Study of the magnetohydrodynamic flow of a Newtonian fluid using Laplace transform

Estudo do escoamento magnetohidrodinâmico de um fluido Newtoniano via transformada de Laplace

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ABSTRACT
The present work studies the magnetohydrodynamic flow of a Newtonian fluid as a complementary case of the simple flow in a parallel plate channel. The effects of kinematic parameters and the influence of the transverse and uniform magnetic field on the fields of speed, flow and shear rate are analyzed. To solve the equations that govern movement, the Laplace Transform technique was used, and the Python programming language and wxMaxima program for the construction of graphs and convergence analysis, respectively. The results obtained were analyzed for different values of the Hartmann's number and the limit case in which this parameter tends to zero compared to the simple flow in the absence of a field. The study showed the effect of the magnetic field on the velocity profile and other derived properties. And the limit case proved to be compatible with the case in which there is no magnetic field applied. In general, the work proved to be valid as a complementary study of the simple case, since the results obtained for the limit case clearly express its tendency towards the simple case presented by the literature.

Keywords: Magnetohydrodynamics, Fluid mechanics, Laplace transform.

RESUMO
O presente trabalho estuda o escoamento magnetohidrodinâmico de um fluido newtoniano como caso complementar do escoamento simples em canal de placas paralelas. Analisam-se, os efeitos dos parâmetros cinemáticos e a influência do campo magnético transversal e uniforme sobre os campos de velocidade, vazão e taxa de cisalhamento. Para solução das equações que regem o movimento foi utilizada a técnica da transformada de Laplace, e a linguagem de programação Python e programa wxMaxima para construção dos gráficos e análise de convergência, respectivamente. Os resultados obtidos foram analisados para diferentes valores do número adimensional Hartmann e o caso limite em que tal parâmetro tende a zero comparado com o escoamento simples na ausência de campo. O estudo evidenciou o efeito do campo magnético sobre o perfil de velocidade e demais propriedades derivadas. E o caso limite mostrou-se compatível com o caso em que não há campo magnético aplicado. No geral, o trabalho apresentou-se válido como estudo complementar do caso ideal, visto que os resultados obtidos para o caso limite expressam claramente sua tendência para o caso simples apresentado pela literatura.

Palavras-chave: Magnetohidrodinâmica, Mecânica dos fluidos, Transformada de Laplace.

1 INTRODUCTION

Fluid mechanics is the study of fluids in motion (fluid dynamics) or at rest (fluid static). A fluid can be a liquid or a gas, thus when we think about it, we realize that everything around us is submerged in fluid [1]. But, what are fluids?

The term "fluid" refers to a state of matter, not a substance. In the study of elementary physics, one learns that there are three states in which matter can be found, they are: solid, liquid and gas. The term "fluid" refers to the second and third of these states collectively [2] A substance in the fluid state deforms continuously when subjected to the action of a tangential force, while it is being applied on it, and in the absence of the force ceases the deformation, however, it does not return to its original configuration, it does not present "memory" as elastic solids [3].

Fluids are classified as the relationship between the applied stress and the strain rate. Being called Newtonian fluids that follow a directly proportional relationship between force and deformation in which viscosity is the proportionality constant, and non-Newtonian sands that have variable proportionality between such parameters [4].

They can also be categorized as to the type of laminar or turbulent flow. Laminar flow can be idealized if we imagine a fluid being composed of several layers, such as the movement of parallel blades that do not mix. In the turbulence, the fluid moves in a whirlwind that move randomly with a mixture between layers.

The study of laminar flow and the properties of incompressible Newtonian fluids in ducts of different geometries (parallel plates, cylinder and others) is already something well defined in the literature. However, the understanding of this flow in the condition in which a transverse and uniform magnetic field has been applied influencing the fluid movement can be studied as an extension of simple flow, and parameters such as velocity field, flow and shear rate can be investigated.

The application of an external magnetic field to a flow allows applications in industry such as heating, pumping, stirring and levitation of liquid metals. Formally this type of flow is called magnetohydrodynamics (MHD), which deals with the mutual interaction of flow and magnetic field. For such flow the fluid must be electrically conductive and non-magnetic, limited to liquid metals, ionized hot gases (plasmas), and strong electrolytes [5].

In this sense, the importance of understanding the implications of the transverse magnetic field in the flow of a conductive Newtonian fluid is evident, as a complementary study of simple flow. Thus, the present work seeks to perform this complementation using the Laplace Transforms method to solve the equations that describe the behavior of the magnetohydrodynamic phenomenon, such as analyzing the effect of kinematic parameters and the influence of the transverse and uniform magnetic field on the velocity, flow and shear rate fields associated with the flow of a Newtonian fluid.

The relevance of the present work is that it is an extension of the investigation of simple flow in parallel plate channel. Implementations were: flow of a conducting fluid and application of an external magnetic field to the flow. An approach is presented that investigates the flow influenced by an applied external magnetic field, which interacts with the conductive fluid flow characterizing a Magnetohydrodynamic flow. It is also noted that the presentation of the stages of development together with the details of the calculations make this work also have a didactic character, since it is possible to follow the development of the mathematical argumentation behind the description of the MHD phenomenon studied. It also serves as a theoretical study base for further implementations.

2 MAGNETOHYDRODYNAMIC

Studies on electricity and magnetism show that magnetic fields interact with many natural and artificial liquids. They are present in various applications in the industry such as heating, pumping and levitation of liquid metals and are generated even in the core of the Earth, in which they are maintained by the incessant movement of fluid that is in this region. This phenomenon of interaction of magnetic fields with fluids is called magnetohydrodynamics or simply MHD [6].

In technical terms, MHD deals with the mutual interaction between the speed field of the moving fluid and the applied magnetic field. For this, the fluid necessarily needs to be electrically conductive and non-magnetic, which limits its use to liquid metals, ionized gases and electrolytes. Quantitatively, this mutual interaction is based on the Laws of Faraday and Ampère and Lorentz forces for a current-carrying body [6] [5].

The first use of the term MHD dates back to the work of the Swedish Hannes Olof Gosta Alfvén published in 1942 whose title is "Existence of Electromagnetic-Hydrodynamic Waves", in which he studied the behavior of a conductive liquid in the presence of a constant magnetic field. The scientist noted that when a conductive liquid is placed in a uniform magnetic field, each movement of the liquid produces an electromotive force that induces electrical current. Moreover, that due to the magnetic field, these currents generate mechanical forces that modify the state of motion of the liquid, which would produce a type of electromagnetic-hydrodynamic wave, whose confirmation of its existence occurred seven years later, through the study of waves in liquid mercury [7].

The Study of MHD has become the focus of many studies in recent decades, mainly in the sense of seeking explanations for the behavior of certain properties inherent to fluids, as well as possible applications for a varied number of processes involving the effect of a constant magnetic field applied on the flow of a conductive fluid.

Alireza and Sahai [8] studied the effect of temperature-dependent transport properties on MHD flow development and heat transfer in a parallel plate channel whose walls are maintained at constant and equal temperatures. They concluded, through representative numerical results, that the variation of properties may have a significant effect on the development of speed and temperature profiles.

Malashetty and Leela [9] conducted an analytical study of the problem of biphasic MHD flow and heat transfer in a horizontal channel. As a result, they verified for the case of open circuit problem for negative values of the electrical charge parameter, that the effect of the increase in the number of Hartmann is to accelerate the speed and increase the temperature field compared to the short circuit situation. They also concluded that, for adequate values of depth, viscosity, thermal and electrical conductivity ratios, speed and temperature may be increased.

Umavathi [10] analytically investigated fully developed flow and heat transfer in a horizontal channel containing electrically conductive fluid sandwiched between two layers of fluid. They verified that, despite the effect of increasing the Hartmann number is to decrease the velocity and temperature field, with adequate values of viscosity and thermal conductivity ratios, velocity and temperature can be increased.

Stamenkovic [11] studied the MHD flow of two electrically immiscible and electrically conductive fluids between isothermal and isolated mobile plates in the presence of an applied and magnetic inclined electric field. They obtained as one of their results that the reduction of the angle of inclination of the magnetic field levels the velocity and temperature profiles.

Kuiry and Bahadur [12] analyzed the constant MHD flow of viscous fluid between two parallel pory plates with heat transfer in the presence of an inclined magnetic field together with the effect of normal influx to the plates, flow pressure gradient and temperature. They observed that the parameters involved in the determination of velocity and temperature profiles play a vital role in the flow. What's more, that fluid with high viscosity causes reverse flow and the magnetic force applied at different inclinations controls this flow. In addition to verifying that the temperature distribution in the fluid layers is directly related to their viscosity.

Campos [13] performed the three-dimensional numerical simulation of a magnetohydrodynamic alternating current pump, whose main objective was to evaluate the implications of using currents of this nature on the flow and compare them with the case of direct current in which the average Lorentz force exerted by the pump is equivalent. As a result, it was observed that from about 10 Hz there is an agreement between the velocity profile for equivalent DC and CA cases, however for lower frequencies some periodicity is evidenced; and that heating due to the Joule effect has a greater significance for the AC configuration, and the pump can operate critically. In addition, current density and pressure were evaluated.

These are some of the work carried out over the last decades, in which the phenomenon of MHD is explored for different conditions of the flow of a fluid, as well as the magnetic field itself applied. What can be seen is that it is still a line of research in the maturing phase.

3 ANALYTICS FEATURES

3.1 PYTHON PROGRAMMING LANGUAGE

Python is a very high-level object-oriented programming language, dynamic and strong typing, interpreted and interactive. It has a clear and concise syntax that favors the readability of source code, which makes the language more productive. It is an open source software, which democratizes access to such language. It was established in 1990 by Guido van Rossum at the Netherlands National Institute of Research for Mathematics and Computer Science (CWI). Initially focused on the use by physicists and engineers, but is currently widely used for industrial and technology companies [14]. For the construction of two-dimensional (2D) graphics, the Matplotlib library was used, which is part of the libraries of the Python programming language. Such graphs will be used as subsidies to understand the effect of the Hartmann dimensionless number on the flow.

3.1 WXMÁXIMA PROGRAM

For the convergence tests of the defined equations, the wxMaxima software was used, consisting of one of the CAS (Computer Algebra System) systems for manipulation of symbolic and numerical expressions, including limits, differentiation and integration, matrices, functions, among others, working their data in two or three dimensions [15]. Created by the MAC group at MIT in the 1960s, it was initially called Macsyma *(Project MAC's SYmbolic MAnipulator). Macsyma* was first developed for large-scale computers that were used in various academic institutions. From the 1980s it was developed for various platforms, and one of these new versions was called máxima. In 2001, the distribution of máxima became free [16].

From this, it is intended to define the equations that govern the behavior of the MHD flow of a Newtonian fluid, using the Laplace Transform technique. Starting from the equations obtained, graphs will be constructed through the Python programming language using the Matplotlib library for this purpose, in addition to the creation of tables to evaluate the convergence of the limit case in which the Hartmann's number (H_a) tends to zero with the simple flow present in the literature. The objective is also to understand the effect of uniform and transverse magnetic field on the profile of speed, maximum velocity, flow rate and shear rate.

4 MATHEMATICAL FORMULATION OF THE PROBLEM

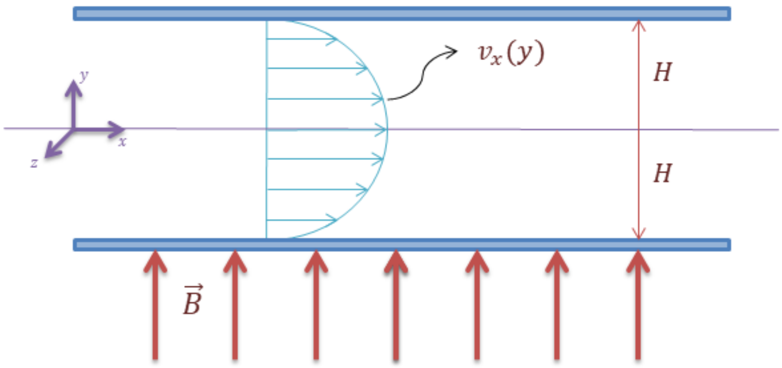
A classic problem in Fluid Mechanics is the flow in parallel plates channels. For the present work, the study of the flow of a conducting Newtonian fluid in the presence of a constant magnetic field normal to the plates will be carried out.

4.1 PROBLEM DESCRIPTION

First among the properties of a flow is the velocity field, the others being (maximum velocity, flow, etc.) as derived properties. Thus, it is initially desired to obtain the velocity profile for a conductive Newtonian fluid that flows between two parallel flat plates, under the effect of constant pressure, and subject to an external magnetic field, uniform and perpendicular to the plates. For the study of this flow, a fully developed velocity profile is considered, and that a pressure difference at the ends (inlet and outlet) of the duct is the cause of the flow, which is stationary (velocity and pressure are constant), incompressible Newtonian fluid and laminar flow. With these conditions the viscosity and density properties are constant [17].

A rectangular coordinate system (x,y,z) is centered in the middle of the space between the two plates: the x coordinate in the direction of the constant pressure gradient and y perpendicular to the plates. **Figure (1)** below illustrates the problem analyzed.

Figure 1 - Scheme of the formulation of the problem.



Source: Authors

The hydraulic diameter for this situation is given by:

$$\boldsymbol{D_H = 4H} \tag{1}$$

Away from the entrance and edges the fluid moves in layers parallel to the plates and in the direction of the pressure gradient. Thus we have that the only component of the nonzero velocity is v_x. It is also said that v_x is a function of only the transverse coordinate y, because $v_x > 0$ between the plates $(-H < y < H)$ and $v_x = 0$ on the surface of the plates [17].

$$v_z = v_y = 0 \quad \text{throughout the channel} \tag{2}$$

$$v_x = v_x(y) \quad \text{to } -H < y < H \tag{3}$$

$$p = constant \quad \text{to } x > 0 \tag{4}$$

It is assumed that the magnetic field is not affected by the runoff field. Thus, the interaction of a single track is analyzed, that is, between the flow of an electrical conductive fluid and a magnetic field, since the magnetic field affects the field of yield, but the field of flow does not affect the magnetic field [18].

With these considerations, and being the transverse area of the duct constant the material balance is automatically satisfied, and the linear momentum balance is reduced to a balance of forces [17]. Thus, we have the following equation:

$$-\frac{\partial \tau_{xy}}{\partial y} = \frac{\partial p}{\partial x} + \sigma {B_0}^2 v_x \tag{5}$$

The one-dimensional stress tensioner of a Newtonian fluid is given by:

$$\tau_{xy} = -\boldsymbol{\eta}\frac{\partial v_x}{\partial y} \tag{6}$$

Where η is viscosity. Replacing equation (6) in equation (5) we have that the equation of movement for this flow is given by:

$$\eta\frac{\partial^2 v_x}{\partial y^2} - \boldsymbol{\sigma B_0{}^2 v_x} = \frac{\partial p}{\partial x} \tag{7}$$

Admitting that $\frac{\partial P_x}{\partial x} = constant = p_x$ and applying the dimensionless groups:

$$U = \frac{v_x}{v_c}, R = \frac{y}{H}, v_c = v_{máx} = -\frac{p_x D_H^2}{32\eta}, H_a = B_0 H\sqrt{\frac{\sigma}{\eta}}$$

We have the adimensionalized equation:

$$\frac{d^2U}{dR^2} - \boldsymbol{H_a^2 U = -2} \tag{8}$$

That corresponds to the differential equation that models the MHD flow of a Newtonian fluid. This equation is dependent only on the dimensionless component R, being invariant in time.

5 SOLUTION METHODS

To solve the dimensionless velocity profile equation, the Laplace Transform technique was used. This technique transforms a differential equation given into an algebraic equation reducing the problem of integrating a differential equation to solving an algebraic equation.

The condition of existence of the transform assumes that the integral improper in its definition converges at least to some value of s, and that $f(t)$ is defined in the interval $[0,\infty)$, and that the function $f(t)$ is continuous by parts in any finite interval of the positive semi-axis $[0,\infty)$, and that does not grow faster than the exponential function, when $x \to \infty$ [19].

It is also noted that the Laplace Transform applies only to time-invariant linear systems, that is, common systems do not change with time (also called systems with constant parameters) [20].

Thus, applying the Laplace Transform to equation (8), we have the following expression:

$$s^2U(s) - sU(R)|_{R=0} - \frac{dU}{dR}|_{R=0} - H_a^2 U(s) = -\frac{2}{s} \tag{9}$$

With the following boundary conditions:

$$\frac{dU}{dR}|_{R=0} = 0 \tag{10}$$

$$\boldsymbol{U(0) = B} \tag{11}$$

In which $U(0) = B$ is an assigned constant that will be determined later. Thus, we come to the following solution for the algebraic equation:

$$\boldsymbol{U(s) = \frac{-2}{s(s^2 - H_a^2)} + B\frac{s}{s^2 - H_a^2}} \tag{12}$$

Applying the inverse transform we will have the solution of the differential equation:

$$\boldsymbol{U(R) = L^{-1}\{U(s)\} = L^{-1}\left\{\frac{-2}{s(s^2 - H_a^2)} + B\frac{s}{s^2 - H_a^2}\right\}} \tag{13}$$

Resulting:

$$\boldsymbol{U(R) = -\frac{2}{H_a^2}[-1 + cosh\,(H_a R)\,] + Bcos\,h(H_a R)} \tag{14}$$

Applying the condition $U(R = 1) = 0$ to equation (14) we have:

$$\boldsymbol{U(R = 1) = 0 = \frac{2}{H_a^2} - \frac{2\,cosh\,(H_a)}{H_a^2} + Bcosh\,(H_a)} \tag{15}$$

Which results that the value of B is:

$$\boldsymbol{B = \frac{2}{H_a^2}\left[1 - \frac{1}{cosh\,(H_a)}\right]} \tag{16}$$

Replacing B in equation (14) we have:

$$\boldsymbol{U(R) = \frac{-2}{H_a^2}[-1 + cosh\,(H_a R)\,] + \frac{2}{H_a^2}\left[1 - \frac{1}{cosh\,(H_a)}\right]cosh\,(H_a R)} \tag{17}$$

Grouping the terms we obtain the equation that describes the flow velocity profile:

$$\boldsymbol{U(R)} = \frac{2}{H_a^2}\left[1 - \frac{\cosh\,(H_a R)}{\cosh\,(H_a)}\right] \tag{18}$$

For the limit case where the Hartmann's number tends to zero in equation (18) we have:

$$\boldsymbol{U(R) = 1 - R^2} \tag{19}$$

Which represents the dimensionless shape of the velocity profile for the case of simple flow in the absence of a magnetic field [17].

From equation (18) we have for the other flow parameters:

MAXIMUM SPEED

The maximum speed is at the center of the geometry at $R = 0$, so starting from equation (18) we have:

$$\boldsymbol{U(R = 0) = \frac{2}{H_a^2}\left[1 - \frac{\cosh\,(0)}{\cosh\,(H_a)}\right]} \tag{20}$$

Being $cosh\ cosh\ (0) = 1$, we have;

$$\boldsymbol{U_{máx} = \frac{2}{H_a^2}\left[1 - \frac{1}{\cosh\,(H_a)}\right]} \tag{21}$$

VOLUMETRIC FLOW

The volumetric flow is given by the integral:

$$\boldsymbol{Q = \int_{-R}^{R} U(R)\, dA} \tag{22}$$

With

$$\boldsymbol{dA = w\, dy} \tag{23}$$

and

$$\boldsymbol{dy = H\, dR} \tag{24}$$

We have:

$$\boldsymbol{Q = wH \int_{-R}^{R} \frac{2}{H_a^2}\left[1 - \frac{\cosh\,(H_a R)}{\cosh\,(H_a)}\right] dR} \tag{25}$$

Organizing:

$$\boldsymbol{Q} = \frac{2wH}{H_a^2}\int_{-1}^{1}\left[1 - \frac{\cosh(H_a R)}{\cosh(H_a)}\right] dR \tag{26}$$

Solving the integration we have:

$$\boldsymbol{Q} = \frac{2wH}{H_a^2}\left[2 - \frac{\sinh(H_a R)}{H_a \cosh(H_a)}\right] \tag{27}$$

SHEAR RATE

Considering Newton's law of viscosity, given by the equation:

$$\tau_{xy} = -\eta \frac{\partial v_x}{\partial y} \tag{28}$$

And the dimensionless group:

$$U = \frac{v_x}{v_c},\ R = \frac{y}{H},\ v_c = v_{máx} = -\frac{p_x D_H^2}{32\eta}$$

With v_c and H constant, the equation (28) becomes:

$$\tau_{xy} = -\frac{\eta v_c}{H}\frac{dU}{dR} \tag{29}$$

Starting from the definition of v_c we have for expression (28):

$$\tau_{xy} = \frac{p_x H}{2}\frac{dU}{dR} \tag{30}$$

Substituting U by equation (18) we have:

$$\tau_{xy} = \frac{p_x H}{2}\frac{d}{dR}\left[\frac{2}{H_a^2} - \frac{2\cosh(H_a R)}{{H_a}^2 \cosh(H_a)}\right] \tag{31}$$

Which after derivation results in:

$$\tau_{xy} = -\frac{p_x H \sinh(H_a R)}{H_a \cosh(H_a)} \tag{32}$$

We have the shear rate given by:

$$\dot{\gamma} = \frac{\partial v_x}{\partial y} = \frac{-p_x H \sinh(H_a R)}{\eta H_a \cosh(H_a)} \tag{33}$$

6 RESULTS AND DISCUSSION

The results that follow were performed for different values of the Hartmann's number, in addition to the comparison between the limit case of $\boldsymbol{H_a \to 0}$ with the flow without the presence of the magnetic field, by means of numerical analysis.

6.1 RELATIONSHIP OF THE MAGNETIC FIELD WITH THE SPEED PROFILE

The numerical analysis of convergence was carried out through the wxmáxima program which was used to generate data which were inserted in a table. Table (1) presents the behavior of the velocity profile defined by equations (18) and (19) for an interval of R with seven partitions and the following values of the Hartmann number: $H_a = 3.000$, $H_a = 1.000$, $H_a = 0.500$ and $H_a = 0.001$.

Table 1. Numerical Analysis.

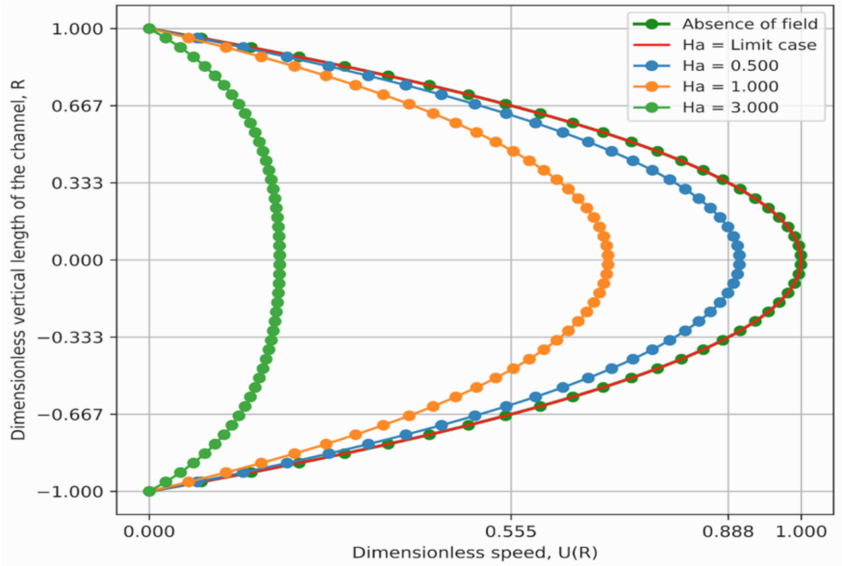
$H_a = 3.000$			$H_a = 1.000$			$H_a = 0.500$			$H_a = 0.001$		
R	Eq. (18)	Eq. (19)	R	Eq. (18)	Eq. (19)	R	Eq. (18)	Eq. (19)	R	Eq. (18)	Eq. (19)
-1.000	0.000	0.000	-1.000	0.000	0.000	-1.000	0.000	0.000	-1.000	0.000	0.000
-0.666	0.139	0.555	-0.666	0.405	0.555	-0.666	0.707	0.555	-0.666	0.555	0.555
-0.333	0.188	0.888	-0.333	0.631	0.888	-0.333	0.806	0.888	-0.333	0.888	0.888
0.000	0.200	1.000	0.000	0.703	1.000	0.000	0.905	1.000	0.000	1.000	1.000
0.333	0.188	0.888	0.333	0.631	0.888	0.333	0.806	0.888	0.333	0.888	0.888
0.666	0.139	0.555	0.666	0.405	0.555	0.666	0.707	0.555	0.666	0.555	0.555
1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000

Source: Authors

With the analysis of the **table (1)**, it is perceived that the velocity profile described by equation (18) converges in the case of absence of magnetic field as H_a decreases. For example, if we compare the velocity in the center of the flow, it is verified that when $H_a = 0.001$ the equation (18) approaches the equation (19) that can be observed in figure (1) in which the two profiles overlap.

Assuming that the fluid is already in motion even before the magnetic field is applied, the MHD phenomenon can be divided into three parts: first, the conducting fluid in motion generates an induced magnetic field; later there is interaction of fields, induced and applied; and finally the combined magnetic field (tax plus induced) interact with the induced current density to give rise to a Lorentz force. This force influences the relative movement of the fluid in the sense of inhibiting it, since the resultant of the interaction between the combined magnetic field and the current density (Lorentz force) is perpendicular to the direction of fluid flow and to the applied external magnetic field [5].

Figure 2 - Flow graph - speed profile.



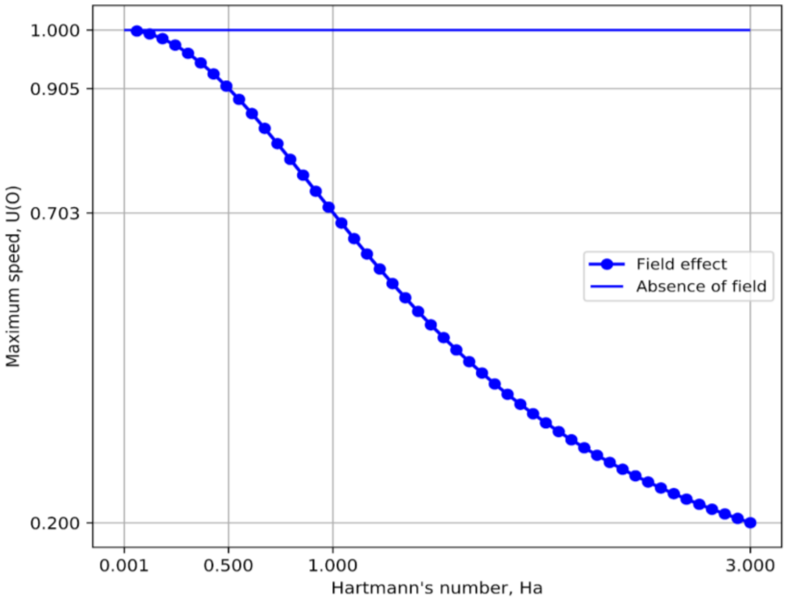
Source: Authors

The **figure (2)** expresses the values of the flow velocity, each parabola represents a profile of that parameter for different values of Hartmann's number. It also exposes the profile of the simple case, in which there is no field, which is shown by the dark green parabola. The representation of the profile in the limit case where the Hartmann's number tends to zero is represented by the red parabola, it can also be seen that this is superimposed on the profile without the influence of the magnetic field. This overlap is also evident when the convergence analysis, expressed in **table (1)**, is verified. Such convergence is mainly due to the fact that the smaller the effect of the applied magnetic field, the smaller the intensity of the Lorentz force that is responsible for damping the fluid movement.

For $\boldsymbol{H_a = 0.001}$ (limit case) when the Hartmann's number tends to zero, it is perceived that the values of the equation (18) which is the speed profile influenced by the magnetic field, are the same as that of the equation (19) velocity profile without the magnetic field.

Analyzing the graph, we have the higher the Hartmann's number, the more evident the effect of the magnetic field acting on the flow, the effect of the field tends to inhibit the movement of the fluid. For example, for $\boldsymbol{H_a = 0.500}$ it was obtained that the velocity in the center of the duct is equal to $\boldsymbol{U(0) = 0.700}$, when compared to the case of $\boldsymbol{H_a} = \mathbf{3}.000$ in which $U(0) = 0.100$, it is verified that the increase in the number of Hartmann has a significant effect on the speed profile slowing it.

Figure 3. The Effect of the magnetic field on the fluid layer in the center of the pipeline.



Source: Authors

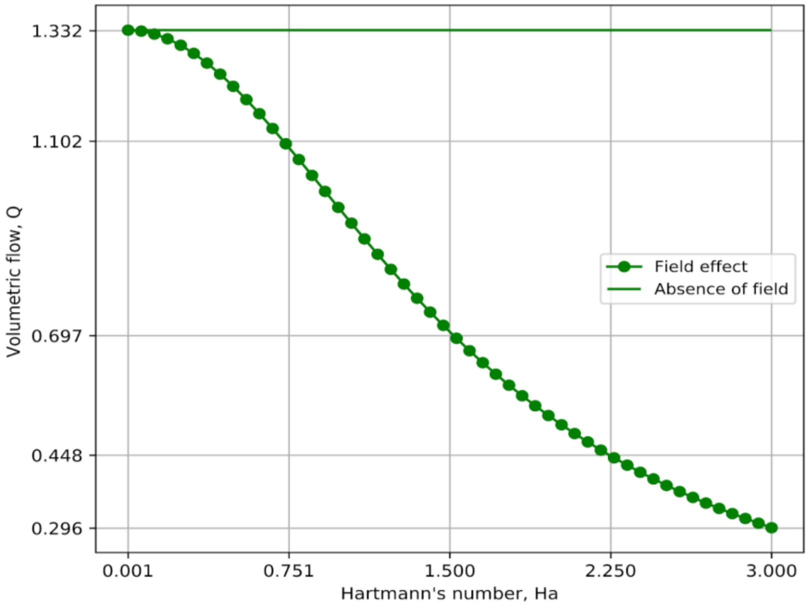
Figure (3) expresses the profile of the maximum speed. The dashed blue line represents the constant profile, which is when the flow is not influenced by the external magnetic field. It also expresses the effect of the field on such a profile, which begins to decline exponentially.

For the case where $\boldsymbol{H_a}$ tends to zero the maximum speed approaches the expected for the situation in which there is no magnetic field. With the increase in Hartmann's number there is a delay of this speed. For example, comparing $\boldsymbol{H_a = 2.000}$ in which $\boldsymbol{U(0)}$ approaches $\boldsymbol{0.400}$ with $\boldsymbol{H_a = 3.000}$ where $\boldsymbol{U(0) = 2.000}$ is verified the decrease in speed with the increase in Hartmann's number. By comparing the two profiles one can perceive how expressive is the effect of the magnetic field on this parameter.

6.2 VOLUMETRIC FLOW

The **figure (4)** shows the volumetric flow profiles. The dashed line represents the flux without magnetic field effect, which is constant, followed by the profile influenced by the magnetic field, which is significantly affected, this is justified by the fact that the volumetric flow is given by the integration of velocity by the area differential and knowing that the velocity is slowed down due to the effect of the magnetic field, the fluid volume flow that crosses a channel cross-section decreases.

Figure 4. Volumetric flow.



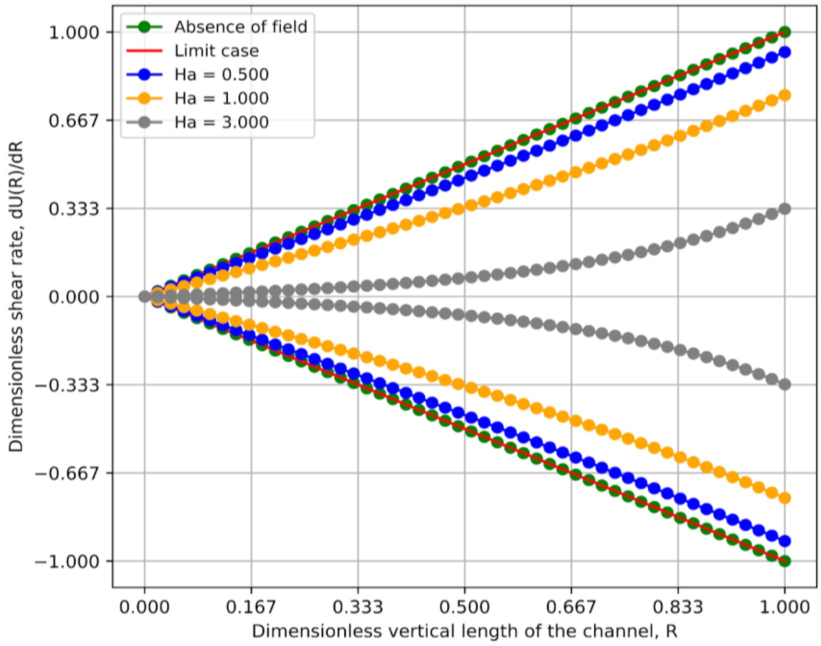
Source: Authors

For the analysis of this property, a range from $\mathbf{0.000}$ to $\mathbf{3.000}$ was used for the parameter that encompasses the effect of the magnetic field. And it was compared with the case in which the flow rate is given by the integral of the equation (19) as a function of $\boldsymbol{dA}$. With the analysis of the graph, it is observed that just as the other properties already presented, the effect of the magnetic field acting on this parameter is also to suppress its progression.

6.3 SHEAR RATE INFLUENCED BY MAGNETIC FIELD

The **figure (5)** shows the relationship between the shear rate and Hartmann's number. For this analysis, four values were used for the dimensionless number: 0.001, 1.000, 2.000, 3.000. And the value that refers to the limit case ($\boldsymbol{H_a = 0.001}$) compared to the case of simple flow in parallel plate channel.

Figure 5. Shear Rate.



Source: Authors

As the **figure (5)** shows, the action of the magnetic field attenuates the shear rate. This effect is evident when we observe the lines and curves present in **figure (5)**. For example, when comparing the lines in red, which are associated with a limit value of the Hartmann number, with the curves in blue, yellow and gray, it is concluded that the higher the value assigned to that number, the greater the tendency of these lines (in red) in approaching the axis associated with the dimensionless vertical length of the channel crossing the point at which the shear rate is equal to zero. And for the other extreme, which approaches zero, this rate tends to be compared with the case presented for the simple flow in the absence of the magnetic field.

7 CONCLUSIONS

Through the study it was possible to define the equations that govern the behavior of the MHD flow of a Newtonian fluid through the application of the Laplace Transform technique to solve the differential equation that governs movement.

With the equations obtained, it was possible to implement a code in Python programming language for the construction of the graphs, from which they served as a database for analysis, which made it possible to understand the effect of the magnetic field applied on the velocity profile for different values of the Hartmann's number, in which the increase of this proved to be an inhibitor of the development of the flow.

From the comparison of the limit case in which the Hartmann's number tends to zero with the case of simple flow, without the presence of magnetic field, it is evident the convergence of the profile obtained with that presented in the literature.

The maximum velocity, flow rate and shear rate were also sensitive to the increase in the Hartmann's number, since the increase of this directly affects the speed profile, which in turn influences all other properties of the flow.

In general, the study proved to be valid as a complementary study of the case of simple flow, since the results obtained clearly express its tendency to that presented by the literature, in the condition that the Hartmann's number approaches zero. Besides showing that the speed profile and other derived properties are affected by the magnetic field.

REFERENCES

[1] WHITE, F. M. Mecânica dos Fluidos. 6. ed. Porto Alegre: AMGH, 2011. 878 p.

[2] ZAMIR, M. **The physics of pulsatile flow**. vol. 3. Springer Science e Business media, 2000. 230 p.

[3] FOX, R. W.; PRITCHARD, P. J.; MCDONALD, A. T. **Introdução à mecânica dos fluidos**. 8. ed. Grupo Gen-LTC, 2014. 884 P.

[4] DAILY, J. W; HARLEMAN, D. R. F. **Dinámica de los fluidos con aplicaciones en ingeniaría.** 1. ed. México: Editorial Trillas, 1975. 512 p.

[5] DAVIDSON, P. A., 2001. An Introduction to Magnetohydrodynamics. s.l.: Cambridge University Press.

[6] AOKI, L. P. **Estudo do efeito magnetohidrodinâmico em um eletrólito a partir do uso de um dispositivo ejetor eletromagnético**. 2011. Dissertação (Mestrado) – Escola de Engenharia de São Carlos, Universidade de São Paulo, São Carlos, Brasil, 2011.

[7] LOBATO, B. **Estudo da magnetohidrodinâmica em dutos usando transformadas integrais.** 2015. 131 f. Tese (doutorado em engenharia de recursos naturais da Amazônia) – Universidade Federal do Pará, Belém, 2015.

[8] ALIREZA, S., SAHAI, V. **"Heat transfer in developing MHD poiseuille flow and variable transport properties"**, Int. J. Heat Mass Transfer, Vol.33, n. 8, (1990), pp. 1711-1720. https://doi.org/10.1016/0017-9310(90)90026-Q.

[9] MALASHETTY, M. S., Leela, V. **"Magnetohydrodynamic heat transfer in two phase flow"**, Int. J. Engg. Sci., Vol. 30, (1992), pp. 371-377. https://doi.org/10.1016/0020-7225(92)90082-R.

[10] UMAVATHI, J. C. et al. **"Fully developed flow and heat transfer in a horizontal channel containing electrically conducting fluid sandwiched between two fluid layer"**, Int. J. Applied Mechanical Engineering, Vol. 9, n. 4, (2004), pp. 781-794.

[11] STAMENKOVIC, M. Z., et al. **"MHD flow and heat transfer of two immiscible fluids between moving plates"**. Transactions of the Canadian Society for Mechanical Engineering, Vol. 34, n. 3-4, (2010), pp. 351-372. https://doi.org/10.1139/tcsme-2010-0021.

[12] KUIRY, D., BAHADUR, S. **Steady MHD Flow of Viscous Fluid between Two Parallel Porous Plates with Heat Transfer in an Inclined Magnetic Field**. Journal of Scientific Research, Vol. 7, n. 3, (2015), pp. 21-31. https://doi.org/10.3329/jsr.v7i3.22574

[13] CAMPOS, M. A. M. **Simulação Numérica Tridimensional de uma Bomba MHD de Corrente Alternada**. 2020. Dissertação (Mestrado) – Instituto Alberto Luiz Coimbra de Pós-graduação e Pesquisa em Engenharia, Universidade Federal do Rio de Janeiro, RJ, Brasil, 2020.

[14] BORGES, L. E. **Python para desenvolvedores: aborda Python 3.3**. Novatec Editora, 2014.

[15] GOULART, L. C. R; OLIVEIRA, S. L. O software máxima e suas aplicações**.** In: V Bienal da SBM, 5, 2010, Paraiba. **Resumo.** Paraíba: UFPB.

[16] VAZ, C. L. D. et al. **O software máxima e aplicações**. 1. ed. Belém: Editaedi, 2016. 186p.

[17] CANEDO, E. L. **Fenômenos de transporte**. 5. ed. Rio de Janeiro: LTC, 2015. 536p.

[18] RÊGO, M. G. O. **Análise da magnetohidrodinâmica com transferência de calor em canais de placas paralelas via transformação integral**. Dissertação (Mestrado) – Universidade Federal do Rio Grande do Norte. Centro de Tecnologia. Programa de Pós-Graduação em Engenharia Mecânica, 2010.

[19] NETO, A. C. W. **Equações Diferenciais**: Uma Aplicação ao Escoamento de Fluidos. 2017. 76 f. Trabalho de Concussão de Curso (Licenciatura em Matemática) – Universidade Federal da Paraíba, João Pessoa, 2017.

[20] SOUZA, F. M. S. **Transformadas de Laplace em circuitos elétricos RLC.** 2017. 52 f. Monografia (Bacharelado em Ciências Exatas e Tecnológicas) – Universidade Federal do Recôncavo da Bahia, Cruz das Almas, 2017.