

## Analytical and finite element analysis of the concrete stress intensity factor with carbon fibers

### Análise analítica e de elementos finitos do fator de intensidade de tensões concretas com fibras de carbono

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#### ABSTRACT

Concrete is a material with low tensile strength and fracture resistance. One way to improve these properties is through the insertion of synthetic fibers such as carbon fibers into the concrete. This work determined the stress intensity factor for concrete with aligned carbon fibers (laminate) using an analytical model and finite element methods – macro-mechanical analysis of the composite laminate using the ABAQUS finite element package. Symmetrical laminate models were used with carbon fibers ranging from  $\pm 15^\circ$  to  $\pm 75^\circ$ . A single-edge notch bending specimen geometry was used to determine the KI of the laminates. The mechanical properties of the laminate were obtained through analytical models of the laminate theory. Results indicate that the fiber angles influence the stress intensity factor. The behavior is practically linear and a strong correlation was observed between the angle of the fibers and the KI through Pearson's  $r$ . The laminate with a fiber angle of  $\pm 75^\circ$  showed a higher KI value compared to the others ( $\times 14$  about  $\pm 15^\circ$ ). Comparisons of the analytical and numerical results showed good agreement in the determined KI.

**Keywords:** laminate, carbon fibers, finite element methods, stress intensity factor.

#### RESUMO

O concreto é um material com baixa resistência à tração e à fratura. Uma forma de melhorar essas propriedades é inserindo no concreto fibras sintéticas como as de carbono. Este trabalho determinou o fator de intensidade de tensão para o betão com fibras de carbono alinhadas (laminado) usando um modelo analítico e métodos de elementos finitos - análise macromecânica do laminado composto usando o pacote de elementos finitos ABAQUS. Modelos de laminados simétricos foram usados com fibras de carbono variando de  $\pm 15^\circ$  a  $\pm 75^\circ$ . Uma geometria de amostra de dobra de um único bordo foi usada para determinar o KI dos laminados. As propriedades mecânicas do laminado foram

obtidas através de modelos analíticos da teoria do laminado. Os resultados indicam que os ângulos de fibra influenciam o fator de intensidade de tensão. O comportamento é praticamente linear e uma forte correlação foi observada entre o ângulo das fibras e o KI através do  $r$  de Pearson. O laminado com um ângulo de fibra de  $\pm 75^\circ$  mostrou um valor KI mais alto em comparação com os outros ( $\times 14$  de  $\pm 15^\circ$ ). Comparações dos resultados analíticos e numéricos mostraram boa concordância no KI determinado.

**Palavras-chave:** laminado, fibras de carbono, métodos de elementos finitos, fator de intensidade de tensão.

## 1 INTRODUCTION

Concrete is a composite material widely used in civil construction due to its mechanical properties, having several structural applications and numerous advantages due to its resistance to compression and durability. However, within its disadvantages, it has a high specific mass, low young modulus, low tensile strength, and low fracture toughness (Barbero, 2017). It is classified as a quasi-brittle material because of the fracture process zone ahead of the real crack tip (Shah *et al.*,1995). As a result of these characteristics, several researchers have shown that fibers can improve concrete's properties, such as the maximum tensile strength and fracture toughness. Fibers, such as glass, carbon, steel, and polymer, are mainly used in more demanding civil structure applications where their higher cost can be justified by the improved performance. The fibers used can be short or long (Bauer,2007).

Regarding continuous long fibers, the composites can have fibers aligned in several directions. A laminate is constructed by stacking several laminas in the thickness (z) direction. Examples of the types of laminates are unidirectional laminate, angle-ply laminate, cross-ply laminate, symmetric laminate, antisymmetric laminate, unsymmetric laminate, and quasi-isotropic laminate. The configuration of the laminate influences the mechanical response of the composite (Neto,2016).

The literature indicates a great potential to use carbon fibers inside concrete because they improve the mechanical strength, fracture toughness, and difficult crack propagation (Liu *et al.*,2020). Cement matrix laminates are still poorly studied. Fibers are generally being used in laminates to reinforce structural elements - carbon fiber-reinforced polymer (CFRP) laminates (Kurniawan *et al.*,2021; Sangi *et al.*,2020).

Most of the existing research related to the determination of KI is on concrete with short fibers (carbon, polymer, glass, or natural). The short fibers improve little stress

concentration factor in relation to long fibers (Jorbat, *et al.*, 2020; Karamloo, *et al.*, 2020; Saad, *et al.*, 2020)

Safiuddin *et al.*, (2018) researched concrete with short carbon fibers ranging from 0% to 1%, where they observed a significant increase in the flexural strength and fracture toughness up to 0.25% of the volumetric fraction of carbon fibers. However, the compressive strength decreases with increased fiber concentration.

The use of short fibers has a limiter, which is the fiber concentration because even a high volumetric fraction of a high fiber concentration creates several fiber contact points that facilitate crack propagation in addition to the difficulty of the fiber distribution in the matrix (Safiuddin *et al.*, 2018; Kimm *et al.*, 2020; Liu *et al.*, 2019).

In contrast, structural composites such as laminates stand out due to varied configurations and mechanical properties and greater control of properties. Several kinds of research are carried out with polymer laminates and synthetic fibers (Dong, 2020). However, cement matrix laminates are still poorly studied. Fibers are generally being used in laminates to reinforce structural elements - carbon fiber-reinforced polymer (CFRP) laminates (Sangi *et al.*, 2020; Kurniawan, *et al.*, 2021). In this type of reinforcement, the chemical interaction between the laminate and the concrete must be taken into account.

The material manufacturing process generates small defects that can range from air bubbles to microcracks. The fracture mechanics aims to study these defects in the materials, as these defects are stress amplifiers that can cause the catastrophic failure of the material from a determined crack size. The insertion of holes and structural discontinuities in the materials increases the state of tension in that region (Anderson, 2017). This level of stress causes the beginning of the propagation of the failure (Neto, 2016).

Materials can be designed using the parameters of fracture mechanics. Griffith (1921) was the first to study the fracture proposing the energy criterion. However, the study developed by Irwin (1956), who was responsible for developing the current version that considers the energy release rate,  $G$ , which is defined as the rate of change in potential energy with a crack area for a linear elastic material. At the time of the fracture,  $G = G_c$ , the critical energy release rate, is a measure of fracture toughness. According to studies, failure occurs when  $KI = K_{Ic}$ . In this respect,  $KI$  is the driving force for material fracture, and  $K_{Ic}$  is a measure of material resistance to failure. As with  $G_c$ , the property of

similitude should apply to  $K_{Ic}$ . In other words,  $K_{Ic}$  is assumed to be a size-independent material property (Anderson, 2017).

The stress intensity factor (SIF) is a widely accepted parameter to determine the material's resistance to crack propagation under external loading. The evaluation of toughness to concrete fracture concerning the stress intensity factor has been studied by several authors (Khitab, 2017; Reis, 2004).

Material failure can happen in several ways. It can occur in laminated composite material within the three types classified in fracture mechanics: mode I, where crack propagation occurs under normal loading conditions in the plane where the crack is propagated perpendicular to the load application. Mode II, where the applied load is transversal to the crack length, is also known as shear. Mode III fracture is the type of failure in which the applied load is parallel to the crack length. Mode I from fracture mechanics is considered the most dangerous (Sangi *et al.*, 2020). Furthermore, in several problems, the propagation starts in mixed mode and then continues in mode I (Bouiadjra *et al.*, 2002; Peres, 2017; Shi, 2009). The stress intensity factor is a widely accepted parameter to determine the material's resistance to crack propagation under external loading. The evaluation of toughness to concrete fracture concerning the stress intensity factor has been studied by several authors (Khitab *et al.*, 2017; Reis *et al.*, 2004).

Fracture mechanics concerns the design and analysis of structures that contain cracks or flaws. On some size-scale, all materials have flaws either microscopic, due to cracked inclusions, debonded fibers, etc., or macroscopic, due to corrosion, fatigue, welding flaws, etc. (Peres, 2017). The fracture mechanics can be linear elastic or Nonlinear. The first applies to materials that present a brittle rupture, while the second considers the area of inelastic processes. The linear elastic fracture mechanics (LEFM) is also used when the energy dissipation effects associated with the inelastic process zone are small enough to be neglected. In the ideal case of LEFM, the process zone is composed of a single point, that is, the crack tip.

Therefore, large process zones usually require nonlinear models of fracture mechanics. Many researchers use these concepts to determine the stress intensity factor (Bauer, 2007). Several methods are used to determine the stress intensity factor. The most common are experimental and analytical. Due to the computational advance, it is already possible to get the material mechanic's behavior through the finite element method, mainly using fracture mechanics. The most used numerical methods for fracture

mechanics problems are: the finite element method (FEM) (Zieba *et al.*,2020), boundary element method (BEM) (Nikbin *et al.*,2020) extended finite element method (XFEM) (Faron and Rombach,2020; Huang *et al.*,2018;), meshfree methods(Rajagopal,2011), and scaled boundary finite element method (SBFEM) (Khaji and Yazdani,2016; Li *et al.*,2018; Yazdani *et al.*,2016). The method provides high precision the stress intensity factor at the crack tip (Bouiadjra *et al.*,2002; Gawil ,2016).

However, there are still few studies to determine the stress intensity factor of concrete with carbon fibers (laminate) using finite elements compared to analytical methods, so studies on this are of fundamental importance. The application of finite elements in composites has been investigated by several authors, such as Ramesh and Nijanathan, (2016) where they studied mechanical property analysis of kenaf – glass fiber reinforced polymer composites with two different fiber orientations of  $0^\circ$  and  $90^\circ$ . Experimental results showed that composites with a  $90^\circ$  orientation showed greater resistance to traction and impact tension than composites with fibers oriented at  $0^\circ$ . The greatest resistance to flexion was for the  $0^\circ$  composite. Regarding the FEA model, the results are very close to the experimental values.

In another study, Ramesh *et al.*, (2019) analyzed the flax-glass fibers reinforced hybrid composites with two different fiber orientations of  $0^\circ$  and  $90^\circ$  experimentally and by the finite element method. The results show that composites with  $0^\circ$  orientation presented higher values of resistance to traction, flexion, and impact in relation to composites with  $90^\circ$ .

In another research, Ramesh *et al.*, (2018) analyzed the effect of hybridization on polymer composites' mechanical properties reinforced with carbon fiber and hemp, making a comparison between experimental and FEM tests. The values obtained in ANSYS were compared with the experimental results. There is a high correlation between alkali-treated and untreated carbon fiber and hemp compounds.

Researchers as Marques *et al.*, (2010) calculate the KI of the concrete using analytical and experimental methods (using single edge notched beam (SENB) specimens) where satisfactory results were obtained.

Carpinteri *et al.*, (2017) studied the fracture behavior of FRC (fiber reinforced concrete) with micro synthetic polypropylene fibrillated. Mode I fracture toughness was calculated using experimental e analytics methods. It was observed that there is an increase in fracture toughness with an increase in the volumetric fraction of fibers.

Chari (2020) studied the mechanical behavior of concrete reinforced with steel fibers and applied LEFM. It was determined the elasticity module, stress Intensity factor, fracture energy both numerically and experimentally. Steel fibers have been found to increase these mechanical properties.

Chauhan *et al.*, (2017) applied linear fracture mechanics to calculate the stress factor of concrete using Abaqus / CAE. It was noted that the SIF increases with a decrease in specimen size. Also, the SIF increases with the increase in the proportion in the notch-to-depth ratio.

Although the finite element method is applied to several fracture mechanics problems, the elements generally used do not generate adequate precision at the crack tip, the number in these places increases. This method requires dense meshes in the region near the crack tip. It makes the algebraic system of equations too large. The increase in the number of elements in these regions increases the computational effort to solve the problem. The increase in the number of elements in these regions increases the computational effort to solve the problem. To solve this problem, researchers such as Henshel and Shaw (1975) developed singular quarter-point elements (or crack-tip elements) by making some changes to the formulation of regular elements (Henshell,1975). Another procedure commonly used in fracture mechanics applications with the use of the finite element method is to ignore the presence of singularity and refine the mesh in the vicinity of the crack tip to reduce its effect (Blandford,1981). The numerical value of the stress components calculated at the crack tip will always be finite, but it can be made as large as desired by increasing the mesh's refinement (Santana,2015). In this way, the FEM can solve problems of fracture mechanics of complex and massive structures.

## 1.1 OBJECTIVES

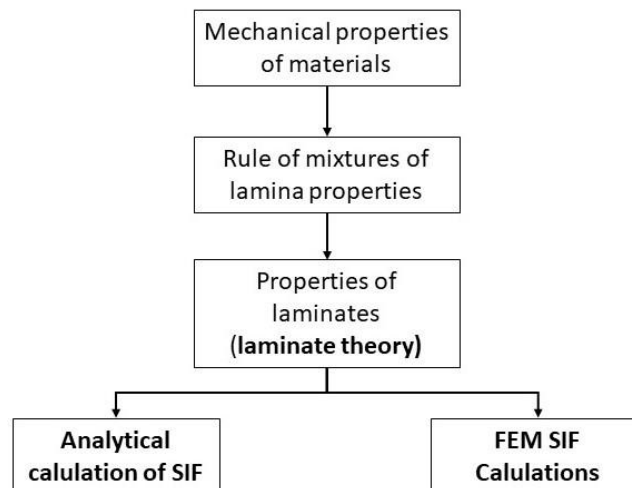
Structural composites such as laminates have mechanical properties influenced by the fiber arrangement in the matrix, the failure being of fundamental importance for applying this material. The laminated composite's fracture resistance is related to the specimen's geometry, the length of the notch, the loading, and the different angles of each lamina. The objectives of the work are to determine the stress concentration factor mode I, using the finite element method of laminates (cementitious matrix with carbon fibers), varying the orientation from  $\pm 15^\circ$  to  $\pm 75^\circ$  of the fibers where it was compared with

analytical results; verify the correlation between fiber orientation and KI numerically and analytically, evaluate the mechanical behavior of the laminates, analyze in the FEM to determine the KI, improve the properties of the concrete and understand the effects of the fibers on the stress intensity factor.

## 2 MATERIAL E METHOD

The research followed the following steps to calculate the mechanical properties of the laminate. The properties of the symmetrical laminate were obtained by laminate theory. The stress intensity factor(SIF) was calculated analytically and numerically, as shown in the flowchart of Figure 1.

Figure 1. Research flowchar.



A single edge notch bending specimen was used with a 20 mm notch, as shown in Figure 2. Laminate presented 40 layers where each layer has 1mm. One support was considered as a pin and the other a roller with a distance between 170mm. The load was applied as a static pressure load middle of the specimen. A micromechanical interlocking between fibers/matrix is considered perfect for analytical analysis according to the theory of laminates (Barbeiro, 2017). The bonding stress transfer mechanism on the interface was perfect (Barbeiro, 2017). The properties of concrete and carbon fiber were obtained through studies of (Gawil ,2016; Liu *et al.*, 2020). The Carbon fiber with a diameter of 7.3  $\mu\text{m}$  was used in this study, whose elastic modulus is 231 GPa, tensile strength is 4558 MPa, elongation at break is 2.05%, and density is 1820  $\text{kg/m}^3$ . The concrete proprieties

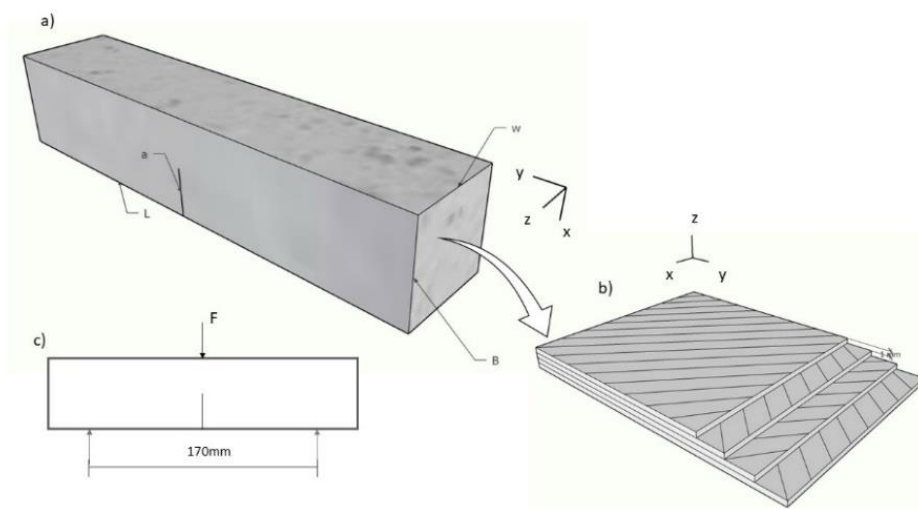
young modulus,  $E$ , (GPa) is 26.1 MPa; compressive strength is 32 MPa, poisson's ratio is 0.25, tensile strength is 3.5. It is average values of the samples. Also, it was considered how a matrix linear isotropic for the calculus analytic.

The composite laminate properties of carbon fiber/concrete were obtained through the rule of mixtures with a 1% volumetric fraction of fibers. The rule of mixtures allows estimating the elasticity modules  $E_1$  and  $E_2$  of a lamina with unidirectional reinforcement, starting from the elasticity modules and the volumetric fractions of the fibers and matrix. From the laminate theory, the laminate presented  $S_{ij}$  (parameters flexibility) that will be used to calculate  $\rho$ . (Measures parameters of plane material orthotropy) e  $Y(\rho)$  (Material orthotropic correction parameter).

The composite lamina's properties parallel and perpendicular to the loading direction (parallel and perpendicular to the plate geometrical axes) for different fiber orientations are obtained using the following transformation relationships (Barbeiro, 2017).

The geometry of the laminate was defined as 40x40x200mm according to Figure 2a, where the depth/thickness of the plate ( $B$ ), Width ( $W$ ), length of the notch ( $a$ ), length ( $L$ ) are shown. Figure 2b is a part of laminate section of the laminate and Figure 1c is schematic representation of test methods for pure Mode I.

Figure 2- The geometry of the laminate(a) part of laminate section (b) Pure Mode I(c)



## 2.1 ANALYTICAL METHOD

The analytical method for determining the mode I stress intensity factor for a single edge crack plate sample (Barbeiro, 2017; El-Hajjar and Haj-Ali,2005). The



calculation of the stress intensity factor for the concrete laminate configurations is obtained by the solutions, with the following form (Equation 1- 4).

$$KI = \frac{P}{B\sqrt{W}} f\left(\frac{a}{W}\right) Y(\rho) \quad (1)$$

Where P = Applied load (N); f(a/w) = Dimensionless geometry function; B = Depth / plate thickness (mm); Width (mm); Crack length (mm); Length (mm); Y(ρ)=Material orthotropy correction parameter; ρ= Measures parameters of plane material orthotropic; Sij= Compliances.

$$\rho = \frac{2S_{12}+S_{66}}{2\sqrt{S_{11}S_{22}}} \quad (2)$$

$$Y(\rho) = \frac{[1+(0.1(\rho-1)-0.016(\rho-1^2)+0.002(\rho-1^3))]}{\left(\frac{1+\rho}{2}\right)^{1/4}} \quad (3)$$

$$f\left(\frac{a}{w}\right) = \frac{3\left(\frac{S}{W}\right)\sqrt{\frac{a}{W}}}{2\left(1+2\left(\frac{a}{W}\right)\right)\left(1-\left(\frac{a}{W}\right)^2\right)} \left[1.99 - \frac{a}{W} \left(1 - \frac{a}{W}\right) \left[2.15 - 3.93 \left(\frac{a}{W}\right) + \left(\frac{a}{W}\right)^2\right]\right] \quad (4)$$

A  $\sigma(2)$  of 0.25 MPa was considered , it is known that  $\sigma(x); \sigma(y); \sigma(xy) = [T - (\theta)] * \sigma(2); \sigma(2); \sigma(12)$  (laminate theory). For this analysis, MATLAB was used, creating a program to calculate the Stress Intensity Factor of each laminate configuration.

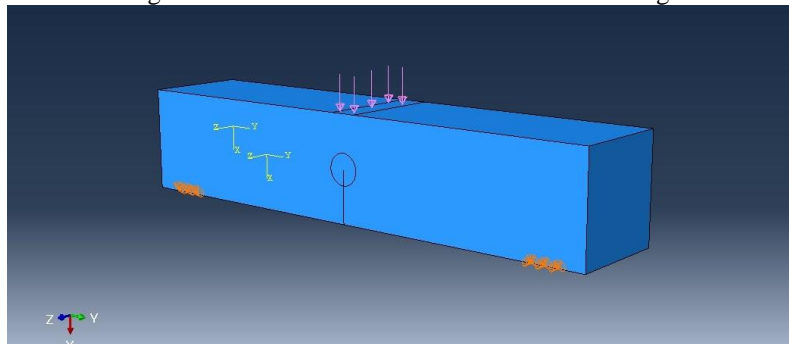
## 2.2 MODELING AND ANALYSIS

The ABAUS/CAE commercial software performed the numerical simulation of the laminate. Laminate presented 40 layers where each layer has 1mm. Each lamina's properties were obtained through the mixtures rule, and the simulation considers the perfect interaction between fiber and matrix. This simulation assumed the following simplifications: perfect bonding exists between the fibers and the matrix. Both fibers and matrix behave as linearly elastic materials, and the matrix is free of voids. The layers of each laminate vary in orientation  $[(\pm 15)_{10}]_s, [(\pm 25)_{10}]_s, [(\pm 35)_{10}]_s, [(\pm 45)_{10}]_s, [(\pm 55)_{10}]_s, [(\pm 65)_{10}]_s, [(\pm 75)_{10}]_s$ . Geometry Single edge notch bending specimen with 20mm notch was used Figure 3.

Static stress of 0.25 MPa (laminated theory) applied to the test body was considered according to Figure 3. The boundary conditions adopted were: one support was considered as a pin and the other a roller.

Figure 2 shows the three-dimensional model of the composite with the notch on one of the edges. The type of structural analysis that was used is the static analysis model. The beginning of the notch length and the end of the single crack and type 0.25 as a middle node parameter, and select the collapsed element side, duplicate nodes. The deformed mesh around the crack tip, Wedge Element Shape, was used. It was adopted in the Abaqus, an integral interaction method (J).

Figure 3- Model of concrete with notch on one edge



Due to the mesh's deformation at the crack tip, the shape of the wedge element was used. The ABAQUS software identifies square elements and analyzes them as degenerate square elements. Another point is that using the 3D specimen geometry is difficult to obtain the mesh to sweep the singularity and then sweep from one face to another. Thus, there must be an element along the length of the line in the circular region. There is a sweep in the mesh; an element number of this line is assigned on one side as the crack propagates to the other side. A selected Quadratic element type has been assigned as Geometric Order Reduced integration C3D20 (A 20-node quadratic tetrahedron element brick, and the mesh size (3x3x3 integration points)). For the mesh outside the tip region of the crack, hex format was used. The chosen technique was scanning and the medial axis algorithm. The "spider web" configuration is the most suitable for problems involving crack, as this mesh design is more efficient in the region of the crack tip. This configuration consists of concentric rings of elements on four sides located in the crack tip region. The closed ring is formed by quadrangular elements degenerated into triangles.

### 2.3 CONVERGENCE ANALYSIS

Convergence analysis was carried out following the methods of (Blandford *et al.*,1981; Henshell and Shaw,1978). According to the method, stress and displacement fields change as a function of the crack tip's radius. When R tends to 0 ( $r \rightarrow 0$ ), the singularity occurs. This analysis used circles at the crack tip with radius: 6mm, 5mm, 4mm, 3mm, 2mm, and 1mm. The number of elements in the crack tip varying from 10 to 30. Also, it changed the number of elements in the crack's external region.

## 3 RESULTS AND DISCUSSION

### 3.1 CONVERGENCE ANALYSIS

Figure 4 shows that as the radius decreases and the number of elements at the tip of the crack increases, there is a tendency for the KI values to converge. The laminate used in this analysis was  $\pm 75^\circ$ .

Figure 4-The convergence of the finite element model for SIF analysis of laminates

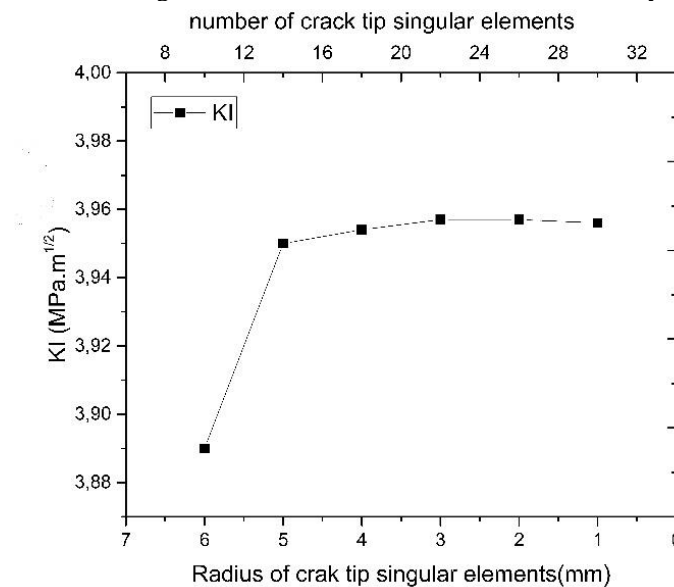


Table 1 shows the values obtained in the simulations. As the radius decreases and the number of elements at the crack tip increases, the KI of interest starts to converge to a particular value. Figure 3 and Table 1 show the mesh refinements. If no more considerable change in the result is observed, one can then assume that the result has converged. The final error was 0.60% for a radius of 1 mm, 24 elements at the crack tip, and for a total number of 22912 elements.

Table 1-Convergence analysis parameters

R Crack(mm)	N° crack tip	N° total	SIF $MPa\sqrt{m}$	erro(%)
6	10	2584	3.890	2.26
5	12	17360	3.950	0.75
4	14	19072	3.954	0.65
3	16	21648	3.957	0.58
2	21	22016	3.957	0.58
1	24	22912	3.956	0.60

### 3.2 NUMERICAL ANALYSIS AND ANALYTICAL ANALYSIS

The numerical analysis result in Figure 5 presents the maximum displacement (U1) values for the single-edge notch bending specimen. The maximum stress value is observed to vary according to the laminate configuration.

Figure 5- Displacement of concrete and carbon fiber laminate with a notch on the edge.

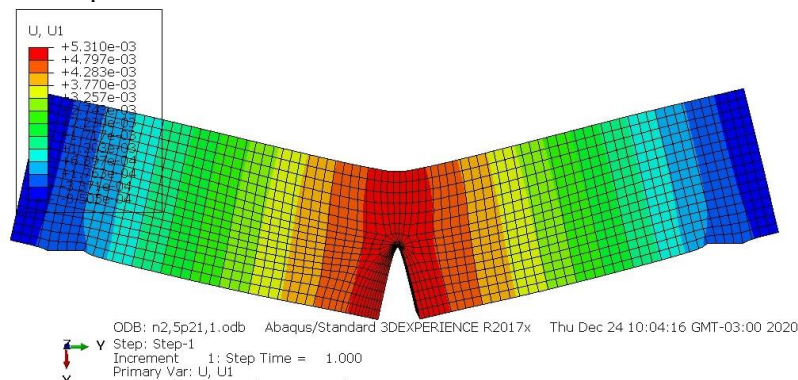


Table 2 shows the stress intensity factor in the crack tip region for different fiber orientations in the laminate obtained through finite element analysis. Laminates with angles of  $\pm 15^\circ$  exhibit analytically and numerically obtained KI values of  $0.28 MPa\sqrt{m}$  and  $0.27 MPa\sqrt{m}$ , respectively (see Table 2). The stress reached 0.75 MPa and the displacement is found to be 0.00045 mm. Comparing this result to that of the laminates with an angle of  $\pm 75^\circ$ , the KI values obtained from the analytical ( $3.96 MPa\sqrt{m}$ ) and numerical model ( $3.98 MPa\sqrt{m}$ ) exhibit an increase of  $\times 14.67$  and  $\times 14.21$ , respectively. For the laminate with an angle of  $\pm 75^\circ$ , the crack tip's stress reached 9.24 MPa and allowed a greater displacement of 0.006 mm. The laminate symmetry with an angle of  $\pm 45^\circ$  gave us an intermediary value of KI, enabling applications with bi-directional loading.

Table 2-Maximum stresses, deformations and FEM result of SIF KI for concrete / carbon single crack with different orientation angles.

Angle (°)	ABAQUS KI	ANAL. KI	Sd	Stress	S ANAL.	U.MAX.	U ANAL.
±15	0.27	0.28	0.0070	0.84	0.87	0.00045	0.0005
±25	0.75	0.76	0.0070	2.12	2.33	0.00121	0.0013
±35	1.38	1.41	0.0212	3.72	4.30	0.0020	0.0023
±45	2.11	2.14	0.0212	5.75	6.53	0.00345	0.00350
±55	2.82	2.87	0.0353	7.60	8.77	0.0040	0.0047
±65	3.46	3.51	0.0353	9.24	10.74	0.0050	0.0058
±75	3.96	3.98	0.0141	11.87	12.20	0.0060	0.0065

Figure 6a and Figure 6b shows the stress distribution at the crack tip for the composite with  $\pm 75^\circ$  fiber orientation. Fibers at this angle allow greater traction effort due to the combination with the concrete matrix.

Figure 6- Model of concrete with notch on one edge(a) and magnification(b)

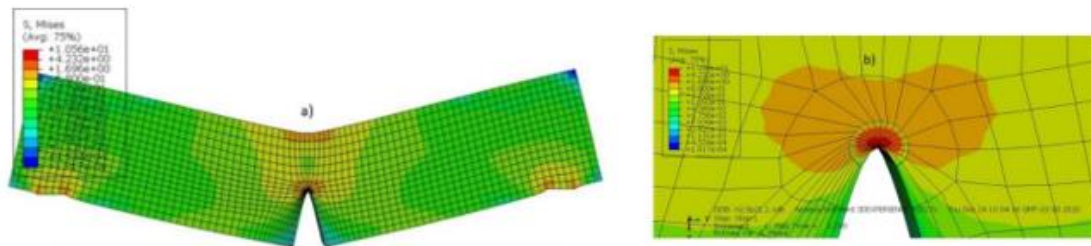
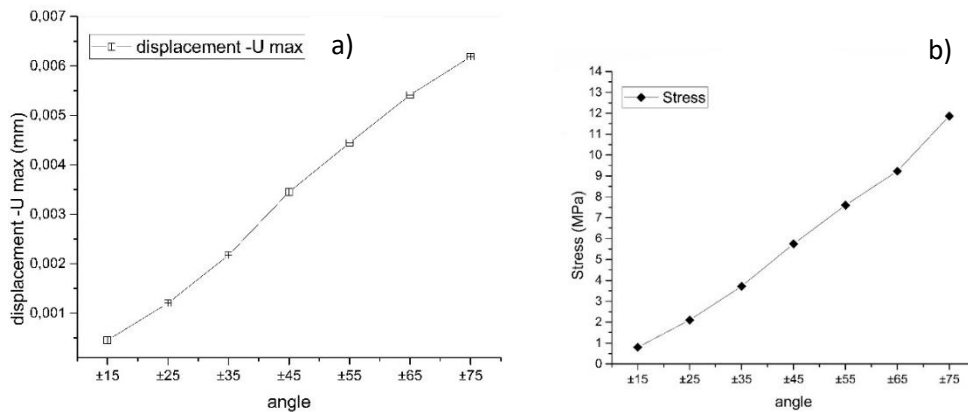


Figure 7a shows the variation in the concrete deformation with the carbon fiber for different fiber orientations. Through the results, it was noted that the maximum deformation occurs with an angle of  $\pm 75^\circ$ . On the other hand, the composite with an angle of  $\pm 15^\circ$  showed a lower strain value.

Figure 7-Displacement x fiber orientation angle of laminates concrete/fiber carbon(a). Stress variations of composite with Fiber Orientation angle(b).



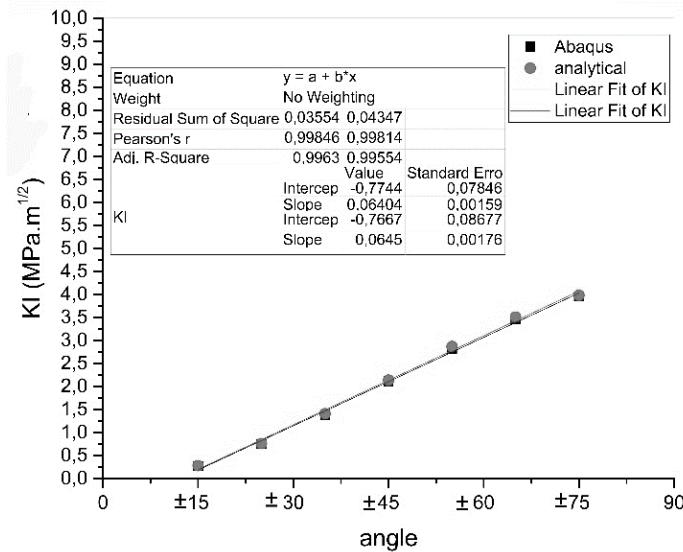
Results reveal that at angles below  $\pm 15^\circ$ , most of the load is absorbed by the matrix. Thus, the tension in the crack region is less since the carbon fiber is responsible for increasing the composite's strength. As the angle is increased, the crack propagation becomes more difficult, as there is a high stress transfer to the fibers (both shear and normal stresses), resulting in a high deformation at the crack tip. In contrast, composites with an angle below  $\pm 75^\circ$  reduce deformations around the crack tip.

Figure 7b shows the variation of stress around the composite's crack tip for different fiber orientations. The results noted that the maximum tension occurs when the fibers' angulation is  $\pm 75^\circ$ . Composites with fibers oriented at  $\pm 15^\circ$  have a lower value since they are practically parallel to the crack propagation, thus offering no impediment to the crack's advance. Observing that with smaller angles, only the matrix practically absorbs the energy, resulting in a low tension distribution around the crack tip.

From  $\pm 75^\circ$  and at greater angles, results reveal a high fiber-matrix interaction, allowing higher shear stresses and normal tensile stresses at the interface and thus creating an increase in tension in the crack tip region. The fibers are more parallel to the loading with smaller angles, setting a more uniform tension around the crack and resulting in a reduction in the crack tension. Figure 8 shows how the stress intensity factor varies according to the variation in fiber orientation. Results show that composite with an angle of  $\pm 75^\circ$  received the maximum stress. As the angulation decreases to  $\pm 15^\circ$ , the fibers are more or less approximately parallel to the front of the crack. When analyzed from the point of view of micromechanics, the crack tip's front does not present difficulties to the advance since the tension is practically transferred to the matrix of the concrete. These results in the distribution of the low-stress intensity factor around the crack tip. In addition, the behavior of KI as a function of the angle of fibers in the laminate is observed

to be linear for both the numerical and analytical model with the slope of the straight line being 0.064 and 0.065, respectively. Through a statistical treatment, the Pearson's correlation coefficient is observed to approach the value of  $r = 1$ , indicating a strong correlation between the two KI values and fiber orientation. These results are satisfactory because the variation between the models is low.

Figure 8- SIF of composite variations with fiber orientation angle.



Regarding the  $\pm 75^\circ$  composite, from a micromechanical point of view, there is a higher chance that the crack will find the fibers transferring the stresses from the matrix to the fiber. These stresses are normal at the fiber/matrix interface and shear stresses, resulting in a high-stress intensity factor around the crack tip. For angles of  $\pm 75^\circ$  and above, the number of continuous fibers sharing the uniformly applied pressure in the distant field also increases the intensity, resulting in a reduction in the stress intensity factor around the crack tip. The observed results were consistent with those found by other authors, such as (Gebru,2018), where increasing the variation of the fiber angle is observed to increase the KI of the laminated composites with polyester matrix and carbon fiber.

Sousa (2019) observed an increase in KI depending on the type of fiber, length, and concentration of fibers in the matrix, adhesion, and interfacial adhesion. The greater the length and the more aligned the fiber in the matrix, the greater the KI in the experimental results. In addition, the crack travels along the defects created at the ends of the fibers and along with the interface. This indicates that when considering the perfect

adhesion between the fiber and the matrix and that has a good correlation with the possible real performance of the composite. With regard to concrete and fiber composites, both short fibers and long fibers increase KI as shown in the studies by (Carpinteri *et al.*,2017; Saad *et al.*,2020, indicating the good potential of using carbon fibers in a cementitious matrix. These researchers observed an increase in the flexural strength and fracture toughness using the aligned fibers, which is the same behavior observed in this study. It has been shown that very simple analytical formulas can give a very accurate description of the numerical results in a wide range of fiber orientations in the laminate for determining the stress intensity factor in composite materials. It is worth mentioning that for this to be possible, it was necessary to consider the perfect adhesion between the fiber and matrix and treating the concrete as isotropic in the analysis. When considering these restrictions, the laminates have anisotropic characteristics with mechanical properties in the x- and y- direction, which were adopted in the model based on the theory of laminates. In this anisotropic model, it is noted that the orientation of the fibers influences the resulting stress at the crack tip. As this study considered a perfect interaction between the layers, the tensions between the layers were not analyzed.

In addition,  $a/W$  has a determining factor in the fracture toughness factor as seen in the studies by (Carpinteri *et al.*,2017). They noted that the critical situation happens when  $a/w = 0.5$ , which was the adopted value in that analysis. In other words, this present study obtains KI values that are close to the failure of the material in this study. The ABAQUS J integral technique proved to be satisfactory in finding the KI in the laminate according to the data found by the analytical method.

## 5 CONCLUSIONS

One of the materials' significant problems is the premature failure of the structural elements due to cracks. Fibers play a fundamental role in increasing the mechanical resistance of concrete elements. Carbon fibers are a potential substitute for steel fibers. Thus, understanding the failure of this material is extremely important. The SIF, as an essential parameter in LEFM, can be obtained through computational models. From the results obtained for the concrete composite with carbon fiber at different notches of fiber with a notch, it appears that the composite with the highest maximum stress value, KI, and deformation are those with angles of  $\pm 75^\circ$ . Carbon fibers can increase the fracture toughness of concrete and make the crack propagation more complicated. The angulation



between the layers is essential for this increase in stress. The Pearson's correlation coefficient indicated a strong correlation between the KI and fiber orientation in the laminate. Results of the finite element modeling technique proved to be satisfactory compared with the theoretical results. In concrete laminates with carbon fiber, it is verified that the variation of the angle modifies its mechanical behavior and directly interferes with the stress intensity factor. Results obtained from the finite element and analytical method are in close agreement with each other.

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