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Analysis of Tutoring in the Professional Development of STEM Teachers

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Abstract: The aim of this work is to explore the impact of the use of five lenses for the professional development of mathematics teachers in a Practicum environment of a master's degree in Teaching in Secondary Schools in Spain, based on based on the video-related model and facilitated by a tutor. This qualitative ethnographic study is part of an action research and focuses on two cases of future secondary mathematics teachers who share a university tutor. The results indicate that the application of the lenses in the video-related model produces signs of changes in professional development, both in terms of the notable events noticed by the teachers and in their narrative, and, in addition, promotes self-regulation of training. It should be noted that, throughout this process, the intervention of the university tutor or facilitator is essential. It is concluded that this model can be replicated in other STEM teacher training specializations.

Keywords: mathematics education; teacher professional development; teacher noticing; STEM; video-related model; reflective practice; facilitators' practices

MSC: 97B50; 97D30; 97D40

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1. Introduction

In the last 20 years, research in mathematics education has focused intensely on the professional training of teachers and, in particular, has had an interest in the design of experiences or initial and continuous training programs that favor professional development, to help teachers identify what is relevant in a teaching and learning process to make decisions and know how to respond to each situation [1].

It should be noted that one of the most used resources to achieve this is video recording. This is used for various purposes that promote it: to show examples of specific professional situations or aspects (new curricular guidelines, pedagogical strategies, activities, etc.) [2–4]; to show models of good practices or exemplary teaching practices [5]; to facilitate and improve understanding of mathematical concepts; to enrich mathematical knowledge for teaching; and to reflect on cultural and contextual aspects of teaching or to develop teaching reflection tools [6].

The benefits and advantages of using this resource in the initial and permanent training of teachers are many. According to Wang and Hartley [7], situations shown by video recording are more efficient than written ones, in terms of developing reflection skills; in fact, video recording promotes great opportunities for deep reflection, as it allows reflection on aspects of the teaching and learning of mathematics based on real practices [8]. In addition, video recording has the unique potential to convey the complexity and context of the interactions between agents that occur in the classroom [9]. Likewise, it helps to identify changes and to transform beliefs, conceptions, and teaching strategies that are usually influenced by previous experiences [7,10]. Analyzing one's own practice or that of

a colleague produces more motivation for reflection [11–13]. Finally, video activities always promote positive attitudes in teachers [14].

This is why, in the initial training and, particularly, in the practice modules, it is used specially to develop reflection skills on one's own teaching practice [5], to assess it and to promote in the self-regulation of training [6], as well as to develop the professional noticing skill [2,3,15–17].

It is true that in the literature there are numerous works focused on developing the noticing skill, based on the use of video recording, but the study we present is important and innovative because it focuses, above all, on facilitators that promote the development of competence; this is unlike most works on it, which are designed to support the development of the teacher's noticing of student thinking [18]. Specifically, this work offers various strategies used for professional development for mathematics educators and STEM [19] in general.

In addition, it should be noted this the research is framed in a context of competence development with future secondary mathematics teachers, which is also unusual [20,21].

Finally, unlike most works focused on teacher noticing, one's own video recording is the type of video recording used [18].

2. Literature Review

In teacher training, there are several models or consolidated learning environments that use video recording as a scaffolding to address competence development and the improvement of teaching practice from reflective processes, although the importance of reflection and its potential value are not always fully recognized and appreciated. In fact, within this area, there is an interest in characterizing the reflective processes carried out by preservice teachers or teachers in mathematics training and to see what opportunities for reflection these environments offer. The video clubs of Sherin and Van Es [4,8], the Video-LM project of Karsenty and Arcavi [6], or the video-related model of Fortuny and Rodríguez [17], which is what is applied in our study, are some examples of these environments. However, simply viewing the recordings does not guarantee competence development [5,22], but these models will be successful if there is a facilitator [7,23–26], who can be a university tutor or the mentor of the center, who manages the corresponding activities, monitors discussions, in particular, and helps preservice teachers or inservice teachers to focus on relevant aspects of learning and teaching processes [26–29].

The analysis of facilitating strategies in video-recording-based learning environments has been the object of study in research [13,22,23], which has identified the contextualization of video practice, asking questions to bring out primary ideas, as well as the revoicing technique, which is understood as the re-expression by the teacher of an idea expressed by a student (which is most important). Rethinking, clarifying, or establishing connections are other ideas that also contribute to meaningful analysis of the video recording.

It should be noted that the main identification and classification of facilitating actions for professional development based on video recording have been carried out with in-service teachers.

One of the most relevant works is that of Van Es et al. [22]. The authors identify and classify the facilitating strategies during the discussions in the video recordings. The categories, which include subcategories, are as follows (p. 347): orienting the group to the video analysis task; sustaining an inquiry stance; maintaining a focus on the video recording; and the mathematics and supporting group's collaboration.

Later, Coles [25] expands the maintaining a focus category with a sequence of strategies: interruption, highlight, and redirect. This strategy refers to interrupting the speaker when they make a comment that is not appropriate due to the sociocultural norms of the activity.

In the work of Tekkumru-Kisa and Stein [30], also with inservice teachers, the researchers propose five facilitating practices to guide competency development, based on video discussions: anticipating responses, sequencing of video recordings, monitoring

responses, selecting participant responses, and connecting participant responses. It should be noted that the first two practices take place before the discussion sessions, and, therefore, this implies that the facilitator is the one who selects the video recordings so that the development occurs. Regarding the other three practices, they are carried out during the discussion sessions. Likewise, it should be noted that the monitoring and selecting practices explicitly include some of Van Es et al.'s [22] subcategories.

3. Conceptual Framework

To achieve professional development, it is important that the tutor conducts some concrete actions [22,25,30] during the discussions. However, it is advisable to provide guidance in the form of questions, items, or scaffolding lenses [6,31]. In this sense, Karsenty et al. [26] propose the conceptual framework Resources Orientations Goals Identity (ROGI) to explain tutor practices and decisions throughout the professional development of future teachers.

Figure 1 shows an adaptation of this conceptual framework in the case of the semi-naris of the video-related model.

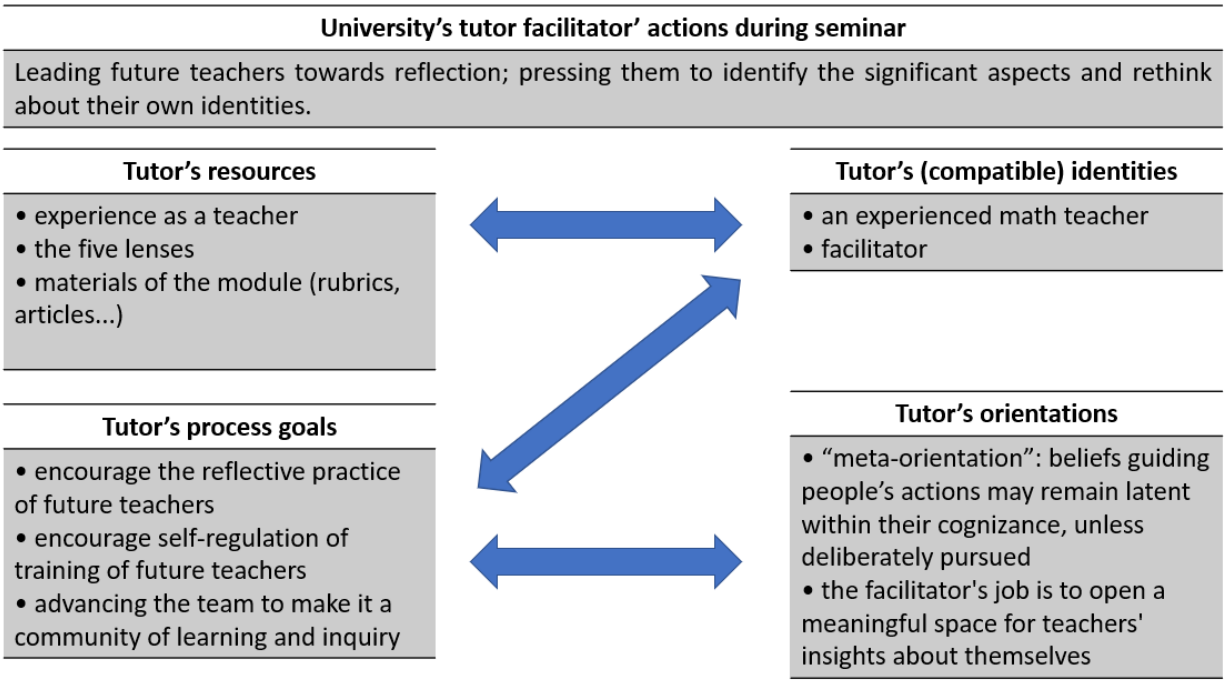

Figure 1. Constructs of the seminars. Adapted from ROGI framework [26].

The video-related instructional model aims to improve the professional competence of preservice teachers through a process of reflection, that is, a detailed and careful analysis of past, present, and future professional experiences that consider plans, intentions, and behaviors in order to obtain information about oneself, about decisions and on actions [6].

Along the lines of Karsenty and Arcavi [6], we adopt the term lens to refer to each of the aspects or components involved in a process of teaching and learning mathematics that can be observed and discussed by future teachers or teachers in service. Unlike these authors, who propose six lenses or components of analysis (mathematical ideas, objectives, tasks, interaction, dilemmas and beliefs), we suggest the selection of lived and videotaped moments based in the following five aspects that we believe are defining in a process of teaching and learning mathematics (Table 1): mathematical thinking of the student, role of the teacher, discourse classroom, tasks and mathematical content.

Table 1. The five lenses for reflection.

Lens	Description	Example of Questions
Student's mathematical thinking	Cognitive aspects. Identifying evidence of a student's mathematical thinking and interpreting their mathematical understanding.	Describe what you think the student did to solve the problem. How do you interpret the student's answer in terms of their mathematical knowledge? What mathematical difficulties do you identify in students?
Role of the teacher	Affective and ecological aspects. Approach to the teacher's role in the interaction with students: asking questions, listening to comments, managing discussions, delegating responsibilities in the knowledge generation process, feedback, etc.	Describe the way the teacher responds and manages the class. Interpret the strategies used by the teacher during the course of the activity. How does the teacher resolve conflicts?
Classroom discourse	Interactive aspects. Focus on classroom interactions that promote mathematical learning opportunities for students.	Identify and describe a specific moment in the classroom discourse in which specific aspects of mathematics are worked on. Identifies and interprets with a theoretical foundation key moments of the discourse regarding the learning of mathematical content. What other learning opportunities have the different solutions to the problem provided the students?
Tasks	Mediation aspects. Analysis of the mathematical tasks or activities proposed to students to achieve the learning objectives.	Identifies and interprets with a theoretical foundation the key activities of the session that lead to the achievement of the mathematical learning objectives.What are the main mathematical ideas behind the task that students are expected to understand? What are the strengths and weaknesses of the task? How could it be improved to move towards the learning objectives?
Mathematical content	Epistemic aspects. Enumeration and analysis of the mathematical contents of the episode (school mathematics).	Identify the mathematics contained in this episode (concepts, procedures, linguistic elements, and properties). It evaluates whether the mathematical contents are worked on adequately. Do you identify in the teacher ambiguities in the meaning of the mathematical elements that emerge? Which is? How could they be avoided?

Note that the five lenses we propose are an adaptation of Karsenty and Arcavi's for initial teacher training. Hence, the beliefs lens, which is more appropriate for teachers in training, is not considered in this study with future teachers.

Likewise, these five lenses must not only be useful for future teachers but must also be considered by the tutor to guide seminar discussions and, therefore, to encourage student reflection. These five lenses also address the didactic suitability criterion of Breda et al. [32] in the following relations: epistemic (mathematical content), cognitive (student's mathematical thinking), interactional (classroom discourse), mediational (tasks), affective and ecological (role of the teacher).

On the other hand, to record some reflections, future teachers are provided with a rubric such as the one in Table 1, where three columns are added to complete. In the first column, the student must present evidence selected from a classroom episode that addresses the corresponding lens. In the second column, they must make a reflection presenting an alternative to improve the practice of the selected moment. The third column

corresponds to a proposal for improvement or final conclusion that the future teacher makes after exposing their evidence in the seminar, to discuss with the group.

4. Learning Design

The video-related model is implemented in the module of teaching practices called Practicum, both for the degree of Primary Education and the master's in Teaching in Secondary Schools at one university of Spain. Therefore, the experiences refer to the training of the Practicum.

In the first period of practice, future teachers observe and reflect on the dynamics of a class, its management, and its intervention in some classroom activities. They focus, above all, on the figure of their mentor, that is to say, on the role of the teaching mentor of the practice center to which they are assigned. Based on their observations and reflections, they design a teaching unit (TU) of mathematics for the previously analyzed context. In the second practice period, the future teachers act as teachers and carry out the implementation of the learning situation, which they record on video recording [17].

Future teachers regularly attend reflective practice seminars, in which the aim is to analyze and guide their teaching practice [33,34] under the tutoring of a university professor and, in which the other classmates with whom they share a tutor can also participate. In these seminars, future teachers select three moments from their videotaped lessons to be discussed and analyzed for 30–40 min.

The actions of the tutor for the professional development process are to lead the future teachers, through different looks or perspectives, towards reflection, pressuring them to identify the significant aspects of their practice by viewing their recordings and rethinking their own identities. These views proposed by the tutor are designed to direct the attention of preservice teachers to the different characteristics of the teacher's actions and decisions, as a vehicle for discussing issues at the core of the teaching–learning practice of both mathematics and STEM.

In this way, the reflection and analysis of one's own practice is enhanced, and, especially, it contributes to teacher noticing and to promote the self-regulation of training, that is to say, to become aware of one's own professional development [35–38].

The main challenge of the initial training of teachers is to achieve the best preparation for the future teachers and, in the case of mathematics, this falls on achieving a sufficient development of the professional noticing skill in future teachers. This competence consists of being able to notice aspects and important factors involved in the process of teaching and learning mathematics, interpret them, and present alternatives to improve practice in these situations [1,39].

It is hoped that, in this way, once they arrive in the classroom, they will be prepared to approach, in the best way possible, the processes of teaching and learning mathematics in terms of the learning of their students. Consequently, it is necessary that initial university training programs contemplate hypothetical training trajectories aimed at developing this competence and showing its evolution, based on the involvement of students or future teachers in a reflective process.

This study aims to provide opportunities for reflection and analyze its effect on the professional development of future secondary mathematics teachers in the Practicum.

In this sense, to enhance the reflection and analysis of their own practice in the Practicum module, the video-related instructional model by Fortuny and Rodríguez [17] is implemented, with the participation of the tutor. In this work, five lenses are applied for reflection that contribute to the noticing and promotion of the self-regulation of training, which is to say, focusing the teacher's gaze on what is really important in the processes of learning–teaching and, thus, promoting professional development. The article provides a tutoring guide and evaluates a professional trajectory of two cases of future teachers for one year.

5. Materials and Methods

5.1. Context and Participants

In Spain, to be able to work as a teacher of any subject in secondary education (12–16 years) and high school (16–18 years), it is mandatory to take an official master's degree in Teaching in Secondary Schools, Vocational Training and Language Centers.

In this professional master's degree, students are expected to acquire the necessary skills to teach on these educational stages. The master's degree in mathematics is divided into six modules, one of which is the Practicum.

The Practicum plays an especially important role in the master's degree, because it represents the module where the theoretical and practical learning of the entire study plan is synthesized.

Before starting this module, each future teacher or student is assigned a mentor, i.e., a teacher from the center with whom they will do the training, as well as a tutor or university professor who evaluates the different tasks and guides and monitors the practices.

When a student does their Practicum in a class, the average number of pupils in the class is 28, which is the usual ratio of students per class group.

The profile of the student of the master's degree in the specialty of mathematics is that of a bachelor's degree or graduate in Mathematics or in some other related discipline such as physics or engineering, where the weight of mathematics is important in their study plan. This means that in Spain, mathematics teachers in the educational stages between 12–16 years of age are not all mathematics graduates, but may be physicists, engineers, or even economists.

Our study was carried out during the 2021–2022 academic year in the Practicum module of this master's in the mathematics specialty of a Spanish university, and, specifically, we focus on the case of two future teachers who share a university tutor (Figure 2).

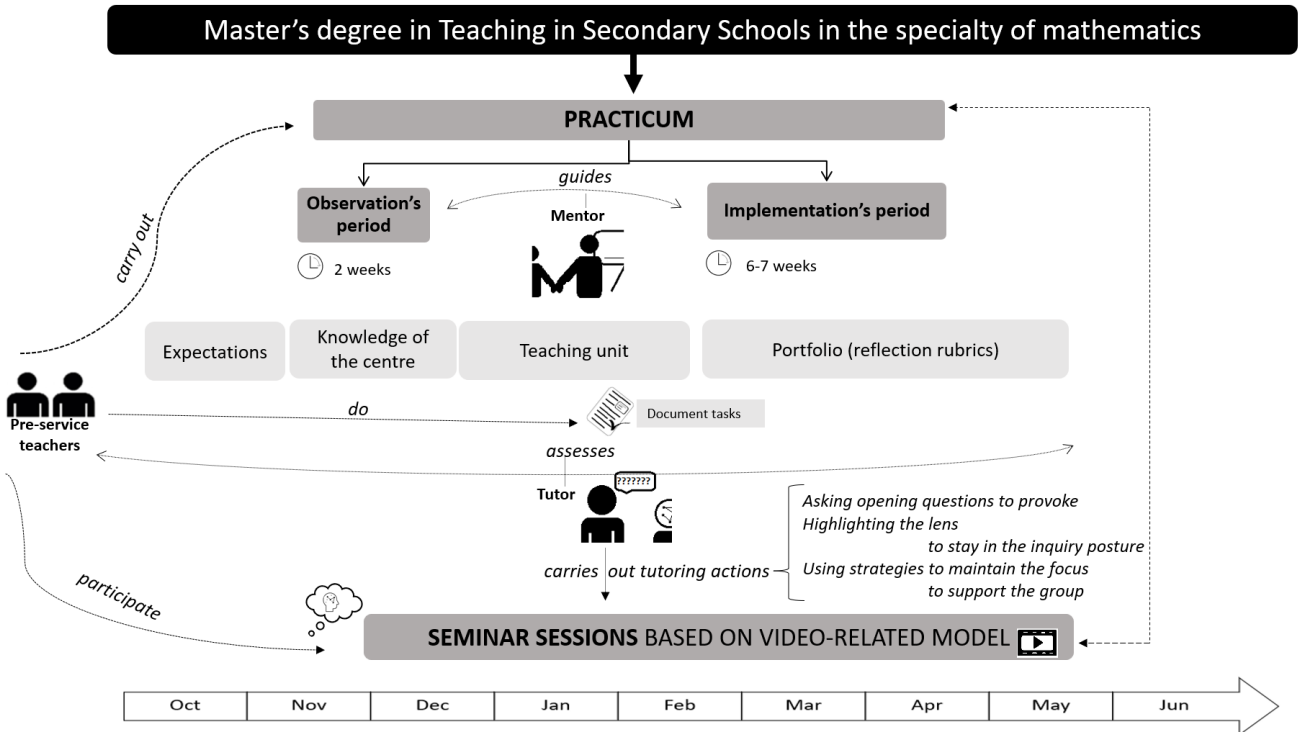

Figure 2. Practicum environment.

With regard to the profile of the future teachers, one of them, Gregory, who is 23 years old and has a clear intention to dedicate himself to teaching, has a degree in Mathematics and stated that he hoped, above all, to learn about the management of the class. He designed and implemented a TU to work on divisors in the 1st year of ESO (12–13 years)

in a high school of maximum complexity. The other teacher, Alex, who is 23 years old, is physicist by training, and has a less clear intention to dedicate himself to teaching, also marked learning about group management as the main objective of the training as well as techniques didactics and interaction with students. His TU dealt with trigonometry in 15–16 years.

Regarding their previous training in didactics, they had only received it in the previous training modules of the master's degree they were studying.

The tutor, who is one of the authors of the paper, is also dedicated to research in the field of mathematics teacher training. In particular, he is interested in improving the Practicum modules to achieve a competence development of future teachers that is sufficient for their future teaching practice in mathematics. This need and interest were transferred to his two master's students, who, motivated to contribute to the research, decided to voluntarily participate in it and gave their consent to use their data.

It should be noted that, in our case, the tutor, particularly, adopts the video-related model to carry out the tutoring. Thus, the two future teachers attended 10 seminars, which were video-recorded, of reflective practice throughout the course. In the seminar sessions, each future teacher selected and showed different video-recorded moments of their intervention in the classroom that corresponded to one of the five lenses, to be discussed and analyzed in 50′–60′.

It should be mentioned that the names of the future teachers, as well as that of their pupils, are fictitious.

5.2. Design, Data Collection, and Analysis

The focus of this ethnographic study is qualitative, and its design is that of action research, since it aims to address a real problem in the educational community and achieve change [40]. In this type of research, the researchers, who act as tutors, are the same natural agents of the community that forms a group to improve a real situation in which they are immersed [41,42].

In our case, the aim is to address the competence development of two future teachers in mathematics training and achieve a change in their professional practice through, above all, discussion seminars led by their university tutor.

The data collection includes notes collected during the seminars, the transcripts of the video recordings of the seminars and the reflection rubrics of the two teachers of the portfolio.

Regarding the content analysis, the following process was mainly carried out, based on the phases of Nurick et al. [43]:

1. Defining units of analysis of the transcripts: the interventions of future teachers were segmented according to each moment selected to comment.
2. Coding the interventions segmented according to the five lenses.
3. Identifying the level of narrative [1,44] of the interventions of future teachers in the seminars and in the reflection rubric, as well as the stages that we consider a development trajectory (description, subjective evaluation, explanation, suggestion, interpretation, alternative, and prediction).
4. Identifying the actions of the tutor according to Van Es et al. [22] and Coles [25].

6. Results

Next, the main results obtained by applying the five lenses for the analysis of teaching practice throughout the video-related method are presented. In the first three subsections, some general results are presented, and, in the following, each of the lenses is focused on. Likewise, through the examples of the lenses, the tutor's actions are identified to facilitate reflection.

6.1. Changes in the Content and Form of the Participants

During the course of the seminars, signs of changes are identified both in the content (what they notice) and in the form (way of talking about it) of the discussions. Specifically, it is observed that a future teacher starts talking about a recorded session in its entirety and, little by little, refers to specific events that, many times, are used to discuss particular ideas. In addition, this goes from commenting on aspects of general pedagogy or on the role of the teacher to doing so on the students' mathematical thinking.

6.2. Changes in the Speech and Language of the Participants

On the other hand, changes are also identified in the speech and language of the participants; first, they adopt a more descriptive narrative "account of", then they interpret, and, finally, they have a more prospective discourse "account for"; even, they are able to understand the consequences that a specific theory can produce in learning. Below are two examples of the narrative of one of the future teachers that come from two different seminars (Tables 2 and 3).

Table 2. The future teacher reproduces some facts without further elaboration.

I1	*[FT]: Here I suggested to Mia that she go out and do her homework on the board instead of me doing it: "Mia, can you do it on the board?" She: "What number?" And me: "20". And she went out and wrote the answer on the board. She did really well, and I was surprised.*
I2	*[T]: Why were you surprised?*
I3	*[FT]: I didn't expect them to listen. She said, I don't remember who she addressed, but we assume it was to John: "John, what are the divisors of 20?" And John answered him. I liked.*

Table 3. Aware of the lost learning opportunity.

I1	*[T]: Whatever, in the written part. Sure, I believe it's an important part and then we can start to formalize.*
I2	*[FT]: Yes, yes, the idea was that . . . that is, I missed this part. Well, I basically did it. It was, like at the end of the session, I made the connection with the ideas of all the children, but it's not the idea, or at least it shouldn't have been like that. But an alternative would have been, which I already did in another session, and it turned out quite well, but not in this one, which was to let the students explain as if they were teachers, that is, the idea is for them to stand up, go to the blackboard with different colors, different tools and explain, in writing and orally, what they have to do to reach the solution. In other words, here the idea would have been: "Toni, okay, you have explained it very well, now you get up to the blackboard and go and explain it to everyone".*

6.3. Account off Narrative

In this fragment (Table 2), we see that the future teacher reproduces some facts without further elaboration, and he even makes a subjective assessment, though he is not able to synthesize or refer explicitly to the importance of the fact that he delegated a responsibility to a student.

6.4. Account for Narrative

In this fragment (Table 3), the future teacher shows he is aware of the lost learning opportunity and produces a not very descriptive, crucial, and prospective account; it is focused on the didactic idea of encouraging mathematical communication in students, and, in addition, it proposes a future improvement action on how the future teacher should have done it. In fact, the future teacher claims that, on some other occasion, he has been able to incorporate it into his practice. The tutor suggests to the future teacher to institutionalize the knowledge after letting the students explain their decisions.

In the first extract, the future teacher focuses on the role of the teacher and only describes in detail a moment experienced in the classroom, in which he proposed to a student to go to the board to share the results of the duties he had given them. The narrative is totally an account of, since, in the discussion, it tries to expose the classroom dialogues

literally; he complements his intervention with some judgments or impressions caused by the moment he lived. It shows his being between the initial stages of description and subjective evaluation of the development trajectory. On the other hand, in the second extract (Table 3), a more prospective discourse of the future teacher is recognized; in fact, he is able to propose an alternative to turn a lost learning opportunity into a learning opportunity. In short, it is recognized that the future teacher is in an alternative stage of the development trajectory.

6.5. Self-Regulation and Metacognition

Another important result is that the video narrative facilitated by the lenses has clearly promoted the self-regulation and metacognition of the students' training. Look at the dialogue in Table 4, which took place in one of the seminars between the tutor and one of the teachers in training.

Table 4. Example of autoregulation.

I1	*[FT]: Yes, I noticed it in the videorecordings. The word "yes" and the word "okay".*
I2	*[T]: You see [. . .]. This is already a significant part of change. That thanks to the videorecording a modification occurs.*
I3	*[FT]: I have noticed these two things: that I talk a lot and that I repeat those words a lot.*
I4	*[T]: It's usual. We all have tics; what happens is that you have to be aware of it.*
I5	*[FT]: Today I realized in class that I was about to say "okay", but I stopped, that is, I noticed it and said: "you were about to say . . . "*

The future teacher confirms that, when he saw himself in the video recordings, he noticed that he has tics when speaking: he very frequently uses the words "yes" and "okay"; that is, it shows that he has carried out a process of metacognition about his training, since he has identified an aspect that he does not do well enough, The tutor identifies, as a positive change in the professional development of the student, that the student realizes that he commits tics. The future teacher comments that there are two aspects of the teacher's role that he needs to improve: tics and that he talks too much. Again, it shows that he has carried out a process of metacognition. At this point in the discussion, it is identified how the future teacher has been able to self-regulate in class.

As it can be seen, the future teacher realizes when he sees himself in the video recordings that he has tics and that, therefore, he is not doing well enough. He becomes aware of it, and, finally, he is able to self-regulate within the intervention period of the Practicum. Consequently, there has also been a change in the intervention of the future teacher in the classroom.

It should be mentioned that, in the seminars, it was possible to see signs of changes in the interventions of the future teachers in the classroom and in the discourse. Specifically, as the training progressed, the future teachers were able to take better advantage of their students' interventions and turn them into learning opportunities. For example, if a student immediately gave the correct answer, the future teacher, instead of accepting it as valid, was able to question it and ask the other students what they thought or if they agreed. Or, another example, is what we have just seen in Table 3. Consequently, it can be affirmed that the future teachers reached a level of practical competence development, that is to say, they modified their way of acting in accordance with new epistemologies, they incorporated new strategies in the classroom and are able to plan and make coherent judgments. In any case, in the future it should be checked whether these modifications persist over time or is it just circumstantial.

Finally, it must be said that the lenses to reflection have helped future teachers to identify some barriers in the environment that hinder universal learning: sensory, emotional, curricular, structural, and communicative. In the seminars, they have reflected on it, and, throughout the practices, the future teacher has tried to overcome them.

Below are examples of how Gregory and Alex applied the five lenses in seminar discussions and how their tutor intervened in these discussions.

6.6. Student's Mathematical Thinking

The following discussion (Table 5) is evidence in which Gregory applies the mathematical thinking perspective and describes what a student, Sara, has done to solve a problem about covering a surface with square floor tiles of different sizes. Nevertheless, the discussion progresses thanks to the intervention of the tutor, who conducts an action of highlighting and causes the future teacher to also focus on his role as a teacher. The future teacher explains how one of his students, Sara, has solved the problem of the floor tiles and describe her resolution strategy. The tutor has a strategy of staying in the inquiry posture, specifically of highlighting [22], and asks Gregory if that resolution strategy was only followed by this student. In his answer, the future teacher explains other resolution strategies that he anticipated and focuses on his role as a teacher; in fact, it links the practice with the five practices proposed by the NCTM [45].

Table 5. Mathematical thinking lens.

I1	*[G]: Sara did this. So, what has she done? She turns 3 × 3 red. 6 × 3 gives 18 and 8 × 3 gives 24. She's done it two ways, the 3 and the 6. And the 6, she's realized that 3 and 3 are 6, and so 4 of those red squares will make one of the 6 and it is fixed that you only need to take the 3 and the 6 as well. She says: "And 6 × 4 gives 24 and 6 × 3 gives 18".*
I2	*[T]: Did it just happen to her?*
I3	*[G]: No, to more people. Here we have Nia. She did it like this (shows the girl's evidence on the board). I used the 5 NCTM practices. The anticipation, I did the work with 4 different forms. A first form, which I think is perhaps the most typical, is me trying, but without making any sense. This was done, for example, by Diana. The other way is I look at the side that measures 18 and I overlap only the side, not my whole rectangle and see if my side fits. If it also fits on both sides, I understand that it works. The third way is, from this second, to vary and say: oh, okay, what does it have to do with the dividers on the sides. This is the third form that I wrote. In other words, here you already realize that it has to do with the dividers and it's not like I'm checking, oh yes, yes look, it works, period. It works, why does it work? Oh well, I noticed that they are the dividers. Noah and Sara did this, they noticed and told me. (Search.) OK, here's the selecting and sequencing part. I select which students I want to speak and sequence which forms I want them to come out first.*

In this transcript extract, it should be noted how the future teacher has taken some theoretical notions from the master's teaching modules, such as the five NCTM practices, and applied them to modify his teaching style. This is evidence that the future teacher has achieved, from our point of view, a level of practical professional development.

In Table 6, a discussion is presented in which Alex wants to focus on the classroom discourse and on a proposal to improve a learning opportunity, but, to achieve this improvement, the future teacher must interpret the mathematical thinking of his students. The tutor takes a launching action to provoke Alex to express his ideas or reflect.

This refers to a class episode in which the similarity of triangles is being worked on; the future teacher explains that two students, Alex, and Judith, almost deduced the answer to the proposed challenge and wants to use their intervention and their arguments to institutionalize the resolution. However, after reflecting on it, he thinks it would have been better if Anna and Martin, two students who had a clear answer, had explained it directly. The future teacher's dilemma is that he does not quite see clearly that it is a desirable choice for Anna and Martin to explain, because they already know. For him, it is to change the transmitter of knowledge, the teacher, to the students, and he believes that the effect it will have on the rest is the same as if he explained it. The tutor explains that, for these two students, it is a good learning opportunity, since having to explain increases understanding. In fact, it is one of the benefits of mathematical communication, as it helps to clarify, order, and refine ideas to those who communicate [46]. The tutor manifests that the effect produced by the classmates' explanations on the classmates is not the same as if the teacher does it, since the students know that if the teacher says something, it is true; on the other hand, if a friend says it, it forces them to think and reason if that is true.

Table 6. Alex's mathematical thinking and teacher's discourse lenses.

I1	*[T]: Now, if you could only make one comment about what you have prepared, what would you like to teach us?*
I2	*[A]: Since Martin and Anna were clear, so let them be the ones who tried to convince the others, instead of . . .*
I3	*[T]: OK*
I4	*[A]: Take advantage of the fact that they are clear and that they explain it. I choose, because I know that Aleix has come to see that they are proportional and that Judith, almost links it to the fact that since the angle coincides, everything grows in the same way, I want these two to act as a bridge for me so that let's go little by little.*
I5	*[T]: But that's very good.*
I6	*[A]: But then what scares me about using Martin and Anna is that if they are also so clear, in the end we change one transmitter of knowledge, which would be me, for two which are them. In other words, I don't know to what extent they will promote that other really become convinced of this or it will simply be: "no because having the same angle, they are proportional".*
I7	*[T]: Yes, but making an idea verbalize to another person increases the value of the idea to the person who verbalizes it. In other words, what changes is the epistemic value of the knowledge, they have more . . . If you have the opportunity for them to explain it, they have to do some mental actions that increase their understanding. In other words, what do Martin, and Anna get from me? If they already know, they understand it much better.*
I8	*[T]: Is it the same, Gregory?*
I9	*[G]: I think it is not the same. It increases the value, it's peer-to-peer learning, it's not the same. When a colleague explains something to you, it doesn't have the same effect as if the teacher explains it to you. And this is particularly important.*
I10	*[A]: Okay, but still . . .*
I11	*[F]: One thing is the teacher, and the other is the students and the role of persuasion that one student has over another is often much higher than the role that the teacher has with respect to the student. It's what he calls a teaching contract. It is already assumed that the teacher knows it and must teach it well and, therefore, the student listens to, but when another colleague, who has no obligation, tells him that this is so, the value that the other receives is much higher. It makes you put in a higher cognitive effort.*

In this intervention, we see how the tutor presents his argument connecting with Brousseau's didactic contract theory.

This discussion shows an account of the discourse of Alex, and it evidences how it moves in the alternative stage of the competence development trajectory, because after his reflection and a comparison with the theory received in the other modules of the Practicum, he realizes that he could have chosen other students to do the institutionalization. Therefore, he also reaches the level of practicality of competence development.

It should be noted that, in this discussion, two tutor strategies or actions are identified that complement the strategies proposed by van Es [22] and Coles [25]: a strategy of giving positive feedback (I5), which would be integrated into a category new teacher support, and a strategy to bring theoretical–didactic ideas to the discussion (I11), which would fit in the category of maintaining the focus on the video recording and mathematics of van Es [22].

6.7. Tasks and Mathematical Content

The discussion in Table 7 looks at the tasks and, in particular, addresses the question of how the activity could be improved to move towards the learning objectives. In the beginning, it seems that Gregory should focus on how the students solve the problem, that is, on the mathematical thinking of the students, but, at a certain moment, the tutor turns his gaze to the resources. Gregory explains that the publisher they work with at the practice center proposes an applet as a resource to help solve the problem; what this applet does is visually show whether a tile type exactly covers the surface or not. The tutor turns to Gregory and asks him directly why he did not use another type of material resource, such as cardboard, to make the tiles and have the students themselves try tiling, as a trial-and-error method.

Table 7. Gregory's task lens.

I1	*[T]: Stop, stop here (In Gregory's videorecording). Why didn't you show the floor tiles?*
I2	*[G]: It's an applet they have and it is used to see if it completes or not . . .*
I3	*[T]: Why didn't you give them cardboard and tile? You make 1 × 1, 2 × 2, 3 × 3 tiles . . . you work the area. And then, well, it's like the Cuisenaire rods, you see how, which is the square and why some don't. And even overlap one with the other and of course, that's what he said, they go with the tablets just by moving the mouse. It's like what I said to the teacher that "What have they learned?" "Move the mouse".*

We notice that the tutor connects mathematical ideas. He draws a parallel between the use of Cuisenaire rods and square tiles and explains that the manipulation of the latter allows students to work on area and square numbers and discover why some tiles work and others do not; instead, with the use of the applet, they only learn to move the mouse.

This action by the tutor should be highlighted because it identifies a new strategy, an interruption strategy (I1); however, unlike the action of interruption proposed by Coles [25], we mean interrupting so as not to lose an idea or a possible point of discussion. This action would also fit into the category of keeping the focus on the video recording and math.

If we enter the content of the discussion, the tutor questions the suitability of the resource used to solve the problem and, instead, proposes the use of manipulable material such as cardboard, so that the students advance towards the goal of learning.

In the future teacher's reflection rubric, it can be seen that, after the seminar, he had reflected on it, since he proposed the use of cardboard and scissors as an improvement to the task, so that his students would use their own tiles; he even included another option in which he himself gave different tiles on paper.

Regarding the lens of mathematical content, Gregory did not refer to it in any discussion. We believe that this fact is a consequence of the good initial mathematical training of the future teacher, which allowed him to design a good and adequate teaching unit.

In Alex's case, he also did not directly address the mathematical content lens, but it emerged when he applied the task lens. Specifically, Alex suggested a change regarding his didactic unit, but thanks to the intervention of the tutor and his colleague, finer issues such as the learning objectives or the mathematical content were addressed. We can see this in Table 8. The tutor clarifies that he did not do it out of conviction or because of his way of being. At this point in the discussion, the institute mentor intervenes to offer his point of view and explain the facts. He explains that, indeed, they realized that they had not scheduled enough sessions to carry out the TU, but they decided not to modify it, since it is a way of learning. However, he instructed the student to reflect on it and think about possible changes. The tutor took a counterpoint action. He intervened to say that he believes it is also important to learn how to reformulate the programs, since, if the students do not learn, the teaching unit they are implementing does not contribute to progress towards the learning objectives.

In the interventions in Table 8, a naive attitude of Alex is evidenced, and, in fact, it can be affirmed that he moves in a suggestion stage of the competence development trajectory, since he is only able to propose a general change alternative (I1), which is unspecified.

6.8. Role of the Teacher

The discussion in Table 9 is an example of applying the lens of the role of the teacher, but it also illustrates how future teachers analyze the practice of another colleague. In this case, the discussion was triggered when the tutor invited Alex to express his ideas about how Gregory represents the students' answers on the board. The tutor carries out an action of permanence in the posture of inquiry, specifically, of pressure [22] and invites Alex to express his ideas about how his partner represents and institutionalizes the answers of his students. The tutor explains and clarifies [22] Alex's comment; the tutor explains to Gregory that the representation scheme he has used communicates little to the students and that he should have used language that was closer to them. At this point, the future

teacher shows that he had partially reflected on it, since when he changed his teaching strategy and had a student come out on the board to do the institutionalization, the rest of the students they paid more attention to her than to him.

Table 8. Alex's task lens.

I1	*[A]: In the middle of TU, I think I could have changed and done a couple of days of exercises, doing definitions and practice and then being able to go back to . . .*
I2	*[G]: And why haven't you done it? What's the point of ending . . .*
I3	*[T]: No, no, he has made up his mind that if this is the TU, this must be done.*
I4	*[A]: We talked to my mentor, and he told me that it is also a way of learning, that is to say, that you made a mistake in this sense, that you thought that, perhaps the students were more capable or had many more facilities than they really have, and this is also a way to learn. Then he told me to continue with this TU as I had planned it and that nothing happened, but to do this reflection, especially important for me I would consider it in a different way, here, in this activity maybe that would change, maybe he wouldn't have done it that way . . . that he thought of it this way, for the sake of improvement, to change the TU he had in mind. And that he considered that the TU itself has enjoyable content and is well structured. The only thing, the "timing" perhaps.*
I5	*[T]: I think it is also important to reformulate because of course, the law is there, and you have planned, but at a given moment you have to know that those students you have in class . . . because as much as he wants to advance, if he is advancing on something that . . . let's see, that, if they are not finding out, it will not help at all. You have to find that balance, but to reformulate, perhaps, at a certain moment, to say, this is my plan, and this is important. This is my plan, but I had to change this. Why I have had to change or how I have adapted, because in the end you have to adapt to them going up, but that happens to us at all educational levels.*

Table 9. Gregory's teacher's role lens.

I1	*[T]: (To Alex) You see his board, what do you see?*
I2	*[A]: Everything is very symbolic . . .*
I3	*[T]: In other words, if this were an advertisement that has to communicate something, the visual language is extremely poor. It's your scheme (Gregory's). Gregory, it doesn't have to be your scheme, it has to be like an advertisement, it has to adapt to others. That's what Alex meant. Or am I interpreting it wrong?*
I4	*[G]: No, no. It is symbolic and since he is already truly clear about what the divisors are, which means that they are common, he makes the list of 18 and 24, see which ones are common and that's it. But this is my . . .*
I5	*[T]: In other words, on the board, it should not be your personal scheme, but it should be something to communicate to others. As Alex says, it's too formal.*
I6	*[G]: Yes, yes. The next class, this homework, which is the same but with different numbers, I told Sara to go out and do it. I mean, she was the one who wrote on the board and . . . no photo, but it was impressive. They paid more attention to her than to me.*

This discussion is reflected in Gregory's reflection rubric. This concludes that, in view of his future practice as a teacher, he must avoid an excess of formal mathematical language at the first ESO level and must use a representation scheme closer to the students and, as much as possible, let them appear at the board.

6.9. Classroom Discourse

The discussion of Table 10 occurs after Gregory shows an excerpt from a class session on divisors. The tutor does a launching [22] with a clear mathematical question to Gregory. The tutor highlights [22] the statement made by student 1 in the video recording and intervenes by clarifying and interpreting his idea. At this point, Alex intervenes to refute [22] the tutor's point of view and offer another one, since he considers that, since the beginning of the session is simply a review or an exercise that Gregory proposed for students to practice divisors, perhaps, at that time, there is no need to enter into mathematical reasoning. The tutor also comments that asking the students why they are working on divisors is a way of

validating their mathematical knowledge. From his point of view, it is necessary to take advantage of all the opportunities in the classroom to make the children argue.

Table 10. Gregory's discourse lens.

I1	*[T]: Let's see. Why are they divisors of 30?*
I2	*[G]: The 1 and the 30?*
I3	*[T]: No, you notice that, as a teacher, you say: "They are divisors of 30", in other words, you are affirming something. I would write and then ask why they are divisors of 30.*
I4	*[A]: I think that in this episode they told you, 1 × 30 gives 30, and then you keep pointing 1 × 30 = 30.*
I5	*[T]: Yes, but notice that it says: "they are divisors of 30". "Maria, 1 and 30 are divisors of 30, why?"*
I6	*[A]: But I understand that this is a reminder, right? The teacher has already institutionalized the divider. And that they told you in the 1 × 30 form. 1 and 30 are numbers that multiply to 30 and therefore divide 30.*
I7	*[T]: That! I want them to tell me all this.*
I8	*[G]: Then let Alex come to class and sit down, because I wouldn't have anyone but him.*
I9	*[T]: Because then you are validating the knowledge. Otherwise, if you say they are dividers, it remains that the teacher said so. And then, you don't make the children, in this case, make it explicit. In other words, it is to take advantage of . . .*

As can be seen, the discussion refers to the teacher's classroom discourse and focuses on a teacher–student interaction that does not end up promoting a mathematical learning opportunity for the students.

When doing the post-seminar reflection, the future teacher proposes, as an improvement to his practice, to try to get the students to argue as much as possible, that is to say, to take advantage of any opportunity to reason and build knowledge; he understands that a teacher must provide tools so that it is the students themselves who do the recapitulation.

Table 11 shows an extract from the application of the discourse lens in the case of Alex. In it, he has a descriptive narrative, and how his thoughts are involved can be seen.

Table 11. Alex's discourse lens.

I1	*[A]: And then what is also true is that David begins by asking that he has not understood the problem. Then we explain it to him and him himself . . .*
I2	*[T]: Do you explain it to him, or do you have Raul explain it to him?*
I3	*[A]: No, I explained it.*
I4	*[T]: You always must let the students do it.*
I5	*[A]: Yes, yes, I agree, but . . .*
I6	*[T]: Don't you mind?*
I7	*[A]: No, no, I just haven't thought about it. At that point . . . you're like chatting with David you know? David says to you "but is that because I don't want Messi to shoot?" and you say "No, no, Messi kicks, as if it were Pepe, what you want is to make it difficult for him." It's like you should stop 3" and say "Ok Raul . . . "*
I8	*[T]: Here is something difficult. Suppose you have a class with 22 students like Gregory has. Then you have to put 22 threads of wool between you and the students and between them, that the conversation is not a thread and you forget about the other threads, but it is a global conversation, it is a network that must pass from one to the others, that is, you can stop at David, but immediately you have to keep in mind, which is difficult, the other 21 . . .*

On the other hand, this discussion should be highlighted because it again identifies the strategy of interruption, in order not to lose sight of an idea (I2). Alex confirms that he explains it to the student. The tutor intervenes to give his point of view: they must let the students intervene and explain. Alex agrees, and, from there, the discussion turns to class interactions; the tutor tells Alex that the teacher must be attentive to all the interventions and comments of the students and must know how to take advantage of them.

Throughout the various evidences, it has been possible to see how the future teachers do not analyze all the lenses with the same depth; in some cases, such as in the role of the teacher, they show that they have more didactic tools or notions that move them to more advanced stages of professional development, while, in others, such as the students' mathematical thinking, we show more descriptive tools than interpretive.

However, the results show that we are facing two future teachers with different styles: Gregory has been more executive in all his interventions, while Alex has been more reflective, and we have even been able to see how their thoughts and beliefs influence the way they act in the classroom.

7. Discussions

In this study, it is found that the video-related model mediated by the five lenses contributes to the future teachers noticing new types of events or specific aspects of the mathematics learning and teaching processes as significant, which, otherwise, it would be difficult to do it. For example, in the beginning, future secondary mathematics teachers tend to focus on class management or the use of material resources; however, as the discussions in the seminars progress, and they become more familiar with the lenses, they place much more importance on the students' mathematical thinking, the type of task or discourse, and, in particular, the opportunities for learning. In any case, future secondary teachers find it easier to analyze some lenses than others: at one end, we find the role of the teacher and, at the other, mathematical thinking.

It should also be said that this model promotes the learning of new professional techniques to give meaning and make better use of classroom interactions and provides strategies to overcome environmental barriers that hinder universal learning. In fact, future secondary teachers become aware of the barriers, especially when they see themselves in the video recordings; in the seminars, they discuss and take away some strategies that they try in the classroom, which usually work. Consequently, at the end of the training they feel more motivated and prepared to develop their profession satisfactorily.

In short, according to Schwarts et al. [47], the reflection produced by the video-related model has been effective for professional development, since conceptual and didactic changes have occurred in the two cases analyzed.

In this way, this instructional design contributes to enriching the self-regulation capacity of future secondary teachers' training, which also becomes a tool or strategy for improving future practice. Now, it is not enough for the trainers to simply provide the framework for the analysis of the episodes, but it is necessary that they teach them to use and interpret the lenses by explaining and exemplifying each one of them. Thus, according to Llinares et al. [48], it is necessary to train future secondary teachers in a specific vocabulary of professional development, so that they can better name and focus on each of the lenses. Consequently, the discussions will also be deeper [49].

Another point that must be considered is that there are also video episodes that lend themselves more than others to being analyzed with a certain lens. In this regard, the trainer may need to rephrase or adapt some of the facilitating questions to the circumstances of the video recording. On the other hand, it should be emphasized that in the face of the new paradigms and challenges of the society of the future, we consider the convenience of opening a new affective lens on the image of mathematics that includes values, a gender perspective, and inclusion.

8. Conclusions

According to Clarke [50], professional growth changes occur in future teachers through a mediation process of reflection and action. In other words, professionalization, as well as the development of the noticing competence, is achieved from the reflection of one's own practice, along with the video reports guided by the tutors and mentors in the trajectories of the practicums and mediated by the five lenses. In fact, the Practicum becomes a true learning situation.

Consequently, we can conclude that this model addresses, with guarantees, Sustainable Development Goal 4 of the 2030 Agenda, as it promotes a lifelong learning opportunity for teachers [51].

In the Practicum seminars, the participants observed and discussed teaching practices in video-recorded lessons, an environment that allowed them to hear and consider ideas and feedback from others. This exhibition stimulated and promoted the reflection of teachers, based on the co-analysis of alternative practices, goals, actions, and more. In line with previous work [6,52,53], we suggest that a Practicum environment based on a video-related method offers a combination of discussions, a vivid object for analysis (the video recording), and the guidance provided by a tutor, which allow for deep reflection.

The final reflection in the participants' portfolios also provides a personal environment, where future secondary teachers can see an authentic representation of their own teaching and examine, analyze, and evaluate situations. The detailed description of one's own actions shown in a video recording helps future secondary teachers to notice situations, including those that were missed in "real time". However, self-observation arouses emotions, and some teachers tend to criticize themselves or, alternatively, justify their actions, rather than productively examine their practice.

The results of this study reveal that beyond the importance of any reflection process per se, the settings (video-related, stimulated recall seminars based on a video-recorded lesson and final reflection flashcards) in which reflection takes place and its specific possibilities make the difference. What is more, from our point of view, the model with its five lenses can be replicated in other STEM teacher training specialties, such as science or technology teacher training.

9. Limitations and Discussing the Future Work

We agree with Nurick et al. [43] that reflective processes require time, resources, and support, but with careful guidance and the appropriate tools (through mentoring), reflection can be learned and put into practice productively. In fact, the value of this research regarding the role of the tutor in discussion seminars should be highlighted. We think the facilitative actions or strategies identified and described, as well as the exemplification of their application by tutors, provide a useful tutoring guide for tutor researchers pursuing the professional development of their students.

As a limitation of this work, motivated by the natural context of research, we emphasize that it is a study with a small sample, both of future secondary teachers and tutors. Therefore, it is necessary to continue investigating the scope of the video-related model with more future secondary mathematics teachers and with more tutors, both from the same profile as the participants in this study as well as from other profiles.

As for future lines of research, it is necessary to investigate the changes that future secondary mathematics teachers experience in their practice during the Practicum based on the video-related method. Moreover, what remains to be explored is what happens when the Practicum ends: what is the teaching learning that they take away and apply as teachers? How is this reflection of initial training cultivated in permanent training?

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