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Research Article

Preparing, leading, and reflecting on whole-class discussions: How prospective mathematics teachers develop their knowledge during lesson study

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ABSTRACT

This research aims to understand how prospective teachers develop their knowledge about whole-class discussions during a lesson study. Following a qualitative approach, we focus on the work of three prospective mathematics teachers preparing, leading, and reflecting on whole-class discussions. Data was collected through participant observation with field notes, audio recordings, and document collection. The results show that a detailed analysis of a lesson plan, previously prepared autonomously by a prospective teacher based on a guide, promotes a focused discussion and a collaborative environment. Additionally, being involved in each other's formative process, through lesson study micro-cycles, created opportunities for prospective teachers to analyze and discuss different classroom situations, which promoted the development of their knowledge. Finally, classroom experiences seemed to be fundamental for the development of prospective teachers' knowledge. They were able to put into practice the work carried out during the lesson study, which allowed them to link theory and practice, and reflect on their own practice. This research also underlines the importance of the teacher educator's role in creating fruitful opportunities for the development of prospective teachers' knowledge.

Keywords: whole-class discussions, initial teacher education, lesson study, mathematics teaching

INTRODUCTION

Whole-class discussions are moments during a lesson in which teachers lead students to present, explain and justify their mathematical ideas, and contrast them with their colleagues, establishing connections between those ideas. These moments allow students to reflect on the work that they did solving the tasks proposed by the teachers, and extend their learning (Ponte & Quaresma, 2016; Stein et al., 2008). Whole-class discussions foster students' learning (NCTM, 2014), and teachers have a key role in creating these opportunities. Hence, it is fundamental that teachers know how to lead whole-class discussions that encourage students to share and discuss their work and reflect on the mathematical ideas presented (Ponte et al., 2017; Stein et al., 2008).

However, leading whole-class discussions is a challenge for teachers, especially for novices (Fujii, 2018; Stein et al., 2008). It requires "an extensive network of content knowledge, pedagogical knowledge, and

knowledge of students as learners" (Stein et al., 2008, p. 320) to be able to know how to act regarding students' work and responses, maintaining the focus of the lesson. As Stein et al. (2008) point out, "without solid expectations for what is likely to happen, novices are regularly surprised by what students say and do, and therefore often do not know how to respond to students in the midst of a discussion" (p. 321).

Several investigations (Fujii, 2018; Ponte et al., 2017; Potari & Ponte, 2017) suggest that planning in detail the lesson helps teachers to know how to lead whole-class discussions. By anticipating the students' solving strategies and difficulties and preparing teachers' interventions, they became more prepared and confident to lead the discussion, minimizing the in-the-moment decisions. Notwithstanding, prospective teachers tend to have limited knowledge about students, so anticipating students' solving strategies or difficulties is also a challenge (Chen & Zhang, 2019; Meiliasari, 2019). Therefore, it is important to understand how initial teacher education courses can contribute to the development of prospective teachers' knowledge about whole-class discussions (Chen & Zhang, 2019; Conceição et al., 2019; Ni Shuilleabhain & Bjuland, 2019), particularly regarding its preparation and leading.

Lesson study has been pointed out as a professional development process, focused on students' learning, with promising results in initial teacher education, namely concerning the development of prospective teachers' knowledge (Ponte, 2017; Willems & van den Bossche, 2019). This process creates opportunities for prospective teachers to collaboratively prepare their interventions for whole-class discussions, based on the anticipation of students' work. The prospective teachers may also have the opportunity to teach the lesson they planned, and to put into practice the strategies they prepared. Furthermore, lesson study promotes the reflection on these classroom experiences, focusing on students' work. These opportunities allow them to connect teaching practice and what they have learnt in previous courses, promoting the development of their knowledge.

Nevertheless, few investigations focus on how initial teacher education courses can help the prospective teachers to develop their knowledge, in particular, so they can understand "how to communicate in the classroom … [and] how the teacher conducts classroom communication" (Conceição et al., 2019, p. 80). Recognizing the importance of a deeper understanding of how prospective teachers may develop their knowledge about whole-class discussions during a lesson study, this research focuses on the work of three prospective teachers, preparing and leading whole-class discussions, to answer the research question: What elements of lesson study provide opportunities for the development of prospective teachers' knowledge about whole-class discussions?

THEORETICAL FRAMEWORK

Teachers' Knowledge

A key feature of initial teacher education is related to the development of prospective teachers' knowledge. *Teacher's professional knowledge* is a complex concept, with many factors at play, and is based on classroom experiences and reflection on these experiences (Ponte, 2012). Being the basis of teaching practice, it is necessary to understand what knowledge teachers effectively need to have for teaching (Ball et al., 2008).

Shulman (1986, 1987) was a pioneer in proposing a theoretical framework for teacher's knowledge, highlighting the *pedagogical content knowledge*, which "represents the blending of content and pedagogy" (Shulman, 1987, p. 8). Ball et al. (2008) refined the dimensions proposed by Shulman (1987), seeking to better understand "what do teachers need to do in teaching mathematics . . . and how does this work demand mathematical reasoning, insight, understanding, and skill" (p. 395). The authors focused on the *work of teaching* (and not on teachers), highlighting the importance of *specialized content knowledge* and *knowledge of content and students*. Ponte (2012), in turn, focused on the *knowledge related to teaching practice*, referring to it as "the fundamental core … where crucial choices are made that guide practice and in which the entire teaching process is regulated" (pp. 88-89). Thus, he proposed four dimensions for what he refers to as *didactic knowledge*, explaining that, differently from other frameworks, these dimensions cannot be separated as they are all constantly present in teacher's practice.

Given the focus of this research, we consider two dimensions of teacher's knowledge, based on the ideas of Ball et al. (2008) and Ponte (2012). The first dimension is related to the *knowledge about students' learning*,

which includes their interests, needs and cultural and social characteristics, as well as their common difficulties and ways of learning. For example, when selecting a task and analyzing its strengths, teachers need to anticipate how the students may solve it, what difficulties they may experience, and whether they will find it interesting. This knowledge about the students will help the teacher to select the strategies to explore and prepare his/her interventions for the whole-class discussion, to foster meaningful learning.

The second dimension refers to the *knowledge about teaching practice* and is related to the didactic aspects that influence students' learning. It involves task design, lesson planning, organization of students' work for the whole-class discussion and the creation of opportunities for students to establish connections between mathematical ideas and to extend their learning. For instance, teachers may adapt a task to meet the lesson goals and organize the whole-class discussion considering their anticipation of students' work. Moreover, when leading a whole-class discussion, teachers need to know when to clarify mathematical ideas, how to support and guide the students, and when and how to challenge them.

Development of Prospective Teachers' Knowledge

Besides characterizing teachers' knowledge, it is also important to understand how it may be developed (Ponte, 2012). Several investigations pointed out activities that promote the development of teachers and prospective teachers' knowledge, such as research on classroom experiences and reflection on practice, focusing on students' learning, as well as opportunities for sharing and discussing ideas (Coenders & Verhoef, 2019; Potari & Ponte, 2017). However, prospective teachers face a great challenge in connecting what they learnt in previous courses with teaching practice (Chen & Zhang, 2019; Conceição et al., 2019). Leading a whole-class discussion based on students' work "can be particularly daunting for teachers who are new to discussion-based pedagogy" (Stein et al., 2008, p. 320). Since prospective teachers' knowledge is developing, and the degree of control of what can happen in the lesson is limited, they often do not know how to act regarding students' responses (Martins et al., 2021; Ni Shuilleabhain & Bjuland, 2019).

Hence, a detailed lesson planning, anticipating students' work and preparing teacher's interventions, may help prospective teachers to feel more prepared and confident to lead whole class discussions (Chen & Zhang, 2019; Meiliasari, 2019). Moreover, it is important that prospective teachers also have the opportunity to lead such discussions in real classroom contexts since "in-school experiences form an important part of student teacher learning" (Ni Shuilleabhain & Bjuland, 2019, p. 2) and promote the development of their knowledge. However, these *in-school experiences* alone may not be sufficient to promote the development of prospective teachers' knowledge, and it is necessary to combine them with structured learning processes that guide this development, essentially through the support of teacher educators and cooperating teachers (Chen & Zhang, 2019; Shaughnessy et al., 2021).

Lesson study is a professional development process where this may happen. The basis of lesson study is the collaborative work of a group of teachers in the preparation, enactment, and reflection on a lesson to respond to a problematic issue emerging from teaching practice. This process differs from other professional development processes due to its reflective nature and its focus on students' learning. After defining a learning goal (step 1), based on a common difficulty among the students or a mathematical topic whose exploration is deemed interesting, the teachers begin to plan a lesson in detail (step 2). They select and adapt tasks, analyzing their strengths and weaknesses, and anticipate the students' work and difficulties, in order to prepare the lesson and their interventions with these aspects in mind. Then, one of the teachers teaches the lesson (step 3), while the others carefully observe the students' work. Finally, the group meets again to reflect on students' learning (step 4) and to draw conclusions for future cycles (step 5) (Fujii, 2018).

Since this process is centered on students' learning, prospective teachers have the opportunity to analyze tasks, adapting them or formulating questions that meet the lesson's goal, to prepare the lesson in detail, anticipating the students' work and their own interventions, to observe or teach that lesson, and to reflect on it by analyzing the students' work, supported by a teacher educator. Besides promoting reflection, lesson study encourages research into one's own practice and promotes the development of prospective teachers' knowledge. For example, in the studies of Chen and Zhang (2019) and Coenders and Verhoef (2019), prospective teachers developed their ability to plan lessons and to anticipate students' responses in solving tasks. Conceição et al. (2019) also found that a prominent factor in their study was the fact that prospective teachers planned the lesson to include the communication features that would be fostered in the classroom.

This experience allowed them to understand that how the teacher leads communication influences students' learning. Thus, lesson study creates opportunities for prospective teachers to lower the theory-practice gap, and to develop knowledge through classroom experiences, while researching and reflecting on their practice.

Teachers' Role in Preparing and Leading Whole-Class Discussions

Creating moments in which students explain their work and justify their ideas promotes discussions where they can establish connections between those ideas and reflect on their work (NCTM, 2014). In an *exploratory lesson* (Ponte et al., 2017), structured into three phases (Stein et al., 2008), the work is based on a task, which can be solved using different strategies and representations, and on the communication prompted through the discussion of these strategies and related mathematical ideas. After the teacher proposes a task, the students work autonomously on solving it. Meanwhile, the teacher monitors the students' work, supporting them in the difficulties that may arise, without reducing the degree of challenge of the task. Then, the teacher leads a whole-class discussion, inviting students to present and explain their solving strategies and justify their ideas. This moment creates opportunities for students to establish connections from their previous work and extend it.

As Stein et al. (2008) state, "such discussions are thought to support students' learning" (p. 315). So, teachers assume an essential role in leading discussions, which highlights the importance of developing their knowledge in this field. There are several investigations focused on whole-class discussions during a mathematics lesson. Stein et al. (2008) suggest five practices for facilitating mathematical discussions to help novice teachers to get better prepared to lead whole-class discussions. The work begins with (i) *anticipating students' responses*, foreseeing how their work "might relate to the mathematical concepts, representations, procedures, and practices" (Stein et al., 2008, p. 323) of the lesson. During the lesson, the teacher (ii) *monitors students'* work to identify their learning potential. With this information, the teacher (iii) *selects the relevant responses* to explore in the discussion, and (iv) *purposefully sequences them* to maximize the chances to achieve the lesson goals. Thus, the teacher may "build [students' presentations] on each other to develop powerful mathematical ideas" (Stein et al., 2008, p. 330), by exploring misconceptions, comparing related or contrasting strategies, and relating concepts, representations, and procedures, challenging the students to (v) *connect the mathematical ideas* that arose from their work.

In the context of lesson study, Fujii (2018) also highlights the importance of preparing the lesson based on anticipating students' solutions and monitoring their work during the lesson, to gather information to lead the whole-class discussion. Therefore, the teacher is able to decide which solving strategies should be presented by the students, and in which order. These practices help the teacher to create situations to compare similar and different strategies, which is an opportunity "to foster mathematical ways of thinking in their students" (Fujii, 2018, p. 7). Thus, the teacher may challenge the students to "move toward their potential from what they came up on their own" (Fujii, 2018, p. 7).

Focused on whole-class discussions, Ponte et al. (2013, 2017) proposed a framework to analyze the patterns of teachers' actions. Based on empirical data, the authors concluded that the teacher usually begins by *inviting* the students to the discussion, asking them to present and explain their work. The teacher may then alternate between *supporting/guiding* the students by making interventions and posing questions, or *informing/suggesting*, giving explicit suggestions, introducing information, and validating answers. The teacher may also *challenge* the students, seeking to encourage them to extend their work.

In initial teacher education, Shaughnessy et al. (2021) sought to decompose the practice of leading discussions "in ways the practice can be taught and learned by [prospective teachers]" (p. 453). The authors point out *discussion-enabling practices*, which involve selecting tasks, anticipating students' answers, and monitoring their work. Shaughnessy et al. (2021) also suggest *discussion-leading practices*, which are related to what teachers do to lead the discussions. As Stein et al. (2008), the authors refer to encouraging students to explain their work and comparing and questioning each's others strategies, by "choosing and using appropriate representations to convey key mathematical ideas" (p. 456). These *discussion-leading practices* also include launching the discussion, similar to the inviting practice of Ponte et al. (2013, 2017), as well as making contributions to keep "the discussion on track … [and] focused on the learning targets" (Shaughnessy et al., 2021, p. 459), eliciting and probing student thinking, and orienting students towards their colleagues' thinking to extend their work.

Considering these authors' ideas, some *practices* are particularly noteworthy in preparing whole-class discussions. The teacher begins to prepare the discussion by selecting the task to propose to students (Fujii, 2018; Shaughnessy et al., 2021). By analyzing the strengths of the task, the teacher may anticipate different solving strategies that the students may use and possible difficulties and misconceptions (Fujii, 2018; Stein et al., 2008). Anticipating students' work allows the teacher to prepare his/her interventions (Fujii, 2018; Ponte et al., 2013, 2017; Shaughnessy et al., 2021; Stein et al., 2008). It is also based on this anticipation and monitoring of the students' work during the lesson that the teacher gathers information to select and sequence the strategies to be explored in the whole-class discussion to foster students' learning (Fujii, 2018; Stein et al., 2008).

By leading whole-class discussions, teachers may develop different *actions*. Some actions support establishing connections between different mathematical ideas shared during the lesson (Shaughnessy et al., 2021; Stein et al., 2008). By inviting some students to present and explain their work (Fujii, 2018; Ponte et al., 2013, 2017), the teacher may support them to compare similar and contrasting strategies (Fujii, 2018; Stein et al., 2008), and relate concepts, procedures, and representations (Stein et al., 2008).

Other actions provide opportunities for students to extend their work (Fujii, 2018; Ponte et al., 2013, 2017; Shaughnessy et al., 2021). The teacher does so guide the students to keep the discussion focused on the lesson goals (Shaughnessy et al., 2021; Stein et al., 2008) without giving explicit suggestions or reducing the degree of challenge of the task. The teacher may elicit the students' reasoning by asking them to justify their mathematical ideas and challenging them to contrast those ideas with their colleagues. This environment allows students to reflect on the work carried out and be involved in meaningful learning (Ponte et al., 2013, 2017; Shaughnessy et al., 2021).

RESEARCH METHODOLOGY

This research followed a qualitative approach (Bogdan & Biklen, 2007), considering the work of three prospective teachers during a lesson study, as they prepared and taught their first lesson, with a focus on the whole-class discussions.

Participants

The lesson study was carried out in Portugal, with three prospective teachers, Sílvia, Maria, and Lila (pseudonyms) who had three years or more of specialized qualification in mathematics and would therefore be comfortable in the content knowledge. Their teaching experiences, however, were different. Sílvia and Maria had never taught classes before, although they had already prepared lesson plans in the previous months. Additionally, Maria had teaching experience in individual tutoring. Lila, in turn, had over ten years of teaching experience. These prospective teachers were supervised by a teacher educator who was interested and available to carry out the lesson study. In addition to the prospective teachers and the teacher educator, the first author also participated as researcher, and facilitated the lesson study together with the teacher educator.

Lesson Study Work

The researcher and the teacher educator met to discuss how they could integrate the lesson study in the formative process of the prospective teachers. They chose to carry out the lesson study with the three prospective teachers supervised by the teacher educator because

(i) several investigations point out that a lesson study should be carried out with a small number of prospective teachers to ensure more engagement and

(ii) the prospective teachers should be engaged in all the steps of the lesson study so that they can connect theory with practice (Chen & Zhang, 2019; Ni Shuilleabhain & Bjuland, 2019).

In order to prepare the lesson study sessions, the researcher held initial one-to-one semi-structured interviews with the prospective teachers, addressing topics on task selection and lesson planning, as well as the reflective and collaborative dynamics that they had already experienced.

Aspects to consider in pre-lesson written reflection

1. Justifying the task, the materials and the strategy.
2. Anticipating possible students' solving strategies. What difficulties may the students display? How can they be supported?
3. What challenges do you think you will experience? How do you expect to overcome them?

Aspects to consider in post-lesson written reflection

1. Make a description of your lesson (strengths and weaknesses).
2. What did you intend for this class? What was accomplished? And what was not? Why?
3. What challenges did you experience in implementing what was planned? How were they overcome?
4. If you taught this lesson again, what would you change? Why? What outcomes do you draw from this lesson for your future practice? Justify.

Figure 1. Reflection guide

After analyzing the content of these interviews, the researcher shared some excerpts with the teacher educator in which aspects of potential interest emerged, particularly those related to the theory-practice gap:

> I thought that [in the previous year] ... it should have been more practical... learning how to manage the classroom would be important (Sílvia).
>
> There's a huge gap between the academic side and practical side... [How] I'm going to build on my skills to be in a classroom, communicating (Maria).
>
> One thing is what we do when we're in a formative process, another is our daily practice, which has to be adapted to the time we have available, the classes, and contexts (Lila).

Furthermore, the prospective teachers also expressed their interest to foster classroom communication in grade 10.

To respond to the prospective teachers' needs, the teacher educator and the researcher encouraged them to critically analyze tasks to promote rich whole-class discussions, suggesting reading several articles from professional journals and books, and research articles (such as Ponte et al., 2013 and Stein et al., 2008). By gathering information about how to prepare and lead whole-class discussions, the prospective teachers would be able to share their conceptions and contrast them with each other's teaching experiences. The teacher educator and the researcher also encouraged them to anticipate the students' possible solving strategies and difficulties, which, for Maria, "is the hardest thing to do". By anticipating those aspects, the prospective teachers should be able to share their ideas on students' work, and collaboratively prepare the lesson and their interventions, which was Sílvia's main concern.

To lower the gap between theory and practice that the prospective teachers pointed out in the initial interviews, the teacher educator and the researcher adapted the lesson study cycle proposed by Fujii (2018) so the prospective teachers could put into practice what they planned and have the opportunity to reflect on their practice and improve it. Therefore, the teacher educator and the researcher organized three cycles for each prospective teacher and structured each cycle into five steps: *lesson plan discussion*, *lesson preparation*, *research lesson*, *post-lesson discussion*, and *written reflection*.

Each prospective teacher began by preparing a first version of the lesson plan, considering the guide proposed by the teacher educator for the pre-lesson written reflection. This reflection guide (**Figure 1**) was created at the university for the initial teacher education program. After sharing the lesson plan with the group, the teacher educator asked the other prospective teachers to suggest improvements to consider in the *lesson plan discussion* session. Thus, the work of this step focused on collaboratively adapting the tasks to the goal of the lesson, considering the work the prospective teachers intended to develop with the students and their possible difficulties.

After reformulating the lesson plan, considering the suggestions provided, each prospective teacher had the opportunity to explain how they intended to structure their lesson in the *lesson preparation* session, and the group could again suggest improvements. Additionally, they could simulate the moment of proposing the task, as well as how they would support the students in the difficulties they anticipated. In this session, aspects related to leading whole-class discussions were also discussed, namely how it would be fostered and the solving strategies that would be interesting to explore.

Due to the COVID-19 pandemic, apart from the prospective teacher who taught the lesson, only the cooperating teacher responsible for the class and the teacher educator could attend the *research lesson* on-site. However, after each lesson, the video recording was shared with the other prospective teachers and the researcher.

For the *post-lesson discussion*, the teacher educator asked the prospective teachers to consider the aspects suggested in the post-lesson written reflection guide (**Figure 1**), and to share their reflections orally. This session began with the intervention of the prospective teacher who had taught the lesson, followed by the other two prospective teachers. Then, the teacher educator and the researcher sought to raise issues that had not been previously mentioned by the prospective teachers. For example, they discussed the contrast between different solving strategies and the tendency to guide students through explicit suggestions before the whole-class discussion.

Following this step, the prospective teachers watched the video recording of the lesson they taught, which gave them the opportunity to reflect on the ideas that emerged in the post-lesson discussion. They also analyzed the students' productions collected and contrasted them with the video recording in a *written reflection*.

Upon of the cycle for the first lesson, each prospective teacher re-drafted a first version of a lesson plan for two further lessons, thus creating two more lesson study cycles.

All nine lessons were prepared following the structure of exploratory lessons (Ponte et al., 2017), which was a choice of the prospective teachers, who were already familiar with the structure, although they had never taught such lessons.

Data Collection and Analysis

The data was collected through participant observation by the researcher, who took descriptive fieldnotes during the lesson study sessions and completed them with reflective field notes after the sessions (Bogdan & Biklen, 2007). She also collected audio recordings of the lesson study sessions (Sx). All the documents produced were also collected, namely the tasks, the lesson plans, and the written reflections.

Considering the aim of this research, the work around a common topic in the first lesson of each prospective teacher was selected: problem-solving involving the quadratic function. The lesson plan discussion and lesson preparation sessions, as well as the post-lesson discussion and the written reflections were considered in order to analyze the work that the prospective teachers developed for and during the whole-class discussions.

The task proposed by Sílvia involved the body temperature variation of a boy with a fever, $T(x)$, modelled as part of a parabola with the concavity facing downwards. To answer the questions correctly, the students were required to draw on the situation to interpret the context correctly. In Maria's case, the context of the task was the growth in the number of followers of a girl's social networks, with questions requiring the students to analyze the graphical representations of the given functions (linear and quadratic) and interpret them according to their domains. Lila proposed two tasks: the first had the algebraic expression of a quadratic function, $a(t)$, representing the jump of a grasshopper from a wall of a certain height, $a(0)$, and whose parabola had a downward concavity; the second task required the students to write the expression of the area of an isosceles rectangle according to the distance between two moving points.

Building on the ideas shared in different investigations on the role of the teacher in preparing and leading whole-class discussions (Fujii, 2018; Ponte et al., 2013, 2017; Shaughnessy et al., 2021; Stein et al., 2008), the prospective teachers work during the lesson study to foster students' learning was analyzed considering the actions in **Table 1**.

Table 1. Categories for data analysis

Category	Sub-category
1. Establishing connections between different mathematical ideas	a. Inviting students to present and explain their work b. Supporting students to compare similar and contrasting strategies c. Supporting students to relate concepts, procedures, and representations
2. Extending the work carried out	d. Guiding students to keep the discussion focused on the lesson goals e. Eliciting students' reasoning by asking them to justify their ideas f. Challenging students to contrast their ideas with their colleagues to reflect on the work carried out

The first author began by categorizing the data collected into the two main actions, selecting the most relevant excerpts and subcategorized them. Thereafter, all authors met regularly to review the subcategorization and discuss discrepancies and reach a consensus. The quality of data analysis was ensured by several discussions among the three authors.

Ethical Aspects

For this research, the authorizations for data collection and research publishing were requested, and the ethical principles of educational research (AERA, 2011) were followed, ensuring confidentiality and participants' anonymity. The researcher created an environment of trust and engagement between the teacher educator and the prospective teachers, in which their wishes and availability were taken into consideration, and their informed and voluntary participation was ensured.

RESULTS

Establishing Connections Between Different Mathematical Ideas

Preparing

In the lesson plan discussion sessions, the prospective teachers explained why they selected particular tasks and their purposes with those tasks. Sílvia explained she intended that "students be able to solve [the task] using different strategies, with or without the graphing calculator… [to, in the whole-class discussion,] compare the different solutions" (lesson plan) [1b, 1c]. For Maria, the purpose of the lesson was "for the students to recognize the common characteristics of the study of a function… to solve problems that model situations from real contexts" (lesson plan), so students could relate the mathematical concepts and their applicability in everyday life [1.c]. With a similar purpose, Lila said that in the tasks that she selected, "there is a relationship between the different representations, which enables the student to work with [them]" (lesson plan) [1c]. By sustaining the tasks that they selected, the prospective teachers highlighted their purpose to establish relations between concepts, procedures, and representations by comparing different solving strategies presented by the students [1b, 1c].

By collaboratively analyzing the strengths of the tasks, the teacher educator and the researcher suggested rethinking the wording of some of the questions. As the prospective teachers intended to support the students to establish relations between different mathematical ideas [1c], the tasks should allow the use of different solving strategies and foster a diversity of representations. Therefore, Sílvia, and Maria added questions so the students could feel the need to represent the situation in a graph and relate it with the values obtained algebraically [1c]. For Lila, it arose the idea of replacing the given algebraic expression with a table with three points belonging to the function graph. Therefore, the students could work with the algebraic and graphical representations, and also with the tabular representation [1c].

After adapting the tasks, each prospective teacher anticipated the solving strategies that the students could use, indicating two types: using only algebraic procedures or using the graphing calculator. From this anticipation, in the lesson preparation sessions, it was discussed the solving strategies each prospective teacher intended to explore in the discussion. In addition, the teacher educator and the researcher suggested preparing the prospective teachers' interventions to support the students to establish connections between the main ideas they foresaw that would be discussed.

Sílvia planned to select students with different strategies to "enrich the discussion" (lesson plan), and then favor those with correct mathematical language. To support students to compare those strategies [1b], she prepared questions to pose, seeking "to foster sharing ideas [1a], to assess each student's reasoning [2e] … [and] to support the construction of strategies [1b]" (lesson plan).

Maria focused on exploring students' common misconceptions regarding the graphic representation of the different functions given [1c]. She explained that her purpose was the students "to realize that [the quadratic function] is a parabola and, therefore, was not always increasing [as the linear one]" (S12, lesson preparation). She also foresaw opportunities to support the students to relate "the concept of the domain to the context of the problem" (S12, lesson preparation) during a particular question [1c]. However, she did not prepare her interventions to support the students to explore those misconceptions or to relate the ideas.

Lila explained that "solving them [the tasks] algebraically will take longer … [by] using the calculator to draw the graphs … It's easier to understand the interpretation of the problem" (S6, lesson plan discussion). So, she would encourage using the graphing calculator to interpret the context of the tasks [1c]. However, the prospective teacher did not prepare how she would support the students to establish connections between those ideas.

Leading and reflecting

The prospective teachers began the whole-class discussions by inviting a few students to present their solving strategies [1a]. Sílvia invited two students who used different algebraic strategies to explain their work [1a]. Then, she invited another student [1a] "with a different strategy … in order to promote a comparison between the strategies [1b] … [which led the students] to share their different ideas and … to justify their reasoning [2e]" (written reflection). Considering the students' explanations, she asked the class to "identify the domain of the quadratic function [in context] in the task" (written reflection) [1c], through the questions she had prepared:

> Sílvia: What details do we have here? Of the domain? … What values may x have?
>
> Student A: It's from 0 to +∞.
>
> Sílvia: So, does that mean he measures his temperature infinitely?
>
> Student A: No, we have to find the other zero!
>
> Sílvia: What do you think the [graphical] representation is? … What are the domain values?
>
> Student D: From 0 to 5.
>
> Sílvia: So, can we continue to draw [the graph beyond 5]?… Why?
>
> Several students: We have to erase it. Because it's not part of the context and so, it's not part of the graph.

Sílvia supported the students in relating the algebraic to the graphical representation [1c], seeking that they were able to establish connections between these representations and the context of the task. Reflecting on the lesson, she acknowledged that "I pose a lot of questions in a row… [and the students] haven't even thought about the first question and are already thinking about the second one" (S14, post-lesson discussion), which influences the connection between the different ideas and, consequently, the students' learning. Accordingly, she said she would try to "be calmer, [and] listen to the students" (S14, post-lesson discussion) in the following lessons.

In Maria's lesson, the whole-class discussion began with a student presenting his solving strategy [1a], which considered a quadratic function as monotonic. The prospective teacher took control of its explanation, guiding the class to check its validity:

> Maria: You considered the variable t to be from 0 to 75 and the maximum number of followers would be *f*(75)... Do you agree with? ... Why?
>
> Student E: Because the graph [of the function] is a parabola.
>
> Maria: And if it's a parabola, its concavity is what?
>
> Several students: Downwards.

To support the students to relate the algebraic and graphic representations [1.c] to correct the strategy followed by the student, Maria encouraged them to use the graphing calculator, which, in her words, "generated a very interesting diversity of representations" (written reflection). However, she did not take advantage of student E's answer or explore the *diversity of representations generated* to support the students to establish connections between the graphical representation and their previous results.

When she reflected on the lesson, Maria said that "there weren't as many interactions as I'd expect", relating this to the "culture of the students ... [as] they have not the habit of explaining their ideas" (S15, post-lesson discussion). The prospective teacher acknowledged that, during the lesson, the students did not have the opportunity to explain and relate the representations that they used, but she did not try to understand the influence of her actions on the students' learning.

Concerning Lila's case, after inviting two students to present their different algebraic strategies [1a], she invited another student to explain how he solved the first task using the graphing calculator, "in order to foster a comparison of the different representations" (written reflection) [1c]. As she intended, she focused on that graphic representation to support the students in interpreting that task correctly [1c]. The prospective teacher considered that the highlight of the lesson was that the students "solved [the tasks] using different solving strategies... [for which] I was already prepared" (S10, post-lesson discussion) thus valuing the anticipation work. However, she did not used the diversity of strategies presented to compare them [1b] and recognized that she "ended up unconsciously guiding the students to the answer" (S10, post-lesson discussion), narrowing the opportunities for them to reflect on the work carried out.

Extending the Work Carried Out

Preparing

In the lesson plan discussion, Sílvia expressed her concern about students' difficulties. Recognizing that she had "trouble in identifying the difficulties ... [because the task] appears to be very simple" (S3, lesson plan discussion) and to put herself in the students' position was a challenge for her. However, following the teacher educator and the researcher's suggestion, she detailed her anticipation work of students' difficulties and solving strategies in each task question with Maria and Lila's collaboration. Additionally, to keep the discussion focused on the lesson goals [2d], without reducing the degree of challenge of the task, Sílvia thought of "tricks to avoid slipping up" (S11, lesson preparation) and prepared questions to avoid giving explicit suggestions when supporting students in interpreting the task. Moreover, Sílvia added questions to the task to elicit students' justifications [2e] and, thus, challenge them to contrast their ideas with their colleagues [2f].

Maria anticipated that "the students need to relate the function $k(t)$ [linear] to the other two [quadratic functions]" and considered that "this information may not be easy to interpret" (lesson plan). However, she did not follow the suggestions of the teacher educator and the researcher and kept the anticipation of difficulties she initially did by herself, indicating three types: "lack of practice with the graphing calculator", "calculation errors" and "difficulties in interpreting the questions" (lesson plan). She also did not prepare how she would support them to relate those functions [1c] nor how she would guide them to the lesson goals [2d]. Whereas she prepared two questions to pose at the end of the whole-class discussion, intending to challenge the students to reflect on their work [2f]:

> How can we use the study of a quadratic function in real contexts? What are the most common characteristics of the study of a quadratic function in problems involving the quadratic function? (lesson plan).

For Maria, the purpose of these questions was to challenge the students to reflect on the work carried out on the study of functions, thus challenging them to go extend the work carried out to problem-solving with real contexts.

Lila's lesson plan discussion session (S6) was carried out following Sílvia's and Maria's sessions (S3, S4). In line with the suggestions given, she anticipated the students' difficulties in each question and prepared her interventions to keep the discussion focused on the lesson goals [2d], considering the students' possible difficulties. For example, one of the questions in the first task asked students to find the height of a wall, which corresponded to the value of the quadratic function, $a(t)$, at t=0. Anticipating that the students would have difficulties in interpreting it correctly, Lila decided that she would "explain that we are trying to determine the height of the grasshopper at the first instant, so that the students understand that the value of t has to be replaced with zero" (lesson plan). The teacher educator and the researcher pointed out that, by explicitly suggesting which procedure to use, the students would not relate what was asked in the question to what would need to be done to solve it. Consequently, they would not feel the need to justify their ideas. Therefore, Lila, Sílvia, and Maria rethought the teacher's interventions to keep the discussion focused without reducing the degree of challenge of the task [2d]:

> Maria: … What could we say… If the student doesn't understand?
>
> Lila: 'What is understood by first instant?'… If I ask that, they'll say straight away 'replace with zero because it's the first instant'. (S6, lesson plan discussion)

Leading and reflecting

During the whole-class discussion, Sílvia invited a student to present his incomplete strategy [1a] and guided him to correctly interpret the question [2d]:

> Sílvia: What does 'maximum temperature' mean? And how is the temperature given by?
>
> Student F: By the expression.
>
> Sílvia: Which is what type of function?
>
> Several students: Quadratic.
>
> Sílvia: What concavity does it have?
>
> Several students: Downwards.
>
> Sílvia: And we want to know when the temperature was maximum. So, the maximum temperature corresponds to what?
>
> Several students: To the vertex.
>
> Sílvia: And if [the parabola concavity] was upwards, would the maximum temperature correspond to the vertex?
>
> Student F: No, the vertex would correspond to the minimum temperature.

Sílvia posed the questions that she had prepared to support the students in their difficulties, without reducing the degree of challenge of the task [2d]. She also supported the students to relate the context of the task to the mathematical concepts that they already knew (algebraic and graphic representation of quadratic functions) [1c], seeking to elicit students' reasoning [2e]. She reflected on the lesson concluding that the students "tried to explain their reasoning [and] they were able to clarify [their questions]" (written reflection). In her view, this would not have occurred if "only the tasks had been corrected" (written reflection) and she did not guide the students to keep the discussion focused on the lesson goals [2d]. However, she considered that she posed "more questions to confirm their [students'] learning, focused on a certain aspect, rather than

questions that required a higher level of reasoning" (written reflection). Therefore, to be able to challenge students to contrast their ideas with their colleagues and to reflect on their work [2f], she would need to pose questions to foster students' justifications [2e] in a "higher level of reasoning".

Maria's lesson was based on exploring students' misconceptions. She had not anticipated in detail the students' difficulties, nor prepared how to guide or support them. During the lesson, her interventions were based on explicit suggestions [2d]:

> Maria: What is the $n(t)$ function?
>
> Student U: We have to use this [value]... And the function was y=67.
>
> Maria: So, $n(t)$=67, write it down... 67 is the value of what?
>
> Student H: Of the graph.
>
> Maria: Of the x.

Maria used mainly guiding actions to correct the students' misconceptions and without inviting other students to the discussion to challenge them to contrast their different ideas, nor trying to elicit their reasoning. Maria prepared questions to challenge the students to reflect on their work [2f] but did not have the opportunity to pose them during the lesson. When reflecting, she considered that "the students' involvement... did not correspond to what I expected [and] most of them showed great difficulties in expressing their ideas orally" (written reflection). When the teacher educator and the researcher suggested to think how to improve her future practice, Maria said that she could "try to motivate interactions among the students... [and] pose more open questions during the lesson" (S15, post-lesson discussion).

In Lila's lesson, preparing how to keep the discussion focused on the lesson goals without giving explicit suggestions to the students appears to have helped her to lead the discussion:

> The anticipation enabled me to be better prepared and to support the student more easily (written reflection).

Particularly, in the second task, the prospective teacher posed the questions she had prepared:

> Lila: What do you need to do next?
>
> Student E: If x=1, the distance from P to Q will be 7.
>
> Lila: And what if x is another value?
>
> Student E: It will be x-8. The area of the triangle will be equal to the base multiplied by x-8, divided by 2.
>
> Lila: Where did you get the x?
>
> Student I: As [PQ] and [AB] are parallel... the triangles will be similar.

Lila tried not to reduce the degree of challenge of the task by keeping the discussion focused [2d], thus valuing the anticipation work. When reflecting on what she could improve, she stated: "I could have questioned more [the students'] justifications of their reasoning [2e]" (written reflection) to challenge them to reflect on their work [2f].

DISCUSSION

Preparing

Following the suggestion of the teacher educator and the researcher, the prospective teachers collaboratively adapted the tasks so the students could establish connections between different mathematical

ideas, particularly, relating different representations. In addition, Sílvia rethought how she could create opportunities for the students to extend the work carried out, which is in line with Coenders and Verhoef's (2019) findings. This work on task design provided opportunities for the prospective teachers to develop their knowledge about teaching practice, namely, about how to adapt tasks to meet the lesson goals.

As highlighted in the literature (Fujii, 2018; Meiliasari, 2019), anticipating students' difficulties was a challenge for Sílvia. However, after sharing this challenge in the lesson plan discussion session, Sílvia sought to anticipate as many different solving strategies as possible with Maria and Lila's help, as suggested by Stein et al. (2008), to anticipate the difficulties that might arise, which is related to the development of *knowledge about students'* learning. The prospective teacher tried to organize the lesson based on the students' work and prepared the whole-class discussion mainly through questions. She prepared her interventions to encourage students to establish connections between mathematical ideas without giving explicit suggestions and to justify their ideas, developing her *knowledge about teaching practice*.

As in the study of Meiliasari (2019), Maria's anticipation of students' work was general rather than detailed, despite the suggestions of the teacher educator and the researcher. She intended to explore the students' misconceptions and prepared questions to challenge them to reflect on their work. However, unlike the participants in Meiliasari (2019), Maria did not prepare her interventions to support students to establish connections between different ideas neither to challenge them to extend their work. In contrast with what was mentioned by Chen and Zhang (2019), Maria did not seem to value the anticipation work or a detailed lesson planning.

Lila, in contrast, valued this anticipation work. Following her engagement in the discussion of Sílvia and Maria's lesson plans, she autonomously anticipated the students' strategies and difficulties in each question, preparing explicit suggestions to guide them. Discussing the implications of these suggestions led the prospective teacher to rethink how she could keep the discussion focused on the lesson goals, without reducing the degree of challenge of the task, developing her *knowledge about teaching practice*.

Chen and Zhang (2019) state that when having to plan a lesson, prospective teachers need "more curricular guidance and structured assistance" (p. 550). In this lesson study, the pre-lesson reflection guide appears to have fulfilled this need. The prospective teachers had to select the tasks to be proposed to the students and explain their purposes in light of the lesson goals autonomously. They also anticipated the students' strategies and difficulties in those tasks and prepared their interventions for the whole-class discussion, which is related to *knowledge about students' learning*. The suggestions of the teacher educator and the researcher led the prospective teachers to share their ideas and rethink their lesson plans, which contributed to the development of prospective teachers' *knowledge about teaching practice*. They collaboratively reviewed the tasks and discussed how to support students to establish connections and how to challenge them to extend their work.

Leading and Reflecting

The prospective teachers began the whole-class discussion by inviting the students to present their strategies. Sílvia posed the questions she prepared to guide the students to interpret the task correctly, by helping them to relate the different representations they used. Moreover, she sought to keep the discussion focused, even in the face of students' questions, encouraging them to justify their ideas, which is in line with the practices suggested by Stein et al. (2008).

Reflecting on the lesson, Sílvia reconsidered the short time she allowed the students to answer, concluding that this is a factor she needs to consider when leading discussions. Additionally, she reflected on the kind of questions she should pose to encourage the students to justify their reasoning and to challenge them to contrast their ideas with their colleagues. Conceição et al. (2019) pointed out that "there is a need to use initial teacher education processes that allow future science teachers … [to understand] how to communicate in the classroom" (p. 80). These reflections from Sílvia show that, by getting involved in a lesson study process, she developed her knowledge about classroom communication, which is related to the *knowledge about teaching practice*.

Maria began the whole-class discussion with students' misconceptions, as the teacher's case in Ponte et al. (2017). However, unlike this teacher, she took control of explanation of the students' strategies. She did

not take advantage of students' answers, with or without errors, to support them in relating the representations they used, and mainly used guiding actions to correct the students' misconceptions. When reflecting on the lesson, she considered that she was not able to involve the students as she intended, associating this with the students' characteristics rather than with her leading practices.

This prospective teacher appears not to value a detailed lesson preparation, which may be related to her view of the student's role (solving tasks) and the teacher's role (correcting students' misconceptions) in the mathematics classroom. Fujii (2018) suggests that a detailed anticipation of students' work enables the teacher to think of ways to guide them in their learning, which is directly related to an improvement in teaching practice. For this prospective teacher, a lack of detailed anticipation influenced her practice, particularly as she was not able to involve the students as she intended.

In Lila's case, she initially prepared explicit suggestions to keep the discussion focused, actions that were discussed and changed during the lesson plan discussion. During the whole-class discussion, she tried not to reduce the degree of challenge of the task but, sometimes, she guided the students with explicit suggestions. She also posed the questions she prepared, which is related to the development of her *knowledge about teaching practice*. Anticipating students' solving strategies and difficulties, as well discussing how to keep the discussion focused without reducing the degree of challenge of the task, seems to have helped the prospective teacher to lead the whole-class discussion.

During reflection, Lila recognized she was not able to elicit the students' reasoning nor challenge them to reflect on the work carried out. However, unlike the Irish group of student teachers in Ni Shuilleabhain and Bjuland's (2019) study, she did not consider how she could improve her practice to foster students' justifications and challenge them to extend their work.

During the research lessons, the prospective teachers had the opportunity to put into practice the work done in the lesson study sessions, namely what they had prepared for the whole-class discussion. Sílvia sought to establish connections between different ideas through the relations between different representations, fostering the students' justifications to keep the discussion focused on the learning goals. Moreover, when reflecting on her practice, Sílvia identified how it could be improved, particularly in relation to her questioning. Knowing when and how to clarify mathematical ideas and challenge the students appears to be Sílvia's concern (*knowledge about teaching practice*).

In the case of Maria and Lila, their tendency to guide the students with explicit suggestions and the way by which they led the whole-class discussions may be related to their own classroom experiences and their views on whole-class discussions. Changing practices appears to be more challenging for prospective teachers with some teaching experience than for those with no experience, which seems to have a strong influence on the development of their knowledge.

CONCLUSION

Leading whole-class discussions is a challenge for prospective teachers. Therefore, initial teacher education courses must promote the development of their knowledge on how to prepare and lead these moments during a lesson.

In this research, lesson study provided opportunities for prospective teachers to develop their knowledge about whole-class discussions. Each prospective teacher autonomously prepared a first version of the lesson plan, based on a reflection guide. Then, each sustained their choices, and the colleagues shared their ideas and suggested improvements. In particular, they collaboratively adapted the tasks that each had previously selected, detailed their anticipation work, and prepared how to organize the whole-class discussion to support students to establish connections and to extend their work. Thus, creating moments in which prospective teachers can discuss the lesson plan to foster a meaningful learning seems to be crucial for the development of their knowledge.

Being involved preparing each other's lessons, through lesson study micro-cycles, also seems to be useful for the development of the prospective teachers' knowledge. In particular, Lila improved her anticipation work before sharing her first version of the lesson plan, which was a result of engaging in Sílvia and Maria's lesson

plan discussions. In addition, Sílvia and Maria played a critical role in questioning and thinking with Lila about how she could keep the discussion focused without reducing the degree of challenge of the task.

Beyond all, putting into practice what they had planned was fundamental for the prospective teachers' development process. They were able to experience what they prepared, linking theory and practice, and leading to reflect on their own practice. In this lesson study, Sílvia reflected on what she should improve and how to do it, as did Lila who identified what to improve. However, despite the teacher educator and the researcher's efforts, Maria did not seem to embrace the challenge of reflecting on her practice and did not seem to understand the influence of her actions on students' learning. Yet, lesson study created several opportunities for prospective teachers to reflect on their own classroom experiences, which is a privileged way to develop their knowledge (Bozkurt & Yetkin-Ozdemir, 2018; Ustuk & Comoglu, 2021).

It is important to note that, sometimes, lesson study cannot achieve all the aims for the prospective teachers' development process (Ponte, 2017). Nonetheless, this research contributes to a better understanding of how to structure a lesson study in initial teacher education. Using reflection guides and promoting moments of sharing and discussion between prospective teachers, as well as creating opportunities for them to put into practice what they prepared to reflect on it, are fundamental dynamics to promote the development of their knowledge.

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