PID and LQR controllers applied to the inverse dynamics of a 3-DOF Manipulator

Controladores PID e LQR aplicados à dinâmica inversa de um Manipulador 3-GDL

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ABSTRACT
The application in the industrial manipulator robots has grown over the years making production systems increasingly efficient. Within this context, the need for efficient controllers is required to perform the control of these manipulators. In this work the PID controller (Proportional-Integral-Derivative) and LQR (Linear Quadratic Regulator) is presented from the inverse dynamics model of a RPP (Rotational - Prismatic - Prismatic) cylindrical manipulator. The inverse dynamic model which is modeled on Simulink together with a cascaded PID controller is presented. The PID and LQR results are also presented for joint independent and joint dependent control, i.e a controlled PID is used for each joint, controlling the trajectories and speeds at the same time. This paper has as

main contributions the development of the manipulator dynamics model and the design of the LQR and PID controllers applied to the inverse dynamics model, which makes the system simpler to control.

Keywords: PID Controller, Inverse Dynamics, PID Cascade, Cylindrical Manipulator, LQR Controller.

RESUMO
A aplicação com robôs manipuladores industriais tem crescido ao longo dos anos, tornando os sistemas de produção cada vez mais eficientes. Nesse contexto, a necessidade de controladores eficientes é necessária para realizar o controle desses manipuladores. Neste trabalho é apresentado o controlador PID (Proporcional-Integral-Derivativo) e LQR (Linear Quadratic Regulator) a partir do modelo de dinâmica inversa de um manipulador cilíndrico RPP (Rotacional - Prismático - Prismático). O modelo dinâmico inverso que é modelado no Simulink junto com um controlador PID em cascata é apresentado. Os resultados de PID e LQR também são apresentados para controle independente de junta e dependente de junta, ou seja, um controlador PID é usado para cada junta, controlando as trajetórias e velocidades ao mesmo tempo. Este trabalho tem como principais contribuições o desenvolvimento do modelo de dinâmica do manipulador e o projeto dos controladores LQR e PID aplicados ao modelo de dinâmica inversa, o que torna o sistema mais simples de controlar.

Palavras-Chave: Controlador PID, dinâmica Inversa, PID em Cascata, Manipulador Cilíndrico, Controlador LQR.

1 INTRODUCTION

The spread of robots in industrial environments has led over the years that various methods were developed to monitor and control mobile robots or manipulator robots, giving them the ability to operate in environments that are dangerous to humans, in addition to the terrestrial atmosphere, in aquatic explorations, in the transport of materials, among others [1]. The problem of tracking control of rigid manipulators has been well known for many years in robotics. Manipulators such as multi-rigid systems are traditionally described by complex second-order nonlinear differential equations. The control through the inverse dynamics leads to a decoupled system, but the dynamic equations of motion are still second order [2].

Many researchers considered the control of manipulators with flexibility as a single model along with their [3] drives. In the literature, there are also studies focusing on inverse dynamics control of parallel manipulators of rigid joints. It is possible to use position control to approximate dynamic equations, inverse dynamics control application, and even robust tracking controllers [4]. The problem of approach to the solution of the linear inverse dynamics method can be used if the model parameters of the plant and the external disturbances are exactly known. Generally, incomplete information about this systems in actual practical tasks may occur. In this case, adaptive control methods or intelligent control clusters can be employed to decide this control problem [5]. Research into the performance of industrial robots has drawn the attention of many researchers and goes in many directions. Many works are devoted to the dynamic analysis of robots [6]. In [7], two dynamic models are developed: dynamic and visual, using two different software packages BondSim and BondSimVisual, respectively. Object-Oriented Software Environment BondSim systematically provides the development of more complex systems. Between two models, bidirectional communication is obtained, providing information needed for exchange. The research performed by [8] is based on the method of position control using the inverse dynamic control method with an external loop structure and the second Lyapunov method. The results are presented to verify the operation of the control used in the 2-DOF manipulator. The work performed by [9], develops dynamic modeling and control for a new 3D pantograph manipulator. The Euler-Lagrange method is used to obtain the dynamic model. However, this dynamic model is too complex to be used in model-based control techniques. Therefore, a simplified nominal plant is proposed, where the inverse dynamic solution is used efficiently. It is also proposed a new controller aplied in the inverse dynamic model called PID with feed control, which is designed in the H∞ structure. In [10] is described the inverse dynamic model of a new hybrid kinematic manipulator. The so-called Epizactor consists of two planar disk systems that together move a connecting element in 6 DOF. For a robust singularity control approach, the inverse dynamic model is derived using the Newton-Euler iterative method. Feasibility is shown by applying the model to an example where excessive actuator speeds and torques are avoided.

In the research conducted by [11] it is used the inverse dynamics in a 3-DOF parallel manipulator in the form of ”U”. The manipulator consists of the prismatic-prismatic-revolutionary articular arrangement (PPR) in each joint that has a reactive prismatic articulation. A Proportional-Derivative (PD) controller is proposed to control the robotic manipulator that is controlled by decoupled dynamics, and the measurement performance is very convenient to quantify. The proposed control operations are compared with traditional controllers. In [12] the inverse dynamics model is used in a quadrotor, where it is possible to identify the rotor aircraft model: under activation and strong coupling in pitch-yaw-roll. This research aims to implement a PID and LQR controllers applied to a cylindrical manipulator using the inverse dynamics, where independent controllers for the joint and the complete system are implemented. As results, the controlled position and speed of each manipulator joint are displayed. The most important contribution of this research are the inverse dynamic manipulator modelling and the cascade PID and LQR controller designs. This paper is organized as follows. Section 2 provides some information about the cylindrical manipulator, forward and inverse kinematics, dynamic modelling and inverse dynamics. Section 3 presents the PID and LQR controller design. The results of PID and LQR controllers are presented in Section 4. Finally, conclusions and future work are mentioned in Section 5.

2 CYLINDRICAL MANIPULATOR

In this research, a cylindrical type manipulator was used, which has its joints driven by three-phase induction motors. In Fig. 1, it is noticed that the first degree of freedom has a rotational movement around the main axis of the structure, while the second and third have prismatic movements, classifying it as a Cylindrical type or it can still it is said to be a Rotational-Prismatic-Prismatic (RPP) manipulator. Through kinematic modelling it is possible to establish appropriate control strategies that result in a better quality of robot movements [13], [14].

2.1 CHARACTERISTICS OF THE MANIPULATOR

Solid Edge c modeling software was used to provide this information on the masses and geometries of each joint [15]. This information is used to calculate the masses of each joint of the manipulator. In Fig. 2 presents the graphical modeling of the manipulator made in the Solid Edge c software. With the graphic model of Fig. 2, some physical properties of the manipulator were found, such as dimensions, masses and moments of inertia of each joint. The lengths (l) and mass values (m) of each joint of the manipulator are shown in the Table 1.

2.2 KINEMATIC MODEL OF THE CYLINDRICAL MANIPULATOR

Fig. 3 presents an idea of what the cylindrical manipulator looks like with its joint variables to determine the direct kinematic model (Kinematic model of an RPP robotic.

Figure 1. Structure of cylindrical manipulator.

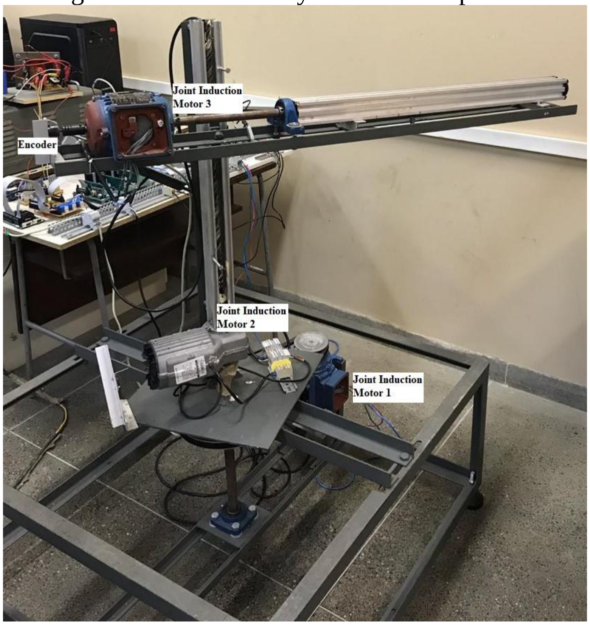
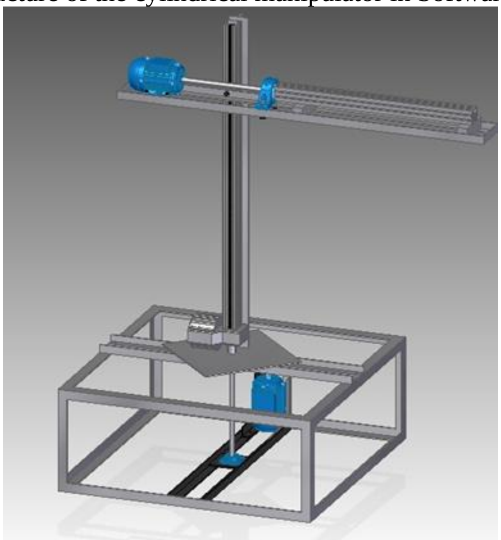


Figure 2. Structure of the cylindrical manipulator in Software Solid Edge.



arm) [16], [17]. A minimal kinematic parameterization of the manipulator according to the well-known Denavit-Hartenberg (DH) in [18] is established and presented in Table 2. The DH parameters are: twist angles αi , link lengths ai , joint displacements qi , and link offsets di , where i = 1,...,n. Obviously, the manipulator has a revolute

Table 1. Values of masses (m) and lengths (l) of each joint.

Joint	l (m)	m [kg]
1	0,050	36,367405
2	0,790	12,632222
3	0,900	23,735183

degree-of-freedom (d.o.f), and two prismatic movements because it is a robotic RPP manipulator.

Figure 3. Schematic of the cylindrical manipulator.

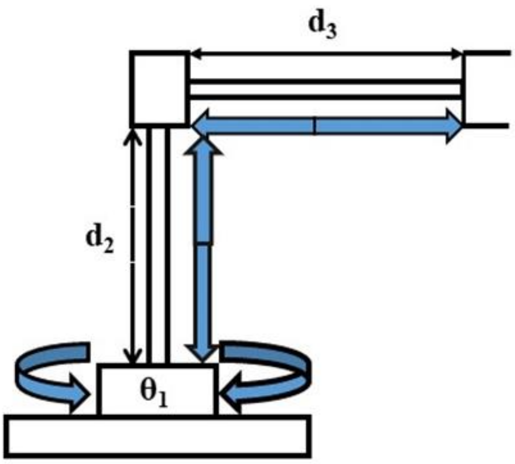


Table 2. DH Parameters of an RPP manipulator

Link	a_i	αi	d_i	θ_1
1	0	0	0,245	θ_1
2	0,11	$-\pi/2$	d_2	0
3	0	0	d_3	0

Applying the DH conversion to the parameters shown in Table 2, the corresponding matrix T [3] is given by:

$$T = \begin{bmatrix} \cos(\theta_1) & 0 & -\sin(\theta_1) & -\sin(\theta_1)(d3 + 0{,}35) \\ \sin(\theta_1) & 0 & \cos(\theta_1) & \cos(\theta_1)(d3 + 0{,}35) \\ 0 & -1 & 0 & 0{,}245 + d2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

2.3 DYNAMIC MODEL OF THE CYLINDRICAL MANIPULATOR

The connection between the position, speed, acceleration and torque of each joint of a manipulator is determined by the dynamic model. Thus, the dynamic model of an industrial robotic manipulator seeks indicate the connection between the forces and the movement of the same that will be needed to perform a given task [13], [19]. The rigid-body model of the robot dynamics can be due using the Euler–Lagrange formulation, [20], [21]. The standard form of the model is as follows:

Partindo das equações da cinemática direta (1) e (2) e aplicando algumas transformações trigonométricas encontra-se as equações da cinemática inversa que são dadas por:

$$L(q, q^{\cdot}) = K(q, q^{\cdot}) - P(q) \ (2)$$

where L is the Lagrangian; K is the kinetic energy and P is the potential energy. To indicate the dynamic model of the manipulator, potential and kinetic energy were first calculated. Then the formulation based on the Lagrangian method was applied for the final calculation. [17]. From the Lagrange equation (2), the equations of the movements of each joint can be determined, as can be seen below

$$\frac{d}{dt}\left[\frac{\partial L}{\partial q}\right] - \frac{\partial L}{\partial q} = \tau \ (3)$$

where τ ∈ R n are the torques of the joints to make a certain trajectory. Then, taking into account the kinetic energy of the manipulator, the dynamic model equation can be written more simply as:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau \qquad (4)$$

where,

– q, $\dot{q}$, $\ddot{q} \in \mathbb{R}^n$ indicate the joint position, speed and acceleration, respectively;

– $M(q)$ is the inertial matrix;

– $C \in \mathbb{R}^n$ is the matrix that account the centripetal and Coriolis forces, and

– $G \frac{\partial g}{\partial q} \in \mathbb{R}^n$ is the gravity matrix.

2.4 INVERSE DYNAMICS

The resolution of the problem of direct dynamics is useful for the purposes of simulating the manipulators to determine which torque is required for a given trajectory. The direct dynamics allows the movement of the system to be described in terms of speeds and accelerations of the joint, when a set of joint torques are assigned to the manipulator. Articular speeds and positions can be calculated from the integration of the system of non-linear differential equations [22] [23]. The analytical relationship between joint positions, speeds and accelerations and joint torques (end-effector forces) can be calculated as [20],

$$\ddot{q} = B^{-1}(q)\,(\tau - \tau') \ (5)$$

where

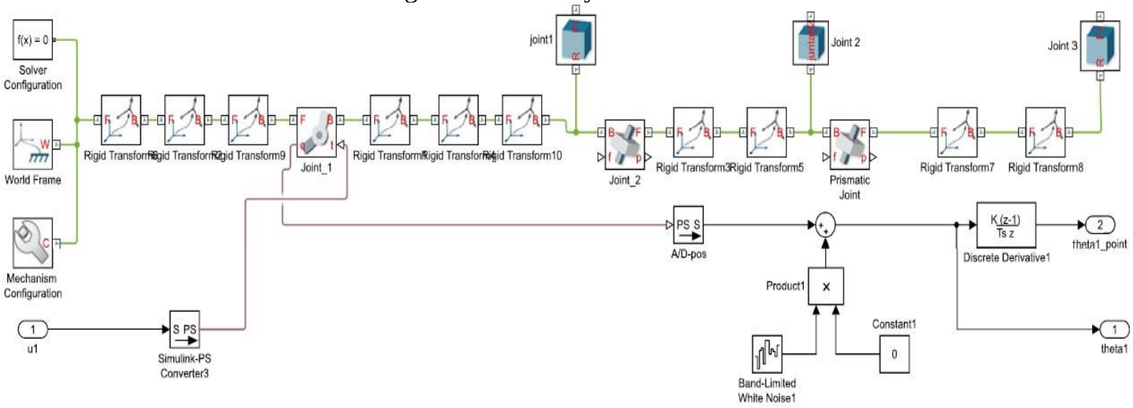
$$\tau'^{(q,\dot{q})} = C(q,\dot{q})\dot{q} + F_v\dot{q} + F_s\dot{q} + g(q) + J^T(q)h_e \quad (6)$$

indicates the torque contributions depending on joint positions and speeds [20]. The form of controlling the inverse dynamics uses the relationship between the torques of the joints as the contact forces of the inputs and the final actuator and the position variables of the end-effector along the restraining surfaces as the outputs [4]. Inverse dynamics is an approach in which a feedback linearization loop is applied to the tracking outputs of interest. Residual dynamics, not directly controlled, are known as internal dynamics. If the internal dynamics are stable, inverse dynamics are successful. Typical use requires the selection of output control variables to ensure that the internal dynamics are stable. This means that tracking cannot always be guaranteed for the desired original outputs [22]. The idea of inverse dynamics is to seek a non-linear feedback control law [20]

$$u = f(q,\dot{q},t) \quad (7)$$

which results in a closed-loop linear system. For general nonlinear systems, such control law can be quite difficult or impossible to find. Figure 4 presents the model of the inverse dynamics of the manipulator used in this research.

Figure 4. Inverse dynamics model.



The inverse dynamics model, shown in Fig. 4, was modeled with the characteristics of the manipulator shown in Table 1. In this model, the controllers proposed in this work will be applied.

3 CONTROLLERS DESIGN

In this section are presented the designer of PID and LQR applied in the control of position and speed to joint manipulator.

3.1 PID CONTROLLER

In dynamic control systems the PID is the most used controller, it has several forms to be developed. The construction of the PID can be through some rules and also analytically [23]. According to [24], the PID controller has a useful architecture, where the procedure can be demonstrated in the equation (8).

$$u(t) = k\left[e(t) + \frac{1}{T_i}\int e(t)dt + T_d\frac{d}{dt}e(t)\right] \quad (8)$$

The control system is cascaded in order to control position and speed. Speeds are additional measurements that can be used in the internal loop. PID controller gains are found through the characteristic equation that is given by [25], [11], [26]

$$f = \ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 \quad (9)$$

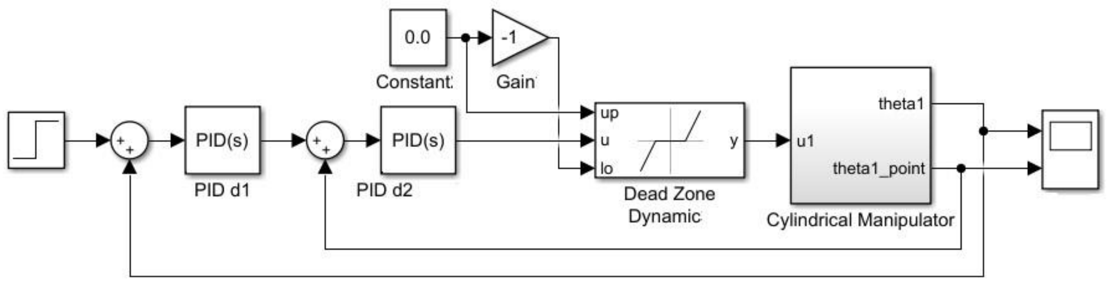
The proportional gain is given by kp = mω 2 n , the derivative gain is calculated by kd = 2mωn, and the full gain, ki , can be determined empirically. The characteristic equation found for the manipulator is

$$f_i = m_i\ddot{q} + \dot{q_i} + q_i \quad (10)$$

where i = 1,2,3 are the joints of the manipulator.

Figure 5 shows the cascaded PID controller design for manipulator joint 1. This project is developed for each board.

Figure 5. PID controller for manipulator joint 1.



Cascading PID controller gains are shown in Table 3. The values kp1 , ki1 and kd1 are for controllerC1 (position control) and the values kp2 , ki2 and kd2 are for controller C2 (speed control).

Table 3. PID controller gains of each joint.

PID/Gains	Kp_1	Ki_1	Kd_1	Kp_2	Ki_2	Kd_2
PID Joint 1	7.5	0.01	-0.1	15	0.05	-0.01
PID Joint 2	0.001	0.01	0.0	0.01	0.0	0.0
PID Joint 3	0.001	0.001	5	0.001	0	0.1

3.2 LQR CONTROLLER

The LQR is a kind of compensator that seeks results in addition to classic and more complex controllers for example, this being the modeling [28]. The purpose of the LQR is to seek a value of u(t) from an initial state to a location in the desired state space. The minimum performance indices can help in the search for the best solution according to the adjustments [29].

According [29],[30] a time-invariant LQR system can be represented by:

$$\dot{x} = Ax(t) + Bu(t) \ (11)$$

$$y(t) = Cx(t) + Du(t) \ (12)$$

The feedback control law is defined as:

$$u(t) = -kx(t) \ (13)$$

being k the states feedback matrix.

Replacing (13) in (11), it is possible to obtain the closed loop system response:

$$\dot{x}(t) = (A - Bk)x(t) \ (14)$$

The optimized LQR control is obtained by minimizing the performance index, given by:

$$J = \int_0^{\infty} [x(t)^T Q x(t) + u(t)^T R u(t)] dt \ (15)$$

being Q a hermitian matrix positive definite (or positive semi-definite) or real symmetric and R is a hermitian positive definite matrix or real symmetric. This is the LQR general form, being Q = C'C, Q'= Q ≥ 0 and R = R' > 0. Its solution is given by:

$$G = R^{-1}B'k \quad (16)$$

Figure 6 shows the LQR controller design for manipulator joint 1. This project is developed for each board. The gains K1 (position) and K2 (speed) found for the design of the joint LQR controllers are presented in Table 4 [31], [32].

Figure 6. LQR controller.

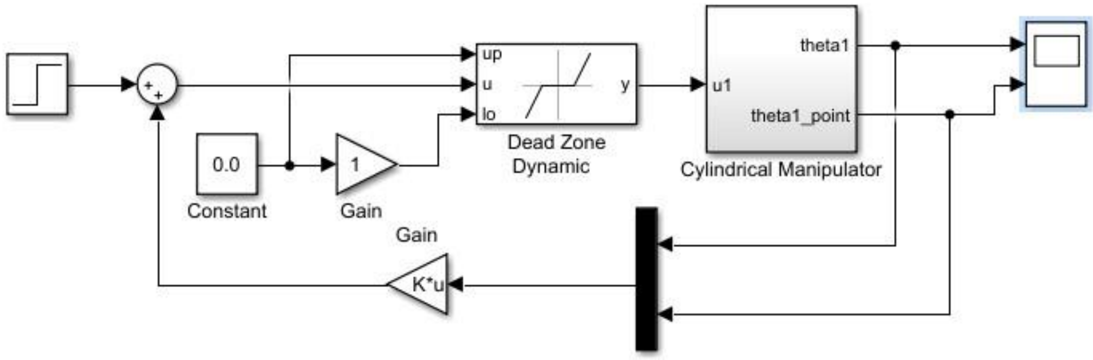


Table 4. LQR controller gains of each joint.

PID/Gains	K_1	K_2
PID Joint 1	1000	1000
PID Joint 2	40	40
PID Joint 3	0.0001	0.0001

4 RESULTADOS

In this section are presented the results of the PID and LQR controllers applied to the position and speed control of the manipulator joints. Controller results are displayed using a step input on each controller (PID and LQR).

4.1 PID RESULTS

Figures 7,8 and 9 show the results of the step response of each manipulator joint to the PID controller. For joint 1, a 60 degree step is applied and for joints 2 and 3 a 0.5m step.

4.2 LQR RESULTS

Figures 10,11 and 12 show the results of the step response of each manipulator joint to the LQR controller. For joint 1, a 60 degree step is applied and for joints 2 and 3 a 0.5 m step

4.3 PERFORMANCE CRITERIA

Based on the controllers responses, the performance criteria for the controllers being studied are presented here. Table 5 shows the performance criteria for the PID and

Figure 7. Position and speed of joint 1 with PID controller.

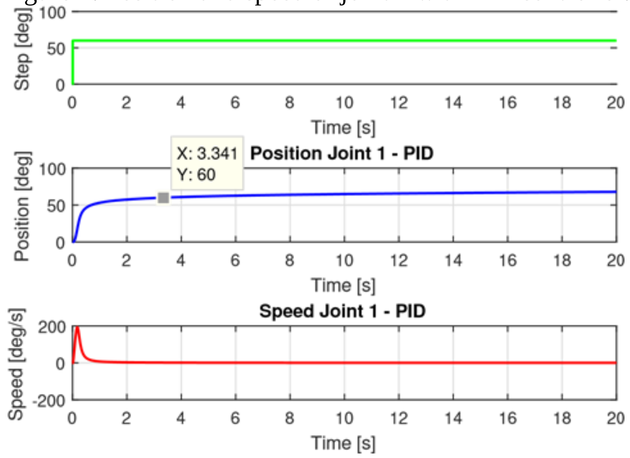
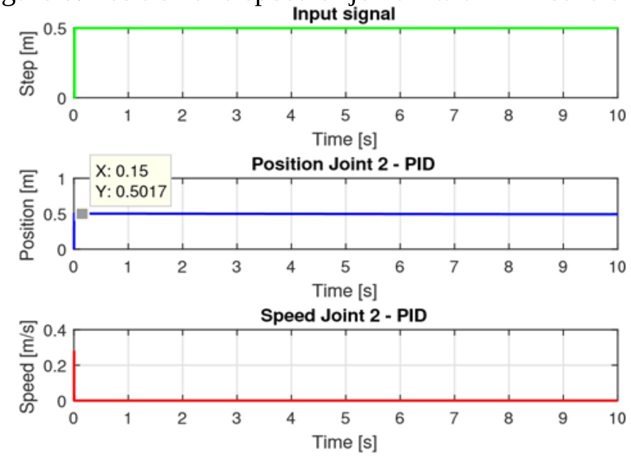


Figure 8. Position and speed of joint 2 with PID controller.



LQR controllers for position control of each joint. The performance criteria analyzed are rise time (tr), settling time (ts) and overshoot.

Figure 9. Position and speed of joint 3 with PID controller.

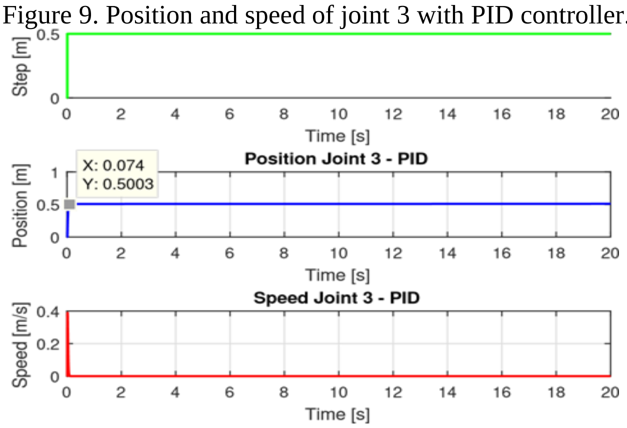


Figure 10. Position and speed of joint 1 with LQR controller.

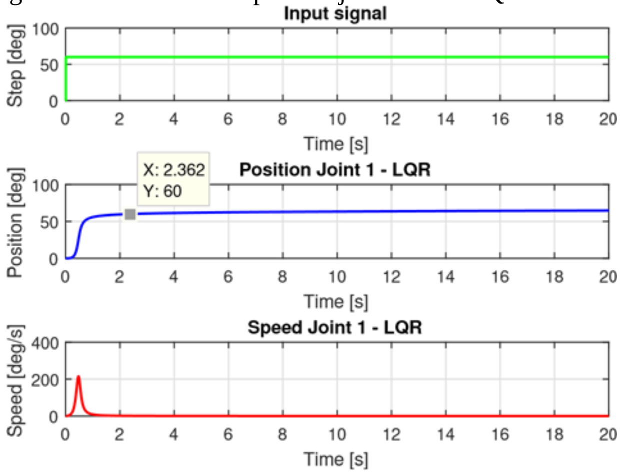


Figure 11. Position and speed of joint 2 with LQR controller.

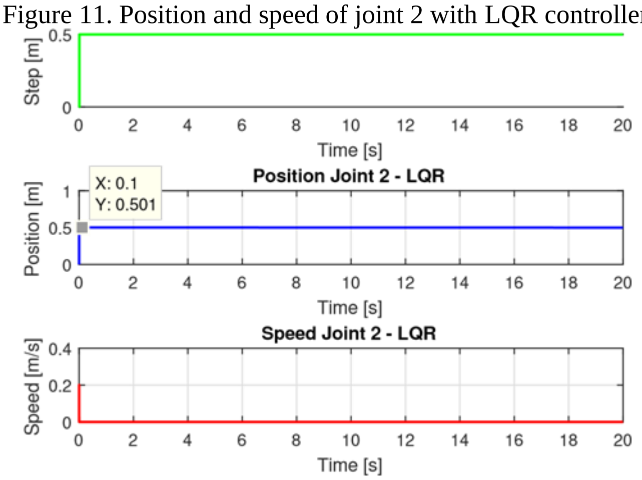


Figure 12. Position and speed of joint 3 with LQR controller.

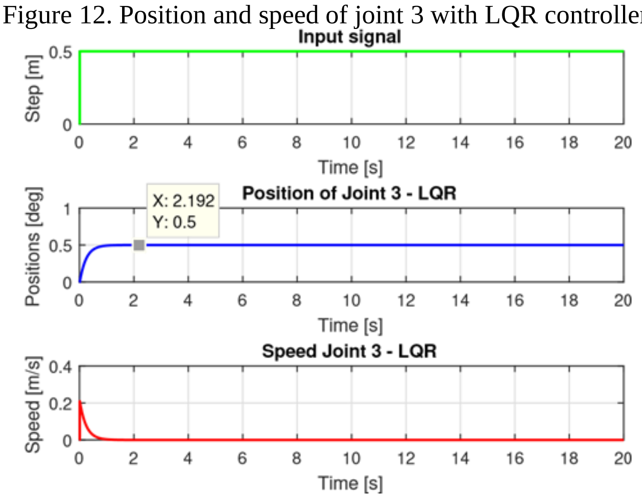


Table 5. PID and PQR Controllers (positions) Performance Criteria for each joint.

PID/Gains	t_r [s]	t_s [s]	Overshoot[%]
PID Joint 1	3.341	3.341	0.00
PID Joint 2	0.150	0.150	0.00

PID Joint 3	0.074	0.074	0.00
PID Joint 1	2.362	2.362	0.00
PID Joint 2	0.100	0.100	0.00
PID Joint 3	2.192	2.192	0.00

5 CONCLUSION

This work presented a model of the inverse dynamics of a cylindrical manipulator and the designs of the PID and LQR controllers applied to the inverse dynamics model of the manipulator for position and speed control. The cascade PID controller were used, where the internal loop is for speed control. The controllers performed satisfactorily to control the position and speed of the manipulator.The results presented demonstrate the efficiency of the controllers for the suggested application, i.e to control the position and speed of the manipulator.

This work had as main contributions the development of the manipulator dynamics model, which can be used as a reference for other manipulator configurations and the design of the PID and LQR controllers applied to the inverse dynamics model, which makes the system simpler to control [12]. The authors are researching other control forms to apply to the model, such as centralized control, where the three joints of the manipulator will be taken into consideration at the same time.

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