frac{u_2^4}{v_2} + \frac{u_3^4}{v_3} + \dots \frac{u_i^4}{v_i}} \tag{6}$$

H. Non – Parametric Bootstrap

The non – parametric bootstrap (Efron, 1979) consists in the resampling of the data collected at each measurement. The entire procedure is shown in Fig. 1, where X is the original sample, $\hat{\mu}$ (X) is the traditional estimator, X_i^* are replicate samples, $\hat{\mu}_i^*$(X) are the traditional estimator applied on replicate samples and $\hat{F}_{\hat{\mu}}$ is the empirical distribution of $\hat{\mu}_i^*$, with $i = 1,2, \dots , B$. In this way, a single sample using the method B generates new samples obtained by sampling with replacement, which reduces the defect parameter estimation uncertainty (standard deviation) in small samples. Through the use of this method, the sample standard deviation value is the nearest of the value of the population standard deviation, a parameter which measures the uncertainty of a specified size variation of the garment. Thus, addiction estimation of measurement uncertainty decreases, especially when the sample is small.

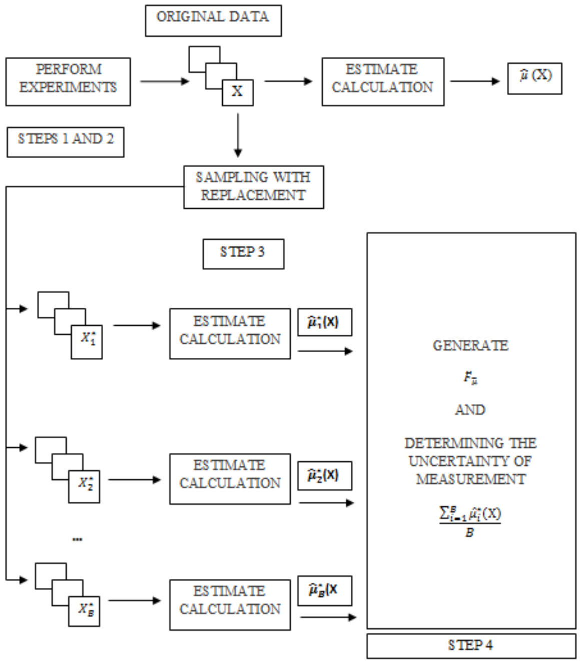

Fig. 1: Procedure for calculating the estimated bootstrap

A greater exposure of this idea will be displayed in the next section, through a simulation study.

3 SIMULATION STUDY

The study's objective was the application of the bootstrap method through a simulation performed using the software R, the data was generated of a normal population with mean ("µ") equal to 10 and a standard deviation (" ") equal to 1. In the same way, thousand samples were taken with specific sample size (= 2, 3…30). Subsequently it was estimated the population standard deviation followed by the calculation of the sample standard deviation, through traditional and bootstrap estimates, each bootstrap estimative was calculated by 999 replications with replacement of each original sample.

After these processes, we calculated the Mean Relative Absolute Error (MRAE):

$$MRAE = \sum_{i=1}^{1000} \left. \frac{\hat{\theta}_i - \theta}{\theta} \middle/ 1000 \right. \qquad (7)$$

where is the real standard deviation (=1) and is the sample standard deviation estimative performed by the corresponding estimator.

To illustrate the efficiency of the method is presented below a graph showing the relationship between the mean relative absolute error and sample size (Fig. 2).

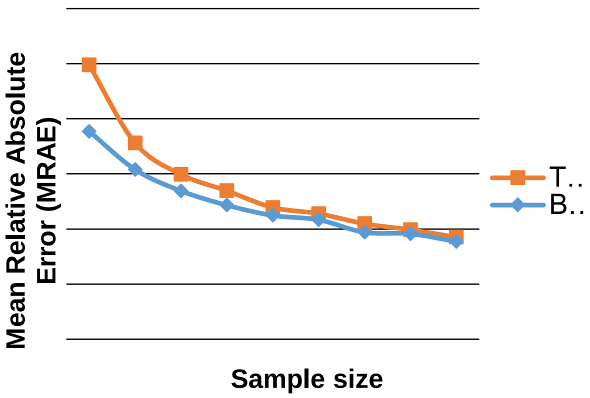

Fig. 2: Mean Relative Absolute Error in the estimation with sample size ranging from 2 to 10.

Through an analysis of 1000 random values, it was observed that bias estimation, reflected in MRAE, is lower in the bootstrap method compared to the traditional method for small samples, especially in samples with sample size lesser then 10. Extending the sample size to 30, the Fig. 3 shows that as larger the sample size is, the bias estimation generated on the bootstrap method approximates to the results generated in the traditional method.

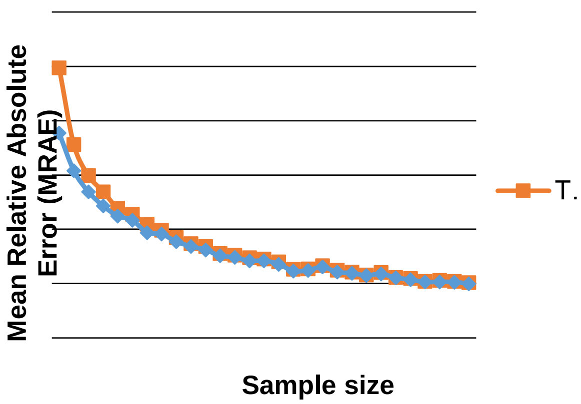

Fig. 3: Mean Relative Absolute Error in the estimation with sample size ranging from 2 to 30.

Thus, it is concluded that the proposed method is effective in cases where the sample size is small, being 10 or less samples. It these situations the gain can be of 95%.

4 A REAL EXPERIMENTAL STUDY

The analysis was performed by three inside diameter measurements Fig. (4) and an outside diameter Fig. (5) of a batch of one hundred parts according to specifications provided by the company. Data were entered into a spreadsheet and resample nine hundred ninety-nine times (Efron, 1979). This process enables a reduction in defect estimation of the standard deviation, the parameter used for analysis of measurement uncertainty. For each resample was averaged and the sample standard deviation, then was calculated measurement uncertainty type A combined uncertainty, expanded uncertainty and effective degrees of freedom, for internal and external measurements. Table 1 shows the results of type A and uncertainties makes the comparison between the traditional method (without resampling the data) and the bootstrap (with resampling of the data), the difference was significant. In the traditional method, it was observed that the measurement uncertainty was overestimated, considering the bootstrap method to true. It can be said that the result of the uncertainty decreases significantly and, in those cases, where the sample is small as in the analysis, so it is recommended to use a method such as the bootstrap for a better estimation of uncertainty of measurement with a lower addiction.

Fig. 4: Piston (Inside Diameter)

Fig. 5: Pin (outside diameter)

Table 1: Comparison of Traditional versus Bootstrap method

Method	INTERNAL		EXTERNAL	
	Traditional	Bootstrap	Traditional	Bootstrap
Combined uncertainty	0,004	0,002	0,003	0,001
Expanded uncertainty	0,007	0,004	0,006	0,003
Degree of effective freedom	37,217	21,722	53,797	12,281

5 CONCLUDING REMARKS

During the preparation of this document, it was possible to have direct contact with the processes of quality control performed in the industries. This observation allowed to know the reality of metrology in industry, as a process that does not generate profit and involves high costs.

Contrary to this reality metrology is one of the most important sectors of the industry, because through it is possible to identify and control problems from manufacturing processes, reduce the incidence of scrap and rework and give quality products.

Within this context, analyzes were made on pieces that had trouble controlling their dimensions, these were based on measurements of internal and external diameters and subsequently calculating the measurement uncertainty was performed in order to verify the maximum variation of the dimensions.

A comparison of the traditional method and the bootstrap method to calculate the uncertainty in the context of metrology was performed. By the results presented, we conclude that the use of the proposed method is most suitable when working with small samples, generating a smallest bias estimation of measurement uncertainty.

All programming code and the spreadsheet used to calculate the bootstrap estimates are available for interested readers.

REFERENCES

Efron, B. 1979. “Bootstrap Methods: Another Look at the J. Ann. Statist”. Volume 7, Number 1, (1979) 1-26.

Incerpi, PH. 2008. “Incerteza de medição – Método proposto para a análise da conformidade do produto”. Universidade Federal de Itajubá. Dissertação de Mestrado. Programa de Pósgraduação em Engenharia de Produção. 2008. Disponível em: http://*adm-net-a.unifei.edu.br/phl/pdf/0032953.pdf*. Acesso em 20 jul. 2012.

INMETRO. 2009. “Vocabulário Internacional de Metrologia: conceitos fundamentais e gerais e termos associados (VIM 2008)”. 1 ed. Rio de Janeiro, 2009.

INMETRO. 2012. “Metrologia Científica”, 2012. Disponível em: < http://www.inmetro.gov.br/metcientifica/>. Acesso em 18 jul. 2012.

Noronha, JL. 2008**. “**Procedimentos de cálculo de incerteza de medição em medições diretas e indiretas”, 2008. Disponível em: < http://www.barca.unifei.edu.br/metrologia.html>. Acesso em 18 jul. 2012.