



# Right-handed neutrinos, domain walls and tadpoles in the superstring inspired $\mu\nu$ SSM

D.E. López-Fogliani<sup>a,b</sup>, C. Muñoz<sup>c,d,\*</sup>

<sup>a</sup> Instituto de Física de Buenos Aires UBA & CONICET, Departamento de Física, Facultad de Ciencia Exactas y Naturales, Universidad de Buenos Aires, 1428 Buenos Aires, Argentina

<sup>b</sup> Pontificia Universidad Católica Argentina, 1107 Buenos Aires, Argentina

<sup>c</sup> Departamento de Física Teórica, Universidad Autónoma de Madrid (UAM), Campus de Cantoblanco, 28049 Madrid, Spain

<sup>d</sup> Instituto de Física Teórica (IFT) UAM-CSIC, Campus de Cantoblanco, 28049 Madrid, Spain

Received 30 July 2022; received in revised form 8 December 2022; accepted 17 December 2022

Available online 23 December 2022

Editor: Hong-Jian He

## Abstract

We discuss the special role of right-handed neutrinos in the  $\mu\nu$ SSM for solving the  $\mu$ - and  $\nu$ -problems, simultaneously avoiding the potential domain wall and tadpole problems. In particular, embedding the  $\mu\nu$ SSM in the framework of superstrings implies that not all gauge invariant terms are necessarily present in the superpotential, and thus there is more flexibility to allow only those that avoid the domain wall and tadpole problems to be present. These can be non-renormalizable terms of dimension 4 or terms of higher dimensions. In addition, non-perturbative effects can also solve both problems. We discuss another implication of the superstring inspired  $\mu\nu$ SSM, since the right-handed neutrino is expected to have extra  $U(1)$  charges at high energies. In this case, the cubic right-handed neutrino terms in the superpotential, helpful for generating Majorana masses and solving domain wall and tadpole problems, can arise through a variety of stringy mechanisms.

© 2022 The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>). Funded by SCOAP<sup>3</sup>.

\* Corresponding author.

E-mail addresses: [daniel.lopez@df.uba.ar](mailto:daniel.lopez@df.uba.ar) (D.E. López-Fogliani), [c.munoz@uam.es](mailto:c.munoz@uam.es) (C. Muñoz).

## Contents

1. Introduction	2
2. The $\mu\nu$ SJM and renormalizable terms	4
3. Domain wall problem and non-renormalizable terms	6
4. Solving the tadpole problem	8
4.1. Non-renormalizable dimension-4 terms	8
4.2. Non-renormalizable terms of higher dimensions	11
4.3. Non-perturbative terms	12
5. Generating the cubic coupling $(\nu^c)^3$	13
5.1. Mixing among fields	14
5.2. Non-renormalizable couplings	14
5.3. Instanton effects	15
6. Conclusions	15
CRediT authorship contribution statement	16
Declaration of competing interest	16
Data availability	16
Acknowledgements	16
References	16

## 1. Introduction

The addition of right-handed (RH) neutrinos  $\nu_R$  to the spectrum of the standard model (SM) provides the light neutrinos with masses at the tree level. In the framework of supersymmetry (SUSY), the ‘ $\mu$  from  $\nu$ ’ Supersymmetric Standard Model ( $\mu\nu$ SJM) [1] (for a recent review, see Ref. [2]), was proposed to solve the  $\mu$ - and  $\nu$ -problems simultaneously without the need to introduce additional energy scales beyond the SUSY-breaking scale, which is the source of the electroweak symmetry breaking (EWSB). To perform it, the presence in the superpotential of trilinear couplings involving RH neutrino superfields,  $\nu_i^c$ , in addition to the usual quark and charged-lepton Yukawa couplings, is crucial [1,3]:

$$\begin{aligned}
 W_{\mu\nu\text{SJM}} = & Y_{ij}^e H_d L_i e_j^c + Y_{ij}^d H_d Q_i d_j^c - Y_{ij}^u H_u Q_i u_j^c \\
 & - Y_{ij}^v H_u L_i \nu_j^c + \lambda_i H_u H_d \nu_i^c + \frac{1}{3} \kappa_{ijk} \nu_i^c \nu_j^c \nu_k^c.
 \end{aligned} \tag{1}$$

Here the summation convention is implied on repeated indexes, with  $i, j, k = 1, 2, 3$  the usual family indexes of the SM.<sup>1</sup> Our convention for the contraction of two  $SU(2)$  doublets is e.g.  $H_u H_d \equiv \epsilon_{ab} H_u^a H_d^b$ ,  $a, b = 1, 2$  and  $\epsilon_{ab}$  the totally antisymmetric tensor with  $\epsilon_{12} = 1$ .

The last three terms containing RH neutrinos in the above superpotential are clearly allowed by gauge invariance, since these fields have vanishing hypercharges by construction. In particular, the fifth term in Eq. (1) is an explicit  $R$ -parity (and lepton-number) violating term which generates dynamically a  $\mu$ -parameter after EWSB,  $\mu = \lambda_i v_{iR}$ , with  $v_{iR}$  the vacuum expectation values

<sup>1</sup> The number of RH neutrinos is in fact a free parameter. These particles are the only SM singlets and therefore their way of arising from a more fundamental theory can be different from the other particles. An analysis using this number as a free parameter can be found in Ref. [4]. Nevertheless, we will assume for simplicity in what follows that their number is three, replicating what happens with the other SM fermions, since this assumption will not modify our results.

(VEVs) of the right sneutrinos. This mechanism solves the so-called  $\mu$ -problem [5] (for a recent review, see Ref. [6]). It is analogue to the solution of the Next-to-Minimal Supersymmetric Standard Model (NMSSM) (for reviews, see Refs. [7,8]), where instead of RH neutrino superfields an extra singlet superfield  $S$  is added to the spectrum with the coupling  $\lambda H_u H_d S$ . By the same EWSB mechanism, the neutrino Yukawa term generates effectively bilinear terms,  $\mu_i = Y_{ij}^\nu v_{jR}$ , as the ones of the Bilinear  $R$ -Parity Violating model (BRPV) (for a review, see Ref. [9]), as well as Dirac masses for neutrinos,  $m_{\mathcal{D}_{ij}} = Y_{ij}^\nu v_u$ , with  $v_u$  the VEV of the Higgs  $H_u$ . In addition, the cubic neutrino term generates electroweak (EW)-scale Majorana masses,  $\mathcal{M}_{ij} = 2\kappa_{ijk} v_{kR}$ , instrumental in generating the correct neutrino masses and mixing angles [10–13], i.e. in solving the  $\nu$  problem through an EW-scale seesaw with  $Y_{ij}^\nu \lesssim 10^{-6}$  [1,3,14–18]. The presence of this cubic term also forbids a Peccei-Quinn-like symmetry  $U(1)_{PQ}$  in the Lagrangian, avoiding therefore the existence of a phenomenologically unacceptable massless Nambu-Goldstone boson once EWSB takes place. Again, this is analogue to what happens in the NMSSM where the cubic term  $\kappa SSS$  is present.

Since  $R$ -parity and lepton number are not conserved in the  $\mu\nu$ S SM in contrast to the NMSSM, this leads to a completely different phenomenology characterized by an enlarged Higgs sector which includes the left and right sneutrinos [3,19–23], and distinct prompt or displaced decays of the lightest supersymmetric particle (LSP) producing multi-leptons/jets/photons with small/moderate missing transverse energy from neutrinos. Studies of these interesting signals at the Large Hadron Collider (LHC) have been carried out in the literature [24–30], taking into account the intimate connection between the LSP lifetime and the size of neutrino Yukawas. The low decay width of the LSP due to the smallness of neutrino masses is also related to the existence of possible candidates for decaying dark matter in the model. This is the case of the gravitino [31–35], or the axino [36], with lifetimes greater than the age of the Universe. It is worth mentioning concerning cosmology, that baryon asymmetry might be realized in the  $\mu\nu$ S SM through EW baryogenesis [37]. See also Refs. [3,38] for vacuum structure analyses of the  $\mu\nu$ S SM. The EW sector of the  $\mu\nu$ S SM can also explain [28,29] the longstanding discrepancy between the experimental result for the anomalous magnetic moment of the muon [39,40] and the SM prediction.

However, despite all these interesting theoretical and phenomenological properties, similar to the NMSSM the  $\mu\nu$ S SM has the potential problem of the existence of a  $Z_3$  discrete symmetry in its superpotential, corresponding to a multiplication of all components of all chiral superfields by a phase  $e^{2\pi i/3}$ . Discrete symmetries when spontaneously broken can give rise to domains of different degenerate vacua separated by domain walls [41–44]. The walls would dominate the energy density in the Universe producing wall driven inflation, unless they are removed well before. Besides, domain walls can also disrupt primordial nucleosynthesis. These cosmological domain wall problems, if present, can be solved if the degeneracy of the vacua is slightly broken, eventually leading to the dominance of the true vacuum [41].

This mechanism to solve the domain wall problem was applied in the NMSSM assuming that the  $Z_3$  discrete symmetry is explicitly broken by gravitationally suppressed interactions producing non-renormalizable terms in the superpotential, which lift the degeneracy of the three original vacua [45–47].<sup>2</sup> Besides, these terms can be chosen small enough as not to alter the low-energy phenomenology. However, another problem arises when working in the framework of NMSSM supergravity. The presence of a linear term  $tS$  in the superpotential (with  $t$  of dimension

<sup>2</sup> Once this solution is present, the domain walls were used in Ref. [48] for the generation of baryogenesis in the NMSSM.

mass squared) would destabilize the hierarchy since the mass scale is expected to be proportional to the cut-off scale of the theory, which is typically the Planck mass  $M_{\text{Pl}} \approx 1.2 \times 10^{19}$  GeV. Although its presence is forbidden by the  $Z_3$  symmetry, the non-renormalizable terms together with soft SUSY breaking can reintroduce it in the effective potential. Such corrections to the potential are quadratically divergent and therefore proportional to powers of  $M_{\text{Pl}}$ , destabilizing the hierarchy [49–51,47,52,53].

Any supergravity model with gauge singlets can have this tadpole problem, since the dangerous diagrams are not excluded by gauge invariance. It has been argued nevertheless in the framework of the NMSSM that the non-renormalizable terms generating them are not necessarily present when the model possesses either target space duality in a string effective action [53] or  $R$ -symmetry [53–55], but other terms that do not destabilize the hierarchy, and solve in addition the domain wall problem, can be present.

The aim of this work is to analyze domain wall and tadpole problems in the framework of the  $\mu\nu\text{SSM}$ , where SM singlets such as RH neutrinos are present. In particular, we will see that these problems can be solved naturally in the context of superstring compactifications, where not all gauge invariant terms are necessarily allowed. Besides, non-perturbative effects can also be relevant. The paper is organized as follows. Sec. 2 will be devoted to discuss the advantage of constructing the  $\mu\nu\text{SSM}$  in the framework of superstring theory. In Sec. 3, we will analyze how the presence of non-renormalizable terms can be helpful for solving the domain wall problem. In Sec. 4, we will see that these terms can induce simultaneously destabilizing tadpole divergences, and we will study how to solve this additional problem. Finally, in Sec. 5 we will explain first that RH neutrinos, whose presence is crucial in the framework of the  $\mu\nu\text{SSM}$ , can have extra  $U(1)$  charges in string compactifications. Then, we will discuss how these charges modify the origin of certain terms in the superpotential with interesting consequences. Our conclusions are left for Sec. 6.

## 2. The $\mu\nu\text{SSM}$ and renormalizable terms

In this section we will discuss how natural is the presence in  $W_{\mu\nu\text{SSM}}$  of only the terms shown in Eq. (1), when RH neutrinos are added to the spectrum of the SUSY SM. Note that the existence of the latter fields imply that the  $SU(3) \times SU(2) \times U(1)_Y$  gauge invariant superpotential containing the most general renormalizable terms is in fact given by:

$$\begin{aligned}
 W_r = & Y_{ij}^e H_d L_i e_j^c + Y_{ij}^d H_d Q_i d_j^c - Y_{ij}^u H_u Q_i u_j^c + \mu H_u H_d \\
 & + \lambda_{ijk} L_i L_j e_k^c + \lambda'_{ijk} L_i Q_j d_k^c + \lambda''_{ijk} u_i^c d_j^c d_k^c + \mu_i H_u L_i \\
 & - Y_{ij}^v H_u L_i \nu_j^c + \lambda_i H_u H_d \nu_i^c + \frac{1}{3} \kappa_{ijk} \nu_i^c \nu_j^c \nu_k^c + \mathcal{M}_{ij} \nu_i^c \nu_j^c + t_i \hat{\nu}_i^c.
 \end{aligned} \tag{2}$$

The four terms in the first line determine the superpotential of the Minimal Supersymmetric Standard Model (MSSM) (for reviews, see e.g. Refs. [56–58]). The four terms in the second line are the conventional trilinear and bilinear  $R$ -parity violating (RPV) couplings (for a review, see Ref. [9]). Finally, all the terms in the last line contain RH neutrino superfields. The first three terms are characteristic of the  $\mu\nu\text{SSM}$  as discussed in the Introduction, the fourth one is a bilinear term giving Majorana masses for RH neutrinos, and the fifth one is a linear (tadpole) term for them.

Given superpotentials (1) and (2), one would like to have an explanation for the absence in  $W_{\mu\nu\text{SSM}}$  of some of the terms present in  $W_r$ . In particular, this is the case of the dangerous

tadpole terms  $t_i v_i^c$ , as well as of the mass terms  $\mu H_u H_d$ ,  $\mu_i H_u L_i$  and  $\mathcal{M}_{ij} v_i^c v_j^c$  which would reintroduce the  $\mu$ -problem and additional naturalness problems if they are of the order of the high-energy scale. Here we are interested in the analysis of the  $\mu\nu$ S<sub>SM</sub> inspired in the construction of superstring models, given the relevance of string theory as a possible underlying unified theory. In this framework, not all gauge invariant terms are necessarily allowed. In particular, at the perturbative level in string constructions the massive modes have typically huge masses of the order of the string scale, whereas the massless ones give rise only to trilinear terms at the renormalizable level. Thus one ends up with an accidental  $Z_3$  symmetry in the low-energy theory. Thanks to it, the presence of the above dangerous terms is automatically forbidden. Let us remark nevertheless that using  $W_{\mu\nu\text{S<sub>SM}}</sub>$  these terms arise dynamically with the correct (EW) scale after EWSB, as discussed in the Introduction. This fact motivates the use of the superpotential (1), where only dimensionless parameters contribute.

On the other hand, it is well known that the simultaneous presence of the terms  $\lambda'_{ijk} L_i Q_j d_k^c$  and  $\lambda''_{ijk} u_i^c d_j^c d_k^c$ , violating lepton and baryon number respectively, is dangerous since they would produce fast proton decay. The usual assumption in the literature of the MSSM is to invoke an *ad hoc*  $Z_2$  discrete symmetry  $R$ -parity [9] to avoid the problem, providing with charge +1 to ordinary particles and  $-1$  to their superpartners. However, this assumption is clearly too stringent, since then other terms like e.g.  $\lambda_{ijk} L_i L_j e_k^c$  or  $\lambda_i v_i^c H_u H_d$ , which are harmless for proton decay, would also be forbidden. Less drastic solutions arise in the context of string theory. In compactifications of the heterotic string on orbifolds [59] the matter fields lie in different sectors of the compact space. For example, for the case of the  $Z_3$  orbifold couplings between three fields in the untwisted sector or three fields in the twisted sector are allowed, but couplings between untwisted and twisted fields are forbidden by the space group selection rules. Similar results about couplings that are forbidden even if they are gauge invariant, can also be obtained in other kind of compactifications and in other types of strings. Thus “stringy” selection rules can forbid couplings that in principle seem to be allowed by gauge invariance, as pointed out in Ref. [3]. As a consequence, the RPV couplings  $\lambda''_{ijk}$  can be naturally forbidden whereas the other RPV couplings are allowed. In fact, the couplings  $\lambda_{ijk}$  and  $\lambda'_{ijk}$  can be naturally added to  $W_{\mu\nu\text{S<sub>SM}}}</sub>$  [1,3,24].

Alternatively, in the context of e.g. heterotic orbifolds or free fermion constructions the typical presence of several  $U(1)$  gauge symmetries beyond the hypercharge (see a more detailed discussion below in Sec. 5), can also be useful. Although these extra  $U(1)$ ’s are spontaneously broken by VEVs of scalar fields through  $D$ - and  $F$ -flat directions of the scalar potential, dangerous couplings allowed by the SM gauge symmetry as those discussed above can be forbidden by the extra  $U(1)$  charges [60–62]. In fact, residual  $Z_N$  symmetries can be left in the low-energy theory depending on the choice of flat direction. In Refs. [63,64], a less model-dependent mechanism was obtained in type IIA intersecting brane constructions, where e.g. models with  $Z_3$  Baryon-parity which forbids  $\lambda''_{ijk}$  couplings were constructed.

We conclude therefore that string constructions facilitate the presence of only certain couplings in  $W_{\mu\nu\text{S<sub>SM}}}</sub>$  solving crucial theoretical problems, such as the  $\mu$ - and  $\nu$ -problems. However, this superpotential has a  $Z_3$  discrete symmetry that can induce a cosmological domain wall problem. This is the subject that we will discuss in the next sections. As we will see, the addition of non-renormalizable terms to  $W_{\mu\nu\text{S<sub>SM}}}</sub>$  breaks the  $Z_3$  symmetry and can generate dynamically the appropriate contributions to solve this problem, simultaneously avoiding the destabilization of the hierarchy due to tadpoles.

Let us finally comment that there is another strategy to explain the absence of linear, bilinear and baryon-number violating terms in  $W_{\mu\nu\text{S<sub>SM}}}</sub>$ , solving simultaneously the domain wall and

tadpole problems, without relying on the superstring framework. It consists of extending the SM gauge group with an extra  $U(1)$  at low energies [65,66,4] (see Ref. [67] for a review in other models).<sup>3</sup> Then, the dangerous operators could be forbidden by gauge invariance, and the domain wall problem disappears once the discrete symmetry is embedded in the broken gauge symmetry [69–71]. After the EW phase transition one expects a network of domain walls bounded by cosmic strings to form and then collapse. Since the RH neutrinos (and the other matter fields) are charged under the extra  $U(1)$  the dangerous tadpole diagrams could also be forbidden. In addition to the presence of the extra  $U(1)$  at low energies, another crucial characteristic of the models in Refs. [65,4] is the presence in their spectrum of exotic matter dictated by the anomaly cancellation conditions, such as extra quark representations or singlets under the SM gauge group. In addition to the non-minimality of the particle spectrum, gauge invariance forbids the cubic term  $\kappa_{ijk} v_i^c v_j^c v_k^c$  that generates Majorana masses, making more involved to obtain the correct neutrino masses and mixing angles.

In the following we will focus our analysis in the minimal  $\mu\nu$ SJM characterized by the SM gauge group and the superpotential (1), in the framework of superstrings.

### 3. Domain wall problem and non-renormalizable terms

The  $Z_3$  discrete symmetry of the superpotential of the  $\mu\nu$ SJM is spontaneously broken due to the EWSB in the early Universe. As discussed in the Introduction, in this case dangerous domain walls separating degenerate vacua can be produced. The strongest constraint on domain walls arises from nucleosynthesis, since the walls must disappear before the nucleosynthesis era beginning at  $T \sim 1$  MeV. If we trust this cosmological argument, to solve the problem requires the presence in the effective potential of  $Z_3$ -breaking terms [47]

$$\delta V \gtrsim 10^{-7} \frac{v_R^3 m_W^2}{M_{\text{Pl}}}, \quad (3)$$

where the surface energy of the domain wall  $\sigma \sim v^3$ , with  $v$  a typical VEV of the fields determining the scale of the EWSB, has been approximated using in the  $\mu\nu$ SJM  $v \sim v_{iR} \sim v_R$ . This condition ensures that the pressure produced by one of the vacua slightly deeper than the others,  $\epsilon \sim \delta V$ , is larger than the surface tension  $\sigma/R$  (where  $R$  is the curvature scale of the wall) before nucleosynthesis, leading to the dominance of the true vacuum.

Now, one can straightforwardly see that non-renormalizable dimension- $n$  terms, with  $n > 3$ , in the superpotential (working with the Kähler potential will not modify our conclusions)

$$W_{\text{nr}} = c_n \frac{\phi_1 \cdots \phi_n}{\Lambda^{n-3}}, \quad (4)$$

break explicitly the discrete  $Z_3$  symmetry inducing the following soft SUSY-breaking contributions to the effective potential

$$\delta V_{\text{nr}} \approx c_n m_{3/2} \frac{v_1 \cdots v_n}{\Lambda^{n-3}}, \quad (5)$$

where  $v_{1,\dots,n}$  are the VEVs of the fields  $\phi_{1,\dots,n}$ , we have assumed soft SUSY-breaking parameters  $\sim m_{3/2}$ , and  $\Lambda$  is the cut-off scale of the high-energy theory. To solve the domain wall problem,  $\delta V_{\text{nr}}$  must be large enough to fulfill requirement (3).

<sup>3</sup> This strategy is also possible in string constructions, where as mentioned above several  $U(1)$ 's are typically present and some of them could survive at low energies (see e.g. the orbifold model with an extra  $U(1)$  of Ref. [68]).

In the case of a supergravity theory where  $\Lambda = M_{Pl}$ , obviously only dimension-4 terms producing

$$\delta V_{nr} \approx c_4 \frac{v_R^5}{M_{Pl}}, \tag{6}$$

can fulfill this requirement with the coupling constant verifying the reasonable bound,  $c_4 \gtrsim 10^{-9}$ , where we have used  $m_{3/2} \sim v_R \sim v_{1,\dots,4} \sim 1$  TeV. Terms of dimension larger than four are too much suppressed by the Planck mass. For example, a dimension-5 term in the superpotential  $c_5 \phi^5 / M_{Pl}^2$  gives rise to  $\delta V_{nr} \approx c_5 v_R^6 / M_{Pl}^2$ , implying the bound  $c_5 \gtrsim 10^7$ , well beyond perturbativity theory.

On the other hand, even in superstrings the relevant scale of the theory can be much lower than  $M_{Pl}$  (see e.g. Ref. [72] and references therein). As pointed out in Ref. [37], as long as  $\Lambda < M_{Pl}$  the above result on the dimensionality of the non-renormalizable terms to solve the domain wall problem,  $n = 4$ , can be relaxed. In particular, from Eqs. (3) and (5) it is straightforward to deduce that in general the coupling constants must verify

$$c_n \gtrsim 10^{-25} \left( \frac{\Lambda}{v_R} \right)^{n-3}. \tag{7}$$

Thus, even for values of the cut-off scale as large as  $\Lambda = 10^{15}$  GeV, one obtains that dimension-5 terms can be used to solve the domain wall problem. For very low values of the scale such as  $\Lambda = 10^4$  GeV, terms of dimension as large as 27 can be used. In the following we will consider only the case  $\Lambda = M_{Pl}$ , but the results can be straightforwardly extrapolated for lower scales.

To solve the domain wall problem in the  $\mu\nu$ S $\overline{S}$ M supergravity model we have available the following dimension-4 terms in the superpotential, with the only contributions of RH neutrino and Higgs superfields:

$$\kappa_{ijkl}^{nr} v_i^c v_j^c v_k^c v_l^c / M_{Pl}, \quad \lambda_{ij}^{nr} v_i^c v_j^c H_u H_d / M_{Pl}, \quad \lambda_{ud}^{nr} (H_u H_d)^2 / M_{Pl}, \tag{8}$$

and the terms

$$Y_{ijk}^{v,nr} v_i^c v_j^c H_u L_k / M_{Pl}, \quad \kappa_{ij}^{nr} H_u L_i H_u L_j / M_{Pl}, \quad Y_i^{v,nr} H_u L_i H_u H_d / M_{Pl}, \tag{9}$$

with the contribution also of lepton doublet superfields  $L_i$ .

The terms in Eq. (8) are similar to the ones available in the NMSSM supergravity model [47], substituting in the first two the RH neutrino superfields  $v_i^c$  by the singlet superfield  $S$ . Now, as discussed above, to solve the domain wall problem with some of these non-renormalizable terms their couplings must fulfill requirement (3), where now

$$\delta V_{nr} \approx \kappa_{ijkl}^{nr} \frac{v_R^5}{M_{Pl}}, \quad \lambda_{ij}^{nr} \frac{v_R^3 v_u v_d}{M_{Pl}}, \quad \lambda_{ud}^{nr} \frac{v_R v_u^2 v_d^2}{M_{Pl}}, \tag{10}$$

implying the reasonable bounds shown at the top of Table 1. Obviously, the bound for the couplings  $\kappa_{ijkl}^{nr} \gtrsim 10^{-9}$  is the same as for the  $c_4$  coupling obtained in the generic discussion of Eq. (6). The stronger constraints for the other couplings,  $\lambda_{ij}^{nr} \gtrsim 10^{-7}$  and  $\lambda_{ud}^{nr} \gtrsim 10^{-5}$ , are due to the smaller values of the VEVs  $v_{u,d} \sim 10^{-1} v_R$ . Let us point out that these values for the couplings are reasonable, since one expects the non-renormalizable couplings to be smaller than the renormalizable ones. This is in fact the natural situation in string compactifications where the former get a suppression factor with respect to the latter (see e.g. the discussion of couplings in the  $Z_3$  orbifold compactification of the heterotic string in Ref. [73]).

Table 1

(top) Bounds on some of the couplings to solve the domain wall (DW) problem with the corresponding non-renormalizable terms of Eqs. (8) and (9), as discussed in Sec. 3. (bottom) Bounds on the associated renormalizable couplings of Eq. (1) to solve simultaneously the tadpole (T) problem, as discussed in Sec. 4.1.

DW	$\kappa_{ijkl}^{\text{nr}} \gtrsim 10^{-9}$	$\lambda_{ij}^{\text{nr}} \gtrsim 10^{-7}$	$\lambda_{ud}^{\text{nr}} \gtrsim 10^{-5}$	$Y_{ijk}^{\nu,\text{nr}} \gtrsim 10^{-1}$
T	$\kappa_{ijk} \lesssim 10^{-3}$	$\lambda_i \lesssim 10^{-5}$	$\kappa_{ijk}\lambda_j\lambda_k \lesssim 10^{-5}$	$Y_{ij}^{\nu} \lesssim 10^{-11}$

Concerning the non-renormalizable terms in Eq. (9), let us remark that because of RPV in the  $\mu\nu$ SJM not only the right sneutrinos and Higgses acquire VEVs, but also the left sneutrinos in  $L_i$ . This is the reason for the presence of these terms in the  $\mu\nu$ SJM unlike the case of the NMSSM. Their contributions to  $\delta V_{\text{nr}}$  depend therefore on the VEVs of the left sneutrinos which are small  $v_{iL} \sim v_L \lesssim 10^{-4}$  GeV [1,3,2], because their minimization equations are determined by the small neutrino Yukawas. We have respectively

$$\delta V_{\text{nr}} \approx Y_{ijk}^{\nu,\text{nr}} \frac{v_R^3 v_L v_u}{M_{\text{Pl}}}, \quad \kappa_{ij}^{\text{nr}} \frac{v_R v_L^2 v_u^2}{M_{\text{Pl}}}, \quad Y_i^{\nu,\text{nr}} \frac{v_R v_L v_u^2 v_d}{M_{\text{Pl}}}, \quad (11)$$

implying that the bounds on the couplings are stronger, with  $Y_{ijk}^{\nu,\text{nr}} \gtrsim 10^{-1}$  as shown at the top of Table 1 and  $\kappa_{ij}^{\text{nr}} \gtrsim 10^7$ ,  $Y_i^{\nu,\text{nr}} \gtrsim 10$ . Because of perturbativity, clearly only the couplings  $Y_{ijk}^{\nu,\text{nr}}$  are useful for solving the domain wall problem.

Thus, we have shown above that the domain wall problem can be solved in the framework of supergravity by dimension-4 terms. However, the latter induce in the effective potential linear terms that generate quadratic tadpole divergences, which destabilize the hierarchy [49–52,47,53]. This will be the subject of the next section, where we will also explore another solution to the domain wall problem through higher dimensional terms harmless to the gauge hierarchy.

## 4. Solving the tadpole problem

### 4.1. Non-renormalizable dimension-4 terms

Given that we are working with a non-renormalizable effective theory valid below the cut-off scale  $M_{\text{Pl}}$ , the presence of quadratically divergent tadpole diagrams giving rise to contributions to the effective potential proportional to  $M_{\text{Pl}}$  can destabilize the hierarchy [49–52,47,53]. In particular, the first and second terms in Eq. (8) give rise to quadratically divergent tadpole diagrams at 2-loop order when SUSY is spontaneously broken, as shown in Fig. 1. In the case of the third term the tadpole divergence arises at 3-loop order. Similar comments apply to the terms in Eq. (9). In particular, the first one which is useful for solving the domain wall problem generates a tadpole divergence at 2-loop order (exchanging in Fig. 1 two propagators  $v^c$  by  $H_u$  and  $L$ ).

To simplify the notation, let us denote generically all the couplings of the 2-loop divergent terms,  $\kappa_{ijkl}^{\text{nr}}$ ,  $\lambda_{ij}^{\text{nr}}$  and  $Y_{ijk}^{\nu,\text{nr}}$  as  $c_4$ , and the related renormalizable couplings contributing to the dangerous Feynman diagrams,  $\kappa_{ijk}$ ,  $\lambda_i$  and  $Y_{ij}^{\nu}$ , respectively, as  $c_3$ . Similarly, for the 3-loop divergent term we denote  $\lambda_{ud}^{\text{nr}}$  as  $c_4$ , but there are now three renormalizable couplings  $c_3$  contributing to the Feynman diagram, two contributions from  $\lambda_i$  and one from  $\kappa_{ijk}$ .

Following the analyses of Refs. [49–52,47,53], the quadratically divergent 2-loop integral of  $\mathcal{O}(M_{\text{Pl}}^2/(16\pi^2)^2)$  generates in the effective potential a linear term of the form

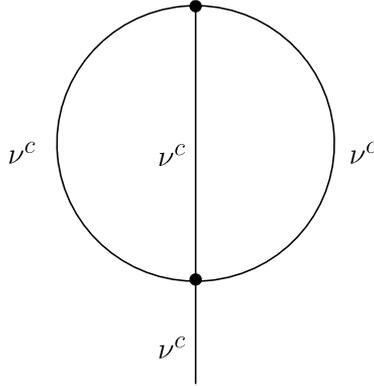


Fig. 1. The dangerous diagram for the first term in Eq. (8) which can destabilize the hierarchy. For the second term the figure is the same exchanging two propagators  $\nu^c$  by  $H_u$  and  $H_d$ .

$$\delta V \sim \frac{c_4 c_3}{(16\pi^2)^2} m_{3/2}^2 M_{\text{Pl}} (\tilde{\nu}_R + \tilde{\nu}_R^*). \tag{12}$$

For the 3-loop divergences, this result is valid but with the substitution  $c_3/(16\pi^2)^2 \rightarrow c_3^3/(16\pi^2)^3$ . Let us remark related to the discussion of Eq. (7) that we are using  $M_{\text{Pl}}$  as the cut-off scale of our theory. Otherwise, we should substitute in Eq. (12)  $M_{\text{Pl}} \rightarrow \Lambda^2/M_{\text{Pl}}$ .

Now, it is straightforward to see that the contribution (12) destabilizes the hierarchy unless the factor in front of the RH sneutrino field is

$$\frac{c_4 c_3}{(16\pi^2)^2} m_{3/2}^2 M_{\text{Pl}} \lesssim m_{3/2}^3, \tag{13}$$

implying the following bound to avoid the tadpole problem:

$$c_4 c_3 \lesssim 10^{-12}. \tag{14}$$

This constraint is compatible with requirements for the non-renormalizable couplings  $c_4$  in Table 1,  $\kappa_{ijkl}^{\text{nr}}$ ,  $\lambda_{ij}^{\text{nr}}$  and  $Y_{ijk}^{\nu,\text{nr}}$ , if the related renormalizable couplings  $c_3$ , i.e.  $\kappa_{ijk}$ ,  $\lambda_i$  and  $Y_{ij}^{\nu}$  respectively, fulfill the bounds shown at the bottom of the table. Note, however, that the bound  $Y_{ij}^{\nu} \lesssim 10^{-11}$  is very unlikely to be compatible with  $Y_{ijk}^{\nu,\text{nr}} \gtrsim 10^{-1}$ , because one expects the renormalizable couplings to be larger than the non-renormalizable ones, as discussed below Eq. (10).

Concerning the other two couplings,  $\kappa_{ijk}$  and  $\lambda_i$ , in the case of the NMSSM their bounds correspond to  $\kappa \lesssim 10^{-3}$  and/or  $\lambda \lesssim 10^{-5}$ . The former would imply a light singlino, which is tightly constrained by the invisible Higgs decay and dark matter searches. The latter would give rise to  $\mu = \lambda \langle S \rangle \sim 10^{-2}$  GeV, which is excluded by experimental bounds on chargino masses. In the  $\mu\nu$ SVM there is nevertheless more flexibility since we have available 3 couplings  $\lambda_i$  and 10 couplings  $\kappa_{ijk}$ . Thus hierarchies among couplings are allowed, still solving the  $\mu$ - and  $\nu$ -problems. For example, if  $\lambda_{1j}^{\text{nr}} \gtrsim 10^{-7}$  is used to solve the domain wall problem, with the corresponding renormalizable coupling  $\lambda_1 \lesssim 10^{-5}$  to avoid the tadpole problem, we can still have  $\lambda_{2,3} \lesssim 1$  available for solving the  $\mu$  problem. If instead we solve the domain wall problem e.g. with  $\kappa_{112l}^{\text{nr}} \gtrsim 10^{-9}$  with the corresponding  $\kappa_{112} \lesssim 10^{-3}$ , we can have the other couplings  $\kappa_{123} \lesssim 1$ ,  $\kappa_{113} \lesssim 1$ , etc., which are useful to solve the  $\nu$ -problem with the EW-scale seesaw of the  $\mu\nu$ SVM.

One might be worried that the presence of the non-renormalizable terms related to these large couplings of order one, e.g.  $\lambda_{2j}^{\text{nr}}$  or  $\kappa_{123j}^{\text{nr}}$ , would reintroduce the tadpole problem via diagrams as in Fig. 1. As already discussed in Sec. 2 not all gauge invariant renormalizable couplings have to be allowed in a particular model due to the stringy selection rules. Similarly, not all gauge invariant non-renormalizable couplings have to be allowed. For example, three untwisted fields in the  $Z_3$  orbifold of the heterotic string can only couple with a particular number of twisted fields, and three twisted fields can only couple with a particular number of twisted or moduli fields (see e.g. Ref. [73] and references therein). For other  $Z_n$  orbifolds the situation is similar. These peculiarities of the superpotential also carry over to the Kähler potential. For example, a non-minimal non-renormalizable Kähler term  $\Phi H_u H_d / M_{\text{Pl}} + \text{hc.}$  mixing the hidden ( $\Phi$ ) and visible ( $H_u, d$ ) sectors and generating after SUSY breaking a Giudice-Masiero  $\mu$ -term [74], is not present in prime orbifolds such as the  $Z_3$  orbifold [75,76]. Thus, embedding the  $\mu\nu\text{SSM}$  in the framework of superstrings, the stringy selections rules can forbid the dangerous non-renormalizable terms related to the above large couplings.

We conclude that dimension-4 terms in Eq. (8) with couplings  $\kappa_{ijkl}^{\text{nr}} \gtrsim 10^{-9}$  solve the domain wall problem, simultaneously avoiding the tadpole problem if the corresponding renormalizable couplings fulfill  $\kappa_{ijk} \lesssim 10^{-3}$ . Similarly, couplings  $\lambda_{ij}^{\text{nr}} \gtrsim 10^{-7}$  with the corresponding  $\lambda_i \lesssim 10^{-5}$  can also be used for this task.

On the other hand, for the case of 3-loop divergences the constraint becomes less stringent

$$c_4 c_3^3 \lesssim 10^{-10}, \quad (15)$$

where now  $c_4$  corresponds to  $\lambda_{ud}^{\text{nr}}$  with the corresponding bound shown in Table 1,  $\lambda_{ud}^{\text{nr}} \gtrsim 10^{-5}$ , and  $c_3^3$  to the product  $\kappa_{ijk} \lambda_j \lambda_k$ . Therefore, this constraint implies the bound  $\kappa_{ijk} \lambda_j \lambda_k \lesssim 10^{-5}$ , as shown also in Table 1. For example, assuming  $\kappa_{ijk} \sim \lambda_j \sim \lambda_k$ , this constraint would imply  $\kappa_{ijk}, \lambda_j, \lambda_k \lesssim 10^{-2}$ . Similarly to above, one could argue that depending on the concrete values of these renormalizable couplings, the other terms generating 2-loop divergences could still destabilize the hierarchy. But again, embedding the  $\mu\nu\text{SSM}$  in the framework of superstring, the non-renormalizable terms that generate 3-loop divergences could be allowed whereas the terms generating 2-loop divergences could be forbidden.

Finally, it is worth noting that the values of the couplings discussed above are compatible with the physical vacuum, including a SM-like Higgs which in turn is compatible with collider constraints [3,23,38]. In Refs. [3,23], it was shown that working with small and moderate values of  $\lambda_i$  and diagonal couplings  $\kappa_{iii}$ , i.e.  $\lambda_i, \kappa_{iii} \lesssim 10^{-1}$ , it is possible to find global electroweak vacua in significant regions of the parameter space of the  $\mu\nu\text{SSM}$ . In Ref. [38], the alignment-without-decoupling limit of the  $\mu\nu\text{SSM}$  was analyzed, which implies that at least one of the  $\lambda_i$  has a value of order one, close to be non-perturbative up to the GUT scale. In particular they used  $\lambda_1 \sim 0.5$ ,  $\lambda_2 \sim 0.4$  and  $\lambda_3 \sim 10^{-3}$ , showing that in this region of the parameter space, with all diagonal couplings  $\kappa_{iii} \sim 0.4$ , the possibility of long-life unstable electroweak vacua becomes viable. In all these results above, negligible off-diagonal couplings  $\kappa_{ijk}$  were used. Thus, the use in the first and third columns of Table 1 of off-diagonal couplings  $\kappa_{ijk}$  is trivially compatible with the physical minima studied in Refs. [3,23,38]. For example, in order to apply the solution of the third column to the vacua of Ref. [38], one can use  $\lambda_1 \sim 0.5$ ,  $\lambda_2 \sim 0.4$ , and  $\kappa_{i12} \lesssim 10^{-4}$ . Diagonal couplings are also compatible, for example one could use  $\kappa_{333} \sim 0.4$  with  $\lambda_3 \sim 10^{-3}$ . On the other hand, the solution of the second column of Table 1 can be present in the vacua of Refs. [3,23]. Also the solution of the first column with diagonal  $\kappa_{iii}$ .

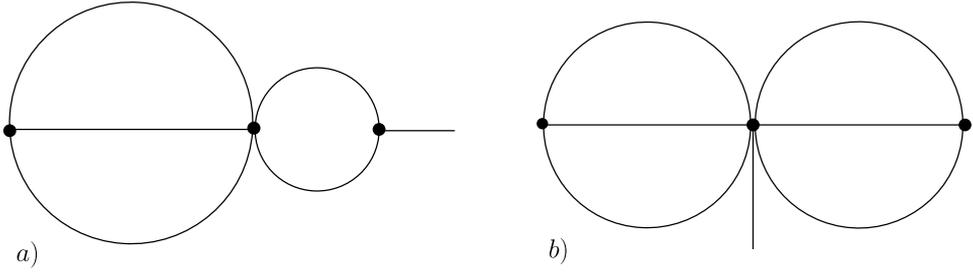


Fig. 2. Tadpole diagrams for (a) dimension-5 terms in the superpotential, and (b) dimension-7 terms in the superpotential.

#### 4.2. Non-renormalizable terms of higher dimensions

We will use here the above simple argument about the flexibility of string constructions concerning the presence of certain couplings, in the context of another solution proposed to solve simultaneously the destabilization and domain wall problems. In particular, it was shown in the framework of the NMSSM [53] that any extra odd-dimension terms in the superpotential (or even-dimension terms in the Kähler potential) are not harmful to the gauge hierarchy, but can avoid the domain wall problem. In the framework of the  $\mu\nu$ S SM these are the following dimension-5 terms with the contribution of RH neutrino superfields:

$$c_5 (v^c)^5 / M_{\text{Pl}}^2, \quad c_5 (v^c)^3 (H_u H_d) / M_{\text{Pl}}^2, \quad c_5 v^c (H_u H_d)^2 / M_{\text{Pl}}^2, \quad (16)$$

and the terms

$$c_5 (v^c)^3 (H_u L) / M_{\text{Pl}}^2, \quad c_5 v^c (H_u L)^2 / M_{\text{Pl}}^2, \quad c_5 v^c (H_u L) (H_u H_d) / M_{\text{Pl}}^2, \quad (17)$$

with the contribution of lepton doublet superfields. In these formulas we have simplified again the notation, denoting  $v^c \equiv v_i^c$ ,  $L \equiv L_i$ , and by  $c_5$  all non-renormalizable couplings. These terms give rise to a divergence at 3-loop order when SUSY is spontaneously broken, as shown in Fig. 2a.

In the case of dimension-7 terms, these are

$$c_7 (v^c)^7 / M_{\text{Pl}}^4, \quad c_7 (v^c)^5 (H_u H_d) / M_{\text{Pl}}^4, \quad c_7 (v^c)^3 (H_u H_d)^2 / M_{\text{Pl}}^4, \quad c_7 v^c (H_u H_d)^3 / M_{\text{Pl}}^4, \quad (18)$$

and

$$c_7 (v^c)^5 (H_u L) / M_{\text{Pl}}^4, \quad c_7 (v^c)^3 (H_u L)^2 / M_{\text{Pl}}^4, \quad c_7 v^c (H_u L)^3 / M_{\text{Pl}}^4, \\ c_7 v^c (H_u L)^2 (H_u H_d) / M_{\text{Pl}}^4, \quad c_7 v^c (H_u L) (H_u H_d)^2 / M_{\text{Pl}}^4, \quad (19)$$

giving rise to a divergence at 4-loop order as shown in Fig. 2b, except the fourth term in Eq. (18), and the last three terms in Eq. (19), which give rise to divergences at 5-loop order.

For example, for the dimension-7 terms the quartically divergent 4-loop integral of  $\mathcal{O}(M_{\text{Pl}}^4 / (16\pi^2)^4)$  generates in the effective potential a linear term of the form

$$\delta V \sim \frac{c_7 c_3^2}{(16\pi^2)^4} m_{3/2}^3 (\tilde{\nu}_R + \tilde{\nu}_R^*), \quad (20)$$

which is obviously not harmful to the gauge hierarchy, but large enough to break the  $Z_3$  symmetry (see Eq. (3)), and therefore to eliminate the domain wall problem. Similar results are obtained for other odd-dimension terms, but with the corresponding  $n$ -loop factors  $(16\pi^2)^{-n}$  in Eq. (20).

In the framework of the NMSSM supergravity, it was further argued that the dangerous even (odd)-dimension terms could be forbidden in the superpotential (Kähler potential) under special conditions. In particular, in Ref. [53] it was shown that this is possible when the models possess either target space duality in a string effective action or gauged  $R$ -symmetry, and they advocated finally for generating  $\mu$ -terms from couplings in the Kähler potential leading to small terms,  $B\mu H_u H_d$  and  $B'\mu' SS$ , in the low-energy theory. In Ref. [54], a  $Z_2$   $R$ -symmetry was used, allowing only the dimension-7 term  $c_7 S^7/M_{\text{Pl}}^4$ . A similar approach was used in Ref. [77] to attack the domain wall problem in the NMSSM extended by an  $A_4 \times Z_3$  flavor symmetry and three RH neutrinos. On the other hand, in Ref. [55] the discrete  $Z_5$   $R$ -symmetry used gave rise to the proposal of the new Minimally-extended Supersymmetric Standard Model (mNSSM) [78]. In this case, the cubic term  $\kappa SSS$  is forbidden by the symmetry, and its role contributing to generate the VEV of the scalar component  $S$  and the breaking of the Peccei-Quinn-like symmetry is played by the tadpole. The latter arises at 6-loop level in the effective potential by combining two non-renormalizable dimension-4 terms of the Kähler potential with the renormalizable superpotential term  $c_3 SH_u H_d$ , and is of order  $c_4^2 c_3^4 m_{3/2}^2 M_{\text{Pl}}^4 (S + S^*) / (16\pi^2)^6$ . The authors adjust  $c_4^2 c_3^4 \sim 10^{-3}$  for the tadpole to have the desired SUSY scale. A similar model using discrete  $Z_5$  and  $Z_7$   $R$ -symmetries, but with the tadpole contribution appearing in the superpotential, was proposed in Ref. [79] with the name of Minimal Non-minimal Supersymmetric Standard Model (MNSSM).<sup>4</sup>

In the case of the  $\mu\nu$ SMS supergravity from superstrings, one has in addition the freedom of playing with the presence or absence of particular types of non-renormalizable terms. This is model dependent, and therefore one can assume a model where only harmless odd (even)-dimension terms are allowed in the superpotential (Kähler potential) by the stringy selection rules, generating contributions in the effective potential of the type (20). These contributions are small enough as not to alter the minimization equations for the right sneutrinos, and therefore one can continue working with the conventional superpotential of the  $\mu\nu$ SMS in Eq. (1), i.e. their impact on the phenomenology can be neglected.

If the non-renormalizable terms allowed by the stringy selection rules are of the type proposed in Ref. [55], then the tadpole contribution in the  $\mu\nu$ SMS is of the form

$$\delta V \sim \frac{c_4^2 c_3^4}{(16\pi^2)^6} m_{3/2}^2 M_{\text{Pl}}^4 (\tilde{\nu}_R + \tilde{\nu}_R^*), \quad (21)$$

where we have exchanged  $S \rightarrow \nu^c$ . In this case one expects  $c_4^2 c_3^4 \ll 10^{-3}$ , being this factor suppressed by a non-renormalizable squared coupling ( $c_4^2$ ), and therefore this tadpole contribution solves the domain wall problem, but again without modifying the  $\mu\nu$ SMS phenomenology. Even if one manages to have in a concrete model  $c_4^2 c_3^4 \sim 10^{-3}$ , this contribution will not essentially modify our numerical results either since we are assuming that the stringy selection rules allow the presence of the cubic term  $\kappa_{ijk} \nu_i^c \nu_j^c \nu_k^c$  in the superpotential (1). If this is not the case and it is not present, then to account for neutrino data becomes more involved.

### 4.3. Non-perturbative terms

In type II superstring another mechanism to solve the domain wall and tadpole problems is available. As discussed in Refs. [84,85], the singlet of the NMSSM carries global  $U(1)$  charge

<sup>4</sup> It is worth noting however that global symmetries are most likely broken by quantum gravity effects [80,81] (for a review, see e.g. Ref. [82] and references therein). This is also one of the swampland conjectures (for a recent discussion, see e.g. Ref. [83] and references therein).

under the branes, and therefore perturbative couplings with several singlets are forbidden. Nevertheless, a D-instanton can generate in the superpotential non-perturbatively terms of the type

$$W_{\text{np}} \sim e^{-S^{\text{cl}}} M_s^{3-n} S^n \equiv c_n S^n, \tag{22}$$

where  $M_s$  is the string scale, and the instanton suppression factor given by the classical instanton effective action  $S^{\text{cl}}$  depends on the volume of the three-cycle which the instanton wraps. In Refs. [86–90], similar instanton effects were used to generate a Majorana mass term for RH neutrinos with  $n = 2$ .

Then, in the case of the  $\mu\nu\text{SSM}$  a term with  $n = 1$  and  $S \rightarrow \nu^c$

$$W_{\text{np}} \sim e^{-S^{\text{cl}}} M_s^2 \nu^c, \tag{23}$$

can solve the domain wall problem without spoiling the hierarchy if the exponential suppression factor is such that  $e^{-S^{\text{cl}}} M_s^2 \lesssim m_{3/2}^2$ .

### 5. Generating the cubic coupling $(\nu^c)^3$

We have discussed in the previous sections the importance of the presence of RH neutrinos in the low-energy spectrum of the  $\mu\nu\text{SSM}$  for solving the  $\mu$ - and  $\nu$ -problems, simultaneously avoiding the potential domain wall and tadpole problems. We would like to discuss in this section another characteristic of RH neutrinos. When added to the SM spectrum they are the only fields with no quantum numbers under the gauge group, however in the heterotic string any SM singlet arises from the  $E_8 \times E_8$  or  $SO(32)$  gauge groups and as a consequence must be charged under some of the groups. This implies that terms of the type  $(\nu^c)^3$  in the superpotential (1) are expected to be forbidden by gauge invariance. See also the discussion in Sec. 4.3 for the type II superstring, where they are also forbidden. On the other hand, as discussed in the Introduction, this type of terms is very useful for solving the  $\nu$ -problem in a natural way through the generation of Majorana masses. We also saw in Secs. 3 and 4 that their presence can be helpful for solving the domain wall and tadpole problems. Let us then analyze how this type of terms can originate in the superstring framework. In Ref. [91], the case of the singlet in the NMSSM was analyzed.

In orbifold compactifications of the heterotic string, models with  $SU(3) \times SU(2) \times U(1)^n \times [\text{hidden sector}]$  as gauge group after the breaking due to the presence of Wilson lines, were built [92]. The matter content of these models is initially very large, including extra representations in addition to SM ones, with typically many singlets under the SM gauge group. All these representations have non-vanishing  $U(1)^n$  charges, including the RH neutrinos if present.

Subsequently, the Fayet-Iliopoulos (FI)  $D$ -term of the “anomalous”  $U(1)^{(a)}$  [93–96] arising from a combination of the  $U(1)$ ’s of the models was used for breaking the gauge group further [60]. In particular, the anomaly is cancelled due to the Green-Schwarz mechanism [97], which determines the following value of the FI  $D$ -term:

$$D^{(a)} = c^{(a)} + \sum_{\alpha} Q_{\alpha}^{(a)} \eta_{\alpha}^* \eta_{\alpha}, \quad c^{(a)} = \frac{\sum_{\alpha} Q_{\alpha}^{(a)}}{192\pi^2} g^{(a)2} M_P^2, \tag{24}$$

where  $\eta_{\alpha}$  are the scalar fields with charges  $Q_{\alpha}^{(a)}$  under the anomalous  $U(1)^{(a)}$  and  $M_P \equiv M_{\text{Pl}}/\sqrt{8\pi}$ . Therefore, in order to have a SUSY vacuum solution it is necessary to give VEVs to particular scalar fields, say  $\chi_{\beta}$ , in such a way that the anomalous  $D$ -term is cancelled:

$$\sum_{\beta} Q_{\beta}^{(a)} |\langle \chi_{\beta} \rangle|^2 = -c^{(a)}. \tag{25}$$

If these fields are charged under the  $U(1)^n$ , except under the  $U(1)_Y$  which arises from a combination of the non-anomalous  $U(1)$ 's, then we are left at low energies with the SM gauge group in the observable sector as desired. In addition, many of the extra matter representations acquire a high mass thus disappearing from the low-energy spectrum, giving rise to an observable sector with the usual three families of matter (plus a few extra representations) [60–62,98]. These extra fields are related to solutions for the generation of terms of the type  $(\hat{\nu}^c)^3$ , as we will discuss below.

### 5.1. Mixing among fields

The extra matter representations discussed above acquire their masses from terms of the type:

$$(\chi \dots \chi)\zeta, \quad (\chi \dots \chi)\zeta\zeta, \quad (26)$$

where  $\chi$  means any field acquiring a large VEV in order to cancel the FI  $D$ -term, and  $\zeta$  is any field. (Of course higher-order operators must also be taken into account here.) The calculation of the massless spectrum shows that some of the massless fields are combinations of several states [60–62]. As a consequence, some of the “old” physical particles have combined with other ones, and the “new” particles may have more couplings allowed. This mechanism, relying on the mixing between fields due to FI breaking was used in Ref. [99] for generating the observed structure of quark and lepton masses and mixing angles. In the case that we are interested here, for example  $\nu^c \rightarrow \nu'^c = (a\nu^c + b\varphi^c + c\psi^c)/\sqrt{a^2 + b^2 + c^2}$ , where  $a, b, c$  are proportional to some VEVs, and  $\varphi$  and  $\psi$  are other singlets under  $SU(3) \times SU(2)$ . Therefore, a term of the type  $(\nu'^c)^3$  becomes present in the superpotential if the term  $\nu^c\varphi^c\psi^c$  is initially allowed.

### 5.2. Non-renormalizable couplings

A second mechanism to generate  $(\nu^c)^3$  in the superpotential is the following. An effective coupling of massless fields which is not  $U(1)^n$  invariant, say  $\zeta_1\zeta_2\zeta_3$ , can be generated by a higher-order operator

$$(\chi \dots \chi)\zeta_1\zeta_2\zeta_3, \quad (27)$$

where as in (26)  $\chi$  means any field acquiring a VEV. (Of course one has to consider here the “new” massless fields again.) One would expect this type of couplings that can generate  $(\nu^c)^3$  as desired, to be very small being non-renormalizable and therefore suppressed by the Planck mass, but actually this is not necessarily what happens. Given that typically  $\sum_\alpha Q_\alpha^{(a)} \gtrsim 100$  in the models built, one obtains from Eqs. (24) and (25) that  $\langle \chi \rangle / M_{\text{Pl}} \lesssim 1$  and as a consequence effective terms  $(\nu^c)^3$  of the desired order of magnitude might be generated in the superpotential of the  $\mu\nu\text{SSM}$ . Of course, another consequence of these mechanisms is that the non-renormalizable terms discussed in the previous sections to solve the domain wall problem must be effectively generated in a similar way.

Related to the above, let us comment the following. All the other terms in the superpotential of the  $\mu\nu\text{SSM}$  (1) can be allowed in principle at the renormalizable level. In that case, the simultaneous presence of the terms  $Y_{ij}^\nu H_u L_i \nu_j^c$  and  $\lambda_i H_u H_d \nu_i^c$  implies that  $H_d$  and  $L_i$  have the same  $SU(3) \times SU(2) \times U(1)^n$  quantum numbers. This seems not to be easy to obtain in the model building since one needs not only two different fields with exactly the same quantum numbers but also having different values for their couplings. We can avoid this situation if any of the above

two terms arises from a non-renormalizable operator which is  $U(1)^n$  invariant. In addition, this mechanism also implies that the conventional lepton-number-violating terms  $\lambda_{ijk} L_i L_j e_k^c$  and  $\lambda'_{ijk} L_i Q_j d_k^c$  are forbidden (at least at the renormalizable level) once we impose the presence of the Yukawa couplings  $Y_{ij}^e H_d L_i e_j^c$  and  $Y_{ij}^d H_d Q_i d_j^c$  in the superpotential.

### 5.3. Instanton effects

As discussed in Sec. 4.3, instanton effects in the type II superstring can generate terms coupling NMSSM singlets among themselves (see Eq. (22)) [84,85]. In Ref. [100], similar instanton effects were used to generate the cubic coupling of the NMSSM in F-theory GUTs. This mechanism, which lies in the violation of the (typically anomalous)  $U(1)$ 's under which  $S$  is charged by the instanton corrections [84], can be straightforwardly applied to the singlet  $v^c$  of the  $\mu\nu$ SSM.

Thus, one can generate couplings of the type

$$W_{\text{np}} \sim e^{-S^{\text{cl}}} (v^c)^3 \equiv \kappa (v^c)^3, \tag{28}$$

with  $\kappa \lesssim 1$ , if one manages to construct a model with  $n = 3$  and the appropriate suppression factor. In fact, the presence of several  $v_i^c$  superfields in the  $\mu\nu$ SSM, in contrast to the NMSSM which contains only one singlet superfield  $S$  makes easier to generate from this mechanism the parameters  $\kappa$  (and  $\lambda$ ) capable to generate a viable mass spectrum. Besides, the simultaneous presence of other instanton-induced terms with differing  $n$ , such as the tadpole term in (23) with  $n = 1$ , can be achieved [84] by superpotential corrections from multiple instantons with different intersection numbers with the gauge branes.

## 6. Conclusions

We have discussed the domain wall and tadpole problems in the  $\mu\nu$ SSM, where RH neutrinos are added to the SM spectrum. Their solutions present peculiarities with respect to the usual ones found in the NMSSM. An important argument concerns the presence of several different couplings involving singlet superfields in the  $\mu\nu$ SSM (the RH neutrinos), unlike the NMSSM where only one extra singlet is present. In addition, embedding the  $\mu\nu$ SSM in the framework of superstring models makes that not all gauge invariant terms are necessarily allowed, providing us with more flexibility for only the interesting terms to be present.

In particular, dimension-4 non-renormalizable terms in the superpotential can be large enough as to solve the domain wall problem through their contributions to the effective potential. Although they also generate tadpole divergences giving rise to linear terms in the effective potential, these contributions can be small enough for several ranges of the couplings avoiding the destabilization of the hierarchy, as discussed in Sec. 4.1.

Alternatively, terms of higher dimensions, odd (even) in the superpotential (the Kähler potential), can also be used to solve these problems in the superstring inspired  $\mu\nu$ SSM without relying in arguments based on target space duality or  $R$ -symmetry. Unlike dimension-4 terms, their contributions to the effective potential are not large enough to solve the domain wall problem (unless the cut-off scale of the high-energy theory is smaller than the Planck scale). Nevertheless, the linear contributions generated from tadpole divergences can be large enough to solve the problem without destabilizing the hierarchy, as explained in Sec. 4.2.

In addition, tadpole terms in the superpotential of the correct order of magnitude to solve the problems can also be generated by instanton effects in type II superstring or F-theory, as discussed in Sec. 4.3.

Finally, given the special role of RH neutrinos in the  $\mu\nu$ SSM for solving the  $\mu$ - and  $\nu$ -problems, simultaneously avoiding the potential domain wall and tadpole problems, we have analyzed one of their relevant features in string models. In particular, they are expected to have typically extra  $U(1)$  charges at high energies. We have discussed in Sec. 5 several mechanisms that still allow the presence of cubic neutrino terms in the low-energy theory, such as the mixing between fields due to Fayet-Iliopoulos breaking, the presence of appropriate non-renormalizable terms, or instanton effects. The presence of cubic neutrino terms together with Dirac neutrino Yukawas, makes very easy the generation of a EW-scale seesaw able to reproduce neutrino masses and mixings in agreement with data. We thus highly motivate our string model builder colleagues to take account of this option seriously.

### CRedit authorship contribution statement

All authors have contributed to the work in the same way.

### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

### Data availability

No data was used for the research described in the article.

### Acknowledgements

We thank Ángel Uranga for useful conversations about instanton-induced operators in strings. The work of D.L. was supported by the Argentinian CONICET, and also through PIP 11220170100154CO. The work of C.M. was partially supported by the Spanish Research Agency AEI through grants IFT Centro de Excelencia Severo Ochoa CEX2020-001007-S, PGC2018-095161-B-I00 and PID2021-125331NB-I00; All of them are funded by MCIN/AEI/10.13039/501100011033 and the second grant also by ERDF “A way of making Europe”.

### References

- [1] D.E. López-Fogliani, C. Muñoz, Proposal for a supersymmetric standard model, *Phys. Rev. Lett.* 97 (2006) 041801, arXiv:hep-ph/0508297 [hep-ph].
- [2] D.E. Lopez-Fogliani, C. Muñoz, Searching for supersymmetry: the  $\mu\nu$ SSM (a short review), *Eur. Phys. J. Spec. Top.* 229 (21) (2020) 3263–3301, arXiv:2009.01380 [hep-ph].
- [3] N. Escudero, D.E. López-Fogliani, C. Muñoz, R.R. de Austri, Analysis of the parameter space and spectrum of the  $\mu\nu$ SSM, *J. High Energy Phys.* 12 (2008) 099, arXiv:0810.1507 [hep-ph].
- [4] J.A. Aguilar-Saavedra, I. Lara, D.E. Lopez-Fogliani, C. Muñoz,  $U(1)'$  extensions of the  $\mu\nu$ SSM, *Eur. Phys. J. C* 81 (5) (2021) 443, arXiv:2101.05565 [hep-ph].
- [5] J.E. Kim, H.P. Nilles, The  $\mu$  problem and the strong CP problem, *Phys. Lett. B* 138 (1984) 150–154.
- [6] K.J. Bae, H. Baer, V. Barger, D. Sengupta, Revisiting the SUSY  $\mu$  problem and its solutions in the LHC era, *Phys. Rev. D* 99 (11) (2019) 115027, arXiv:1902.10748 [hep-ph].
- [7] M. Maniatis, The next-to-minimal supersymmetric extension of the Standard Model reviewed, *Int. J. Mod. Phys. A* 25 (2010) 3505–3602, arXiv:0906.0777 [hep-ph].

- [8] U. Ellwanger, C. Hugonie, A.M. Teixeira, The next-to-minimal supersymmetric standard model, *Phys. Rep.* 496 (2010) 1–77, arXiv:0910.1785 [hep-ph].
- [9] R. Barbier, et al., R-parity violating supersymmetry, *Phys. Rep.* 420 (2005) 1–202, arXiv:hep-ph/0406039 [hep-ph].
- [10] F. Capozzi, E. Di Valentino, E. Lisi, A. Marrone, A. Melchiorri, A. Palazzo, Global constraints on absolute neutrino masses and their ordering, *Phys. Rev. D* 95 (9) (2017) 096014, arXiv:1703.04471 [hep-ph].
- [11] P.F. de Salas, D.V. Forero, C.A. Ternes, M. Tortola, J.W.F. Valle, Status of neutrino oscillations 2018:  $3\sigma$  hint for normal mass ordering and improved CP sensitivity, *Phys. Lett. B* 782 (2018) 633–640, arXiv:1708.01186 [hep-ph].
- [12] P.F. De Salas, S. Gariazzo, O. Mena, C.A. Ternes, M. Tórtola, Neutrino mass ordering from oscillations and beyond: 2018 status and future prospects, *Front. Astron. Space Sci.* 5 (2018) 36, arXiv:1806.11051 [hep-ph].
- [13] I. Esteban, M.C. Gonzalez-Garcia, A. Hernandez-Cabezudo, M. Maltoni, T. Schwetz, Global analysis of three-flavour neutrino oscillations: synergies and tensions in the determination of  $\theta_{23}$ ,  $\delta_{CP}$ , and the mass ordering, *J. High Energy Phys.* 01 (2019) 106, arXiv:1811.05487 [hep-ph].
- [14] P. Ghosh, S. Roy, Neutrino masses and mixing, lightest neutralino decays and a solution to the  $\mu$  problem in supersymmetry, *J. High Energy Phys.* 04 (2009) 069, arXiv:0812.0084 [hep-ph].
- [15] A. Bartl, M. Hirsch, A. Vicente, S. Liebler, W. Porod, LHC phenomenology of the  $\mu\nu$ SSM, *J. High Energy Phys.* 05 (2009) 120, arXiv:0903.3596 [hep-ph].
- [16] J. Fidalgo, D.E. López-Fogliani, C. Muñoz, R.R. de Austri, Neutrino physics and spontaneous CP violation in the  $\mu\nu$ SSM, *J. High Energy Phys.* 08 (2009) 105, arXiv:0904.3112 [hep-ph].
- [17] P. Ghosh, P. Dey, B. Mukhopadhyaya, S. Roy, Radiative contribution to neutrino masses and mixing in  $\mu\nu$ SSM, *J. High Energy Phys.* 05 (2010) 087, arXiv:1002.2705 [hep-ph].
- [18] S. Liebler, W. Porod, On-shell renormalization of neutralino and chargino mass matrices in R-parity violating models - correlation between LSP decays and neutrino mixing angles revisited, *Nucl. Phys. B* 855 (2012) 774–800, arXiv:1106.2921 [hep-ph].
- [19] J. Fidalgo, D.E. López-Fogliani, C. Muñoz, R. Ruiz de Austri, The Higgs sector of the  $\mu\nu$ SSM and collider physics, *J. High Energy Phys.* 10 (2011) 020, arXiv:1107.4614 [hep-ph].
- [20] P. Ghosh, D.E. López-Fogliani, V.A. Mitsou, C. Muñoz, R. Ruiz de Austri, Probing the  $\mu\nu$ SSM with light scalars, pseudoscalars and neutralinos from the decay of a SM-like Higgs boson at the LHC, *J. High Energy Phys.* 11 (2014) 102, arXiv:1410.2070 [hep-ph].
- [21] T. Biekotter, S. Heinemeyer, C. Muñoz, Precise prediction for the Higgs-boson masses in the  $\mu\nu$ SSM, *Eur. Phys. J. C* 78 (6) (2018) 504, arXiv:1712.07475 [hep-ph].
- [22] T. Biekotter, S. Heinemeyer, C. Muñoz, Precise prediction for the Higgs-Boson masses in the  $\mu\nu$ SSM with three right-handed neutrino superfields, *Eur. Phys. J. C* 79 (8) (2019) 667, arXiv:1906.06173 [hep-ph].
- [23] E. Kpatcha, D.E. López-Fogliani, C. Muñoz, R. Ruiz de Austri, Impact of Higgs physics on the parameter space of the  $\mu\nu$ SSM, *Eur. Phys. J. C* 80 (4) (2020) 336, arXiv:1910.08062 [hep-ph].
- [24] P. Ghosh, I. Lara, D.E. López-Fogliani, C. Muñoz, R. Ruiz de Austri, Searching for left sneutrino LSP at the LHC, *Int. J. Mod. Phys. A* 33 (18n19) (2018) 1850110, arXiv:1707.02471 [hep-ph].
- [25] I. Lara, D.E. López-Fogliani, C. Muñoz, N. Nagata, H. Otono, R. Ruiz De Austri, Looking for the left sneutrino LSP with displaced-vertex searches, *Phys. Rev. D* 98 (7) (2018) 075004, arXiv:1804.00067 [hep-ph].
- [26] I. Lara, D.E. López-Fogliani, C. Muñoz, Electroweak superpartners scrutinized at the LHC in events with multi-leptons, *Phys. Lett. B* 790 (2019) 176–183, arXiv:1810.12455 [hep-ph].
- [27] E. Kpatcha, I. Lara, D.E. López-Fogliani, C. Muñoz, N. Nagata, H. Otono, R. Ruiz De Austri, Sampling the  $\mu\nu$ SSM for displaced decays of the tau left sneutrino LSP at the LHC, *Eur. Phys. J. C* 79 (11) (2019) 934, arXiv:1907.02092 [hep-ph].
- [28] E. Kpatcha, I. Lara, D.E. López-Fogliani, C. Muñoz, N. Nagata, Explaining muon  $g - 2$  data in the  $\mu\nu$ SSM, *Eur. Phys. J. C* 81 (2) (2021) 154, arXiv:1912.04163 [hep-ph].
- [29] S. Heinemeyer, E. Kpatcha, I. Lara, D.E. López-Fogliani, C. Muñoz, N. Nagata, The new  $(g - 2)_\mu$  result and the  $\mu\nu$ SSM, *Eur. Phys. J. C* 81 (9) (2021) 802, arXiv:2104.03294 [hep-ph].
- [30] E. Kpatcha, I. Lara, D.E. López-Fogliani, C. Muñoz, N. Nagata, H. Otono, Searching for stop LSP at the LHC, *Eur. Phys. J. C* 82 (3) (2022) 261, arXiv:2111.13212 [hep-ph].
- [31] K.-Y. Choi, D.E. Lopez-Fogliani, C. Muñoz, R.R. de Austri, Gamma-ray detection from gravitino dark matter decay in the  $\mu\nu$ SSM, *J. Cosmol. Astropart. Phys.* 03 (2010) 028, arXiv:0906.3681 [hep-ph].
- [32] G.A. Gomez-Vargas, M. Fornasa, F. Zandanel, A.J. Cuesta, C. Muñoz, F. Prada, G. Yepes, CLUES on Fermi-LAT prospects for the extragalactic detection of  $\mu\nu$ SSM gravitino dark matter, *J. Cosmol. Astropart. Phys.* 02 (2012) 001, arXiv:1110.3305 [astro-ph.HE].

- [33] A. Albert, G. Gomez-Vargas, M. Grefe, C. Muñoz, C. Weniger, et al., Search for 100 MeV to 10 GeV  $\gamma$ -ray lines in the Fermi-LAT data and implications for gravitino dark matter in  $\mu\nu$ SSM, *J. Cosmol. Astropart. Phys.* 10 (2014) 023, arXiv:1406.3430 [astro-ph.HE].
- [34] G. Gomez-Vargas, D.E. Lopez-Fogliani, C. Muñoz, A.D. Perez, R.R. de Austri, Search for sharp and smooth spectral signatures of  $\mu\nu$ SSM gravitino dark matter with Fermi-LAT, *J. Cosmol. Astropart. Phys.* 1703 (03) (2017) 047, arXiv:1608.08640 [hep-ph].
- [35] G.A. Gómez-Vargas, D.E. López-Fogliani, C. Muñoz, A.D. Perez, MeV-GeV  $\gamma$ -ray telescopes probing gravitino LSP with coexisting axino NLSP as dark matter in the  $\mu\nu$  SSM, *Astropart. Phys.* 125 (2021) 102506, arXiv:1911.08550 [hep-ph].
- [36] G.A. Gómez-Vargas, D.E. López-Fogliani, C. Muñoz, A.D. Perez, MeV-GeV  $\gamma$ -ray telescopes probing axino LSP/gravitino NLSP as dark matter in the  $\mu\nu$ SSM, *J. Cosmol. Astropart. Phys.* 2001 (01) (2020) 058, arXiv:1911.03191 [hep-ph].
- [37] D.J.H. Chung, A.J. Long, Electroweak phase transition in the  $\mu\nu$ SSM, *Phys. Rev. D* 81 (2010) 123531, arXiv:1004.0942 [hep-ph].
- [38] T. Biekötter, S. Heinemeyer, G. Weiglein, Vacuum (meta-)stability in the  $\mu\nu$ SSM, *Eur. Phys. J. C* 82 (4) (2022) 301, arXiv:2112.12132 [hep-ph].
- [39] Muon g-2 Collaboration, B. Abi, et al., Measurement of the positive muon anomalous magnetic moment to 0.46 ppm, *Phys. Rev. Lett.* 126 (14) (2021) 141801, arXiv:2104.03281 [hep-ex].
- [40] Muon g-2 Collaboration, T. Albahri, et al., Measurement of the anomalous precession frequency of the muon in the Fermilab Muon  $g - 2$  Experiment, *Phys. Rev. D* 103 (7) (2021) 072002, arXiv:2104.03247 [hep-ex].
- [41] Y.B. Zeldovich, I.Y. Kobzarev, L.B. Okun, Cosmological consequences of the spontaneous breakdown of discrete symmetry, *Zh. Eksp. Teor. Fiz.* 67 (1974) 3–11.
- [42] T.W.B. Kibble, Topology of cosmic domains and strings, *J. Phys. A* 9 (1976) 1387–1398.
- [43] T.W.B. Kibble, Some implications of a cosmological phase transition, *Phys. Rep.* 67 (1980) 183.
- [44] A. Vilenkin, Cosmic strings and domain walls, *Phys. Rep.* 121 (1985) 263–315.
- [45] J.R. Ellis, K. Enqvist, D.V. Nanopoulos, K.A. Olive, M. Quiros, F. Zwirner, Problems for (2,0) compactifications, *Phys. Lett. B* 176 (1986) 403–408.
- [46] B. Rai, G. Senjanovic, Gravity and domain wall problem, *Phys. Rev. D* 49 (1994) 2729–2733, arXiv:hep-ph/9301240.
- [47] S. Abel, S. Sarkar, P. White, On the cosmological domain wall problem for the minimally extended supersymmetric standard model, *Nucl. Phys. B* 454 (1995) 663–684, arXiv:hep-ph/9506359.
- [48] S.A. Abel, P.L. White, Baryogenesis from domain walls in the next-to-minimal supersymmetric standard model, *Phys. Rev. D* 52 (1995) 4371–4379, arXiv:hep-ph/9505241.
- [49] U. Ellwanger, Nonrenormalizable interactions from supergravity, quantum corrections and effective low-energy theories, *Phys. Lett. B* 133 (1983) 187–191.
- [50] J. Bagger, E. Poppitz, Destabilizing divergences in supergravity coupled supersymmetric theories, *Phys. Rev. Lett.* 71 (1993) 2380–2382, arXiv:hep-ph/9307317.
- [51] V. Jain, On destabilizing divergencies in supergravity models, *Phys. Lett. B* 351 (1995) 481–486, arXiv:hep-ph/9407382.
- [52] J. Bagger, E. Poppitz, L. Randall, Destabilizing divergences in supergravity theories at two loops, *Nucl. Phys. B* 455 (1995) 59–82, arXiv:hep-ph/9505244.
- [53] S. Abel, Destabilizing divergences in the NMSSM, *Nucl. Phys. B* 480 (1996) 55–72, arXiv:hep-ph/9609323.
- [54] C. Panagiotakopoulos, K. Tamvakis, Stabilized NMSSM without domain walls, *Phys. Lett. B* 446 (1999) 224–227, arXiv:hep-ph/9809475.
- [55] C. Panagiotakopoulos, K. Tamvakis, New minimal extension of MSSM, *Phys. Lett. B* 469 (1999) 145–148, arXiv:hep-ph/9908351.
- [56] H.P. Nilles, Supersymmetry, supergravity and particle physics, *Phys. Rep.* 110 (1984) 1–162.
- [57] H.E. Haber, G.L. Kane, The search for supersymmetry: probing physics beyond the standard model, *Phys. Rep.* 117 (1985) 75–263.
- [58] S.P. Martin, A supersymmetry primer, arXiv:hep-ph/9709356 [hep-ph], *Adv. Ser. Dir. High Energy Phys.* 18 (1998) 1.
- [59] L.J. Dixon, J.A. Harvey, C. Vafa, E. Witten, Strings on orbifolds, *Nucl. Phys. B* 261 (1985) 678–686.
- [60] J. Casas, E. Katehou, C. Muñoz,  $U(1)$  charges in orbifolds: anomaly cancellation and phenomenological consequences, *Nucl. Phys. B* 317 (1989) 171–186.
- [61] J.A. Casas, C. Muñoz, Three generation  $SU(3) \times SU(2) \times U(1)_Y$  models from orbifolds, *Phys. Lett. B* 214 (1988) 63–69.

- [62] J.A. Casas, C. Muñoz, Yukawa couplings in  $SU(3) \times SU(2) \times U(1)_Y$  orbifold models, *Phys. Lett. B* 212 (1988) 343–350.
- [63] M. Berasaluce-Gonzalez, L.E. Ibañez, P. Soler, A.M. Uranga, Discrete gauge symmetries in D-brane models, *J. High Energy Phys.* 12 (2011) 113, arXiv:1106.4169 [hep-th].
- [64] L.E. Ibañez, A.N. Schellekens, A.M. Uranga, Discrete gauge symmetries in discrete MSSM-like orientifolds, *Nucl. Phys. B* 865 (2012) 509–540, arXiv:1205.5364 [hep-th].
- [65] J. Fidalgo, C. Muñoz, The  $\mu\nu$ S $\overline{S}$ M with an Extra  $U(1)$ , *J. High Energy Phys.* 04 (2012) 090, arXiv:1111.2836 [hep-ph].
- [66] V. Martín-Lozano, S. Oviedo-Casado, The non-Universal  $U(1)$  gauge extended  $\mu\nu$ S $\overline{S}$ M: anomalies cancellation and singular phenomenology, *J. High Energy Phys.* 09 (2018) 102, arXiv:1804.02378 [hep-ph].
- [67] P. Langacker, The physics of heavy  $Z'$  gauge bosons, *Rev. Mod. Phys.* 81 (2009) 1199–1228, arXiv:0801.1345 [hep-ph].
- [68] J. Casas, C. Muñoz, Three generation  $SU(3) \times SU(2) \times U(1)_Y \times U(1)$  orbifold models through Fayet-Iliopoulos terms, *Phys. Lett. B* 209 (1988) 214–220.
- [69] G. Lazarides, Q. Shafi, Axion models with no domain wall problem, *Phys. Lett. B* 115 (1982) 21–25.
- [70] T. Kibble, G. Lazarides, Q. Shafi, Walls bounded by strings, *Phys. Rev. D* 26 (1982) 435.
- [71] S.M. Barr, D. Reiss, A. Zee, Families, the invisible axion, and domain walls, *Phys. Lett. B* 116 (1982) 227–230.
- [72] L.E. Ibañez, C. Muñoz, S. Rigolin, Aspect of type I string phenomenology, *Nucl. Phys. B* 553 (1999) 43–80, arXiv:hep-ph/9812397.
- [73] J.A. Casas, C. Muñoz, Fermion masses and mixing angles: a test for string vacua, *Nucl. Phys. B* 332 (1990) 189–208. Erratum: *Nucl. Phys. B* 340 (1990) 280.
- [74] G.F. Giudice, A. Masiero, A natural solution to the  $\mu$  problem in supergravity theories, *Phys. Lett. B* 206 (1988) 480–484.
- [75] G. Lopes Cardoso, D. Lust, T. Mohaupt, Moduli spaces and target space duality symmetries in (0,2)  $Z(N)$  orbifold theories with continuous Wilson lines, *Nucl. Phys. B* 432 (1994) 68–108, arXiv:hep-th/9405002.
- [76] I. Antoniadis, S. Ferrara, E. Gava, K.S. Narain, T.R. Taylor, Perturbative prepotential and monodromies in  $N=2$  heterotic superstring, *Nucl. Phys. B* 447 (1995) 35–61, arXiv:hep-th/9504034.
- [77] M.A. Ouahid, M.A. Loulidi, R.A. Laamara, E.H. Saidi, Neutrino phenomenology in the flavored NMSSM without domain wall problems, *Phys. Rev. D* 102 (11) (2020) 115023, arXiv:1810.10753 [hep-ph].
- [78] A. Dedes, C. Hugonie, S. Moretti, K. Tamvakis, Phenomenology of a new minimal supersymmetric extension of the standard model, *Phys. Rev. D* 63 (2001) 055009, arXiv:hep-ph/0009125.
- [79] C. Panagiotakopoulos, A. Pilaftsis, Higgs scalars in the minimal nonminimal supersymmetric standard model, *Phys. Rev. D* 63 (2001) 055003, arXiv:hep-ph/0008268.
- [80] G. Gilbert, Wormhole induced proton decay, *Nucl. Phys. B* 328 (1989) 159–170.
- [81] L.M. Krauss, F. Wilczek, Discrete gauge symmetry in continuum theories, *Phys. Rev. Lett.* 62 (1989) 1221.
- [82] T. Banks, N. Seiberg, Symmetries and strings in field theory and gravity, *Phys. Rev. D* 83 (2011) 084019, arXiv:1011.5120 [hep-th].
- [83] M. van Beest, J. Calderón-Infante, D. Mirfendereski, I. Valenzuela, Lectures on the Swampland program in string compactifications, arXiv:2102.01111 [hep-th].
- [84] R. Blumenhagen, M. Cvetič, S. Kachru, T. Weigand, D-brane instantons in type II orientifolds, *Annu. Rev. Nucl. Part. Sci.* 59 (2009) 269–296, arXiv:0902.3251 [hep-th].
- [85] M. Cvetič, J. Halverson, P. Langacker, Singlet extensions of the MSSM in the quiver landscape, *J. High Energy Phys.* 09 (2010) 076, arXiv:1006.3341 [hep-th].
- [86] R. Blumenhagen, M. Cvetič, T. Weigand, Spacetime instanton corrections in 4D string vacua: the seesaw mechanism for D-brane models, *Nucl. Phys. B* 771 (2007) 113–142, arXiv:hep-th/0609191.
- [87] L.E. Ibañez, A.M. Uranga, Neutrino Majorana masses from string theory instanton effects, *J. High Energy Phys.* 03 (2007) 052, arXiv:hep-th/0609213.
- [88] M. Cvetič, R. Richter, T. Weigand, Computation of D-brane instanton induced superpotential couplings: Majorana masses from string theory, *Phys. Rev. D* 76 (2007) 086002, arXiv:hep-th/0703028.
- [89] L.E. Ibañez, A.N. Schellekens, A.M. Uranga, Instanton induced neutrino Majorana masses in CFT orientifolds with MSSM-like spectra, *J. High Energy Phys.* 06 (2007) 011, arXiv:0704.1079 [hep-th].
- [90] M. Cvetič, P. Langacker, D-instanton generated Dirac neutrino masses, *Phys. Rev. D* 78 (2008) 066012, arXiv:0803.2876 [hep-th].
- [91] O. Lebedev, S. Ramos-Sanchez, The NMSSM and string theory, *Phys. Lett. B* 684 (2010) 48–51, arXiv:0912.0477 [hep-ph].
- [92] L.E. Ibañez, J.E. Kim, H.P. Nilles, F. Quevedo, Orbifold compactifications with three families of  $SU(3) \times SU(2) \times U(1)^n$ , *Phys. Lett. B* 191 (1987) 282–286.

- [93] E. Witten, Some properties of  $O(32)$  superstrings, *Phys. Lett. B* 149 (1984) 351–356.
- [94] M. Dine, N. Seiberg, E. Witten, Fayet-Iliopoulos terms in string theory, *Nucl. Phys. B* 289 (1987) 589–598.
- [95] J.J. Atick, L.J. Dixon, A. Sen, String calculation of Fayet-Iliopoulos  $d$  terms in arbitrary supersymmetric compactifications, *Nucl. Phys. B* 292 (1987) 109–149.
- [96] M. Dine, I. Ichinose, N. Seiberg,  $F$  terms and  $d$  terms in string theory, *Nucl. Phys. B* 293 (1987) 253–265.
- [97] M.B. Green, J.H. Schwarz, Anomaly cancellation in supersymmetric  $D=10$  gauge theory and superstring theory, *Phys. Lett. B* 149 (1984) 117–122.
- [98] A. Font, L.E. Ibañez, H.P. Nilles, F. Quevedo, Yukawa couplings in degenerate orbifolds: towards a realistic  $SU(3) \times SU(2) \times U(1)$  superstring, *Phys. Lett. B* 210 (1988) 101. Erratum: *Phys. Lett. B* 213 (1988) 564.
- [99] S.A. Abel, C. Muñoz, Quark and lepton masses and mixing angles from superstring constructions, *J. High Energy Phys.* 02 (2003) 010, arXiv:hep-ph/0212258.
- [100] L. Aparicio, P.G. Camara, D.G. Cerdeno, L.E. Ibañez, I. Valenzuela, The NMSSM with F-theory unified boundary conditions, *J. High Energy Phys.* 02 (2013) 084, arXiv:1212.4808 [hep-ph].