Leading whole-class discussions: from participating in a lesson study to teaching practice

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Abstract

Purpose – This article aims to analyse how a teacher leads whole-class discussions during and after participating in lesson studies and to what extent that participation influences her teaching practice.

Design/methodology/approach – This is a qualitative/interpretative research with a case study design, carried out with a secondary school mathematics teacher who participated in two lesson studies. Data were collected from participant observation, audio recording of lesson study (LS) sessions and discussions with the teacher, video recording of lessons and semi-structured interviews. Frameworks regarding the teachers' actions are used in the analysis.

Findings – The results suggest that in her teaching practice, the teacher led students to explain their strategies with *supporting/guiding* actions, but she also *challenged* the students to justify their productions, ensuring that the students' ideas were clear. Additionally, the teacher *explored incorrect strategies* and *disagreements, inviting* and *challenging* other students to intervene or react and involved students in *drawing connections,* as discussed in the LS. Therefore, the teacher put into practice several actions teachers can do in leading whole-class discussions to promote students' learning. Participating in LS was an opportunity to rethink her teaching practice, as the teacher pointed out, bringing her a new perspective on leading discussions in which students play an active role in learning mathematics, creating opportunities for the students to explain and react to their colleagues' ideas.

Originality/value – This article examines an under-researched issue: the influence of LS on the way a teacher leads whole-class discussions, during and after participating in lesson studies.

Keywords Whole-class discussions, Lesson study, Teaching practice, Teachers' actions, Mathematics, Secondary education

Paper type Research paper

1. Introduction

Whole-class discussions are a particular form of mathematical communication, where students have the opportunity to present and explain their strategies, building on the work they have done, listen to their peers' explanations and reflect on them. The moments of whole-class discussions are especially fruitful for students' learning when they work on tasks where they can follow different strategies (Ponte *et al.*, 2013; Takahashi, 2008). However, leading such discussions poses several challenges to teachers, such as deciding which strategies to share or how to manage the discussion to promote the mathematical learning of the class, and requires teachers to have knowledge of mathematics, students, and how to teach mathematics (Stein *et al.*, 2008; Takahashi, 2008). To reduce this challenge, it is important that teachers prepare whole-class discussions by anticipating solving strategies and considering which are the most important ones to discuss with the students, how they can be sequenced and what connections they can help the students make (Stein *et al.*, 2008; Takahashi, 2008). This can be done when teachers participate in a lesson study (LS), during

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the planning of the research lesson. The observation of that lesson, conducted by one of the teachers, is an essential part of LS, allowing teachers to reflect on students' learning.

Participating in a LS, as a professional development process centred on teaching practice, is an opportunity for teachers to develop their knowledge, namely in leading whole-class discussions (Gomes *et al.*, 2021; Inoue, 2011; Quaresma and Ponte, 2016). However, an understudied issue is the contribution of this participation to teachers' subsequent practice (Richit *et al.*, 2021). Thus, in this article, the following research question is addressed: What is the contribution of participating in lesson studies to the development of the teacher's practice on leading whole-class discussions?

2. Theoretical framework

2.1 Planning and leading whole-class discussion

In the mathematics classroom, the students' work and learning depend on the role carried out by the teacher and the pupils themselves, the tasks set and the communication established (Ponte and Quaresma, 2016), and this is an essential part of mathematics learning. The whole-class discussion is a particular aspect of mathematical communication and may occur at various points in the lesson and for different purposes. In a lesson in which the teacher presents a task and then gives students time to solve it, usually in pairs or small groups, the whole-class discussion is the phase of the lesson in which students present and explain their strategies, listen to their peers' explanations, and reflect on them (Ponte *et al.*, 2017; Stein *et al.*, 2008).This discussion is an important moment for the negotiation of meanings and the construction of new knowledge that follows on from the students' autonomous work, involving tasks for which they did not initially find a solution and for which different strategies or representations may be used.

Clivaz and Miyakawa (2020) state that Swiss teachers consider the students' autonomous work the most important part of the lesson and the *mise en commun* serves to conclude this work. However, Japanese teachers consider the autonomous work phase as preparation for the whole-class discussion, called *neriage,* which they consider the most important part of a problem-solving lesson (Clivaz and Miyakawa, 2020; Fujii, 2018; Inoue, 2011; Takahashi, 2008).

During whole-class discussions, the teachers orchestrate students' ideas, but they also polish their solutions, helping them to clarify their ideas, extend their mathematical content and achieve the goals of the lesson (Inoue, 2011; Takahashi, 2008). For this to be productive, the teacher should encourage the students to present and justify their solving strategies or to formulate alternative justifications, and listen to and analyse other students' strategies. It is also important for the teacher to accept and value incorrect or incomplete contributions and explore disagreements among students, challenging them to present arguments or formulate new questions (Ponte *et al.*, 2017). Thus, the teacher needs to give students a voice to present and justify their reasoning, leading to relevant learning (Stein *et al.*, 2008; Inoue, 2011; Takahashi, 2008).

However, leading whole-class discussions is challenging for teachers (Stein *et al.*, 2008; Takahashi, 2008). Why? As a whole-class discussion is more than "show and tell", in the busy pace of classroom work, teachers have to monitor the students' work to understand the strategies they are following and how they think, and carefully select and order the students' strategies (Fujii, 2018; Stein *et al.*, 2008; Takahashi, 2008). This is even more challenging in secondary school when students work on tasks whose solution involves a greater number of steps than in lower secondary school (Kooloos *et al.*, 2020). Also, to manage the discussion, teachers need to combine their knowledge of mathematics, knowledge about teaching mathematics, knowledge of the students, and skills to lead the discussion (Takahashi, 2008).

Fujii (2018) and Takahashi (2008) argue that the quality of the discussion depends on the lesson planning and, like Stein *et al.* (2008), highlight the need for teachers to anticipate students' possible strategies and difficulties during planning. Stein *et al.* (2008) also suggest that teachers monitor students' work, select and order the strategies they consider important to discuss with the class, and establish connections between them. However, they do not propose possible actions to explore incorrect answers or disagreements among students or how to deal with unforeseen situations (Ponte *et al.*, 2017).

In LS, teachers collaboratively plan a research lesson and spend a significant part of that planning anticipating students' strategies, correct and incorrect, and difficulties, thinking of ways to help them overcome them. Based on this work, they prepare the whole-class discussion, namely how to begin, which strategies to include and how to compare them to achieve the lesson goals (Fujii, 2018).

LS is a professional development process in which teachers can develop their knowledge and beliefs, professional routines and norms, and instructional materials (Lewis *et al.*, 2019). Particularly, they can develop knowledge on leading whole-class discussions (e.g. Gomes *et al.*, 2021; Inoue, 2011; Quaresma and Ponte, 2016). Inoue (2011) reports that during the LS, the teachers moved into the role of facilitators of the discussion, rather than explaining the strategy presented, as they did at first. In the study by Quaresma and Ponte (2016), after the LS, the teacher began to allow more space for students' participation, namely in the whole-class discussions, and was surprised by the quality of their participation. This element of surprise was also highlighted by Gomes *et al.* (2021), stating that the participating teachers valued their students' detailed explanations during the discussion. In their whole-class discussion, they considered incomplete answers and encouraged the comparison of different strategies instead of all the students presenting their answers, as they planned.

2.2 Teachers' actions when leading whole-class discussion

During the whole-class discussion, the teacher takes different actions for different purposes, such as encouraging the students to participate and to explain their thinking processes or highlighting important information. The efficacy of the students' work and learning depends on how the teacher manages the whole-class discussion, whether by giving them more opportunities to explain their strategies or by taking control of the discussion (Kooloos *et al.*, 2020; Ponte and Quaresma, 2016). So, one of the challenges for teachers when leading whole-class discussions is understanding what actions they can take to make it productive, as it is impossible to use all the students' contributions or ask them to explain everything in detail. Moreover, although *challenging* (Ponte and Quaresma, 2016) or *requesting explanation* (Kooloos *et al.*, 2020) allows the students to take a central role in the discussion, to make it productive, teachers may also need to introduce information that they consider important or encourage students to continue to participate.

Focusing on the teacher's actions in leading whole-class discussions, Ponte *et al.* (2013) identify four types of actions related to mathematical topics and processes: *inviting*, leading students to present their solving strategies; *supporting/guiding*, seeking to guide the students to solve the task; *informing/suggesting*, introducing ideas into the discussion or validating students' answers; and *challenging*, encouraging the students to introduce representations, interpret and establish connections, argue and evaluate their peers' arguments. The authors found sequences of actions that were often repeated: supporting/suggesting and challenging; supporting and suggesting; and inviting and suggesting/challenging. Using this framework, Ponte and Quaresma (2016) state that *challenging* actions occur mainly when seeking to establish connections, making generalisations and presenting justifications. *Informing/suggesting* and *guiding* actions mainly occur when concepts and procedures are introduced.

Drageset (2015) also proposes an analytical framework for teachers' actions, focussing on the effect of these actions: *redirecting*, to change the strategies students are following; *progressing*, to help them progress in solving the task; and *focussing* "used by teachers to stop progress, in order to look into details or reasons behind" (p. 260). Stating that the teacher's interventions are related to those of the students, the author proposes five categories for the students' actions: *explanations, initiatives, partial answers, teacher-led responses*, and *unexplained answers*. Drageset (2015) reports that *teacher-led responses* are the most frequent interventions and identifies two repeating patterns, one between *teacher-led responses* and *progressing* and another between *explanation* and *focussing actions*.

Based on Drageset's (2015) analytical framework, Kooloos *et al.* (2020) analyse a secondary school teacher's actions in leading whole-class discussions, distinguishing *convergent* from *divergent actions*. In the former, the teacher takes control of the discussion, discarding (*set aside*) or *reformulating* (*reformulate*) the students' ideas, explaining (*demonstration*), or asking for *closed progress details*. In the latter, the teacher leads the discussion based on students' shared ideas, *inviting* one (*external directed*) or *several* students (*external general*) to participate or to react to a peer's explanation, asking them to explain (*request explanation*) or clarify (*request clarification*) their ideas or through *open progress initiatives*. The authors consider two other categories: *encouraging actions*, to encourage students to continue their explanation (*confirmation* or *encouragement*); and *regulating actions*, related to the management of classroom communication. They also propose eight student actions: (*steps of*) *solution method, explanation, question, external, partial answer, teacher-led answer, evade answer* and *remark about solution method*. Kooloos collaborated with a secondary school teacher in planning four lessons, focussing on leading whole-class discussions. Comparing how she managed the discussion in these lessons, the authors report that over time the teacher gradually included incomplete or inaccurate answers in the discussion, using fewer *convergent actions* and giving students more opportunities to explain their answers.

3. Research methodology

3.1 Participants

In this article, we present the case of Sofia, a teacher who participated in two LS, teaching and observing research lessons. In a first LS (LS1), two more teachers were involved who, like Sofia (all names used in this article are pseudonyms), taught mathematics to secondary school students (grades 10–12) and had more than 25 years of experience. In the following academic year, Sofia and one of the above-mentioned teachers participated in a second LS (LS2). Sofia was chosen for this study as she was the single teacher from whom it was possible to observe lessons after the LS. Both LS were facilitated by the first author of this article, who had already worked with the teachers.

3.2 The lesson studies

Mathematical communication is a crosscutting dimension in the Portuguese curriculum, and the development of students' mathematical communication was an important topic of discussion in the teachers' curriculum group. So, in both LS, the teachers decided to focus on managing classroom communication and encouraging the students to communicate their ideas.

The LS were organised into five steps (Fujii, 2018): goal setting, planning the research lesson, research lesson, post-lesson discussion and reflection. In this last stage, the teachers shared their work with other teachers from their school, in LS1, and from other schools, in LS2.

In the first sessions of each LS, the teachers received articles about communication and teachers' practices in lessons that value whole-class discussions, based on students' work. The articles were analysed in the LS sessions, paying particular attention to the role of the teacher and possible actions at different moments of the discussion. When analysing Stein *et al.* (2008), we paid particular attention to the organisation of the lesson presented by the authors and the practices they suggested to prepare the whole-class discussion.

Considering the goal of enhancing students' mathematical communication, the teachers were challenged to plan lessons organised in three phases: setting the task, students' autonomous work and whole-class discussion and synthesis. Sofia was familiar with this kind of lessons and had already taught them. However, her interventions and reflections suggest that she frequently took the role of explaining the strategies presented by the students, asking them if they had any questions.

Part of the planning sessions was spent on designing tasks in which students could follow different strategies, which they could then present and explain to their classmates during the discussion. To prepare for this, the five practices suggested by Stein *et al.* (2008) were considered, namely anticipating students' strategies, selecting and sequencing them, and drawing connections between the students' answers. The teachers also thought about ways to support students in the difficulties that might emerge.

LS1 was held between November 2019 and February 2020. The teachers planned a research lesson on Functions (RL1) to grade 11 students. After conducting the lesson and reflecting on the students' work, they planned another lesson (RL2) on the same topic. As the schools were closed in March 2020 due to the pandemic, lessons were conducted remotely (online) until the end of the academic year, and the teachers were no longer available to continue working on the LS and it was not possible to observe their practices.

LS2 was held the following school year, between September and November 2020. The teachers decided to plan a lesson (RL3) on Probability to grade 12 students. Considering the classroom circulation restrictions, the teachers anticipated increased challenges in monitoring, selecting and sequencing answers for whole-class discussions. As teachers were unable to monitor the students' work, they decided that after working on a task, the students should send them a photograph of their answers. This lesson was followed by two LS sessions to plan the whole-class discussion: in the first, the teachers analysed the students' answers; in the second, they discussed the representations and strategies they deemed important to further explore with the students and how they could be ordered. In the next lesson, they led the discussion as planned. With yet another school closure in January 2021, it was impossible to continue this work.

3.3 Teaching practice

In April 2021, face-to-face lessons were resumed and the school year ended in mid-June. Therefore, it was only possible to observe the lessons conducted by Sofia after LS2. This observation occurred on dates previously arranged, between mid-April and early May, as the students would have to sit national final exams at the end of the school year. Before and after each lesson, the researcher (first author) met the teacher to discuss her planning or to reflect on the students' work.

In these lessons, Functions were being worked on and the teacher set tasks where students could use various strategies. In the first lesson (A1), the teacher assigned the students to groups and monitored their work, and in the second half of the lesson, she asked them to present and explain their answers. In the other lessons (A2 and A3), the students were seated at individual tables and although the teacher circulated around the room to monitor the students' work, circulation was still limited. Therefore, whenever a student explained his/her answer, the teacher recorded it on the board, checking that what was written was in line with the explanation.

3.4 Data collection and analysis

This research follows a qualitative and interpretative approach (Bogdan and Biklen, 1994), with a case study design, drawing on the work of teachers in lesson studies held in their school.

The data collection included document collection (tasks and lesson plans), audio recording of sessions, conversations before and after the lessons we observed after the LS, semi-structured interviews at the end of each LS, and video recording of lessons during and after the LS. We requested permission for data collection.

When analysing the LS discussions, the conversations before and after observing the lessons after LS, and the interviews, we focussed on the teacher's practice in leading whole-class discussions, considering that as "putting into practice" the knowledge she might have developed in LS. This analysis was informed by the theoretical framework (Fujii, 2018; Inoue, 2011; Ponte *et al.*, 2017; Stein *et al.*, 2008; Takahashi, 2008), which gives rise to a set of practices that teachers can use to lead discussions: (1) *eliciting student thinking*, encouraging students to share several strategies or representations, even if these are not the most effective; (2) *probing student thinking*, asking for explanations and justifications; (3) *exploring incorrect strategies*, valuing the presentation of incorrect/incomplete answers as a way to engage all students in the discussion and help them overcome difficulties they may have; (4) *exploring disagreements*, asking students to justify their positions and encouraging others to get involved; and (5) *drawing connections*, asking/supporting students to compare solving strategies and discuss their similarities and differences, co-determining which might be most effective. Elements of these practices also emerged in the articles discussed in the LS sessions.

Considering the importance of the teacher's actions in supporting students' engagement and learning through whole-class discussion, we analysed Sofia's actions in leading whole-class discussions in the research lessons and after the LS, using the framework proposed by Ponte *et al.* (2013). Aimed at understanding whether, as the teachers intended, they gave students a voice to explain their thinking and build the discussion on their ideas (*divergent actions*), we analysed the data using the framework proposed by Kooloos *et al.* (2020). This analysis allowed us to identify a relationship between the actions proposed in both frameworks, resulting in Figure 1. We observed that with *inviting* or *challenging* actions, the teacher encourages the students to explain their strategies and leads the discussion from their ideas (*divergent actions*). So, starting from Ponte *et al.*'s (2013) framework, we observed that: (1) similar to *inviting* actions, *external general*, *external directed* and *open progress initiatives*

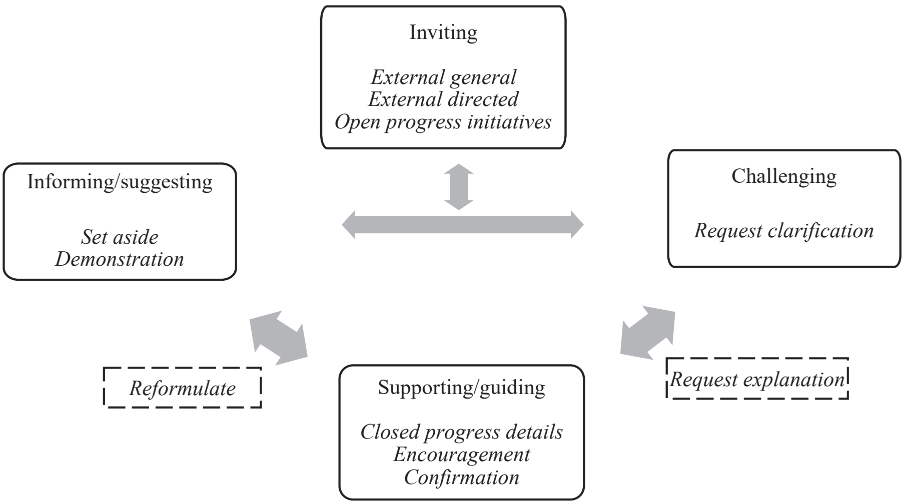

Figure 1.
Teacher actions in mathematical discussions

Source(s): Adapted from Ponte *et al*. (2013) and Kooloos *et al*. (2020)

are teachers' actions to get students to share their strategy or to react to a peer's explanation; (2) when the teacher *requests explanations* or *clarifications*, he/she *challenges* students; (3) the teacher may *support/guide* students by asking for *closed progress details*, or by *encouraging* them to continue their explanation; and (4) with *informing/suggesting* actions, the teacher introduces ideas into the discussion, as with *set aside* and *demonstration*.

Additionally, a connection emerged between the *supporting/guiding* and *informing* actions, given that when the teacher reformulated students' statements she did so to encourage them to continue to participate or to draw attention to aspects that they should understand and use in other situations. In addition, *request explanation* can be used to *challenge* the students to go beyond what they have written or thought, or to *guide* them by helping them explain their answer.

Seeking to understand the purpose of teacher's actions in leading whole-class discussions and connections between different actions, we also analysed students' actions using the framework proposed by Kooloos *et al.* (2020). The *remark about solution method* was omitted as they were not found in our data. *Change answer* was introduced as inviting students to participate or challenging them to clarify their answers led them to correct or reformulate their ideas, without the teacher telling them they made a mistake.

4. Results

4.1 Teacher's practice in lesson study

Planning the lesson. In LS1, the teachers discussed the importance of *eliciting* and *probing student thinking*. When discussing how to select and sequence students' strategies, Sofia asked: "are we also going to present the wrong solutions? [To] draw attention to errors" (Session 5, LS1). After some discussion, the teachers decided to *explore incorrect strategies* as well, and ask the students for explanations, as this might help their peers.

In LS2, they also decided to explore incorrect/incomplete answers. After deciding to start it by an answer with both natural and algebraic language, Sofia asked: "The student will present, and explain the thinking process. OK . . . what other questions can I ask?" (Session 8, LS2). This was another opportunity for discussing possible teacher actions in whole-class discussions to promote students' learning, such as exploring disagreements or asking other students if they would like to clarify or add anything.

For RL3, the teachers selected a task on conditional probability. The task mentioned booking accommodation on four online platforms and the students had the following information: part of the accommodation was booked on platform A (1/8); Maria rated the reservations on that platform Very Good in half of the cases; and for reservations not booked on platform A, Maria rated the accommodation Very Good in only one out of seven cases. The students were asked to answer the question "We randomly choose an accommodation that Maria gave a Very Good rating to. Determine the probability of this accommodation being booked using platform A". The teachers planned to start the discussion with an unanticipated answer, from Gabriel, which they considered "more intuitive", and end it with another answer using a tree diagram.

Leading the whole-class discussion. In the research lessons conducted by Sofia, she started the whole-class discussions with *inviting* actions, especially through *open progress initiatives*. In the discussions we analysed, these actions prompted the students to share and explain their strategies, as in the following segment:

> Gabriel: The reservations on platform A correspond to 12.5% . . . Then, they say that $\frac{1}{2}$ of 12.5%, that is, 6.25% are the Very Good accommodation . . . (*explanation*)
>
> Teacher: That's because they tell us that it is in half of the cases, is not it? (*reformulate/supporting*) You did $\frac{1}{2}$ which corresponds to half, yes . . . (*confirmation/supporting*)

Gabriel: If 12.5% are the reservations on platform A, 87.5% are reservations on the remaining platforms ... $\frac{1}{7}$ of these reservations are very good accommodation. $\frac{1}{7}$ of 87.5% is 12.5% ... So, 18.75% corresponds to the total number of very good accommodation. We know that 6.25% are very good on platform A, so I divided that by the total value of the very good accommodation facilities. (*explanation*)

Teacher: Exactly .... We have the total of those that are Very Good on platform *A* and the total of being very good and so, we just need to calculate the ratio ... The universe becomes the set of the very good facilities ... Well done. (*reformulate/informing*) (video recording RL3)

During the student's explanation, Sofia clarified what "$\frac{1}{2}$ of 12.5%" represents and encouraged him to continue explaining, ending with an *informing* action, drawing attention to aspects she considered important.

After *inviting* other students and they *explained* their strategies without her intervention, Sofia felt the need to *support* a student who used a tree diagram and asked her to explain aspects that might be unclear to her peers:

Teacher: We know then that if the probability of A is $\frac{1}{8}$... (*request explanation/guiding*)

Alissa: The probability of $\overline{A}$ is $\frac{7}{8}$. We also know the probability of B if A is $\frac{1}{2}$, and then the probability of $\overline{B}$ if A is $\frac{1}{2}$ and B if $\overline{A}$ is $\frac{1}{7}$, and then $\overline{B}$ if $\overline{A}$ is $\frac{6}{7}$. (*explanation*)

Teacher: What was the probability requested? (*request clarification*)

Alissa: The probability of the accommodation being evaluated as Very Good and being on platform A. *(steps of) solution method*

Teacher: Is it "and"? (*request clarification/challenging*)

Alissa: It's not "and". If the accommodation is very good, see if the reservation was made on platform A. (*change answer*)

Teacher: We want the probability of ... ? (*closed progress details/guiding*)

Alissa: *A* if *B*. (*teacher-led response*)

Teacher: The probability of *A* if *B*. To do this ... (*confirmation/supporting*) (video recording RL3)

Sofia led Alissa to correct her mistake by asking her a question ("Is it 'and'?"). In RL3, when the teacher *challenged* the students, they mainly responded with *explanations*. However, students' explanations, like Gabriel's, resulted mostly from *open progress initiatives* (7 out of 19 *explanations*) or *confirmation/demonstration* (6 out of 19 *explanations*). In this lesson, as in this segment, half of the 16 *closed progress details/teacher-led response* sequences were followed by *confirmation* or *reformulation* actions, encouraging the student to continue explaining.

Therefore, as discussed in the LS sessions, Sofia *probed students' thinking*, *requesting explanation* and *clarification*, and ended the whole-class discussion by *drawing connections*, as is RL1, discussing with the students the strategies that might be most effective: "There are tasks where it's easier to use a tree diagram. There are others where it's easier to use a table. There are others where it's easier to just use properties. If you know the various strategies, if you can't do it one way, you can do it the other" (video recording RL3).

In the post-lesson reflection, Sofia acknowledged the potential of whole-class discussion to enrich all the students' range of strategies and to enhance their learning, stating that "The [students] with difficulties saw the others' explanation ... It's interesting that they understand ... That they can take several paths, and for some situations there are easier paths than others, and they have to decide the best for each situation" (post-lesson reflection).

4.2 Teacher's practice after the lesson study

Planning the lesson. For the lessons we observed, Sofia selected tasks that students could solve using different strategies and tried to anticipate them. In one of these (A1), a function f was given, defined by $f(x) = x - ln(2x+1)$, and students had to study the function for monotonicity, extrema, and asymptotes to its graph. Before A1, Sofia proposed this task in another class and one of the students followed a strategy she did not anticipate, leading her to change the lesson plan: "They surprised me because they did not go for any of the solutions I had thought of. I added them [to the lesson plan] because I observed that they could go in that direction" (audio recording, preparation of A1). Considering the students' previous knowledge and her work with them, Sofia also mentioned valuing using a table to study the monotonicity of the function, considering it the most effective strategy and the one that is most valued in the national exam.

Leading the whole-class discussion. In lesson A1, Sofia started the discussion *inviting* the students through *open progress initiatives*, like in the research lessons:

> Teacher: Can you explain how did you thought?
>
> Alissa: We calculated the derivative, from the derivative we calculated the zeros, we made the table, and we reached the conclusion that it is strictly increasing in $\frac{-1}{2}$ to $\frac{1}{2}$ and strictly decreasing in $\frac{1}{2}$ to $+\infty$ and . . . (*explanation*)
>
> Teacher: How did you reach that conclusion here of the [signs] + and –? How did you analyse this? (*request explanation/challenging*)
>
> Alissa: Like this: $2x$ is a positive number . . . it's a straight line . . . how can I explain? (*partial answer*)
>
> Teacher: With a positive slope, is that what you mean? (*reformulate/supporting/guiding*)
>
> Alissa: Yes, exactly. With a positive slope. So, it had to be positive before the zero and negative after the zero. (*partial answer*)
>
> Teacher: So, but not with a positive slope. (s*et aside*) I suggest you try to simplify this fraction $[1 - \frac{2}{2x+1}]$ . . . or reduce it to the same denominator. (*closed progress details/guiding*) (video recording A1)

Sofia *explored an incorrect strategy* and sought to support and guide the student after challenging her. As Alissa did not correct her answer, Sofia invited other students to contribute, seeking to promote the emergence of disagreements:

> Teacher: Would anyone from the other groups like to comment? Did you all do it in the same way? (*external general/inviting*)
>
> Flora: No, teacher. (*external*)
>
> Teacher: Tell me, Flora. (*open progress initiatives/inviting*)
>
> Flora: We also considered the denominator sign, but I realise now that it must always be positive. (*change answer*) In the table, we have the $2x - 1$, the $2x + 1$, then we did the operation with the signs and only then did we try to understand whether the function was increasing or decreasing. (*explanation*)
>
> Teacher: OK. (*confirmation*) And what did you find? (*closed progress details/supporting/guiding*)
>
> Flora: We found that f is strictly increasing in $\frac{1}{2}$ to $+\infty$ and strictly decreasing in $-\frac{1}{2}$ to $\frac{1}{2}$. (*(steps of) solution method*) (video recording A1)

After *inviting* and *supporting* Flora to share her answer, and without validating it, Sofia *invited* Alissa to react. Since she was still unable to answer, Sofia *invited* students from Alissa's group to react, *challenging* them to connect their answers to those of their peers:

Teacher: Do you agree, Alissa? (*external directed/inviting*) [silence]

Alissa: I do not know, teacher. (*partial answer*)

Teacher: Do the members of Alissa's group agree with Flora? (*external directed/inviting*) Because Flora said exactly the opposite to what you did. (*challenging*)

Eva: Yes, teacher, we made a mistake. I had actually simplified as you said to do earlier, $\frac{2x-1}{2x+1}$, and . . . And ended up with the opposite of that. But then we changed it because we thought we made a mistake. (*explanation*) (video recording A1)

In the end, Sofia resumed Alissa's explanation, informing students that they can use the graphical representation of the polynomial functions in the numerator and denominator of the fraction to solve tasks like this. In the observed lessons, the students' *explanations* resulted mostly from *challenging* actions (7 of 16 *explanations*), as in Eva's case, but also through *open progress initiatives* or *encouraging* actions (9 of 16 *explanations*).

So in leading the whole-class discussion, in addition to *eliciting students' thinking*, *inviting* them to share their strategies, Sofia *probed students' thinking*, with *supporting* and *challenging* actions, and *explored an incorrect answer* (from Alissa), like in the research lessons. When Alissa explained her answer, Sofia *invited* other students to react (*external general*), seeking to *explore disagreements*, as discussed in the LS sessions.

In the interview, Sofia acknowledged the importance of *exploring incorrect strategies*, as discussed in the LS, stating that if students: "understood how they can overcome that difficulty, they can explain to their peers how they did it . . .". She added that the way she leads the discussion is influenced by her knowledge of the students, adjusting her actions to those who are speaking: "'Am I on the right path?' If you say 'OK', 'So, what now?' [The student] feels more confident to continue . . ." (interview). She, therefore, acknowledged the importance of the teacher's actions to foster student learning. In fact, in the lessons we observed, the teacher supported students who seemed less confident or were shyer, helping them to engage in the discussion.

We also saw Sofia creating several opportunities to give students the floor, *probing their thinking*, as she mentioned when reflecting on the influence that participating in the LS may have had on her practice:

> I have this concern of giving the student a voice . . . it is a concern that I felt more keenly after the discussions we had . . . We are aware that it is important that the student explains what he/she is thinking . . . I find myself, many times, thinking "let me ask them to explain. Let them explain it to each other instead of me" . . . I'm trying to get them to explain, because I know that by explaining, knowledge is more solid . . . did the work we did [in LS] have an impact? I think so. Because it makes this concern about having students explain what they did more visible, for a deeper learning. (interview)

Thus, reflecting on her practice, Sofia mentioned that after participating in LS, she is more concerned about giving students a voice, acknowledging the importance of students explaining their ideas to foster their learning. With an example, Sofia illustrated how she has been *eliciting* and *probing student thinking*:

> I'm always asking them "how did you think this through?" . . . "$\binom{5}{2}$ combinations of 5, 2 to 2" . . . They say and I repeat or correct them "How did you think to get to this?" and the student explains "So, we had to choose two of the 3 rows . . ." I am constantly trying to get them to explain what they are doing. (interview)

The work carried out during LS helped Sofia value and develop new practices in which students had a more active role during the whole-class discussions.

5. Discussion

In the research lessons, Sofia *elicited* and *probed students' thinking* with *inviting*, *challenging* and also supporting/guiding actions, leading students to explain (*explanation*) their answers. She ended the whole-class discussion in RL1 and RL3 *inviting* and *challenging* students to *draw connections* between the strategies presented, *reformulating* their interventions, when appropriate, thus broadening the range of strategies students could follow.

In A1, the teacher *elicited students' thinking* through *open progress initiatives*, as in RL3, and involved other students in the discussion. As discussed in the LS sessions, Sofia decided to *explore an incorrect strategy* by inviting a student and challenging her to explain her answer. She invited other students to engage in the discussion, seeking to promote the emergence of *disagreements*. This led another student to explain her answer, supported by the teacher. Her actions are in line with Ponte *et al.* (2017), who highlight the importance of exploring disagreements among students when leading whole-class discussions, an aspect discussed in the LS planning sessions.

Therefore, during and after the LS, Sofia not only *elicited students' ideas*, mainly through *open progress initiatives*, but she also *requested explanations*, focussing on the solving process and not just on the students' answers, as discussed in the LS sessions. However, in contrast to Kooloos *et al.* (2020), she used more *convergent* than *divergent* actions, except for A2. *Closed progress details* were the most frequent, followed mainly by *teacher-led response*. However, she often interrupted these sequences, supporting students to continue their explanation with *reformulate*, as in Drageset (2015), or *confirmation*. By reformulating a student's statement, she highlighted important aspects of the solution to the whole-class (*informing*), as in RL3, or helped the student continue to participate (*supporting*), as in A1. *Encouraging* actions helped students explain their ideas, especially those who were less confident, as we observed in Sofia's lessons and as she mentioned in the interview, stating that these actions were intentional and influenced by her knowledge about the students. In her practice, she also asked students to explain or clarify their ideas (*request clarification/explanation*), as they had discussed in the LS2 sessions, based on a question she posed. These actions allowed her to *probe students' thinking*, even though some just responded with partial answers.

Another question from Sofia, in LS1, prompted discussions about exploring incorrect answers and disagreements. In the discussions she led, Sofia invited students to present and explain *incorrect answers*, as in A1. She discarded a student's answer, but instead of correcting it, she sought to *explore disagreements* among students by inviting other students to engage in discussion with *external general/directed*, and *request explanation*. To end the discussions, Sofia mainly used *demonstration* or *reformulate* actions to highlight what the students should have learnt (as in Ponte *et al.* (2013)). However, she also *invited* and *challenged* the students to identify similarities and differences between the different strategies, and discussed if there were other more effective strategies, as in RL3. Although *drawing connections* is not as visible in the episodes we present from A1, the students compared their strategies, for example when a student mentioned she did not "do it the same way".

In the research lessons and in her subsequent practice, Sofia, therefore, gave students the floor to explain and justify their strategies, in a different way than the *mise en commun* in Clivaz and Miyakawa (2020).

This analysis shows the importance of teachers making different actions to lead whole-class discussions, adjusting them to students' interventions. While *informing/suggesting* and *supporting/guiding* puts the teacher in a central role, it is possible to transfer this role to the students by *eliciting* and *probing* their thinking and *exploring incorrect strategies* or *disagreements*. However, it is not possible to ask everything or involve all the students in the discussion, and it is up to the teacher to decide when to challenge or support them, when to introduce information or when to lead the discourse so that the discussion is productive and leads to relevant mathematical learning.

To address the research question, we drew on the uncertainties that Sofia expressed in the LS sessions about which answers to share during the whole-class discussion, namely about exploring incorrect answers. Sofia further questioned which interventions she could make to support and challenge the students. Based on these questions and the articles analysed, in the LS sessions we discussed different teachers' interventions and actions in leading this moment of the lesson, namely about exploring disagreements or incorrect answers. We also discussed possible teachers' interventions and actions to make the discussion productive.

During and after the LS, we saw Sofia's different actions to get students to present and justify their answers, including *exploring incorrect answers* and *disagreements*, and also *drawing connections* between different strategies. Thus, Sofia was able to "put into practice" a set of actions teachers can implement in leading whole-class discussions to enhance student learning, as discussed in the LS sessions. Reflecting on her practice, Sofia said that participating in LS was an opportunity to rethink her role in the classroom, understand the benefit of giving students the floor to explain their ideas and reflect on their peers' ideas to foster student learning. Participating in LS, therefore, gave her a new perspective on leading whole-class discussions, realising that it is up to the teacher to organise a discussion where students play an active role and which allows them to learn mathematical concepts and procedures and develop mathematical communication.

6. Conclusion

Planning and leading the whole-class discussions and reflecting on them, during and after the LS, resulted in Sofia valuing them as moments that enhance students' learning, even among the students with more difficulties, broadening the range of strategies they can follow, and leading them to understand unclear aspects, influencing her *knowledge and beliefs* (Lewis *et al.*, 2019). This work also impacted on *instructional routines and tools* (Lewis *et al.*, 2019), namely on how Sofia managed her lessons, with greater emphasis on giving the floor to the students and asking them for their explanations, supporting them when she felt appropriate.

Learning to lead whole-class discussions is a complex process which requires time (Kooloos *et al.*, 2020). This article seeks to identify possible teacher actions in leading whole-class discussions to enhance student learning, not only whilst participating in a LS but also in her subsequent teaching practice. Analysing this teacher's actions, we showed how discussions held in LS may influence and change teachers' practices. It might be important to continue observing Sofia and other teachers' lessons to understand what tasks they propose, whether they include whole-class discussions and how they lead them, the acknowledged potential of such strategies to enhance students' learning and what constraints they identify.

Planning and teaching several research lessons provided several opportunities for discussion and reflection and prompted Sofia to think about different aspects of her practice. She planned the whole-class discussion in detail and reflected on how she led it, and on students' explanations and learning. Although this planning differed considerably in the two LS, it provided important learning for Sofia, who stated that she now places a greater emphasis on giving students a voice, *eliciting* and *probing their thinking*. Sofia continues to participate in LS, held at her school, which highlights the potential she acknowledges in this professional development process.

This article helps understand how participation in LS influences the way teachers lead whole-class discussions, not only during but also after the LS. Creating opportunities for teachers to plan and teach several lessons and reflect on them seems to be crucial to help them to rethink their teaching practice during and after LS. Also, it shows that it is fruitful to analyse their actions and practices in several research lessons with teachers, providing them with several opportunities to discuss how to lead whole-class discussions, supported by the other participants and the facilitator.

References

Bogdan, R. and Biklen, S. (1994), *Investigação Qualitativa Em Educação*, Porto Editora, Porto.

Clivaz, S. and Miyakawa, T. (2020), "The effects of culture on mathematics lessons: an international comparative study of a collaboratively designed lesson", *Educational Studies in Mathematics*, Vol. 105 No. 1, pp. 53-70.

Drageset, O.G. (2015), "Student and teacher interventions: a framework for analysing mathematical discourse in the classroom", *Journal of Mathematics Teacher Education*, Vol. 18, pp. 253-272.

Fujii, T. (2018), "Lesson study and teaching mathematics through problem solving: the two wheels of a cart", in Quaresma, M., Winsløw, C., Clivaz, S., Ponte, J.P., Ní Shúilleabháin, A. and Takahashi, A. (Eds), *Mathematics Lesson Study Around the World*, Springer, Cham, pp. 1-21.

Gomes, P., Quaresma, M. and Ponte, J.P. (2021), "A lesson study with mathematics teachers: learning about communication in the classroom", *Acta Scientiae*, Vol. 23 No. 5, pp. 126-152.

Inoue, N. (2011), "Zen and the art of neriage: facilitating consensus building in mathematics inquiry lessons through lesson study", *Journal of Mathematics Teacher Education*, Vol. 14 No. 1, pp. 5-23.

Kooloos, C., Oolbekkink-Marchand, H., Kaenders, R. and Heckman, G. (2020), "Orchestrating mathematical classroom discourse about various solution methods: case study of a teacher's development", *Journal Für Mathematik-Didaktik*, Vol. 41 No. 2, pp. 357-389.

Lewis, C., Friedkin, S., Emerson, K., Henn, L. and Goldsmith, L. (2019), "How does lesson study work? Toward a theory of lesson study process and impact", in Huang, R., Takahashi and Ponte, J.P. (Eds), *Theory and Practice of Lesson Study in Mathematics: an International Perspective*, Springer, Cham, pp. 13-37.

Ponte, J.P., Mata-Pereira, J. and Quaresma, M. (2013), "Ações do professor na condução de discussões matemáticas", *Quadrante*, Vol. 22 No. 1, pp. 55-81.

Ponte, J.P. and Quaresma, M. (2016), "Teachers' professional practice conducting mathematical discussions", *Educational Studies in Mathematics*, Vol. 93 No. 1, pp. 51-66.

Ponte, J.P., Quaresma, M. and Mata-Pereira, J. (2017), "The challenge of mathematical discussions in teachers' professional practice", *Didacticae*, Vol. 1, pp. 45-59.

Quaresma, M. and Ponte, J.P. (2016), "Comunicação, tarefas e raciocínio: aprendizagens profissionais proporcionadas por um estudo de aula", *Zetetike*, Vol. 23 No. 2, pp. 297-310.

Richit, A., Ponte, J.P. and Quaresma, M. (2021), "Aprendizagens profissionais de professores evidenciadas em pesquisas sobre estudos de aula", *Bolema: Boletim de Educação Matemática*, Vol. 35 No. 70, pp. 1107-1137.

Stein, M.K., Engle, R.A., Smith, M.S. and Hughes, E.K. (2008), "Orchestrating productive mathematical discussions: five practices for helping teachers move beyond show and tell", *Mathematical Thinking and Learning*, Vol. 10 No. 4, pp. 313-340.

Takahashi, A. (2008), "Beyond show and tell: Neriage for teaching through problem-solving: ideas from Japanese problem-solving approaches for teaching mathematics", *Paper presented at 11th International Congress on Mathematics Education*, Monterey.

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