Validation and optimization of the GSAMQ analytical propagator for spin-stabilized satellite

Validação e otimização do propagador analítico GSAMQ para satélite spin-stabilizado

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Maria Cecília Zanardi
Livre docente em Dinâmica Orbital pela Universidade Estadual Paulista Júlio de Mesquita Filho (UNESP), Guaratinguetá - SP
Institution: Universidade Estadual Paulista Júlio de Mesquita Filho, Guaratinguetá, SP
Address: Av. Dr Ariberto Pereira da Cunha, 333, Guaratinguetá – SP, Brasil,
CEP: 12516-410
E-mail: mceciliazanardi@gmail.com

Thales D Monea Oliveira
Bacharel em Ciência e Tecnologia pela Universidade Federal do ABC (UFABC), Santo André - SP
Institution: Universidade Federal do ABC (UFABC), Santo André - SP
Address: Av. dos Estados, 5007, Bangú, Santo André - SP, Brasil, CEP: 09210-580
E-mail: thales.monea@gmail.com

Claudia Celeste Celestino
Doutora em Engenharia Aeroespacial pela Instituto Nacional de Pesquisas Espaciais (INPE), São José dos Campos - SP
Institution: Universidade Federal do ABC (UFABC), Santo André - SP
Address: Av. dos Estados, 5007, Bangú, Santo André - SP, Brasil, CEP: 09210-580
E-mail: claudia.celeste@ufabc.edu.br

ABSTRACT

The aim of this paper is to validate the analytical propagator through simulations comparing the results with the real data and to optimize the work's source code so that it is processed faster. This work analyzes the use of the quadrupole model in the Earth's magnetic field when an analytical propagator calculates eddy current and residual magnetic torques in spin-stabilized satellites. For that, the components of the gravity gradient torque, the solar radiation torque and aerodynamics torque are also included. The simulations are carried out for a predetermined period and use data from the SCD1 Brazilian satellite, provided by the National Institute for Space Research (INPE). The results are then compared with the real data to validate the propagator. The propagator with the quadrupole model is called GSAMQ and uses the mean deviations of the magnitude of the spin velocity, and the angles of right ascension and declination of the spin axis as evaluation parameters. A statistical approach is included in this analysis and shows the model's accuracy.

Keywords: spin stabilized satellite, analytical propagators, gravity gradiente torque, aerodynamic torque, magnetic torques, solar radiation torque.

RESUMO

O objetivo deste trabalho é validar o propagador analítico através de simulações comparando os resultados com os dados reais e otimizar o código fonte do trabalho para que ele seja processado mais rapidamente. Este trabalho analisa o uso do modelo quadrupole no campo magnético da Terra quando um propagador analítico calcula a corrente parasita e os torques magnéticos residuais em satélites estabilizados por spin. Para isso, os componentes do torque do gradiente de gravidade, o torque da radiação solar e o torque da aerodinâmica também estão incluídos. As simulações são realizadas por um período pré-determinado e utilizam dados do satélite brasileiro SCD1, fornecidos pelo Instituto Nacional de Pesquisas Espaciais (INPE). Os resultados são então comparados com os dados reais para validar o propagador. O propagador com o modelo quadrupolar é chamado de GSAMQ e utiliza como parâmetros de avaliação os desvios médios da magnitude da velocidade de spin, e os ângulos de ascensão reta e declinação do eixo spin. Uma abordagem estatística é incluída nesta análise e mostra a precisão do modelo.

Palavras-chave: satélites estabilizados por rotação, propagador analítico, torque de gradiente de gravidade.

1 INTRODUCTION

Once in space, satellites are subjected to different types of external forces that can alter their motions and their applicability depends directly on their position and space orientation. The focus of this paper are spin-stabilized satellites with low orbit and low eccentricity, which is characterized by rotating around the axis of greatest moment of inertia.

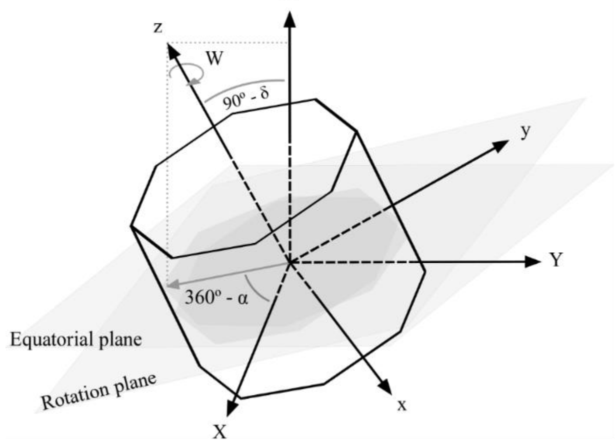
The main sources of disturbance for these satellites (Zanardi et al., 2016) are: Aerodynamic Torque, Solar Radiation Torque, Gravity Gradient Torque, Residual Magnetic and Magnetic Induced Torque due to eddy currents.

The components of each torque must be included in the equations of motion for the attitude of spin-stabilized satellites, and to calculate them, a mathematical model is required. In this work, the geomagnetic field is modeled using the Dipole or the Quadrupole models based on the truncation of the series of spherical harmonics that describe the potential (Garcia, 2007). The models for each torque are introduced and discussed in (Wertz, 1978) and (Zanardi et al., 2018).

The two attitude propagators used here include the five torques mentioned: **GSAM** - **G** for gravity torque, **S** for solar radiation torque, **A** for aerodynamic torque and **M** for magnetic torques with **dipole** model for the geomagnetic field (Motta,2014); **GSAMQ** - with the same nomenclature as GSAM but with the **Q** indicating the quadrupole model for the geomagnetic field (Zanardi et al., 2018).

The equations describing the rotational motion of spin-stabilized satellites (Wertz, 1978; Kuga et al., 1987) are given in terms of the right ascension angle α and the declination angle δ of the spin axis, and absolute value of the spin velocity W. These variables are defined based on the satellites and the equatorial reference frames, as shown in Figure 1.

Figure 1: Representation of the right ascension angle α and declination angle δ of the spin axis and spin velocity W.



The equations of rotational motion depend on the external torque components on the fixed system of the satellite and are given by (Kuga et al., 1987):

$$\frac{d\alpha}{dt} = \frac{N_x}{I_z\, W\cos(\delta)} \qquad (1)$$

$$\frac{d\delta}{dt} = \frac{N_y}{I_z\, W} \qquad (2)$$

$$\frac{dW}{dt} = \frac{N_z}{I_z} \qquad (3)$$

Where I_z is the axis of the greatest principal moment of inertia around the satellite spin axis and N_x, N_y and N_z are the torque components in the three axes of the coordinate system of the satellite.

Motta (Motta, 2014) introduces an analytical solution to the equations of rotational motion, valid for an orbital period and composed of the average components of aerodynamic torque **TA**, gravity gradient torque **GGT**, solar radiation torque **SRT**, eddy current torque **ECT**, and residual magnetic torque **RMT**.

$$W = \left(W_0 + \frac{N_{GGz}}{N_{ECz}}\right) e^{\frac{N_{ECz}}{I_z} t} - \frac{N_{GGz}}{N_{ECz}} \tag{4}$$

$$\delta = \frac{t}{I_z}\left(N_{ECy} - \frac{N_{ys} N_{ECz}}{N_{GGz}}\right) + \frac{N_{ys}}{N_{GGz}} ln\, ln \left(\frac{W}{W_0}\right) + \delta_0 \tag{5}$$

$$\alpha = \frac{t}{I_z \cos\bar{\delta}}\left(N_{ECx} - \frac{N_{xs} N_{ECz}}{N_{GGz}}\right) + \frac{N_{xs}}{N_{GGz} \cos\bar{\delta}} ln\, ln \left(\frac{W}{W_0}\right) + \alpha_0 \tag{6}$$

Where W is the rotational velocity at rad/s, W_0 is the initial value of the rotational velocity, N_{GGz} and N_{ECz} are the components on the z-axis for the torque of gravity gradient and eddy currents, respectively, N_{ECy} and N_{ys} are the components on y-axis for eddy current torque and the sum of the components of the torques on the y-axis that are not multiplied by W, respectively, δ_0 is the initial value of the declination of the spin axis, $\bar{\delta}$ is the average between the calculated slope and its initial value, N_{xs} is the sum of the components of the torques on the x-axis which are not multiplied by W and α_0 is the initial value for the right ascension of the spin axis.

Based on the data provided by INPE, the aim of this paper is to verify the validity of the propagated results for the new simulated periods. To incorporate the variables data input, the routines for defining orbital and attitude parameters were reworked (Motta, 2014). At the end of the investigation, it was found that the propagator can simulate the attitude of the satellites within the required accuracy for several years and days, showing that the results depend directly on the input data. INPE's required precision for validation is ±0.5° for the right ascension angle α and for declination δ and ±0.5(rpm) for the modulus of the spin velocity around the z-axis of the satellite. Therefore, for a result to be validated it should satisfy all 3 criteria simultaneously.

The daily update is about how the propagator will use the data provided. If there is a daily update, the satellite attitude will be propagated for 24 hours and, after this period, data referring to the new day will be used to perform a new propagation, and so on until the end of the simulation. If we decide not to update the data daily, after the first day of propagated attitude, the program will use its own results to start the next day's propagation, loading errors and consequently increasing the inaccuracy of the results. In this paper only the results for daily update are presented for SCD1 Brazilian satellite and to standardize the simulations, all daily updated intervals are fixed at 20 days.

2 METHODOLOGY

To perform the simulations, it is used an already developed program that provides as output results for the Brazilian Data Collection Satellites SCD1 or SCD2, using daily data update or not. In the GSAMQ propagator an automation was introduced, to get the results faster.

The input variables of the propagator are:

Satellite: 1 for SCD1 or 2 for SCD2

Year: from 1993 to 2013 for SCD1 or from 1998 to 2013 for SCD2.

Day: from 1 to 365 or 366 for leap years.

Note: SCD1 data for 1993 starts on February 11 and SCD2 data for 1998 starts on October 24.

Daily update: 1 for yes, 2 for no. If you select option 2 you must enter the number of days of the interval without updating will have.

Save: 1 for yes, 2 for no.

The output of the propagator are the averages of the errors obtained for right ascension α, declination δ, and the spin velocity W. It is also stored the results for each of these variables.

This paper presents the results regarding the simulations performed for SCD1, with daily data update for different time intervals.

3 RESULTS

The simulations were performed with MATLAB software and the results are compiled in the following tables, using the data provided by INPE for SCD1 satellite, with daily data update for different time intervals. Despite the automation performed in GSAMQ, it still requires a longer time to perform the simulations than in the GSAM propagator. This is due to the magnetic torque computed with the quadrupole model in GSAMQ.

3.1 ANNUAL STATISTICAL ANALYSIS

A statistical analysis is performed to present the relationship between the number of simulations with positive results and the total number of simulations performed in the period of one year. The analysis is performed considering that the SCD1 simulation requires data entry referring for a period of 20 days.

To better visualize the behavior of each simulated variable, the three parameters (right ascension, declination, and spin velocity) are separately accounted for. Each percentage represents the relation of the number of simulations to the total procedures which the obtained mean deviations were satisfactory comparing to the required accuracy established by INPE.

This statistical analysis has some advantages. It allows to see which parameters have met the pre-established criteria and to verify for which intervals in the year the proposed model has had a better performance.

The annual statistical analysis for the simulations performed for the satellite SCD1 for the period of 20 days for each simulation are presented in Table 1, highlighting the percentage of the simulations performed for each variable that satisfied INPE's required accuracy.

Table 1: Annual statistical analysis, SCD1 with 20-days interval with daily update.

Year	**Number of simulations performed**	**Annual Percentage (%)**		
		$\Delta\alpha$ (◦)	**$\Delta\delta$ (◦)**	**ΔW (rpm)**
1994	46	39.13	100.00	100.00
1995	33	9.09	75.76	100.00
1996	16	0.00	100.00	100.00
1997	38	0.00	94.74	100.00
1998	15	0.00	93.33	100.00
1999	34	0.00	91.18	100.00
2000	37	0.00	78.38	100.00
2001	29	0.00	100.00	100.00
2002	30	0.00	86.67	100.00
2003	17	0.00	76.47	100.00
2004	16	0.00	75.00	100.00
2005	30	0.00	70.00	100.00
2006	26	15.38	73.08	100.00

From these results it can be observed that the simulations by GSAMQ are accurate and reliable for the spin velocity, as all the simulations met the desired accuracy. For the declination δ it also obtained good results with a relatively high accuracy. However, for the right ascension α, it is necessary that the GSAMQ propagator be redesigned to reduce the error of this parameter. The good performance for the spin velocity is expected, since the model for the Earth's magnetic field is improved in GSAMQ, and this field directly affects the magnetic torque due to eddy currents and the analytical solution of the spin velocity and it can be seen in equation 4 in the introduction of this paper.

To analyze the accuracy of the results that did not meet the first criterion, Table 2 expands the maximum uncertainty criterion previously adopted from ±0.5◦ to ±1.0 for α and δ and ±0.5(rpm) for ±1.0$(r\ pm)$ for W.

With these new results in Table 2, it can be observed that the error found for the right ascension with GSAMQ still does not satisfy the new criterion established, but it is relatively close to this range. Then, the results have shown that the model is quite robust, and that only a fine tuning is necessary to guarantee an improvement in accuracy.

Table 2: Annual statistical analysis, SCD1 with 20-days interval with daily update for the new criterion.

Year	**Number of simulations performed**	**Annual Percentage (%)**		
		Δα (◦)	**Δδ (◦)**	**ΔW (rpm)**
1994	46	71.74	100.00	100.00
1995	33	57.58	93.94	100.00
1996	16	57.58	100.00	100.00
1997	38	84.21	100.00	100.00
1998	15	80.00	100.00	100.00
1999	34	73.53	97.06	100.00
2000	37	70.27	94.59	100.00
2001	29	93.10	100.00	100.00
2002	30	53.33	100.00	100.00
2003	17	64.71	100.00	100.00
2004	16	68.75	100.00	100.00
2005	30	40.00	100.00	100.00
2006	26	42.31	100.00	100.00

3.2 COMPARISON GSAMQ AND GSAM

Using the results previously obtained with the GSAM propagator and the GSAMQ results, it can be analyzed the results obtained for each simulation in the same period and with the same initial input data. The Comparison between the two propagators allows to verify which parameters have been better in each propagator.

3.2.1 Gsamq Results Within The Required Accuracy

Table 3 shows the results for the mean errors of each variable which were simulated by GSAMQ and provided as output the three variables within the pre-established accuracy of $\pm 0.5^{\circ}$ for declination and right ascension and $\pm 0.5\ rpm$ for the spin velocity. It can be observed a significant improvement in the mean of the errors for all variables except for a few days, especially for the spin velocity. It should also be noted that in the last 7 days the GSAMQ propagator obtained results within the required accuracy while GSAM propagator exceeded the accuracy.

Figures 2, 3, and 4 show the results obtained in the simulations on January 1, 1994 for SCD1. In these figures it is possible to observe the differences between the results obtained and the data from INPE for each day, the mean of the differences and the standard deviation obtained in the simulations for the entire 20-day period.

Table 3: GSAMQ and GSAM results

Start	**End**	**GSAM**	**GSAMQ**	**GSAM**	**GSAMQ**	**GSAM**	**GSAMQ**
		Δα(∘)		**Δδ(∘)**		**ΔW (rpm)**	
01/01/1994	01/20/1994	0.73162	0.09596	0.04645	0.22295	-0.98876	-0.12481
01/02/1994	01/21/1994	0.69771	0.13251	0.03562	0.25204	-1.00610	-0.12710
01/03/1994	01/22/1994	0.63089	0.17420	0.01239	0.27502	-1.09630	-0.13018
01/04/1994	01/23/1994	0.80904	0.26048	0.44788	0.30031	-1.68110	-0.13352
01/05/1994	01/24/1994	0.67944	0.30720	0.39741	0.32005	-1.82670	-0.13441
01/06/1994	01/25/1994	0.53319	0.35788	0.34840	0.33586	-1.92130	-0.13446
01/07/1994	01/26/1994	0.39824	0.41317	0.31367	0.35442	-1.96840	-0.12996
01/08/1994	01/27/1994	0.29808	0.47268	0.27354	0.35937	-2.01780	-0.12518
02/06/1994	02/25/1994	-0.03324	0.44747	-0.39970	-0.42966	-0.00904	-0.04125
02/07/1994	02/26/1994	-0.08833	0.36806	-0.41215	-0.42117	-0.01193	-0.04275
02/08/1994	02/27/1994	-0.11034	0.32026	-0.39355	-0.39628	0.10154	-0.04628
02/09/1994	02/28/1994	-0.13641	0.26849	-0.40323	-0.37272	0.10408	-0.04919
02/10/1994	03/01/1994	-0.18544	0.18870	-0.41395	-0.34978	0.10251	-0.05206
02/11/1994	03/02/1994	-0.18892	0.14613	-0.41327	-0.31874	0.09058	-0.05436
02/12/1994	03/03/1994	-0.17846	0.10812	-0.40472	-0.28695	0.06965	-0.05663
02/13/1994	03/04/1994	-0.15553	0.07463	-0.39247	-0.25714	0.04299	-0.05887
02/14/1994	03/05/1994	-0.14570	0.02279	-0.38023	-0.22462	0.00581	-0.06527
02/15/1994	03/06/1994	-0.10523	0.00108	-0.37055	-0.18812	-0.03258	-0.06637
02/06/1995	02/25/1995	-1.61890	0.47819	2.31750	-0.16812	-3.39570	-0.02108
02/09/1995	02/28/1995	-1.69100	0.48356	2.80930	-0.01495	-3.91800	-0.02385

02/12/1995	03/03/1995	-1.97420	0.48758	2.97370	0.10917	-3.60790	-0.01977
03/12/2006	03/31/2006	-0.98687	0.31356	-1.30980	-0.38089	-0.00409	-0.00444
03/17/2006	04/05/2006	-0.99471	0.21463	-1.36340	-0.13076	0.01021	0.00153
03/22/2006	04/10/2006	-0.98015	0.09477	-1.47650	0.00340	0.04473	0.00160
03/27/2006	04/15/2006	6.23630	0.02996	7.35890	-0.10630	0.04601	-0.00295

Figure 2: Time behavior of the mean right ascension error from 01/01/1994

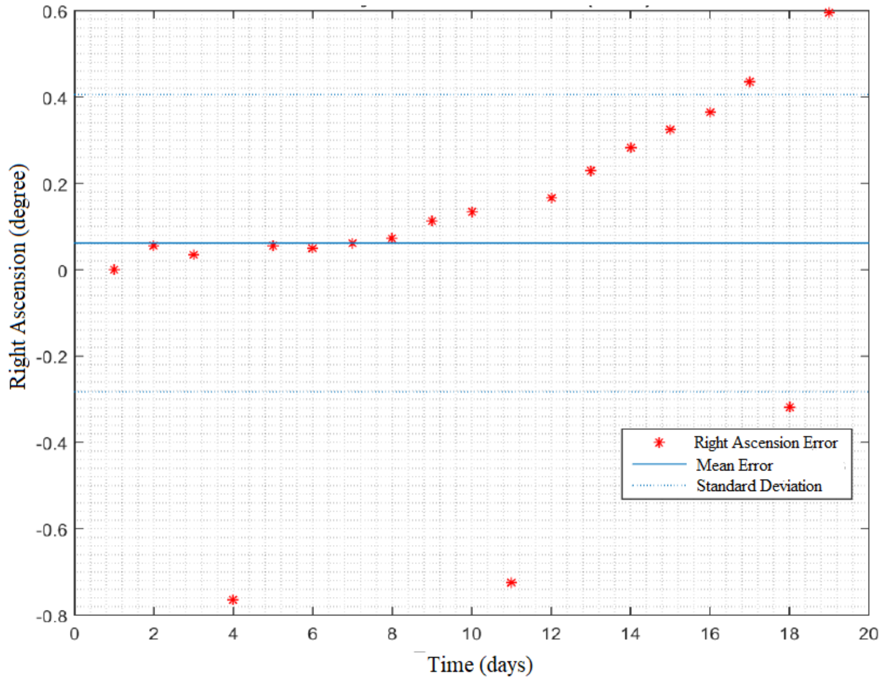


Figure 3: Time behavior of the mean declination error from 01/01/1994

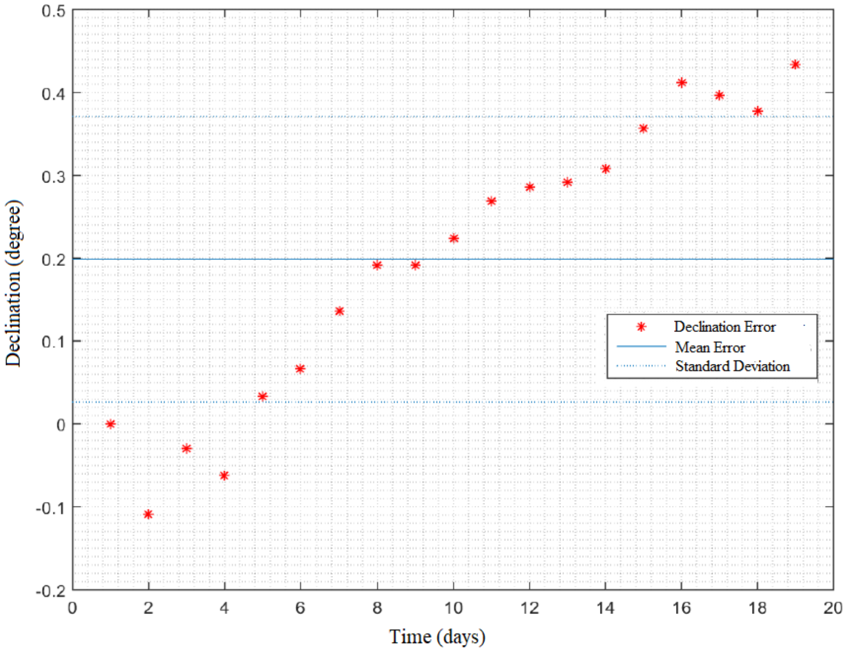
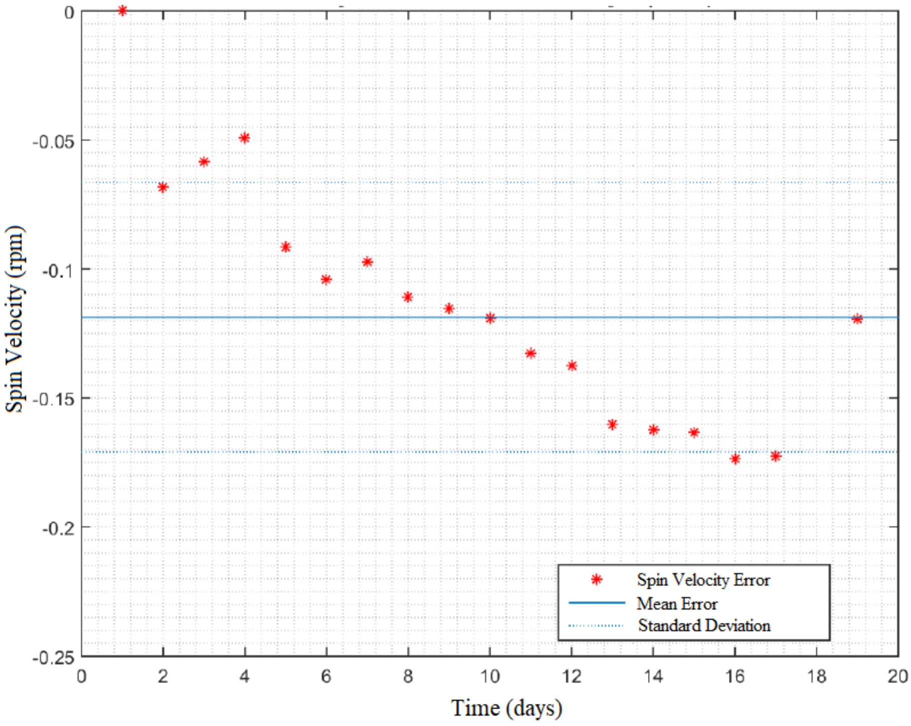


Figure 4: Time behavior of the mean spin velocity error from 01/01/1994



3.2.2 SCD1 simulation with update for 20days and 1 result by year

Table 4 shows the best results for the mean of the errors for each variable in each year in which they were simulated and whose results are within the required accuracy.

From the results in Table 4 it can be observed that there is an increase in accurate results in comparison with the results presented in Zanardi et al (2018) and Zanardi and Mota (2020), obtaining some cases in which the accuracy for right ascension is satisfy.

It is also observed that for the considered intervals in Table 4:

- For the spin velocity only in one simulation (03/17/2001) the accuracy of the GSAM propagator was better than GSAMQ, with a significant decrease in the mean in the mean errors for GSAMQ.
- For the declination of the spin axis only two simulations show better accuracy for GSAM (on 10/07/1998 and 04/30/2001) than the GSAMQ, while all the others significantly decreased the mean errors for GSAMQ, with highlight for the days 02/09/1995, 03/07/2002, 01/006/2003 and 03/22/2006.
- For the right ascension of spin axis, the results for the GSAM propagator are better than for the GSAMQ. Except for 3 days (02/09/1995, 04/30/2000, 03/22/2006) the GSAM results get the required accuracy. However, the GSAMQ propagator get the accuracy only for 3 days (02/12/1994, 02/09/1995 and

03/22/2006), although Table 3 shows other good results with GSAMQ for 1994, 1995 and 2006.

Table 4: SCD1 accurate results for year.

Start	**End**	**GSAM**	**GSAMQ**	**GSAM**	**GSAMQ**	**GSAM**	**GSAMQ**
		Δα(∘)		**Δδ(∘)**		**ΔW (rpm)**	
02/12/1994	03/03/1994	-0.17846	0.10812	-0.40472	-0.28695	0.06965	-0.05663
02/09/1995	02/28/1995	-1.69100	0.48356	2.80930	-0.01495	0.06965	-0.02385
02/06/1996	02/25/1996	0.06623	0.69710	-0. 53371	0.30211	-0.04583	0.00590
03/23/1997	04/11/1997	-0.09355	0.75286	-0.60282	0.44890	0.02350	0.01586
10/07/1998	10/26/1998	-0.22206	0.76857	0.28846	0.30528	0.01140	0.00620
01/16/1999	02/04/1999	-0.21899	0.79699	-0.52220	0.35957	-0.04913	-0.00188
04/30/2000	05/19/2000	-0.55251	0.70297	-0.00520	-0.18481	-0.08446	-0.01768
03/17/2001	04/05/2001	-0.22952	0.68625	-0.57298	-0.29527	-0.00662	0.08250
03/07/2002	03/26/2002	-0.33671	0.88720	0.01584	0.01584	-0.03448	-0.00042
01/06/2003	01/25/2003	-0.07683	0.92577	-0.69741	0.05818	0.161513	0.01744
01/01/2004	01/20/2004	-0.06391	0.94855	-0.52483	0.42569	0.08570	0.01160
04/11/2005	04/30/2005	0.23988	0.90535	-0.23964	0.21314	0.07657	0.00875
03/27/2006	04/15/2006	6.23630	0.02996	7,35890	-0.10630	0.04601	-0.00295

4 CONCLUSIONS

By the results, it is possible to observe that the precision required by INPE was obtained for several analyzed intervals, and that with the optimization of the program, reducing the processing time, several simulations were carried out.

Comparing the GSAMQ results with the GSAM results (which considered the dipole model for the Earth's magnetic field) the results show:

- that with the use of the GSAMQ there was a significant improvement in the mean errors for rotation speed, with the quadrupole model showing itself to be more adequate. It should be noted that the torque with the greatest influence on the analytical solution of the rotational speed is the magnetic torque due to eddy currents, which causes the exponential decay in the rotational speed.
- for the rotation axis declination, the error accuracy was also better for the GSAMQ.
- for the right ascension of the rotation axis, the GSAM results are better. It is justified due to the right ascension angle being most influenced by the solar radiation torque, the aerodynamic torque and gravity gradient torque (Motta,

2014). In the propagator GSAM propagator all the torques are computed simultaneously, so that the influence of all the torques are computed jointly. However, in the GSAMQ propagator, due to the large volume of terms associated with the quadrupole model, the residual magnetic torques and eddy currents are computed outside the main program and this is affecting the behavior of the right ascension angle. Thus, the GSAMQ implementation can be improved by including the calculation of these torques within the main program, but it requires time and computational power.

So, the GSAMQ propagator still needs to be analyzed to improve the behavior of all the variables involved.

The simulations also indicate a good behavior of the GSAM propagator, which is faster and obtained good results, that can be used to predict the attitude behavior of spin stabilized satellites in the early stages of the projects and to verify the need for control intervention during the useful life of the satellites

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